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USING GOAL PROGRAMMING METHOD TO SOLVE DEA PROBLEMS WITH VALUE JUDGMENTS

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Abstract: Data envelopment analysis (DEA) is a linear programming approach for measuring relative efficiency of peer decision making units that have multiple inputs and outputs. DEA was developed without consideration of the decision maker's preference structures. DEA and multiple objective linear programming are tools that can be used in management control and planning. This paper shows how a data envelopment analysis problem can be solved by transforming it into MOLP formulation. We use the goal programming method to reflect the decision making preferences in the process of assessing efficiency, such that the value judgments of the decision maker are considered. Therefore, the proposed method can find a solution that satisfies the decision maker's goal levels. A case study is provided to illustrate how data envelopment oriented efficiency analysis can be conducted by using goal programming method.

Keywords: Data envelopment analysis (DEA), Multiple objective linear programming (MOLP), Goalprogramming

MSC: 90B50.

1. INTRODUCTION

Data envelopment analysis (DEA), originally presented by Charnes et al. (1978), is a well-known family of mathematical programming tools for assessing the relative efficiency of a set of comparable processing decision making units (DMUs). The number of applications of DEA is large, covering diverse fields such as finance, health, education, manufacturing, transportation, etc. Conventional DEA models do not consider decision maker's (DM) preferences, and do not present the decision maker the opportunity to include her/his judgments about the relative importance of inputs and outputs considered. To incorporate DM’s preference information in DEA, various techniques have been proposed such as the goal and target setting models of (Golany,1988; Thanassoulis and Dyson,1992; Athanassopoulis,1995,1998) and weight restrictions models, imposing bounds on individual weights(Dyson and Thanassoulis,1998), assurance region (Thompson et al.,1990), restricting composite inputs and outputs, weight ratios and proportions (Wong and Beasley,1990) and the cone ratio concept by adjusting the observed input–output levels or weights to capture value judgment to belong to a given closed cone (Charnes and Cooper ,1990; Charnes et al.,1994). Alternative approaches include (Thanassoulis and Allen, 1998) the model which adopts unobserved DMUs derived from pareto-efficient observed DMUs, and which incorporates value judgments; Zhu (1996) also integrates preference information into a modified DEA formulation, while Golany and Rol (1994), use hypothetical DMUs to represent preference information. However, all the above-mentioned techniques would require prior articulated preference knowledge from the DM, which in most cases can be subjective and difficult to obtain. In manufacturing or service organizations, decision making can become more complex and often inherently uncertain due to multiple attributes and conflicting objectives. Multiobjective programming methods such as multiple objectives linear programming (MOLP) are techniques used to solve such multiple criteria decision making (MCDM) problems. Many decision making problems can be formulated as multiobjectiveoptimization problems. There hardly exists the solution that optimizes all objective functions in multiobjective optimization problems, and then the concept of Pareto optimal solution (or efficient solution) is introduced (Sawaragi et al. 1985). Usually, a number of Pareto optimal solutions are considered as candidates of final decision making solution (Koopmans, 1951). Now, the main question is how decisionmakers choose the final solution from the set of Pareto optimal solutions. In order to solve this problem, interactive multiobjective optimization methods have been developed, see (Gal et al. 1999; Sawaragi et al. 1985; Wierzbicki et al. 2000). These methods find a decision making solution by processing the following two stages repeatedly: (1) solving auxiliary optimization problem to obtain a Pareto optimal solution closest to the given aspiration level, and (2) revising their aspiration levels by making trade-off analysis. An appealing method to incorporate preference information, without necessary prior judgment or target setting is the use of an interactive decision making technique that encompasses both DEA and MOLP. Golany (1988) first proposed an interactive model combining these approaches, where the DM allocates a set of level of inputs as resources to select the most preferred set of level of outputs from alternative points on the efficient frontier. Post and Spronk (1999) combined the use of DEA and interactive multiple goal programming where preference information are incorporated interactively with the DM by adjusting the upper and lower feasible boundaries of the input and output levels. Then, Joro et al. (1998) showed that there are synergies from both DEA and MOLP, and showed that the DEA formulation is structurally similar to the reference point approach of the MOLP formulation. Halme et al. (1999); Joro et al. (2003); Korhonen (2002); Yang et al. (2009) investigated equivalence models and interactive trade-off analysis procedures in MOLP, such that DEA-oriented performance assessment and target setting can be integrated in a way that the decision makers' preferences can be taken into account in an interactive fashion. In fact, they investigated three equivalence models between the output-oriented dual DEA model and the min-max reference point formulations, namely the super-ideal point model, the ideal point model, and the shortest distance model. Wang et al. (2009) and Hosseinzadeh Lotfi et al. (2010) proposed a method to establish an equivalence model between DEA and multiple objective linear programming, and showed how a DEA problem can be solved interactively without any prior judgments by transforming it into an MOLP formulation. Hosseinzadeh Lotfi et al. used Zionts–Wallenius (Z–W) method to reflect the DM's preferences in the process of assessing efficiency.

In this paper, we propose an interactive model combining DEA and goal programming approaches where the DM allocate a set of level of inputs as resources and select the most preferred set of level of outputs from alternative points on the efficient frontier. We use an equivalence model between DEA and MOLP presented in Wong et al. (2009) and then, we show how a DEA problem can be solved interactively by transforming it into MOLP formulation. For this purpose, we use goal programming method to reflect the DM's preferences in the process of assessing efficiency. The current article proceeds as follows. In section 2, we present the output oriented CCR model, multiobjective linear programming method, goal programming and the equivalence between DEA and MOLP. In section 3, we develop our method for using goal programming method to solve DEA problems with value judgments. A case study is considered in section 4 which illustrates the proposed method. Conclusions are given in section 5.

PRELIMINARIES

In this section, we briefly present some required concepts.

Data envelopment analysis

DEA is a post ante analysis based on the past performance cross-sectional view of several organizational units in a given single period, as measured by their multiple inputs and outputs. We classify the units into two groups, efficient and inefficient, in the pareto sense. DEA does not use common weights, as do Multiple Criteria Decision Theory models, which usually rank the elements based on the multiple criteria (inputs and outputs), and usually provide common weights. In DEA, the weights vary among the units: this variability is the essence of DEA. As a performance measurement and analysis technique, DEA is a nonparametric frontier estimation methodology based on linear programming for evaluating relative efficiency of a set of comparable DMU that share common functional goals. Assume that there are n DMUs, where each DMU_j, $(j=1,...,n)$ uses m different inputs, x_{ij}, $(i=1,...,m)$, to produce s different outputs, y_{rj}, $(r=1,...,s)$. We assume that the data set are positive. We denote by

y_{rj}, $(r=1,...,s)$ the level of the r-th output from unit j(j=1,. . . ,n), and by x_{ij}, $(i=1,...,m)$ the level of the i-th input to the j-th unit.

The relative efficiency score of the DMU_o obtained from the following model is called output-oriented CCR envelopment model.

$$\begin{aligned}
&Max \quad h_o = \varphi_o \\
&S.t. \\
&\sum_{j=1}^{n} \lambda_j y_{rj} \geq \varphi_o y_{ro}, \qquad r=1,...,s, \\
&\sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{io}, \qquad i=1,...,m, \\
&\lambda_j \geq 0, \qquad\qquad j=1,....,n.
\end{aligned} \tag{2.1}$$

Model (2.1) can be used to generate the efficiency score of DMU_o. Also, we use the model (2.1) to generate composite input and composite output levels of a virtual DMU that the observed DMU_o wants to reach. In optimality, λ_j means that DMU_j is used to construct the virtual DMU, that DMU_o should benchmark against it.

Multi objective linear programming

Managerial problems are seldom evaluated with a single or simple goal like profit maximization. Today's management systems are much more complex, and managers want to attain simultaneous goals, in which some of them conflict. In the other words, decisions in the real world contexts are often made in the presence of multiple, conflicting, and incommensurate criteria. Multi-criteria decision making (MCDM) refers to making decision in the presence of multiple and conflicting criteria. Problems for MCDM may range from our daily life, such as the purchase of a car, to those affecting entire nations, as in the judicious use of money for the preservation of national security. There are two types of criteria: objectives and attributes. Therefore, the MCDM problems can be broadly classified into two categories:

- Multi-objective decision making (MODM)
- Multi-attribute decision making (MADM)

The main difference between MODM and MADM is that the former concentrates on continuous decision spaces, primarily on mathematical programming with several objective functions, the latter focuses on problems with discrete decision spaces.

Multi-objective decision making is known as the continuous type of the MCDM. The main characteristics of MODM problems are that decision makers need to achieve multiple objectives while these multiple objectives are non-commensurable and conflict with each other.

An MODM model considers a vector of decision variables, objective functions, and constrains. Decision makers attempt to maximize (or minimize) the objective functions. Since this problem has rarely a unique solution, decision makers are expected to choose a solution from among the set of efficient solutions (as alternatives). Generally, the multiple objective decision making problem can be formulated as follows:

$$\begin{aligned} &Max \ \ \{f_1(x),..., f_k(x)\} \\ &s.t. \\ &\quad x \in X \end{aligned} \tag{2.2}$$

where $f_1,..., f_k$ are n conflicting objective functions and X is non-empty feasible region. Model (2.2) is called MODM problem. If all objective functions and all constraints in above MODM problem are in linear form, we have MOLP problem.

Goal Programming

Goal programming (GP) is now an important area of multiple criteria optimization. The idea of goal programming is to establish a goal level of achievement for each criterion. Goal programming method requires the decision maker to set goals for each objective that he/she wishes to obtain. A preferred solution is then defined as the one which minimizes the deviations from the set goals. Then GP can be formulated as the following achievement function.

$$\begin{aligned} &Min \quad \sum_{i=1}^{k}(d_i^+ + d_i^-) \\ &s.t. \\ &\quad f_i(x) + d_i^- - d_i^+ = b_i, \qquad i = 1,...,k, \\ &\quad x \in X, \\ &\quad d_i^- d_i^+ = 0, \qquad\qquad i = 1,...,k, \\ &\quad d_i^-, d_i^+ \geq 0, \qquad\qquad i = 1,....,k. \end{aligned} \tag{2.3}$$

The DMs for their goals set some acceptable aspiration levels, $b_i \ (i = 1,...,k)$, for these goals, and try to achieve a set of goals as closely as possible. The purpose of GP is to minimize the deviations between the achievement of goals, $f_i(x)$, and these acceptable aspiration levels, $b_i \ (i = 1,...,k)$ .Also, d_i^-, d_i^+ are, respectively, under- and over-achievement of the i th goal.

Equivalence between DEA and MOLP

In a DEA model, an efficiency score is generated for a DMU by maximizing outputs with limited inputs, or minimizing inputs with desired or fixed outputs, or simultaneously maximizing outputs and minimizing inputs. Either way, this can be regarded as a kind of multiple objective optimization problem. In this section, the theoretical considerations of combining MOLP and DEA are presented. Suppose an optimization problem has s objectives reflecting the different purposes and desires of the decision maker. Such a problem can be represented in a general form as follows:

$$\begin{aligned} & max \ \ \{f_1(\lambda),..., f_s(\lambda)\} \\ & s.t. \\ & \quad \lambda \in \Omega_o \end{aligned} \tag{2.4}$$

Where

$$\Omega_o = \{\lambda \mid \sum_{j=1}^{n} \lambda_j x_{ij} \le x_{io}, \ \ i = 1,...,m; \ \lambda_j \ge 0, \ \ \ j = 1,....,n.\} \tag{2.5}$$

In order to reach to a special nondominated extreme point, the MOLP formulation (2.4) can bewritten in a weighted minimax approach as follows:

$$\begin{aligned} & \min_{\lambda} \ \ \max_{1 \le r \le s} \ \ w_r (f_r^* - f_r(\lambda)) \\ & s.t. \\ & \quad \lambda \in \Omega_o \end{aligned} \tag{2.6}$$

The weighted minimax formulation is a special case of the reference point approach with f_r^* as the ideal point, and calculates the minimum of the maximum distance between f_r^*, the maximum value of objective r, and $f_r(\lambda)$, the observed value of objective r.

If we define $f_r(\lambda) = \sum_{j=1}^{n} \lambda_j y_{rj}, \ \ r = 1,...,s,$ Wong et al. (2009) show that output-oriented CCR model (2.1) can be equivalently rewritten as a following MOLP formulation.

$$\begin{aligned} & max \quad \left\{ \sum_{j=1}^{n} \lambda_j y_{1j}, ..., \sum_{j=1}^{n} \lambda_j y_{sj} \right\} \\ & s.t. \\ & \quad \sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{io}, \qquad i = 1,...,m, \\ & \quad \lambda_j \geq 0, \qquad\qquad j = 1,....,n. \end{aligned} \tag{2.7}$$

USING GOAL PROGRAMMING METHOD TO SOLVE DEA PROBLEMS WITH VALUE JUDGMENTS

In this section we use goal programming method to design a procedure for searching for mostpreferred solution thatmaximizes the decision maker's implicit utility function. This method isdesign to derive the decision maker towards his most preferred solution, or at least, to a goodsolution, in the sense that it is acceptable by the DM.
The proposed method has twostages.

First, we calculate the relative efficiency score of the DMU_o by using the output-oriented CCR envelopment model (2.1). After this, we can identifythe composite inputs and composite outputsthat the observed DMU_o should benchmark against. Let (λ^*, φ_o^*) be optimal solution of model(2.1)in evaluating DMU_o. Therefore by this optimal solution the output targets for DMU_o are defined as $(\bar{y}_1, ..., \bar{y}_s)$ and calculated as follows:

$$\bar{y}_r = \sum_{j=1}^{n} \lambda_j^* y_{rj}, \quad r = 1,...,s, \tag{3.1}$$

Similarly we can consider the input targets. However, the decision maker may not satisfywith theidentified composite inputs or composite outputs as target of DMU_o. Nonetheless, thegenerated DEA efficiencyresults do not consider the value judgments of the decision maker.

In order to incorporate such value judgments into DEA model we can transform it into MOLP model(2.7). Then the goal programming method will be used to search for the most preferred solution alongthe efficient frontier for each DMU.
Suppose a set of priority goal rules on outputs is introduced by decision maker. Without loss ofgenerality, consider the outputs $1,..., p$ will be targeted at below a value of $\tilde{y}_1,...,\tilde{y}_p$, respectivelyand the levels of outputs $p+1,...,q$ will be improved to above a value of $\tilde{y}_{p+1},...,\tilde{y}_q$, respectivelyand finally the level of outputs $q+1,...,s$ will be maintain at their current levels $\bar{y}_{q+1},...,\bar{y}_s$, i.e. $\tilde{y}_r = \bar{y}_r, \quad r = q+1,...,s.$

Therefore, we have a multiple objectives decision making problem as follows:

$$
\begin{aligned}
&\mathit{Optimize} && \sum_{j=1}^{n} \lambda_j y_{rj}, && r = 1,\dots, p,\\
&\mathit{Optimize} && \sum_{j=1}^{n} \lambda_j y_{rj}, && r = p+1,\dots, q,\\
&\mathit{Optimize} && \sum_{j=1}^{n} \lambda_j y_{rj}, && r = q+1,\dots, s,\\
&s.t. && && \\
& && \sum_{j=1}^{n} \lambda_j x_{ij} \le x_{io}, && i = 1,\dots, m\\
& && \lambda_j \ge 0, && j = 1,\dots, n
\end{aligned}
\tag{3.2}
$$

Now we use the equivalence MOLP model (2.7) to allow us to incorporate the DMs preferences into our model and generate solutions according to the above preferences, and for performing this purpose we employ the goal programming method.

We set $\tilde{y}_1,\dots,\tilde{y}_n$ as target levels for the above goals (objectives), that is:

I. $\sum_{j=1}^{n} \lambda_j y_{rj} \le \tilde{y}_r, \quad r \in \{1,\dots, p\}$

II. $\sum_{j=1}^{n} \lambda_j y_{rj} \ge \tilde{y}_r, \quad r \in \{p+1,\dots, q\}$

III. $\sum_{j=1}^{n} \lambda_j y_{rj} = \tilde{y}_r, \quad r \in \{q+1,\dots, s\}$

Now, we introduce deviation variables d_r^+ and d_r^-, where d_r^- and d_r^+ are both nonnegative real umbers,but can't be positive at the same time, i.e. $d_r^+ d_r^- = 0$. It is desirable that the deviation variables d_r^- and d_r^+ are kept to be small as possible, which leads to the following goal programming model:

$$
\begin{aligned}
d^* = min \quad & \sum_{r\in\{A\}} d_r^+ + \sum_{r\in\{B\}} d_r^- + \sum_{r\in\{C\}} (d_r^- + d_r^+) \\
s.t. \quad & \\
& \sum_{j=1}^{n} \lambda_j y_{rj} + d_r^- - d_r^+ = \tilde{y}_r, \qquad r \in \{A\}, \\
& \sum_{j=1}^{n} \lambda_j y_{rj} + d_r^- - d_r^+ = \tilde{y}_r, \qquad r \in \{B\}, \\
& \sum_{j=1}^{n} \lambda_j y_{rj} + d_r^- - d_r^+ = \tilde{y}_r, \qquad r \in \{C\}, \\
& \sum_{j=1}^{n} \lambda_j x_{ij} \le x_{io}, \qquad i = 1,...,m, \\
& \lambda_j, d_r^-, d_r^+ \ge 0, \qquad j = 1,...,n,\ r = 1,...,s.
\end{aligned}
\tag{3.3}
$$

where $\{A\},\{B\}$ and $\{C\}$ indicate fixed index sets independent of j, such that $\{A\} = \{1,..., p\}$, $\{B\} = \{p+1,...,q\}$ and $\{C\} = \{q+1,...,s\}$.

Theorem 3.1.*Model (3.3) is always feasible.*

Proof. Suppose $\hat{\lambda} = (\hat{\lambda}_1,...,\hat{\lambda}_n)$ is such that $\sum_{j=1}^{n} \hat{\lambda}_j x_{ij} \le x_{io}, \quad i = 1,...,m,$. Then if for $r = 1,...,s$ we define

$$
\begin{cases}
\hat{d}_r^- = \max\{(\tilde{y}_r - \sum_{j=1}^{n} \hat{\lambda}_j y_{rj}), 0\}, \\
\hat{d}_r^+ = \max\{-(\tilde{y}_r - \sum_{j=1}^{n} \hat{\lambda}_j y_{rj}), 0\}.
\end{cases}
$$

then it is obvious that $(\hat{\lambda}, \hat{d}^-, \hat{d}^+)$ is a feasible solution for model (3.3). □

In model (3.3) if $d^* = 0$ then all goals are achieved, and the decision maker is fully satisfied with the obtained solution. Otherwise if $d^* \ne 0$ then some of his/her wishes goal rules are not satisfied. In order to remove this difficulty, the decision maker has two following ways.

- Method 1: The decision maker must sacrificesome of the unimportant goal rules to improve important goal rules.
- Method 2: This method requires that the decision maker, in addition to setting the goals for the
  objectives, also be able to give an ordinal ranking of the objectives.

We follow the second method. Without loss of generality, suppose that the following goal rules arenecessary, that is, they must be hold.

$$\sum_{j=1}^{n} \lambda_j y_{rj} \leq \tilde{y}_r, \qquad r \in \{A'\},$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \geq \tilde{y}_r, \qquad r \in \{B'\},$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} = \tilde{y}_r, \qquad r \in \{C'\},$$

where $A' \subseteq A$, $B' \subseteq B$ and $C' \subseteq C$. Therefore we set these goals in rank order one and other goals in the second rank order. Hence the goal programming formulation of the problem is as follows:

$$d^* = min \left\{ \begin{array}{l} P_1\left(\sum_{r\in\{A'\}} d_r^+ + \sum_{r\in\{B'\}} d_r^- + \sum_{r\in\{C'\}} (d_r^- + d_r^+) \right), \\ P_2\left(\sum_{r\in\{A''\}} d_r^+ + \sum_{r\in\{B''\}} d_r^- + \sum_{r\in\{C''\}} (d_r^- + d_r^+) \right) \end{array} \right\}$$

$$\begin{array}{lll} s.t. & & \\ \sum_{j=1}^{n} \lambda_j y_{rj} + d_r^- - d_r^+ = \tilde{y}_r, & r \in \{A\}, & \\ \sum_{j=1}^{n} \lambda_j y_{rj} + d_r^- - d_r^+ = \tilde{y}_r, & r \in \{B\}, & \\ \sum_{j=1}^{n} \lambda_j y_{rj} + d_r^- - d_r^+ = \tilde{y}_r, & r \in \{C\}, & \\ \sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{io}, & i = 1,\ldots,m, & \\ \lambda_j, d_r^-, d_r^+ \geq 0, & j = 1,\ldots,n,\ r = 1,\ldots,s. & \end{array} \tag{3.4}$$

where $\{A''\} = \{A\} - \{A'\}$, $\{B''\} = \{B\} - \{B'\}$ and $\{C''\} = \{C\} - \{C'\}$. The P_i' sispreemptive weights; that is $P_1 \gg P_2$. This implies that no number w, however large, can make $wP_2 > P_1$. The solution algorithm for model (3.4) is that $\left(\sum_{r\in\{A'\}} d_r^+ + \sum_{r\in\{B'\}} d_r^- + \sum_{r\in\{C'\}} (d_r^- + d_r^+) \right)$ is minimized first; let $\bar{d} = \min\left(\sum_{r\in\{A'\}} d_r^+ + \sum_{r\in\{B'\}} d_r^- + \sum_{r\in\{C'\}} (d_r^- + d_r^+) \right)$.

Next $\left(\sum_{r\in\{A''\}} d_r^+ + \sum_{r\in\{B''\}} d_r^- + \sum_{r\in\{C''\}} (d_r^- + d_r^+)\right)$ is minimized, but in no circumstances can $\left(\sum_{r\in\{A'\}} d_r^+ + \sum_{r\in\{B'\}} d_r^- + \sum_{r\in\{C'\}} (d_r^- + d_r^+)\right)$ be greater than $\bar{d}$ .Then in this method, first, the most important goals of DM are satisfied, and then it is try to satisfythe other goal rules as much as possible.

A CASE STUDY

In this section we evaluate 7 branches of retail bank industry to demonstrate the proposed approach to search for the most preferred solution on the efficient frontier. The data set is obtained from Wong et al. (2004). Each branch uses three inputs in order to produce three outputs. The labels of inputs and outputs are presented in Table 1.

Table 1: The labels of inputs and outputs

	Input	**Output**
1	The number of branches (‘000)	Total revenue
2	The number of ATM’s (‘000)	Corporate image
3	The number of staff (‘0,000)	Customer satisfaction

The data set for these 7 branches is given in Table 2.

Table 2: The data set of the example

DMU	**Input 1**	**Input 2**	**Input 3**	**Output 1**	**Output 2**	**Output 3**
1	2	2.18	2.35	10.57	3.4	6.79
2	1.95	3.19	8.43	13.35	6.66	2.55
3	0.80	2.1	3.21	8.14	1.92	9.17
4	1.75	4	13.30	23.67	8.47	5.82
5	2.5	4.30	9.27	14.01	3.44	6.57
6	1.73	3.3	7.7	12.04	2.53	4.86
7	0.65	1.53	2.67	7.36	1.26	7.28

The result of output-orientated CCR model is shown in Table 3 which is maximizing the amount bywhich outputs must be proportionally increased for the observed DMU to be efficient. For example,the efficiency score for branch 6 is 1.4578069, implying that it is operating as an inefficient branch withrespect to all 7 branches. The composite outputs that the observed DMUs should benchmark againstare shown in Table 4. For example the vector of composite outputs for DMU_6 is (17.55, 5.11, 9.74)and shows that DMU_6 must improves its output levels from current level (12.04,2.53,4.86) to level (17.55,5.11,9.74). As can be seen in Table 3, DMU_1, DMU_4 and DMU_7 are reference sets for DMU_6 .

Table 3: DEA efficiency result.

DMU	Efficiency	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7
1	1	1	0	0	0	0	0	0
2	1	0	1	0	0	0	0	0
3	1	0	0	1	0	0	0	0
4	1	0	0	0	1	0	0	0
5	1.61	0.61	0	0	0.42	0	0	0.88
6	1.46	0.29	0	0	0.31	0	0	0.71
7	1	1	0	0	0	0	0	1

However the DM may not be satisfied with this identified target for output levels of DMU_6, because he/she wishes to attain certain goal levels as output targets. These certain goal levels are identified by DM according to his/her priority. Therefore we must found a solution such that his/her goal levels are satisfied.

Table 4: The composite outputs for DMUs.

DMU	Composite output 1	Composite output 2	Composite output 3
1	10.57	3.40	6.79
2	13.35	6.66	2.55
3	8.14	1.92	9.17
4	23.67	8.47	5.82
5	22.59	6.68	12.75
6	17.55	5.11	9.74
7	7.36	1.26	7.28

However the DEA efficiency results generated do not consider the value judgments of the DM. Hence, we use goal programming method to design a procedure for searching for most preferred solution, that maximizes the DMs implicit utility function, in the sense that it is acceptable by the DM.

Consider DMU_6 and suppose that, the decision maker is not satisfied with the identified composite outputs as target of DMU_6. In order to derive the most preferred solution a set of priority goal rules will be introduced. We consider two cases:

- **The case when the satisfactory solution is found**
  In this case, suppose that the decision maker gives the set of priority goal rules as follows:
  The first rule is revenue will be targeted at least 16, the second rule is corporate image will betargeted at least 4.5 and the final rule is to improve the level of customer satisfaction to above avalue of 10.5. Now we apply the goal programming method to incorporate the DM's preferencesinto our model and generate solutions according to the above preferences. Therefore we solvethe following goal programming model:

$$
\begin{aligned}
&d^* = \min \; d_1^- + d_2^- + d_3^- \\
&s.t. \\
&\quad 10.57\lambda_1 + 13.35\lambda_2 + 8.14\lambda_3 + 23.67\lambda_4 + 14.01\lambda_5 + 12.04\lambda_6 + 7.36\lambda_7 + d_1^- - d_1^+ = 16, \\
&\quad 3.4\lambda_1 + 6.66\lambda_2 + 1.92\lambda_3 + 8.47\lambda_4 + 3.44\lambda_5 + 2.53\lambda_6 + 1.26\lambda_7 + d_2^- - d_2^+ = 4.5, \\
&\quad 6.79\lambda_1 + 2.55\lambda_2 + 9.17\lambda_3 + 5.82\lambda_4 + 6.57\lambda_5 + 4.86\lambda_6 + 7.28\lambda_7 + d_3^- - d_3^+ = 10.5, \qquad (4.1) \\
&\quad 2\lambda_1 + 1.95\lambda_2 + 0.8\lambda_3 + 1.75\lambda_4 + 2.5\lambda_5 + 1.73\lambda_6 + 0.63\lambda_7 \le 1.73, \\
&\quad 2.18\lambda_1 + 3.19\lambda_2 + 2.1\lambda_3 + 4\lambda_4 + 4.3\lambda_5 + 3.3\lambda_6 + 1.53\lambda_7 \le 3.3, \\
&\quad 2.35\lambda_1 + 8.43\lambda_2 + 3.21\lambda_3 + 13.3\lambda_4 + 9.27\lambda_5 + 7.7\lambda_6 + 2.67\lambda_7 \le 7.7, \\
&\quad \lambda_j, d_r^-, d_r^+ \ge 0, \; j = 1,\dots,7, \; r = 1,2,3.
\end{aligned}
$$

By solving model (4.1) the following solution is obtained

$$
\begin{aligned}
&d^* = 0, \\
&\lambda_1 = 0, \; \lambda_2 = 0, \; \lambda_3 = 0.723093334528948750, \; \lambda_4 = 0.348715380081326076, \\
&\lambda_5 = 0, \; \lambda_6 = 0, \; \lambda_7 = 0.252707501414317248,
\end{aligned}
$$

And the proposed composite outputs for DMU_6 is as

$$
\begin{cases}
\tilde{y}_1 = 16.00, \\
\tilde{y}_2 = 4.66, \\
\tilde{y}_3 = 10.50.
\end{cases}
$$

With this solution all the conditions are met, and the decision maker is fully satisfied with the indifference tradeoffs between the objectives, the procedure will terminate and the most preferred solution is found.

- **The case when the satisfactory solution is not found**
  In this case, suppose that the decision maker gives the set of priority goal rules as follows:
  The first rule is revenue will be targeted at least 19, the second rule is corporate image be maintain at its current target at level 4.5 and the final rule is to improve the level of customersatisfaction to above a value of 11. Now we apply the goal programming method to incorporatethe DM's preferences into our model and generate solutions according to the above preferences.
  Therefore we solve the following goal programming model:

$$
\begin{aligned}
& d^{*} = \min \ d_1^- + d_2^- + d_2^+ + d_3^- \\
& s.t. \\
& \quad 10.57\lambda_1 + 13.35\lambda_2 + 8.14\lambda_3 + 23.67\lambda_4 + 14.01\lambda_5 + 12.04\lambda_6 + 7.36\lambda_7 + d_1^- - d_1^+ = 19, \\
& \quad 3.4\lambda_1 + 6.66\lambda_2 + 1.92\lambda_3 + 8.47\lambda_4 + 3.44\lambda_5 + 2.53\lambda_6 + 1.26\lambda_7 + d_2^- - d_2^+ = 4.5, \\
& \quad 6.79\lambda_1 + 2.55\lambda_2 + 9.17\lambda_3 + 5.82\lambda_4 + 6.57\lambda_5 + 4.86\lambda_6 + 7.28\lambda_7 + d_3^- - d_3^+ = 11, \\
& \quad 2\lambda_1 + 1.95\lambda_2 + 0.8\lambda_3 + 1.75\lambda_4 + 2.5\lambda_5 + 1.73\lambda_6 + 0.63\lambda_7 \le 1.73, \\
& \quad 2.18\lambda_1 + 3.19\lambda_2 + 2.1\lambda_3 + 4\lambda_4 + 4.3\lambda_5 + 3.3\lambda_6 + 1.53\lambda_7 \le 3.3, \\
& \quad 2.35\lambda_1 + 8.43\lambda_2 + 3.21\lambda_3 + 13.3\lambda_4 + 9.27\lambda_5 + 7.7\lambda_6 + 2.67\lambda_7 \le 7.7, \\
& \quad \lambda_j, d_r^-, d_r^+ \ge 0, \ j = 1, \ldots, 7, \ \ r = 1,2,3.
\end{aligned}
\tag{4.2}
$$

The optimal objective of model (4.2) is $d^* = 1.689$ and shows that the decision maker is not satisfied with this solution. Now we asked the decision maker to give an ordinal ranking of the goals. Suppose the third rule, i.e. improve the level of customer satisfaction to above a value of 11, is necessary and we set this goal in rank order one and other goals in the second rank order.
Hence the goal programming formulation of the problem is as follows:

$$
\begin{aligned}
& d^{*} = \min \ \{\mathrm{P}_1(d_1^-), P_2(d_2^- + d_2^+ + d_3^-)\} \\
& s.t. \\
& \quad 10.57\lambda_1 + 13.35\lambda_2 + 8.14\lambda_3 + 23.67\lambda_4 + 14.01\lambda_5 + 12.04\lambda_6 + 7.36\lambda_7 + d_1^- - d_1^+ = 19, \\
& \quad 3.4\lambda_1 + 6.66\lambda_2 + 1.92\lambda_3 + 8.47\lambda_4 + 3.44\lambda_5 + 2.53\lambda_6 + 1.26\lambda_7 + d_2^- - d_2^+ = 4.5, \\
& \quad 6.79\lambda_1 + 2.55\lambda_2 + 9.17\lambda_3 + 5.82\lambda_4 + 6.57\lambda_5 + 4.86\lambda_6 + 7.28\lambda_7 + d_3^- - d_3^+ = 11, \\
& \quad 2\lambda_1 + 1.95\lambda_2 + 0.8\lambda_3 + 1.75\lambda_4 + 2.5\lambda_5 + 1.73\lambda_6 + 0.63\lambda_7 \le 1.73, \\
& \quad 2.18\lambda_1 + 3.19\lambda_2 + 2.1\lambda_3 + 4\lambda_4 + 4.3\lambda_5 + 3.3\lambda_6 + 1.53\lambda_7 \le 3.3, \\
& \quad 2.35\lambda_1 + 8.43\lambda_2 + 3.21\lambda_3 + 13.3\lambda_4 + 9.27\lambda_5 + 7.7\lambda_6 + 2.67\lambda_7 \le 7.7, \\
& \quad \lambda_j, d_r^-, d_r^+ \ge 0, \ j = 1, \ldots, 7, \ \ r = 1,2,3.
\end{aligned}
\tag{4.3}
$$

model (4.2) is
By solving model (4.3) the following solution is obtained

$$
\begin{aligned}
& d^* = 1.689, \\
& \lambda_1 = 0.0688498032220683232, \ \ \lambda_2 = 0, \ \ \lambda_3 = 0, \ \ \lambda_4 = 0.323011448665779688, \\
& \lambda_5 = 0, \ \ \lambda_6 = 0, \ \ \lambda_7 = 1.21428864987762841,
\end{aligned}
$$

And the proposed composite outputs for DMU_6 is as

$$
\begin{cases}
\tilde{y}_1 = 17.31, \\
\tilde{y}_2 = 4.50, \\
\tilde{y}_3 = 11.19.
\end{cases}
$$

With this solution the decision maker is satisfied with the indifference tradeoffs between theobjectives, the procedure will terminate and the preferred solution is found.

CONCLUSION

DEA is a well-known family of mathematical programming tools for assessing the relative efficiencythat was developed without consideration of the decision maker'spreference structures. DEA andmultiple objective linear programming (MOLP) are tools that can be used in management controland planning. In this paper we use the equivalence relationship between the output oriented CCRenvelopment and MOLP (Wong et al., 2009) and show how a DEA problem can be solved by transforming it intoMOLP formulation. We use the goal programming method to reflecting the DMs preferences in theprocess of assessing efficiency. The case study illustrated how this hybrid method can be implementedto support integrated efficiency analysis and target setting.For this reason, we used a data set from Wong et al. 2004. These data analyzed in two different cases: (a) The case when the satisfactory solution is found and (b) The case when the satisfactory solution is not found. The analysis showed that by using a proposed method we can attain a certain goal levels as output targets. Therefore, the presented method can consider the value judgments of the DM and also can find a solution such that the DM's goal levels are satisfied.

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