

IMPACT OF DEFECTIVE ITEMS ON (Q,r,L) INVENTORY MODEL INVOLVING CONTROLLABLE SETUP COST

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Abstract: In a recent paper, Ouyang *et al.* [10] proposed a (Q,r,L) inventory model with defective items in an arrival lot. The purpose of this study is to generalize Ouyang *et al.*'s [10] model by allowing setup cost (A) as a decision variable in conjunction with order quantity (Q), reorder point (r) and lead time (L). In this study, we first assume that the lead time demand follows a normal distribution, and then relax this assumption by only assuming that the first two moments of the lead time demand are given. For each case, an algorithm procedure of finding the optimal solution is developed.

Keywords: Inventory, defective items, setup cost, lead time, minimax distribution free procedure.

1. INTRODUCTION

In traditional economic order quantity (EOQ) and economic production quantity (EPQ) models, setup cost is treated as a constant. However, in practice, setup cost can be controlled and reduced through various efforts such as worker training, procedural changes and specialized equipment acquisition. Through the Japanese experience of using Just-In-Time (JIT) production, the advantages and benefits associated with efforts to reduce the setup cost can be clearly perceived.

In the inventory literature, setup cost reduction models have been continually modified so as to achieve the real inventory situation. The initial result in the development of setup cost reduction model is that of Porteus [15] who introduced the concept and developed a framework of investing in reducing setup cost on EOQ model. Since this introduction, a lot of studies such as Nasri *et al.* [9], Kim *et al.* [5], Paknejad *et al.* [14] and Sarker and Coates [16] have been done on the related researches.

The underlying assumption in above models is that the lead time is prescribed constant or a random variable, which therefore, is not subject to control (see, e.g. Naddor [8] and Silver and Peterson [19]). In fact, lead time usually consists of the following components (Tersine [20]): order preparation, order transit, supplier lead time, delivery time, and setup time. In many practical situations, lead time can be reduced at an added crashing cost; in other words, it is controllable. By shortening lead time, we can lower the safety stock, reduce the stockout loss and improve the customer service level so as to gain competitive edges in business. Inventory models considering lead time as a decision variable have been developed by several researchers recently. Liao and Shyu [6] first presented a probability inventory model in which lead time is a unique decision variable and order quantity is predetermined. Ben-Daya and Raouf [1] extended Liao and Shyu's [6] model by considering both lead time and order quantity as decision variables. Later, some studies [7, 10-13] in the field of lead time reduction generalized Ben-Daya and Raouf's [1] model by allowing reorder point as one of the decision variables. In a recent article, Ouyang *et al.* [10] proposed two general models that even contain some defective items in an arrival order lot. We note that these papers are focusing on the benefits from lead time reduction in which setup cost is treated as a fixed constant.

In this paper, using the same assumptions as in Ouyang *et al.* [10], we formulate a modified continuous review model including defective items to extend Ouyang *et al.*'s [10] model by simultaneously optimizing the order quantity (Q), setup cost (A), reorder point (r) and lead time (L); that is, our goal is to establish a (Q, A, r, L) inventory model with defective items to accommodate more practical features of the real inventory systems. From the numerical examples provided, we can show that our new models are better than that of Ouyang *et al.* [10]. In our study, we first start with a lead time demand that follows a normal distribution, and determine the optimal order policy. Next, we relax the normal distributional form of the lead time demand by only assuming that the first and second moments of the distribution function of the lead time demand are known and finite, and then solve this inventory model by using the minimax distribution free approach. Furthermore, two numerical examples are provided.

2. NOTATIONS AND ASSUMPTIONS

In order to develop the proposed models, we adopt the following notations and assumptions used in Ouyang *et al.* [10] in this paper.

Notations:

D	= expected demand per year for non-defective items
h	= non-defective holding cost per unit per year
h'	= defective treatment cost per unit per year
π_1	= shortage cost per unit short
π_2	= lost sales per unit
ν	= inspecting cost for each item in an arrival order
β	= the fraction of the demand during the stock-out period that will be backordered, $\beta \in [0,1]$

- p = defective rate in an order lot, $p \in [0,1)$, a random variable
 $g(p)$ = the probability density function (*p.d.f.*) of p with finite mean M_p and finite variance V_p
 σ^2 = variance of the demand per year during lead time
 Q = order quantity including defective items, a decision variable
 A = setup cost per setup, a decision variable
 r = reorder point
 L = length of lead time, a decision variable, a decision variable
 X = the lead time demand which has a distribution function (*d.f.*) F with finite mean DL and standard deviation $\sigma\sqrt{L}$
 $E(\cdot)$ = mathematical expectation
 x^+ = maximum value of x and 0, i.e., $x^+ = \text{Max}\{x, 0\}$.

Assumptions:

- Inventory is continuously reviewed. Replenishments are made whenever the inventory level (based on the number of non-defective items) falls to the reorder point r .
- The reorder point $r =$ expected demand during lead time + safety stock (SS), and $SS = k \times$ (standard deviation of lead time demand), i.e., $r = DL + k\sigma\sqrt{L}$, where k is the safety factor.
- The lead time L consists of n mutually independent components. The i th component has a minimum duration a_i and normal duration b_i , and a crashing cost per unit time c_i . Further, for convenience, we rearrange c_i such that $c_1 \leq c_2 \leq \dots \leq c_n$. Then, it is clear that the reduction of lead time should be first on component 1 because it has the minimum unit crashing cost, and then component 2, and so on.
- If we let $L_0 = \sum_{j=1}^n b_j$ and L_i be the length of lead time with components 1, 2, ..., i crashed to their minimum duration, then L_i can be expressed as $L_i = \sum_{j=1}^n b_j - \sum_{j=1}^i (b_j - a_j)$, $i=1, 2, \dots, n$; and the lead time crashing cost $C(L)$ per cycle for a given $L \in [L_i, L_{i-1}]$ is given by $C(L) = c_i(L_{i-1} - L) + \sum_{j=1}^{i-1} c_j(b_j - a_j)$.
- Upon an arrival order lot Q with a defective rate p , the entire items are inspected and all defective items are assumed to be discovered and removed from order quantity Q . And thus, the effective order quantity (i.e., the quantity of non-defective or salable items) is reduced to an amount equal to $Q(1-p)$, and defective items in each lot will be returned to the supplier at the time of delivery of the next lot.
- Inspection is non-destructive and error-free.

3. REVIEW OF OUYANG *ET AL.*'S MODEL

Ouyang *et al.* [10] considered a (Q, r, L) inventory model with defective items in an arrival lot, and asserted the following function of expected total annual cost which is composed of setup cost, non-defective holding cost, defective treatment cost, stock-out cost, inspecting cost, and lead time crashing cost. Symbolically, the problem is given by

$$EAC(Q, r, L) = \frac{D\{A + C(L) + [\pi_1 + \pi_2(1 - \beta)]E(X - r)^+\}}{Q(1 - M_p)} + h[r - DL + (1 - \beta)E(X - r)^+] + \frac{Q\gamma}{2(1 - M_p)} + \frac{vD}{1 - M_p}, \quad (1)$$

where $\frac{D}{Q(1 - M_p)}$ is the expected order number per year (see, e.g. Schwaller [17] or Shih [18]); $M_p = \int_0^1 pg(p)dp$ is the mean of random variable p , $E(X - r)^+$ is the expected demand shortage at the end of cycle,

$$\begin{aligned} \gamma &= h \int_0^1 (1 - p)^2 g(p)dp + 2h' \int_0^1 p(1 - p)g(p)dp \\ &= h + 2(h' - h)M_p + (h - 2h')(M_p^2 + V_p) > 0. \end{aligned} \quad (2)$$

In addition, since the lead time demand X follows a normal *d.f.* $F(x)$ with mean DL and standard deviation $\sigma\sqrt{L}$, and the reorder point $r = DL + k\sigma\sqrt{L}$, where k is the safety factor, we can consider the safety factor k as a decision variable instead of r . Thus, the expected shortage quantity $E(X - r)^+$ at the end of the cycle can be expressed as a function of safety factor k ; that is,

$$E(X - r)^+ = \int_r^\infty (x - r)dF(x) = \int_k^\infty \sigma\sqrt{L}(z - k)dF_z(z) = \sigma\sqrt{L}G(k) > 0,$$

where $G(k) = \int_k^\infty (z - k)dF_z(z)$ and $F_z(z)$ is the *d.f.* of the standard normal variable Z .

Therefore, problem (1) can be transformed to

$$EAC(Q, k, L) = \frac{D\{A + C(L) + [\pi_1 + \pi_2(1 - \beta)]\sigma\sqrt{L}G(k)\}}{Q(1 - M_p)} + h\sigma\sqrt{L}[k + (1 - \beta)G(k)] + \frac{Q\gamma}{2(1 - M_p)} + \frac{vD}{1 - M_p}. \quad (3)$$

4. MODEL EXTENSION

In contrast to Ouyang *et al.*'s [10] model, we consider the setup cost A as a decision variable and seek to minimize the sum of the capital investment cost of reducing setup cost A and the inventory related costs (as express in problem (3)) by optimizing over Q , A , k and L constrained on $0 < A \leq A_0$, where A_0 is the original setup cost. That is, the objective of our problem is to minimize the following expected total annual cost

$$EAC(Q, A, k, L) = \eta\Psi(A) + EAC(Q, k, L) \tag{4}$$

over $A \in (0, A_0]$, where η is the fractional opportunity cost of capital per year, $\Psi(A)$ follows a logarithmic investment function given by

$$\Psi(A) = b \ln\left(\frac{A_0}{A}\right) \text{ for } A \in (0, A_0], \tag{5}$$

$1/b$ is the fraction of the reduction in A per dollar increase in investment. This logarithmic investment function is consistent with the Japanese experience as reported in Hall [4]; and has been used by Nasri *et al.* [9] and others.

From function (5), we note that the setup cost level $A \in (0, A_0]$. It implies that if the optimal setup cost obtained does not satisfy the restriction on A , then no setup cost reduction investment is made. For this special case, the optimal setup cost is the original setup cost.

Substitute (5) and (3) into (4) and minimize the resulting equation; we suffice to minimize

$$EAC(Q, A, k, L) = \eta b \ln\left(\frac{A_0}{A}\right) + \frac{D\{A + C(L) + [\pi_1 + \pi_2(1 - \beta)]\sigma\sqrt{L}G(k)\}}{Q(1 - M_p)} + h\sigma\sqrt{L}[k + (1 - \beta)G(k)] + \frac{Q\gamma}{2(1 - M_p)} + \frac{vD}{1 - M_p}, \tag{6}$$

over $A \in (0, A_0]$.

In order to solve this nonlinear programming problem, we first ignore the restriction $A \in (0, A_0]$ and take the first partial derivatives of $EAC(Q, A, k, L)$ with respect to Q , A , k and $L \in [L_i, L_{i-1}]$, respectively.

$$\frac{\partial EAC(Q, A, k, L)}{\partial Q} = -\frac{D\{A + C(L) + [\pi_1 + \pi_2(1 - \beta)]\sigma\sqrt{L}G(k)\}}{Q^2(1 - M_p)} + \frac{\gamma}{2(1 - M_p)}, \tag{7}$$

$$\frac{\partial EAC(Q, A, k, L)}{\partial A} = -\frac{\eta b}{A} + \frac{D}{Q(1 - M_p)}, \tag{8}$$

$$\frac{\partial EAC(Q, A, k, L)}{\partial k} = -\frac{D[\pi_1 + \pi_2(1-\beta)]\sigma\sqrt{L}P_z(k)}{Q(1-M_p)} + h\sigma\sqrt{L}[1-(1-\beta)P_z(k)], \quad (9)$$

where $P_z(k) = P_z(Z \geq k)$, and

$$\begin{aligned} \frac{\partial EAC(Q, A, k, L)}{\partial L} &= \frac{D[\pi_1 + \pi_2(1-\beta)]\sigma L^{-1/2}G(k)}{2Q(1-M_p)} + \frac{1}{2}h\sigma L^{-1/2}[k + (1-\beta)G(k)] \\ &\quad - \frac{Dc_i}{Q(1-M_p)}. \end{aligned} \quad (10)$$

By examining the second order sufficient conditions, it can be easily verified that $EAC(Q, A, k, L)$ is not a convex function of (Q, A, k, L) . However, for fixed Q, A and k , $EAC(Q, A, k, L)$ is concave in $L \in [L_i, L_{i-1}]$, because

$$\begin{aligned} \frac{\partial^2 EAC(Q, A, k, L)}{\partial L^2} &= -\frac{D[\pi_1 + \pi_2(1-\beta)]\sigma L^{-3/2}G(k)}{4Q(1-M_p)} \\ &\quad - \frac{1}{4}h\sigma L^{-3/2}[k + (1-\beta)G(k)] < 0. \end{aligned}$$

Hence, for fixed Q, A and k , the minimum expected total annual cost will occur at the end points of the interval $[L_i, L_{i-1}]$. On the other hand, it can be shown that, for a given value of $L \in [L_i, L_{i-1}]$, $EAC(Q, A, k, L)$ is a convex function of (Q, A, k) . Thus, for fixed $L \in [L_i, L_{i-1}]$, the minimum value of $EAC(Q, A, k, L)$ will occur at the point (Q, A, k) which satisfies $\frac{\partial EAC(Q, A, k, L)}{\partial Q} = 0$, $\frac{\partial EAC(Q, A, k, L)}{\partial A} = 0$ and $\frac{\partial EAC(Q, A, k, L)}{\partial k} = 0$.

Solving above equations for Q, A and $P_z(k)$ respectively, produces

$$Q = \left\{ \frac{2D\{A + C(L) + [\pi_1 + \pi_2(1-\beta)]\sigma\sqrt{L}G(k)\}}{\gamma} \right\}^{1/2}, \quad (11)$$

$$A = \frac{\eta b Q (1 - M_p)}{D} \quad (12)$$

and

$$P_z(k) = \frac{hQ(1-M_p)}{hQ(1-M_p)(1-\beta) + D[\pi_1 + \pi_2(1-\beta)]}. \quad (13)$$

From equations (11)-(13), we note that it is difficult to find an explicit general solution for (Q, A, k) . Consequently, we establish the following algorithm to find the optimal (Q, A, k, L) .

Algorithm

Step1. For each $L_i, i=0,1,2,\dots,n$, perform (i)-(v).

- (i) Start with $A_{i1} = A_0$ and $k_{i1} = 0$ and get $G(k_{i1})=0.3989$ by checking the table from Silver and Peterson [19, pp. 779-786] or Brown [2, pp. 95-103].
- (ii) Substituting A_{i1} and $G(k_{i1})$ into equation (11) evaluates Q_{i1} .
- (iii) Utilizing Q_{i1} determines A_{i2} from equation (12) and $P_z(k_{i2})$ from equation (13).
- (iv) By checking $P_z(k_{i2})$ from Silver and Peterson [19] or Brown [2] finds k_{i2} , and hence $G(k_{i2})$.
- (v) Repeat (ii)-(iv) until no change occurs in the values of Q_i, A_i and k_i .

Step2. Compare A_i and A_0 .

- (i) If $A_i \leq A_0$, A_i is feasible, then go to Step3.
- (ii) If $A_i > A_0$, A_i is not feasible. For given L_i , take $A_i = A_0$ and solve the corresponding values of (Q_i, k_i) from equations (11) and (13) iteratively until convergence (the solution procedure is similar to that given in Step1), then go to Step3.

Step3. For each $(Q_i, A_i, k_i, L_i), i=0,1,2,\dots,n$, compute the corresponding expected total annual cost $EAC(Q_i, A_i, k_i, L_i)$ utilizing (6).

Step4. Find $\text{Min}_{i=0,1,2,\dots,n} EAC(Q_i, A_i, k_i, L_i)$.

If $EAC(Q^*, A^*, k^*, L^*) = \text{Min}_{i=0,1,2,\dots,n} EAC(Q_i, A_i, k_i, L_i)$, then (Q^*, A^*, k^*, L^*) is the optimal solution. And hence, the optimal reorder point is $r^* = DL^* + k^* \sigma \sqrt{L^*}$.

5. DISTRIBUTION FREE MODEL

In many practical situations, the distributional information of lead time demand is often quite limited. In this section, as in Ouyang *et al.*'s [10] model, the assumption that the lead time demand is normally distributed is relaxed and only assume that the *d.f.* F of X belongs to the class Ω of *d.f.*'s with finite mean DL and standard deviation $\sigma\sqrt{L}$. Since the form of the distribution function of lead time demand X is unknown, the exact value of the expected demand shortage $E(X-r)^+$ cannot be determined. Therefore, we use the minimax distribution free procedure to solve this problem. The minimax distribution free approach for this problem is to find the "most unfavorable" *d.f.* F in Ω for each (Q, A, r, L) and then minimize over (Q, A, r, L) ; that is, our problem is to solve

$$\underset{Q,A,r,L}{\text{Min Max}} EAC(Q, A, r, L), \tag{14}$$

over $A \in (0, A_0]$.

For this purpose, we need the following proposition which was asserted by Gallego and Moon [3].

Proposition. For any $F \in \Omega$,

$$E(X - r)^+ \leq \frac{1}{2} \left[\sqrt{\sigma^2 L + (r - DL)^2} - (r - DL) \right]. \tag{15}$$

Moreover, the upper bound of (15) is tight.

Since $r = DL + k\sigma\sqrt{L}$ as mentioned previously, and for any probability distribution of the lead time demand X , the above inequality always holds. Then, using inequality (15) and model (6), the problem (14) is reduced to minimize

$$\begin{aligned} EAC_U(Q, A, k, L) = & \eta b \ln\left(\frac{A_0}{A}\right) + \\ & \frac{D \left\{ A + C(L) + \frac{1}{2} \sigma \sqrt{L} [\pi_1 + \pi_2 (1 - \beta)] (\sqrt{1 + k^2} - k) \right\}}{Q(1 - M_p)} \\ & + h\sigma\sqrt{L} \left[k + \frac{1}{2} (1 - \beta) (\sqrt{1 + k^2} - k) \right] + \frac{Q\gamma}{2(1 - M_p)} + \frac{vD}{1 - M_p}, \end{aligned} \tag{16}$$

over $A \in (0, A_0]$, where $EAC_U(Q, A, k, L)$ is the least upper bound of $EAC(Q, A, k, L)$.

By analogous arguments in the normal distribution demand case, we can show that $EAC_U(Q, A, k, L)$ is a concave function of $L \in [L_i, L_{i-1}]$ for fixed (Q, A, k) . Thus, for fixed (Q, A, k) , the minimum value of $EAC_U(Q, A, k, L)$ will occur at the end points of the interval $[L_i, L_{i-1}]$. On the other hand, for a given value of $L \in [L_i, L_{i-1}]$, $EAC_U(Q, A, k, L)$ is convex in (Q, A, k) . Hence, for fixed $L \in [L_i, L_{i-1}]$, the minimum value of (16) will occur at the point (Q, A, k) which satisfies $\frac{\partial EAC_U(Q, A, k, L)}{\partial Q} = 0$,

$\frac{\partial EAC_U(Q, A, k, L)}{\partial A} = 0$ and $\frac{\partial EAC_U(Q, A, k, L)}{\partial k} = 0$, simultaneously. The resulting solutions are

$$Q = \left\{ \frac{2D \left\{ A + C(L) + \frac{1}{2} \sigma \sqrt{L} [\pi_1 + \pi_2 (1 - \beta)] (\sqrt{1 + k^2} - k) \right\}}{\gamma} \right\}^{1/2}, \tag{17}$$

$$A = \frac{\eta b Q (1 - M_p)}{D} \tag{18}$$

and

$$\frac{2\sqrt{1+k^2}}{\sqrt{1+k^2}-k} = \frac{D[\pi_1 + \pi_2(1-\beta)]}{hQ(1-M_p)} + (1-\beta) . \tag{19}$$

The similar algorithm procedure as proposed in the previous section can be performed to obtain the optimal solutions for the order quantity, setup cost, reorder point and lead time.

6. NUMERICAL EXAMPLES

In order to illustrate the above solution procedure and the effects of setup cost reduction, let us consider an inventory system with the following data used in Ouyang *et al.* [10]: $D = 600$ units/year, $A_0 = \$200$ per setup, $h = 20$ \$/unit/year, $h' = 10$ \$/unit/year, $\sigma = 7$ units/week, $v = 1.6$ \$/unit, $\pi_1 = \$50$ /unit, $\pi_2 = \$150$ /unit. The lead time has three components with data as shown in Table 1, and the defective rate p in an order lot has a Beta distribution with parameters $s = 1$ and $t = 4$; i.e., the *p.d.f.* of p is

$$\begin{cases} g(p) = 4(1-p)^3, & 0 < p < 1 \\ 0, & \text{otherwise} \end{cases}$$

Hence, the mean of p is $M_p = s/(s+t) = 0.2$, and the variance of p is $V_p = st/[(s+t)^2(s+t+1)] = 0.02667$. Therefore, from equation (2), we can get $\gamma = 16$. Besides, for setup cost reduction, we take $\eta = 0.1$ and $b = 5,800$.

Table 1: Lead time data

Lead time component	Normal Duration	Minimum duration	Unit crashing cost
i	b_i (days)	a_i (days)	c_i (\$/day)
1	20	6	0.4
2	20	6	1.2
3	16	9	5.0

Example 1: Suppose that the lead time demand follows a normal distribution. We solve the cases when $\beta = 0, 0.5, 0.8$ and 1 . Applying the **Algorithm** procedure, we summarize the optimal solutions as shown in Table 2. Furthermore, to see the effects of setup cost reduction, we list the results of fixed setup cost model [10] in the same table.

Table 2: Summary of the optimal solutions for normal distribution case (L_i in week)

β	Setup cost reduction model		Fixed setup cost model		Savings %
	(Q^*, A^*, r^*, L^*)	$EAC(\cdot)$	(Q^*, r^*, L^*)	$EAC(\cdot)$	
0.0	(87, 67.17, 78, 4)	\$ 4,210	(134, 73, 4)	\$ 4,476	5.9
0.5	(76, 58.55, 106, 6)	4,162	(135, 71, 4)	4,427	6.0
0.8	(76, 59.09, 103, 6)	4,105	(135, 68, 4)	4,376	6.2
1.0	(77, 59.81, 99, 6)	4,044	(136, 64, 4)	4,319	6.4

Note: $Savings \% = \{[EAC(Q^*, r^*, L^*) - EAC(Q^*, A^*, r^*, L^*)] / EAC(Q^*, r^*, L^*)\} \times 100\%$

From the results shown in Table 2, comparing our new model with that of fixed setup cost case, we observe the savings which range from 5.9% to 6.4%. It implies that significant savings can be easily achieved due to controlling the setup cost.

Example 2: The assumption and data are as Example 1, except that the probability distribution of the lead time demand is unknown. Using the similar procedure as **Algorithm**, the summarized optimal values are tabulated in Table 3.

Table 3: Summary of the optimal solutions (L_i in week)

β	(Q_*, A_*, r_*, L_*)	$EAC_U(Q_*, A_*, r_*, L_*)$
0.0	(166, 128.06, 75, 3)	\$ 5,586
0.5	(154, 118.76, 67, 3)	5,227
0.8	(137, 105.95, 77, 4)	4,928
1.0	(127, 98.18, 70, 4)	4,633

Moreover, we can compare the procedures for the worst case distribution against the normal distribution. For example, in the case of $\beta = 1$, the optimal solutions of normal distribution and distribution free model are $(Q^*, A^*, r^*, L^*) = (77, 59.81, 99, 6)$ and $(Q_*, A_*, r_*, L_*) = (127, 98.18, 70, 4)$, respectively. Substituting them into (6), we have $EAC(Q^*, A^*, r^*, L^*) = \$4,044$ and $EAC(Q_*, A_*, r_*, L_*) = \$4,148$. Hence, $EAC(Q_*, A_*, r_*, L_*) - EAC(Q^*, A^*, r^*, L^*) = \$4,148 - 4,044 = \$104$ is the largest amount that we would be willing to pay for the knowledge of the probability distribution of demand. This quantity can be regarded as the expected value of additional information (EVAI).

7. CONCLUDING REMARKS

The purpose of this study is to extend Ouyang *et al.*'s [10] model by simultaneously optimizing the order quantity, setup cost, reorder point and lead time, and further examine the effect of defective items on an inventory model. In this paper, we first assume that the lead time demand follows a normal distribution, and determine the optimal order policy. Then, we relax the assumption about the form of the distribution function of lead time demand by applying the minimax distribution free procedure to solve the problem.

In future research on this problem, it would be interesting to adopt the random sub-lot sampled inspection policy to inspect the selected items. Another extension of this work may be conducted by considering the effects of investing in quality improvement.

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