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LOGIC-BASED AGGREGATION METHODS FOR RANKING STUDENT APPLICANTS

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Abstract: In this paper, we present logic-based aggregation models used for ranking student applicants and we compare them with a number of existing aggregation methods, each more complex than the previous one. The proposed models aim to include dependencies in the data using Logical aggregation (LA). LA is a aggregation method based on interpolative Boolean algebra (IBA), a consistent multi-valued realization of Boolean algebra. This technique is used for a Boolean consistent aggregation of attributes that are logically dependent. The comparison is performed in the case of student applicants for master programs at the University of Belgrade. We have shown that LA has some advantage over other presented aggregation methods. The software realization of all applied aggregation methods is also provided. This paper may be of interest not only for student ranking, but

also for similar problems of ranking people e.g. employees, team members, etc.

Keywords: Aggregation, Logical aggregation, Interpolative Boolean Algebra, Fuzzy Logic, Ranking Students.

MSC: 47S40.

1. INTRODUCTION

This paper examines the problem of ranking people in multi-attribute environment. It typically includes aggregation of various attributes, e.g. skills, knowledge, personal characteristics, that may not be easily calculated or estimated. Whether it is a problem of ranking job candidates [10, 22], doctorial students [5, 7, 30], scholarship approval [36], vacancies at university [5] or selection of nominees for the award, it is the people and their characteristics to be valued and estimated. As a consequence, regardless the method used for ranking, the imprecision and logical dependencies of the data are built-in as subjective nature of one's assessment is always present. So, the problem of ranking people should rely on human reasoning and logic, best described by soft computing methods. These methods are not strict in their nature but can model uncertainty [38].

Although there are more sophisticated methods, the techniques used for student applicants ranking in practice are rather simple. They usually include one or two attributes, e.g. entrance exam score, and average grade. However, this may not be appropriate in case of the specific knowledge and skills required by various master programs. In order to treat the problem adequately, it should be perceived in the multi-attribute environment (e.g. [32]). In the literature, linear aggregation functions are appealing because of theirs simplicity and transparency, which enhances the confidence in the results [17]. However, there is considerable imprecision in the way that people reason and make decisions. In other words, it is not always easy to describe people characteristics by crisp numbers. Also, neither logical and statistical dependence of attributes involved in ranking nor compensation effect can be described with linear models.

There are several approaches that incorporate logic and imprecision in the process of multiple attributes aggregation [4]. Fuzzy set theory/fuzzy logic is a powerful tool for modeling imprecision [37]. It is a generalization of classical set theory in which a particular element belongs to some set with a certain degree of membership that is valued in [0,1] interval. Fuzzy sets are able to handle ambiguity (uncertainty) by using their overlapping boundaries, and fuzzy logic relationships among attributes can be described by various logical relations, e.g. disjunctions, conjunctions, negation, or their combination. Fuzzy sets and fuzzy logic are used for modeling and aggregating assessments in education [1, 8, 11, 34]. On the other hand, fuzzy logic, in general, is not Boolean consistent [25], what may affect final rankings/assessments. Logical aggregation (LA) introduced by Radojevic [26] is Boolean consistent and transparent procedure for aggregating different attributes based on interpolative Boolean algebra (IBA). LA is able to model relationships among attributes used for aggregation, which was not possible by weighted average. A linear convex combination of LA is called pseudo LA, and it is applied in different areas [16, 18, 23, 27].

This research continues the work in [20], where we first considered LA as a method for ranking student applicants. Now, we propose models for student applicants ranking based on pseudo LA. These models take into account logical dependencies of attributes, so providing more options for modeling. Further, the purpose of this paper is to compare the existing methods used for ranking student applicants. We aim to provide a critical review of their theoretical background and to propose their application to appropriate problem setting. The comparison of the presented models is performed in the case of student applicants for master programs at the Faculty of Organizational Sciences, University of Belgrade. We applied two models based on weighted average, one model based on fuzzy logic, and two models based on pseudo LA for ranking student applicants. We argue that a linear aggregation is not entirely suitable for the problem of evaluation and ranking and we demonstrate that pseudo LA has some advantage over other presented aggregation methods. The software realization of all applied aggregation methods is also provided.

The paper is organized as follows. In the next section we give a short review of aggregation methods used in this paper. Special attention is given to LA based on IBA. In Section 3, the problem of ranking student applicants is described and the proposed models used for ranking are presented from the simplest to the more complex ones. The results for each model are presented and discussed in Section 4. In the final section, we outline the main conclusions and the guidelines for a future work.

2. METHODS

The fusion of primary attributes in order to obtain new, aggregated information that is more suitable for further processing is known as the aggregation. This process plays a crucial role in multi-attribute decision making, pattern recognition, etc. Formally, an aggregation procedure is based on a mathematical aggregation operator.

Definition 1 ([4]). *A function* $A : [0,1]^n \to [0,1]$ *is called aggregation operator if, for all* $x_i, y_i \in [0,1], i = 1, ..., n$*, there holds:*

- $A(0, ..., 0) = 0$ *(lower boundary condition);*
- $A(1, ..., 1) = 1$ *(upper boundary condition);*
- $A(x_1, ..., x_n) \leq A(y_1, ..., y_n)$ *when* $x_i \leq y_i, i = 1, ..., n$ *(monotonicity).*

There are numerous approaches to aggregation of multiple attributes into a combined score for each alternative [2]. A common approach uses quasi arithmetic means as a family of algebraic aggregation methods [12]. This family of means can be divided into four types of means [30]: generalized mean, geometric mean, harmonic mean, arithmetic mean. In order to average values with different importance, the quasi-linear means are introduced. Another well-known aggregation technique, based on weighting operators, is ordered weighted averaging (OWA) introduced by Yager [35], where linguistic quantifiers are used in the aggregation function.

The weighted sum is considered as the simplest and most frequently used aggregation tool [33]. The weighted sum is additive and it may not be appropriate for the problems which are non-additive in their nature. In the multi-attribute decision making community fuzzy measure and fuzzy integrals are used, where additivity is relaxed by monotonicity. For instance, Choquet integral is appropriate for monotone continuous logical and/or pseudo-logical functions. A generalized discrete Choquet integral [24] is defined as a generalized measure that is non-monotone in a general case. This approach includes all logical and/or pseudo-logical functions but only for one arithmetic operator - min function. However, logical aggregation based on interpolative Boolean algebra [26] can treat all logical and/or pseudo-logical functions, as well as generalized Choquet integral, using all possible interpolative operators. Therefore, the possible domain of application is much wider from the standpoint of logic.

Models based on weighted sums, fuzzy logic, and pseudo-logical aggregation are included in this comparison.

2.1. Weighted sum

The weighted sum (WS) method is the common approach for the multi-attribute decision making. It provides the way to weight and combine normalized attributes in order to obtain the resulting value, which can be used for ranking. Weights in the model can be static or movable. The attributes normalization ensures their comparability, as otherwise, high numbered attributes would make disproportionate contribution to the overall score [32]. WS is defined as follows:

$$\sum_{i=1}^{n} w_i \cdot u_i = score, \sum_{i=1}^{n} w_i = 1$$

where w_i is weight for attribute u_i.

WS is used as the aggregation function within many different algorithms for decision making and optimization - PROMETHEE [3], Analytic Hierarchy Process (AHP) [29], genetic optimization algorithms [15], etc. It is also the basis of many aggregation techniques, e.g. OWA. WS can be used independently as one of the basic measures for aggregation in quite different areas, from finance [19] to software evaluation systems [31].

2.2. Fuzzy logic

Fuzzy logic (FL) is a generalization of classical logic - it can process all values in the unit interval [0,1]. FL is not fuzzy, it is a precise logic of imprecision and approximate reasoning [39]. FL is developed on fuzzy sets theory, so it is particularly suitable for dealing with linguistic variables like *hot*, *very good*, *around twenty*, *not that busy*, etc. Compared with conventional approaches, fuzzy control utilizes more information from the experts' domain and relies less on mathematical modeling a physical system.

Functions that qualify as fuzzy intersections and fuzzy unions are referred as t-norms and t-conorms, respectively.

Definition 2 ([13]). *A function* $T : [0,1]^2 \to [0,1]$ *is called* t*-norm if, for all* $x, y, z \in [0,1], i = 1, ..., n$*, there holds:*

- $T(x, y) = T(y, x)$ *(commutativity);*
- $T(x, T(y, z)) = T(T(x, y), z)$ *(associativity);*
- $x \le y \Rightarrow T(x, y) \le T(x, z)$ *(monotonicity);*
- $T(x, 1) = 1 \wedge T(1, y) = 1$ *(boundary condition / 1 is neutral element).*

The t-conorm is a function $S : [0,1]^2 \to [0,1]$ that satisfies properties of commutativity, associativity, monotonicity, and 0 is its neutral element.

The standard fuzzy intersection is min operator, and it produces the largest membership value of all the t-norms [13]. On top of that, algebraic product is used often as fuzzy intersection. The standard fuzzy union is max operator, and it produces the smallest membership value of all the t-conorms. Probabilistic sum is another operator often used as the t-conorm. These functions, along with the negation operator, are commonly used as a fuzzy aggregation function. It is obvious that every t-norm and t-conorm satisfies conditions to be an aggregation function.

FL is based on principle of truth functionality: The truth valued of a complex formula is uniquely determined by the truth values of its sub formulas [9]. This principle does not take into account the nature of the variables but only their values.

2.3. Interpolative Boolean algebra and logical aggregation

Interpolative Boolean algebra is a consistent [0,1]-valued realization of Boolean algebra in the sense that it preserves all the laws which Boolean algebra relies on. It has two levels - symbolic and valued. On the symbolic level, all laws of Boolean algebra are valued indifferent. The elements of IBA (attributes) $a_1, ..., a_n \in \Omega$ on the symbolic level are treated independently of their realizations, and they represent attributes. Expressions are calculated based on the principle of structural functionality: Structure of any IBA element can be directly calculated on the basis of its components structures [25]. This principle treats negation differently from the classical principle of structure functionality, which allows preservation of all Boolean laws.

IBA is technically based on generalized Boolean polynomials (GBPs). Any logical function $F(a_1, ..., a_n)$ can be transformed into the corresponding GBP

$F(a_1,...,a_n)^{\otimes}$ using a set of IBA transformation rules, and a GBP uniquely corresponds to some logical function. IBA transformation rules on IBA valued level, i.e. the realization of the principle of structural functionality, are the following [26]:

$$(F(a_1,...,a_n) \wedge G(a_1,...,a_n))^{\otimes} = F(a_1,...,a_n)^{\otimes} \otimes G(a_1,...,a_n)^{\otimes}$$

$$(F(a_1,...,a_n) \vee G(a_1,...,a_n))^{\otimes} = F(a_1,...,a_n)^{\otimes} + G(a_1,...,a_n)^{\otimes} - \\ -(F(a_1,...,a_n) \wedge G(a_1,...,a_n))^{\otimes}$$

$$(\neg F(a_1,...,a_n))^{\otimes} = 1 - F(a_1,...,a_n)^{\otimes}$$

$$(a_i \wedge a_j)^{\otimes} = \begin{cases} a_i \otimes a_j, i \neq j \\ a_i, i = j \end{cases}$$

$$(a_i \vee a_j)^{\otimes} = a_i + a_j - (a_i \wedge a_j)^{\otimes}$$

$$(\neg a_i)^{\otimes} = 1 - a_i$$

Operators allowed in GBP are standard + and -, and generalized product $\otimes$. Generalized product (GP) is any function $\otimes : [0,1]^2 \to [0,1]$ that satisfies all four conditions of t-norms: commutativity, associativity, monotonicity, boundary condition (see Definition 2.), and additional non-negativity condition [26]:

$$\sum_{K \in P(\Omega/S)} = (-1)^{|K|} \otimes a_i^v \geq 0, S \in P(\Omega), a_i \in \Omega, a_i^v \in [0,1]$$

where is $P(\Omega)$ a partition of a set Ω and a_i^v is realization of a_i.

In the case of the two-element Boolean algebra $\Omega = \{a, b\}$, GP can be any t-norm that generates the result from the following interval:

$$max(a + b - 1, 0) \leq a \otimes b \leq min(a, b)$$

Depending on the nature of the attributes, we can distinguish three marginal cases for operator selection: 1) attributes of the same/similar nature should be aggregated using min function; 2) for attributes of the same/similar nature but inversely (negatively) correlated, GP should be Lukasiewicz t-norm; 3) independent attributes (different by nature) should be aggregated using ordinary product.

Logical aggregation

Logical aggregation is Boolean consistent and transparent procedure for aggregating factors based on IBA [26]. The task of LA is the fusion of primary attributes values into one resulting globally representative value using logical tools. In other words, any logical function transformed to GBP may be used as LA. LA has two steps:

- Normalization of attributes' values:

  $$|| \cdot || : \Omega \to [0, 1]$$

- Aggregation of normalized values of features into one resulting value by logical or pseudo-logical function as a LA operator:

  $$Aggr[0, 1]^n \to [0, 1]$$

Some logical functions, i.e. exclusive disjunction and equivalence, do not satisfy the property of monotonicity. Therefore, these functions are not aggregating operators in a traditional sense (see Definition 1.). On the other hand, the capability to model non-monotone interactions is one of the crucial advantages of LA.

A pseudo-logical function, called pseudo GBP, is a linear convex combination of generalized Boolean polynomials. Its usage allows aggregation of partial requirements expressed as logical expressions using weighted sum. Thus, arithmetic mean, weighted sum, min function, discrete and generalized Choquet integral can be obtained as special cases of LA operator [26]. Hence, LA is a very powerful tool for modeling vast number of both monotone and non-monotone dependencies. LA and pseudo LA are applied in various areas, e.g. as the base of routing algorithm [16], for the supplier ranking and selection [18], as a part of the case-based reasoning algorithm [23], in financial ratio analysis [27], for solving the problem of web service selection [14], etc.

3. PROBLEM OF RANKING APPLICANTS FOR MASTER OF SCIENCE STUDIES

In this section we use the above described aggregation methods for dealing with a problem of ranking applicants for a Master of Science studies at Faculty of Organizational Sciences (FOS).

Eight different study programs are offered at FOS, considering the proposition for applicant's enrollment in Master of Science studies 2014/2015. Every program is divided into modules or study groups. There are over thirty study groups that are highly specialized in quite different areas, e.g. software engineering, e-business, business intelligence and decision making, etc. Students are ranked in one of six ranking lists depending on the chosen Master of Science program. Enrollment in one of the study groups is used as an illustrative example.

The present ranking criteria for all study departments are the number of points scored at the entrance exam, and the average grade at undergraduate level. The number of points at the entrance examination is more important than the average grade in 3/2 ratio.

In this paper, ranking models are presented and analyzed in case of Operations Research study group. This study group is analyzed due to certain similarities with Business Statistics, the other study groups on the same study program. The qualification exam for the selected groups consists of questions considering operations research, statistics, management science, database administration, and information systems. Student applicants who enroll in this module are ranked on a ranking list for the particular study department.

3.1. Aggregation using weighted sum

Model 1. For the presented problem, the weighted sum of attributes used as criterion for ranking students is:

$$score_1 = w_1 \cdot g + w_2 \cdot p$$

where g is the average grade gained at undergraduate level and p is the number of points on the entrance exam for Master of Science studies. Both values are normalized in the interval [0,1]. Values w_1 and w_2 are weights predefined at the enrolment proposition, so the aggregation function used for ranking has the following form:

$$score_1 = 0.4 \cdot g + 0.6 \cdot p$$

Criticism. The qualification exam to module Operations Research consists of questions considering different domains: information systems, computer sciences, operations research, and statistics. All the questions are valued in the same way. This is not appropriate because it is desired that the applicant has knowledge considering all of these domains, but knowledge considering operations research should be the most important.

Average grade shows the general picture about applicant's success at the undergraduate level. However, the grades in subjects in the domain of operation research, statistics, mathematics, and system theory should be more important for enrolling in this module. Similarly, subjects in the domain of information systems, programming, databases and data structures, etc. should be underlined for enrolling in the study program Information Systems and Technologies.

The other flaw is the restriction of WS itself - it can neither be used to model logical expressions nor interaction among variables. Weighted sum ranking models consider only values of variables, but they cannot model conditionality, compensative or logical relations between the attributes. A certain number of methods for multi-attribute decision making, e.g. weighted product model, AHP, etc, have this kind of restriction.

Model 2. The first two obstacles (a) grades aggregated using arithmetic mean and b) a total score at the qualification exam do not underline characteristics important for the particular master program) can be resolved using a new model based on a weighted sum:

$$score_2 = w_1 \cdot g + w_2 \cdot g_{orsmst} + w_3 \cdot p + w_4 \cdot p_{ors}$$

Two new components appear in this expression: g_{orsmt} - average grade at undergraduate level at subjects in the domain of operations research, statistics, mathematics, and system theory; p_{ors} - points gained at the qualification exam, but only on questions considering operations research and statistics. Both values are normalized in the unit interval. Values w_1, w_2, w_3, and w_4 are the weights in this model. The values of these factors are 0.3, 0.1, 0.5 and 0.1, respectively, so the aggregation function is:

$$score_2 = 0.3 \cdot g + 0.1 \cdot g_{orsmst} + 0.5 \cdot p + 0.1 \cdot p_{ors}$$

The significance of the specific knowledge that is important for the enrolling into a chosen master program is underlined by new variables. Other requirements can be modeled by introducing new variables and setting the corresponding weights. However, the aggregation function defined in this way does not solve the problem - the subjects of interest are both the qualification exam achievement and the appropriate subjects accomplishment, but not their logical dependence.

3.2. Aggregation using fuzzy logic

The reason for introducing logic into consideration is the need of modeling conditions like following: "Applicant's ranking should depend on the number of points on the entrance exam p and the average grade at undergraduate level g". This condition implies that the values of predefined variables p and g should be aggregated using a relation of conjunction. Weighted sum considers variables separately and can't model interaction between them, so it is not an appropriate method to model this problem. It is necessary to introduce logic and logical operators, which can provide more options for aggregation.

Classical logic deals with variables that are either true or false, e.i. that are either 0 or 1. Values of attributes in the observed problem are from the [0,1] interval, so classical logic is not appropriate, too. That is the reason for the usage of models based on fuzzy logic.

Model 3. The modified weighted sum may be used to rank student applicants. The third element of weighted sum is amended, and it is defined as: "Conjunction of average grade at undergraduate level for subjects in the domain of operations research, statistics, mathematics, and systems theory g_{orsmst} and the number of points on the entrance exam for Master of Science studies obtained on matters related to operations research and statistics p_{ors}". Accordingly, weighted sum is:

$$score_3 = w_1 \cdot g + w_2 \cdot p + w_3 \cdot (g_{orsmst} \wedge p_{ors})$$

The values of weights w_1, w_2, and w_3 are: 0.3, 0.5 and 0.2. The algebraic product is used as t-norm. In accordance with that, weighted sum has the following form:

$$score_3 = 0.3 \cdot g + 0.5 \cdot p + 0.2 \cdot g_{orsmst} \cdot p_{ors}$$

Criticism. As it is mentioned, conventional fuzzy logic is based on principle of truth functionality. In general, it doesn't follow the law of excluded middle, one of the Boolean laws. Some authors state that the fuzzy set theory simply does not have an axiom of the excluded middle since it is not suitable for multi-valued case [28]. On the other hand, this inconsistency is considered as a fundamental problem.

3.3. Aggregation using pseudo-logic functions

The two different functions are proposed as the pseudo-logical aggregations for ranking student applicants. Both of them are similar to previously presented model based on the weighted sum operator: the first two elements and weights in the sum are the same (0.3, 0.5, and 0.2 respectively), while the third element in the sum differs.

Model 4. In this model, student applicants who have successfully mastered the material on operations research have advantage. On the contrary, the important thing is that they learned operations research and statistics for the entrance exam, and they have good prior knowledge in mathematics. In this way, knowledge in these areas may compensate the lack of knowledge on operations research.

The third element of weighted sum is defined in the following manner: "If the applicant's average grade at the undergraduate level in subjects in the domain of operations research and statistics g_{ors} is high, we are interested only in it. If it is not good, we are interested in the average grade at the undergraduate level for subjects in the domain of mathematics and systems theory g_{mst} and the number of points at the entrance exam obtained on questions related to operations research and statistics p_{ors}."

This verbal condition may be modeled as the following pseudo-logical function:

$$score_4 = w_1 \cdot g + w_2 \cdot p + w_3 \cdot (g_{ors} \vee (\neg g_{ors} \wedge g_{mst} \wedge p_{ors}))$$

The third element should be transformed to a suitable GPB on IBA symbolic level according to IBA transformation rules:

$$g_{ors} \vee (\neg g_{ors} \wedge g_{mst} \wedge p_{ors}) = g_{ors} + g_{mst} \otimes p_{ors} - g_{ors} \otimes g_{mst} \otimes p_{ors}$$

and after that, the value of expression can be calculated. In this case, the operator of generalized product $\otimes$ is product ($\otimes = \cdot$) because attributes are not of the same nature:

$$score_4 = 0.3 \cdot g + 0.5 \cdot p + 0.2 \cdot (g_{ors} + g_{mst} \cdot p_{ors} - g_{ors} \cdot g_{mst} \cdot p_{ors})$$

In this logic-base aggregation model, lower average grade for subjects in the domain of operations research and statistics may be compensated to a certain extent if the applicant has good grades in subjects in the domain of mathematics and systems theory and learn operations research and statistics for qualification exam. Also, the nature of variables, i.e. their statistical dependencies are included using the algebraic product as GP. The proposed aggregation function is monotone although LA functions are not monotone in the general case. Further, it should be noted that the transformation from requirements expressed verbally to mathematical model is easy and understandable. The logic-based part of the aggregation function is Boolean consistent which is extremely important from the point of validation.

Model 5. In this model, student applicants who have successfully mastered the material on operations research and statistics at the undergraduate studies are in advantage comparing to others. On the other hand, if they haven't mastered operations research, the important thing is that they learned operations research and statistics for the entrance exam, as well as they have good prior knowledge in mathematics. This model is interesting because results considering operations research and statistics are separated and interpreted in different ways. The third variable of weighted sum is defined as follows: "If the applicant's average grade at the undergraduate level for subjects in the domain of operations research g_{or} is high, we are interested in the average grade at undergraduate level in subjects in the domain of statistics g_s, too. If it is not high, we are interested in the average grade at the undergraduate level for subjects related to mathematics and systems theory g_{mst} and the number of points at the entrance exam for Master of Science studies obtained on questions related to operations research and statistics p_{ors}."

This verbal condition may be modeled as the following pseudo-logical function:

$$score_5 = w_1 \cdot g + w_2 \cdot p + w_3 \cdot ((g_{or} \wedge g_s) \vee (\neg g_{or} \wedge g_{mst} \wedge p_{ors}))$$

where the third element should be transformed to a suitable GPB:

$$(g_{or} \wedge g_s) \vee (\neg g_{or} \wedge g_{mst} \wedge p_{ors}) = g_{or} \otimes g_s + g_{mst} \otimes p_{ors} - g_{or} \otimes g_{mst} \otimes p_{ors}$$

The operator of generalized products $\otimes$ is product ($\otimes = \cdot$) since the nature of the variables to be aggregated:

$$score_5 = 0.3 \cdot g + 0.5 \cdot p + 0.2 \cdot (g_{or} \cdot g_s + g_{mst} \cdot p_{ors} - g_{or} \cdot g_{mst} \cdot p_{ors})$$

The values of all attributes are normalized in the unit interval. In this aggregation model, grades for subjects in the domain of operations research and statistics are observed separately. If the applicant has shown good knowledge of operations research, these grades are aggregated using logical conjunction. Everything else claimed for the Model 4. holds for this model, too.

4. APPLICATION AND DISCUSSION

Since there are over 700 applicants enrolling FOS Master of Science studies every year, manual data processing is impractical and it would take a lot of time. That is the main reason for implementing a software solution that can handle analyzed ranking models. Further, the proposed aggregation models are applied to the problem of ranking student applicants for the study group Operations Research and the results are compared.

4.1. Software realization

The software has a main form that contains a text field for entering new aggregation function, a list of the functions in the model pool, a table for entering input data, and a table for output data, e.i. the results calculated using the models from the model pool. There are four user operations that are implemented: adding an aggregation function into the model pool, deleting function from it, defining variables' values, and calculating final scores for each model.

As it is noted, LA is based on IBA that has specific rules of transformation from logical expression to generalized Boolean polynomials. JFuzzyIBATranslator [21] is the software component that handles GBP's transformation and calculation. That program is incorporated in this software system, so the users that are not familiar with IBA in details can simulate data. This program solution can be linked to excel file and collect data from it. Also, an excel file can be selected as an output. In that manner transparency is achieved and the comparison of final scores is simplified and facilitated.

4.2. Results and discussion

For the purpose of ranking, the data on 50 students are used for the calculation. It should be emphasized that the exact number of points at the entrance exam obtained on matters related to operations research and statistics is unknown because it is not important for the current ranking model. This variable, referred as p_{ors}, is defined as linguistic variable with suggested values: *very low*, *low*, *medium*, *high*, and *very high*.

In this research, students were asked to write their hypotheses about scored points in that study fields. Further, their hypotheses are converted to crisp values according to Chen and Hwang's conversion scale [6] with five different linguistic terms. More precisely, the linguistic terms are first converted to fuzzy numbers using a conversion scale, and then the fuzzy scoring method is used to convert each fuzzy number to a corresponding crisp value. According to Chen and Hwang's five-scale fuzzy linguistic scaling, listed linguistic variables are converted to following crisp values: 0.091, 0.283, 0.5, 0.717, 0.909.

Table 1. shows the data on 10 chosen student applicants whose characteristics were used to point out the differences in rankings provided by the proposed models.

Student	g	p	g_{orsmst}	p_{ors}	g_{ors}	g_{mst}	g_{or}	g_s
Student A	1	1	1	1	1	1	1	1
Student B	0.99	1	0.94	1	0.94	0.9	1	0.92
Student C	0.986	1	1	1	1	1	1	1
Student D	0.907	1	0.8	1	0.88	1	0.7	0.7
Student E	0.89	1	0.88	1	0.9	0.9	0.9	0.83
Student F	0.82	1	0.93	1	1	1	1	0.8
Student G	0.77	0.98	0.64	1	0.62	0.6	0.65	0.68
Student H	0.88	0.85	0.9	0.95	0.9	0.9	0.9	0.9
Student I	0.82	1	0.75	1	0.77	0.8	0.75	0.72
Student J	0.77	0.8	0.75	0.9	0.72	0.7	0.75	0.8

Table 1: The input data - characteristics of student applicants

Aggregation scores for each student applicant are shown in Table 2 and Figure 1. Rankings presented in Table 3 and Figure 2 differ for each model that is applied. The initial model is widely applied in practice. The second and the third naturally followed the previous model. Finally, the last two are logic-based models proposed in this paper. Differences in the applicants' rankings are the result of applying different aggregation methods, as well as the need to highlight specific factors. Student applicants A and J are always the first and the last because they are best/worst regarding all considered attributes. It is noticeable that applicants B and C, applicants D, E, and F, and applicants G, H, and I change their ranks when different models are applied.

Student	WS Model 1.	WS Model 2.	FL Model 3.	Pseudo LA Model 4.	Pseudo LA Model 5.
Student A	1	1	1	1	1
Student B	0.9960	0.9910	0.9850	0.9960	0.9954
Student C	0.9944	0.9958	0.9958	0.9958	0.9958
Student D	0.9628	0.9521	0.9321	0.9649	0.9121
Student E	0.9560	0.9547	0.9423	0.9636	0.9456
Student F	0.9280	0.9393	0.9327	0.9460	0.9460
Student G	0.8960	0.8853	0.8497	0.8967	0.8534
Student H	0.8620	0.8740	0.8600	0.8861	0.8681
Student I	0.8880	0.8910	0.8660	0.9022	0.8640
Student J	0.7880	0.7960	0.7660	0.8153	0.7792

Table 2: The outputs - aggregation scores for applied models

Student applicant B has a higher average grade and he/she is better than applicant C according to the first model. However, student B has much lower grade at group of subjects related to operations research than applicant C. As a result, the rest of the models favor student C. In Model 4, which is based on pseudo

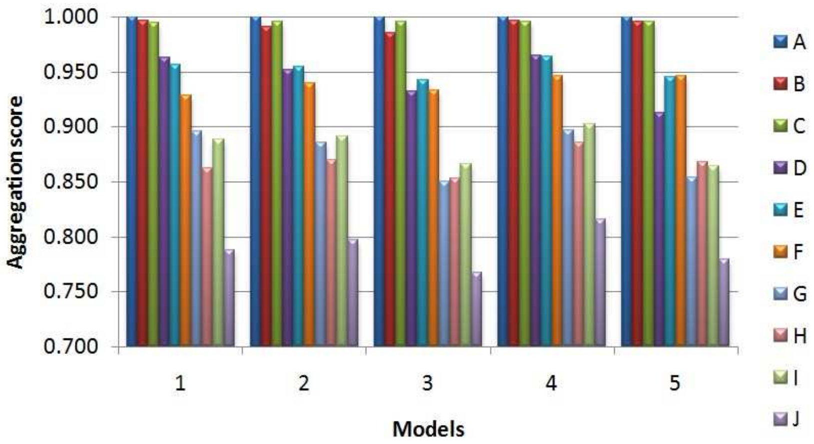

Figure 1: Student applicants' aggregation scores for different models

Student	WS Model 1.	WS Model 2.	FL Model 3.	Pseudo LA Model 4.	Pseudo LA Model 5.
Student A	1	1	1	1	1
Student B	2	3	3	2	3
Student C	3	2	2	3	2
Student D	4	5	6	4	6
Student E	5	4	4	5	5
Student F	6	6	5	6	4
Student G	7	8	9	8	9
Student H	9	9	8	9	7
Student I	8	7	7	7	8
Student J	10	10	10	10	10

Table 3: Student applicants' rankings for applied models

LA, the previously mentioned attribute is not taken into account alone but within the arithmetic mean of grades i.e. for subjects from both operations research and statistics area.

Student D has the highest average grade among student applicants D, E, and F. Therefore, he is ranked as the best among them according to Model 1. Similarly, student F has the lowest average grade, so he is the worst according to Model 1. On the other hand, student F has shown great results in subjects of interest, and that is pointed out by Model 5. The aggregation scores for students E and F, and student D are significantly different when the last model is applied. The main reason for this fact is that student D has lower grades at subjects in the domain of statistics as well as mathematics and system theory.

Student H has shown an average level of knowledge in operations research and

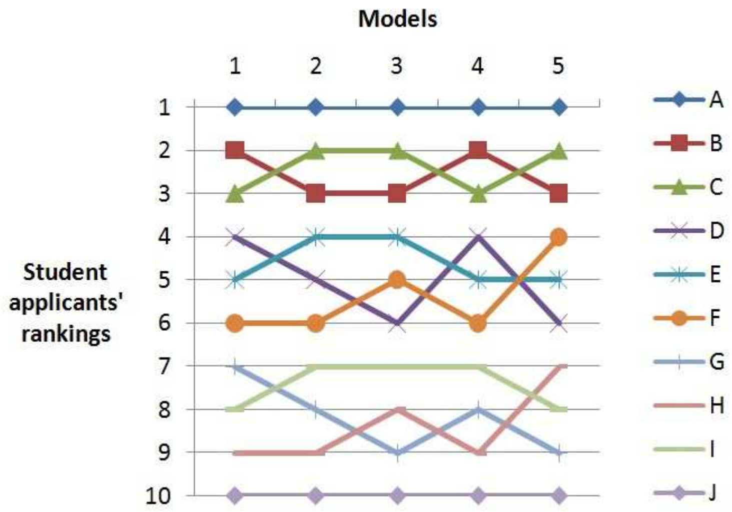

Figure 2: Student applicants' rankings scores for different models

statistics during the undergraduate studies. This affected his score calculated by the first three models. However, student H managed to compensate lower level of knowledge by doing well on the entrance exam. Only models based on pseudo LA could take the compensation into account, so the student's score is higher when these models are applied. Only for the initial model, student G outscored student H although student H had more success regarding specific knowledge of interest.

5. CONCLUSION

In this paper, we proposed a logic-based aggregation methods for ranking student applicants for the Master of Science programs at the University of Belgrade. The programs allows students to get specialized in different occupations' areas - business analysts, programmers, database specialists, managers, etc. Given the diversity of the existing programs, the advantage at ranking should be given to student applicants who have shown better results at subjects related to chosen domains as well as good grade average. It is important to identify and value even the subtle differences between students' characteristics, which can be recognized using models based on LA. LA is a technique which gives the user lots of descriptive power for modeling various problem situations. Due to its ability to model relationships between the attributes with logical functions, LA incorporates logic in the process of multiple attributes aggregation. The nature of variables is also included in modeling by selection of a suitable operator for the generalized product. Only models based on pseudo LA could distinct cases where one characteristic can or cannot compensate for another, which is particularly important in the problem of ranking students. Users can make their own pseudo-logical aggregation functions from their point of view. To support the application of LA, transparent and user-friendly software tool is also provided.

In this paper, we presented the results of a number of existing aggregation techniques and compared them with LA approach. We identified critical cases where LA provided different student rankings compared to other methods. It is shown that LA is able to treat these situations in appropriate manner. Since LA-based ranking models are not limited to student ranking, this paper may be valuable in analyzing and solving similar problems of ranking people, e.g. employees, team members, etc.

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