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A METHOD FOR SOLVING INTERVAL TYPE-2 TRIANGULAR FUZZY BILEVEL LINEAR PROGRAMMING PROBLEM

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Abstract: In this paper, we consider the bilevel linear programming problem (BLPP) where all the coefficients in the problem are interval type-2 triangular fuzzy numbers (IT2TFNs). First, we convert a BLPP with IT2TFN parameters to an interval BLPP. In the next step, we solve BLPPs and obtain optimal solution as an IT2TFN.

Keywords: Type-2 fuzzy sets, interval type-2 triangular fuzzy number, bilevel linear programming problem.

MSC: 90C26, 90C70.

1. INTRODUCTION

The BLPP was formulated by Bracken and McGill [1], which is very similar to the problem first introduced by Stackelberg [2]. So far, several methods have been proposed to solve the BLPP [3, 4, 5, 6, 7, 8, 9].

In the real world, the parameters of a problem may not be exact values; instead, they may be expressed as fuzzy numbers or intervals. Zadeh first introduced the theory of fuzzy sets in 1965 [10]. Lai et al. [11, 12] presented a fuzzy method for solving the crisp BLPP. Then Zhang et al. [13, 14, 15, 16] generalized crisp solving techniques for BLPPs with fuzzy parameters.

On the other hand, interval programming is a method for modeling problems under uncertainty. Recently, many researchers have studied interval linear programming [17, 18, 19, 20, 21, 22]. Calvete et al. [23] proposed two algorithms to find the worst and the best optimal values of the leader objective function in the BLPP when the coefficients of the leader and the follower objective functions are intervals. Mishmast Nehi and Hamidi [24] generalized the algorithm provided by Calvete et al. [23] for the BLPP when all coefficients were intervals and provided corrections to improve their algorithm. Li and Fang [25] used a heuristic algorithm for interval BLPP.

Zadeh introduced type-2 fuzzy sets (T2FS) [26, 27, 28]. Hisdal [29] presented interval T2FS. Figueroa [30] proposed two methods for solving linear programming problems that the right-hand side parameters were interval type-2 fuzzy sets (IT2FSs). One of them presented a Type-reduction method based on finding a fuzzy set embedded on the FOU of the IT2FS right-hand side parameters of the problem. Other proposed method provided pre-defuzzification procedure to find a solution by α-cut of the IT2FS. [31, 32, 33] presented methods for solving linear programming problems with interval type-2 fuzzy constraints and obtained solution problems as crisp numbers. In [34], linear programming with technological coefficients IT2FSs has been solved through an α-cuts approach. Kundu et al. [35] used the chance-constraint programming method for solving linear programming with interval type-2 fuzzy variables.

Also, many studies have been done on multi-criteria decision-making (MCDM) approaches based on IT2FSs so far. For example, an approach for solving MCDM problems, which is based on new arithmetic operations and the ranking rules of interval type-2 fuzzy number (IT2FN) was expressed in [36]. Chen [37] presented an algorithmic approach for analyzing the MCDM problems described by interval type-2 trapezoidal fuzzy numbers. Chen [38] developed interval type-2 fuzzy PROMETHEE I and II methods to manage the MCDM analysis problems in the context of interval type-2 trapezoidal fuzzy numbers. A signed-distance-based approach to importance assessment and MCDM analysis based on IT2FS is proposed in [39]. Chen in [40] expressed an Elimination Et Choice Translating Reality based outranking method for MCDM within the environment of IT2FS. An interactive method for MCDM analysis based on IT2FSs is presented in [41]. Chen [42] proposed the interval type-2 fuzzy TOPSIS method for addressing MCDM analysis problems in the interval type-2 trapezoidal fuzzy framework. Chen in [43] presented the extended QUALIFLEX method for MCDM analysis based on IT2FSs. Hu et al. [44] introduced deviation degree for trapezoidal IT2FN, and then they proposed a new approach based on possibility degree to solve MCDM problems that the criteria value was IT2FN. Sinha et al. [45] used the expected value of trapezoidal IT2FN defined by Hu et al. [44]. They maximized the profit and minimized the transportation time for two transportation models. In one of models, the unit purchase cost, unit selling price, unit transportation cost, and transportation time were trapezoidal IT2FNs, while in other models, all the parameters were trapezoidal IT2FNs.

In this paper, we express a BLPP with IT2TFN parameters that to the best of our knowledge, this problem has not been investigated so far. The use of ranking functions in optimization problems with IT2TFN parameters where each IT2FN is approximated with a crisp number [40, 44, 46], is relatively efficient and appropriate. But there is always a general defect of losing problem information due to ranking in all ranking methods. Using a well-defined approximate closed interval for an IT2FN is more acceptable and logical than replacing it with a crisp number. In this article, we convert any IT2TFN into an interval.

In many above cases IT2FS or IT2FN is approximated by a crisp number and the optimal value and the optimal solution are obtained as crisp number. One of the problems in the defuzzification procedures is the loss of information. Although this exists in all defuzzification procedures, but using a well-defined approximate interval is more acceptable and logical than replacing it with a crisp number.

The general structure of this article is as follows: After stating the basic concepts of BLPP and the type-2 fuzzy, we consider BLPP with IT2TFN parameters. We use an approximate interval for IT2TFN and convert BLPP with IT2TFN parameters to interval BLPP. Then we achieve five BLPPs that by solving them, the optimal value and the optimal solution obtain as IT2TFNs.

2. PRELIMINARIES

This section provides a brief overview and the key concepts required for understanding this paper.

2.1. BLPP

The general form of the BLPP is as (1):

$$\begin{aligned} &\max_{\mathbf{x}_1 \in X_1} F_1(\mathbf{x}_1, \mathbf{x}_2) = \mathbf{c}_1\mathbf{x}_1 + \mathbf{d}_1\mathbf{x}_2 \\ &\text{s.t.} \max_{\mathbf{x}_2 \in X_2} F_2(\mathbf{x}_1, \mathbf{x}_2) = \mathbf{c}_2\mathbf{x}_1 + \mathbf{d}_2\mathbf{x}_2 \\ &\qquad \text{s.t.} \quad A\mathbf{x}_1 + B\mathbf{x}_2 \le \mathbf{b}, \\ &\qquad\qquad \mathbf{x}_1, \mathbf{x}_2 \ge 0, \end{aligned} \tag{1}$$

where $\mathbf{x}_1$ and $\mathbf{x}_2$ are the leader and the follower decision variables, respectively. $F_1(\mathbf{x}_1, \mathbf{x}_2)$ and $F_2(\mathbf{x}_1, \mathbf{x}_2)$ are called the leader objective function and the follower objective function, respectively. Also, $\mathbf{x}_1 \in X_1 \subset \mathbb{R}^n$, $\mathbf{x}_2 \in X_2 \subset \mathbb{R}^m$, $F_1, F_2 : X_1 \times X_2 \to \mathbb{R}^1$, $\mathbf{c}_1, \mathbf{c}_2 \in \mathbb{R}^n$, $\mathbf{d}_1, \mathbf{d}_2 \in \mathbb{R}^m$, $\mathbf{b} \in \mathbb{R}^p$, $A \in \mathbb{R}^{p \times n}$ and $B \in \mathbb{R}^{p \times m}$. The BLPP involves two problems, (a) the problem of the leader level and (b) the problem of the follower level. First the leader tries to optimize $F_1(\mathbf{x}_1, \mathbf{x}_2)$ under some constraints by selecting $\mathbf{x}_1$. Then, the follower tries to optimize $F_2(\mathbf{x}_1, \mathbf{x}_2)$ by choosing $\mathbf{x}_2$ for the specific value of $\mathbf{x}_1$ [47].

Remark 1. *Suppose $I = \{\boldsymbol{d}_2 \in \mathbb{R}^m : d_{2i} \in [\check{d}_{2i}, \hat{d}_{2i}]; i = 1, ..., m\}$ and I_E represents the set of extreme points of I. Throughout this paper, it is assumed that inducible region [3] does not change for any $\boldsymbol{d}_2 \in I_E$.*

2.2. IT2TFN

Definition 2. *[10] Let X be a universal set. A fuzzy set $\tilde{A}$ is defined as an ordered pairs $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)|x \in X, \mu_{\tilde{A}}(x) \in [0,1]\}$ where $\mu_{\tilde{A}}(x)$ is the membership function of $x \in X$.*

Definition 3. *[48] Let $\tilde{A}$ be a fuzzy set in X. The support of $\tilde{A}$ is the crisp set given by $\{x \in X : \mu_{\tilde{A}}(x) > 0\}$.*

Definition 4. *[48] Let $\tilde{A}$ be a fuzzy set in $\mathbb{R}$. $\tilde{A}$ is called a fuzzy number (FN) if*
(i) $\tilde{A}$ is normal,
(ii) $\tilde{A}$ is convex,
(iii) $\mu_{\tilde{A}}$ is upper semicontinuous, and,
(iv) the support of $\tilde{A}$ is bounded.

Definition 5. *[48] A FN $\tilde{A} = (a, b, c)$ is called a triangular FN (TFN) if its membership function $\mu_{\tilde{A}}(x)$ is defined as follows:*

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a}{b-a} & a \leq x \leq b \\ \frac{c-x}{c-b} & b \leq x \leq c \\ 0 & O.W. \end{cases}$$

Definition 6. *[48] The α-cut of a FN $\tilde{A}$ is a set defined as $\tilde{A}_\alpha = \{x \in \mathbb{R}|\mu_{\tilde{A}}(x) \geq \alpha\} = [A^l(\alpha), A^u(\alpha)]$ where $A^l(\alpha) = inf\{x \in \mathbb{R}|\mu_{\tilde{A}}(x) \geq \alpha\}$ and $A^u(\alpha) = sup\{x \in \mathbb{R}|\mu_{\tilde{A}}(x) \geq \alpha\}$.*

Definition 7. *[48] The core of a FN $\tilde{A}$ is defined as $Core(\tilde{A}) = \{x \in \mathbb{R}|\mu_{\tilde{A}}(x) = 1\}$.*

Definition 8. *[49] A T2FS $\tilde{\tilde{A}}$, is characterized by a type-2 membership function $\mu_{\tilde{\tilde{A}}}(x, u)$ where $x \in X$ and $u \in J_x \subset [0,1]$:*

$$\tilde{\tilde{A}} = \{((x, u), \mu_{\tilde{\tilde{A}}}(x, u))|\forall x \in X, \forall u \in J_x \subset [0,1],\ 0 \leq \mu_{\tilde{\tilde{A}}}(x, u) \leq 1\},$$

where J_x is called the primary membership of x and $\mu_{\tilde{\tilde{A}}}(x, u)$ for $x \in X$ and $u \in J_x$ is called a secondary membership.

Definition 9. *[50] In Definition 8, when $\mu_{\tilde{\tilde{A}}}(x, u) = 1, \forall x \in X, u \in J_x \subset [0,1]$, $\tilde{\tilde{A}}$ is called interval T2FS.*

Definition 10. *[51] Uncertainty in the primary membership of an interval T2FS, consists of a bounded region called the footprint of uncertainty (FOU). It is the union of all primary memberships, i.e., FOU$= \cup_{x \in X} J_x$.*

Definition 11. *[51] Interval T2FS $\tilde{\tilde{A}} = (\tilde{A}^l, \tilde{A}^u)$ is bounded by two fuzzy sets $\tilde{A}^l$ and $\tilde{A}^u$, the named lower membership function and upper membership function, respectively.*

Definition 12. *[46] An IT2TFN $\tilde{\tilde{A}} = (\tilde{A}^l, \tilde{A}^u) = \big((\check{a}^l, \bar{a}, \hat{a}^l), (\check{a}^u, \bar{a}, \hat{a}^u)\big)$ is defined on the interval $[\check{a}^u, \hat{a}^u]$. Its lower membership function and upper membership function takes the value equal to 1 in the point $\bar{a}$, where $\check{a}^u \leq \check{a}^l \leq \bar{a} \leq \hat{a}^l \leq \hat{a}^u$. Fig. 1 shows an IT2TFN.*

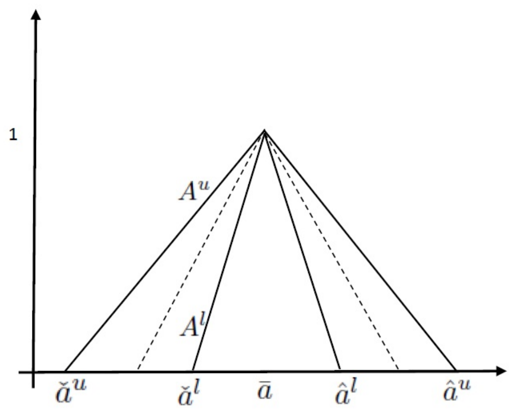

Figure 1: The interval type-2 triangular fuzzy number

3. BLPP WITH IT2TFN PARAMETERS

Consider the BLPP wherein all the parameters (the objective function, technological coefficients and the resource values) are expressed as IT2TFNs:

$$\begin{aligned} &\max_{\mathbf{x}_1 \in X_1} \; \tilde{\tilde{Z}} = \tilde{\tilde{\mathbf{c}}}_1 \mathbf{x}_1 + \tilde{\tilde{\mathbf{d}}}_1 \mathbf{x}_2 \\ &\qquad \text{s.t. } \max_{x_2 \in X_2} \; \tilde{\tilde{z}} = \tilde{\tilde{\mathbf{c}}}_2 \mathbf{x}_1 + \tilde{\tilde{\mathbf{d}}}_2 \mathbf{x}_2 \\ &\qquad\quad \text{s.t. } \tilde{\tilde{A}} \mathbf{x}_1 + \tilde{\tilde{B}} \mathbf{x}_2 \leq \tilde{\tilde{\mathbf{b}}} \\ &\qquad\quad \mathbf{x}_1, \mathbf{x}_2 \geq 0, \end{aligned} \tag{2}$$

where $\tilde{\tilde{\mathbf{c}}}_1 = \big((\check{\mathbf{c}}_1^l, \bar{\mathbf{c}}_1, \hat{\mathbf{c}}_1^l), (\check{\mathbf{c}}_1^u, \bar{\mathbf{c}}_1, \hat{\mathbf{c}}_1^u)\big)$, $\tilde{\tilde{\mathbf{d}}}_1 = \big((\check{\mathbf{d}}_1^l, \bar{\mathbf{d}}_1, \hat{\mathbf{d}}_1^l), (\check{\mathbf{d}}_1^u, \bar{\mathbf{d}}_1, \hat{\mathbf{d}}_1^u)\big)$,
$\tilde{\tilde{\mathbf{c}}}_2 = \big((\check{\mathbf{c}}_2^l, \bar{\mathbf{c}}_2, \hat{\mathbf{c}}_2^l), (\check{\mathbf{c}}_{21}^u, \bar{\mathbf{c}}_2, \hat{\mathbf{c}}_2^u)\big)$, $\tilde{\tilde{\mathbf{d}}}_2 = \big((\check{\mathbf{d}}_2^l, \bar{\mathbf{d}}_2, \hat{\mathbf{d}}_2^l), (\check{\mathbf{d}}_2^u, \bar{\mathbf{d}}_2, \hat{\mathbf{d}}_2^u)\big)$,
$\tilde{\tilde{A}} = \big((\check{A}^l, \bar{A}, \hat{A}^l), (\check{A}^u, \bar{A}, \hat{A}^u)\big)$, $\tilde{\tilde{B}} = \big((\check{B}^l, \bar{B}, \hat{B}^l), (\check{B}^u, \bar{B}, \hat{B}^u)\big)$ and
$\tilde{\tilde{\mathbf{b}}} = \big((\check{\mathbf{b}}^l, \bar{\mathbf{b}}, \hat{\mathbf{b}}^l), (\check{\mathbf{b}}^u, \bar{\mathbf{b}}, \hat{\mathbf{b}}^u)\big)$.

In our proposed algorithm, the solution of (2) is obtained by solving five BLPPs and finally, by solving them, we get the optimal solution as IT2TFNs $\tilde{\tilde{Z}} = \big((\check{Z}^l, \bar{Z}, \hat{Z}^l), (\check{Z}^u, \bar{Z}, \hat{Z}^u)\big)$ for leader objective function and $\tilde{\tilde{z}} = \big((\check{z}^l, \bar{z}, \hat{z}^l), (\check{z}^u, \bar{z}, \hat{z}^u)\big)$ for follower objective function.

According to the proposed method, we first solve problem (3) to obtain $\bar{Z}$ and $\bar{z}$, which we call the middle problem,

$$\begin{aligned}
&\max_{\mathbf{x}_1 \in X_1} \ \bar{\mathbf{c}}_1\mathbf{x}_1 + \bar{\mathbf{d}}_1\mathbf{x}_2 \\
&\quad \text{s.t.} \quad \max_{\mathbf{x}_2 \in X_2} \ \bar{\mathbf{c}}_2\mathbf{x}_1 + \bar{\mathbf{d}}_2\mathbf{x}_2 \\
&\qquad\qquad \text{s.t.} \quad \bar{A}\mathbf{x}_1 + \bar{B}\mathbf{x}_2 \leq \bar{\mathbf{b}} \\
&\qquad\qquad \mathbf{x}_1, \mathbf{x}_2 \geq 0,
\end{aligned} \tag{3}$$

which is a BLPP and can be easily solved using the K-th best method and $\bar{\mathbf{x}}_{1\ opt}$, $\bar{\mathbf{x}}_{2\ opt}$, $\bar{Z}_{opt}$ and $\bar{z}_{opt}$ are obtained.

Since an IT2TFN is formed of the infinite union of the characteristic TFNs, the traditional embedded TFNs are used for all the IT2TFN in (2). So (2) can be written as following type-1 triangular fuzzy BLPP :

$$\begin{aligned}
&\max_{\mathbf{x}_1 \in X_1} \ \tilde{\mathbf{c}}_1^e\mathbf{x}_1 + \tilde{\mathbf{d}}_1^e\mathbf{x}_2 \\
&\text{s.t.} \quad \max_{\mathbf{x}_2 \in X_2} \ \tilde{\mathbf{c}}_2^e\mathbf{x}_1 + \tilde{\mathbf{d}}_2^e\mathbf{x}_2 \\
&\qquad \text{s.t.} \quad \tilde{A}^e\mathbf{x}_1 + \tilde{B}^e\mathbf{x}_2 \leq \mathbf{b}^e \\
&\qquad \mathbf{x}_1, \mathbf{x}_2 \geq 0,
\end{aligned} \tag{4}$$

that $\tilde{\mathbf{c}}_1^e \in \text{FOU}(\tilde{\tilde{\mathbf{c}}}_1)$, $\tilde{\mathbf{d}}_1^e \in \text{FOU}(\tilde{\tilde{\mathbf{d}}}_1)$, $\tilde{\mathbf{c}}_2^e \in \text{FOU}(\tilde{\tilde{\mathbf{c}}}_2)$, $\tilde{\mathbf{d}}_2^e \in \text{FOU}(\tilde{\tilde{\mathbf{d}}}_2)$, $\tilde{A}^e \in \text{FOU}(\tilde{\tilde{A}})$, $\tilde{B}^e \in \text{FOU}(\tilde{\tilde{B}})$ and $\tilde{\mathbf{b}}^e \in \text{FOU}(\tilde{\tilde{\mathbf{b}}})$ are the TFNs (See Fig. 2).

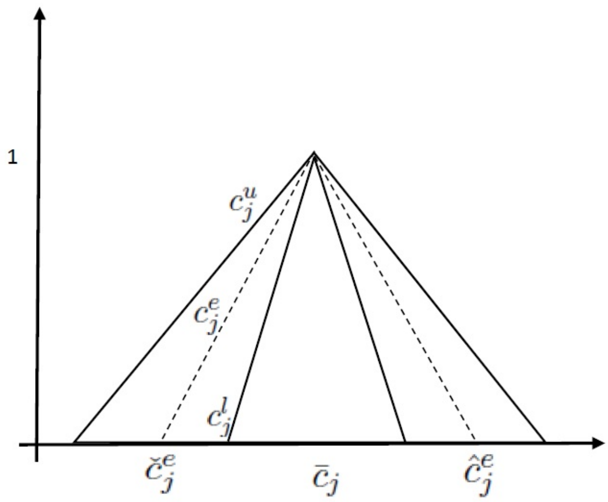

Figure 2: A FN $\tilde{c}_j^e = (\check{c}_j^e, \bar{c}_j, \hat{c}_j^e)$

First, we transform problem (4) into a problem where all the coefficients are intervals and then solve it.

Ban and Coroianu presented an approximation in [52] as follows:

Theorem 13. *[52] Suppose $\tilde{A}$ be a fuzzy number. Ban and Coroianu's nearest interval approximation for $\tilde{A}$ is given by $I_{\tilde{A}} = [v, w]$ where*

$$v = \int_0^1 (\alpha + \frac{1}{2})A^L(\alpha)d\alpha + \int_0^1 (-\alpha + \frac{1}{2})A^U(\alpha)d\alpha,$$
$$w = \int_0^1 (-\alpha + \frac{1}{2})A^L(\alpha)d\alpha + \int_0^1 (\alpha + \frac{1}{2})A^U(\alpha)d\alpha.$$

Theorem 14. *If $\tilde{A} = (a, b, c)$ be a TFN, then Ban and Coroianu's nearest interval approximation for $\tilde{A}$ is given by $I_{\tilde{A}} = [v, w]$ where*

$$v = \frac{5}{12}a + \frac{7}{12}b + \frac{1}{12}(c - b),$$
$$w = \frac{7}{12}b + \frac{5}{12}c + \frac{1}{12}(a - b).$$

Proof. Using the membership function of TFN and Theorem 13 we have:

$$v = \int_0^1 (\alpha + \frac{1}{2})\tilde{A}^L(\alpha)d\alpha + \int_0^1 (-\alpha + \frac{1}{2})\tilde{A}^U(\alpha)d\alpha = \int_0^1 (\alpha + \frac{1}{2})\Big(\alpha(b - a) + a\Big)d\alpha + \int_0^1 (-\alpha + \frac{1}{2})\Big(c - \alpha(c - b)\Big)d\alpha = \frac{5}{12}a + \frac{7}{12}b + \frac{1}{12}(c - b),$$

$$v = \int_0^1 (-\alpha + \frac{1}{2})\tilde{A}^L(\alpha)d\alpha + \int_0^1 (\alpha + \frac{1}{2})\tilde{A}^U(\alpha)d\alpha = \int_0^1 (-\alpha + \frac{1}{2})\Big(\alpha(b - a) + a\Big)d\alpha + \int_0^1 (\alpha + \frac{1}{2})\Big(c - \alpha(c - b)\Big)d\alpha = \frac{7}{12}b + \frac{5}{12}c + \frac{1}{12}(a - b).$$

□

Now, by using Ban and Coroianu's approximation for (4) we have:

$$\begin{aligned}
&\max_{\mathbf{x}_1 \in X_1} [\frac{5}{12}\check{\mathbf{c}}_1^e + \frac{7}{12}\bar{\mathbf{c}}_1 + \frac{1}{12}(\hat{\mathbf{c}}_1^e - \bar{\mathbf{c}}_1), \frac{7}{12}\bar{\mathbf{c}}_1 + \frac{5}{12}\hat{\mathbf{c}}_1^e + \frac{1}{12}(\check{\mathbf{c}}_1^e - \bar{\mathbf{c}}_1)]\mathbf{x}_1 \\
&+ [\frac{5}{12}\check{\mathbf{d}}_1^e + \frac{7}{12}\bar{\mathbf{d}}_1 + \frac{1}{12}(\hat{\mathbf{d}}_1^e - \bar{\mathbf{d}}_1), \frac{7}{12}\bar{\mathbf{d}}_1 + \frac{5}{12}\hat{\mathbf{d}}_1^e + \frac{1}{12}(\check{\mathbf{d}}_1^e - \bar{\mathbf{d}}_1)\mathbf{x}_2 \\
&\text{s.t.} \max_{\mathbf{x}_2 \in X_2} [\frac{5}{12}\check{\mathbf{c}}_2^e + \frac{7}{12}\bar{\mathbf{c}}_2 + \frac{1}{12}(\hat{\mathbf{c}}_2^e - \bar{\mathbf{c}}_2), \frac{7}{12}\bar{\mathbf{c}}_2 + \frac{5}{12}\hat{\mathbf{c}}_2^e + \frac{1}{12}(\check{\mathbf{c}}_2^e - \bar{\mathbf{c}}_2)]\mathbf{x}_1 \\
&\quad + [\frac{5}{12}\check{\mathbf{d}}_2^e + \frac{7}{12}\bar{\mathbf{d}}_2 + \frac{1}{12}(\hat{\mathbf{d}}_2^e - \bar{\mathbf{d}}_2), \frac{7}{12}\bar{\mathbf{d}}_2 + \frac{5}{12}\hat{\mathbf{d}}_2^e + \frac{1}{12}(\check{\mathbf{d}}_2^e - \bar{\mathbf{d}}_2)]\mathbf{x}_2 \\
&\quad \text{s.t.} \quad [\frac{5}{12}\check{A}^e + \frac{7}{12}\bar{A} + \frac{1}{12}(\hat{A}^e - \bar{A}), \frac{7}{12}\bar{A} + \frac{5}{12}\hat{A}^e + \frac{1}{12}(\check{A}^e - \bar{A})]\mathbf{x}_1 \\
&\qquad + [\frac{5}{12}\check{B}^e + \frac{7}{12}\bar{B} + \frac{1}{12}(\hat{B}^e - \bar{B}), \frac{7}{12}\bar{B} + \frac{5}{12}\hat{B}^e + \frac{1}{12}(\check{B}^e - \bar{B})]\mathbf{x}_2 \\
&\qquad \leq [\frac{5}{12}\check{\mathbf{b}}^e + \frac{7}{12}\bar{\mathbf{b}} + \frac{1}{12}(\hat{\mathbf{b}}^e - \bar{\mathbf{b}}), \frac{7}{12}\bar{\mathbf{b}} + \frac{5}{12}\hat{\mathbf{b}}^e + \frac{1}{12}(\check{\mathbf{b}}^e - \bar{\mathbf{b}})] \\
&\qquad \mathbf{x}_1, \mathbf{x}_2 \geq 0,
\end{aligned} \tag{5}$$

where $\check{\mathbf{c}}_1^e \in [\check{\mathbf{c}}_1^u, \check{\mathbf{c}}_1^l]$, $\hat{\mathbf{c}}_1^e \in [\hat{\mathbf{c}}_1^l, \hat{\mathbf{c}}_1^u]$, $\check{\mathbf{d}}_1^e \in [\check{\mathbf{d}}_1^u, \check{\mathbf{d}}_1^l]$, $\hat{\mathbf{d}}_1^e \in [\hat{\mathbf{d}}_1^l, \hat{\mathbf{d}}_1^u]$, $\check{\mathbf{c}}_2^e \in [\check{\mathbf{c}}_2^u, \check{\mathbf{c}}_2^l]$, $\hat{\mathbf{c}}_2^e \in [\hat{\mathbf{c}}_2^l, \hat{\mathbf{c}}_2^u]$, $\check{\mathbf{d}}_2^e \in [\check{\mathbf{d}}_2^u, \check{\mathbf{d}}_2^l]$, $\hat{\mathbf{d}}_2^e \in [\hat{\mathbf{d}}_2^l, \hat{\mathbf{d}}_2^u]$, $\check{A}^e \in [\check{A}^u, \check{A}^l]$, $\hat{A}^e \in [\hat{A}^l, \hat{A}^u]$, $\check{B}^e \in [\check{B}^u, \check{B}^l]$, $\hat{B}^e \in [\hat{B}^l, \hat{B}^u]$, $\check{\mathbf{b}}^e \in [\check{\mathbf{b}}^u, \check{\mathbf{b}}^l]$ and $\hat{\mathbf{b}}^e \in [\hat{\mathbf{b}}^l, \hat{\mathbf{b}}^u]$.

According to [53], (5) can be divided into the worst and the best subproblems, respectively:

$$
\begin{aligned}
\max_{\mathbf{x}_1 \in X_1} \quad & \big(\tfrac{5}{12}\check{\mathbf{c}}_1^e + \tfrac{7}{12}\bar{\mathbf{c}}_1 + \tfrac{1}{12}(\hat{\mathbf{c}}_1^e - \bar{\mathbf{c}}_1)\big)\mathbf{x}_1 + \big(\tfrac{5}{12}\check{\mathbf{d}}_1^e + \tfrac{7}{12}\bar{\mathbf{d}}_1 + \tfrac{1}{12}(\hat{\mathbf{d}}_1^e - \bar{\mathbf{d}}_1)\big)\mathbf{x}_2 \\
\text{s.t.} \quad & \max_{\mathbf{x}_2 \in X_2} \quad \big(\tfrac{5}{12}\check{\mathbf{c}}_2^e + \tfrac{7}{12}\bar{\mathbf{c}}_2 + \tfrac{1}{12}(\hat{\mathbf{c}}_2^e - \bar{\mathbf{c}}_2)\big)\mathbf{x}_1 + \big(\tfrac{5}{12}\check{\mathbf{d}}_2^e + \tfrac{7}{12}\bar{\mathbf{d}}_2 + \tfrac{1}{12}(\hat{\mathbf{d}}_2^e - \bar{\mathbf{d}}_2)\big)\mathbf{x}_2 \\
& \quad \text{s.t.} \quad \big(\tfrac{7}{12}\bar{A} + \tfrac{5}{12}\hat{A}^e + \tfrac{1}{12}(\check{A}^e - \bar{A})\big)\mathbf{x}_1 + \big(\tfrac{7}{12}\bar{B} + \tfrac{5}{12}\hat{B}^e + \tfrac{1}{12}(\check{B}^e - \bar{B})\big)\mathbf{x}_2 \\
& \quad \leq \tfrac{5}{12}\check{\mathbf{b}}^e + \tfrac{7}{12}\bar{\mathbf{b}} + \tfrac{1}{12}(\hat{\mathbf{b}}^e - \bar{\mathbf{b}}) \\
& \quad \mathbf{x}_1, \mathbf{x}_2 \geq 0 \text{ and}
\end{aligned}
\tag{6}
$$

$$
\begin{aligned}
\max_{\mathbf{x}_1 \in X_1} \quad & \big(\tfrac{7}{12}\bar{\mathbf{c}}_1 + \tfrac{5}{12}\hat{\mathbf{c}}_1^e + \tfrac{1}{12}(\check{\mathbf{c}}_1^e - \bar{\mathbf{c}}_1)\big)\mathbf{x}_1 + \big(\tfrac{7}{12}\bar{\mathbf{d}}_1 + \tfrac{5}{12}\hat{\mathbf{d}}_1^e + \tfrac{1}{12}(\check{\mathbf{d}}_1^e - \bar{\mathbf{d}}_1)\big)\mathbf{x}_2 \\
\text{s.t.} \quad & \max_{\mathbf{x}_2 \in X_2} \quad \big(\tfrac{7}{12}\bar{\mathbf{c}}_2 + \tfrac{5}{12}\hat{\mathbf{c}}_2^e + \tfrac{1}{12}(\check{\mathbf{c}}_2^e - \bar{\mathbf{c}}_2)\big)\mathbf{x}_1 + \big(\tfrac{7}{12}\bar{\mathbf{d}}_2 + \tfrac{5}{12}\hat{\mathbf{d}}_2^e + \tfrac{1}{12}(\check{\mathbf{d}}_2^e - \bar{\mathbf{d}}_2)\big)\mathbf{x}_2 \\
& \quad \text{s.t.} \big(\tfrac{5}{12}\check{A}^e + \tfrac{7}{12}\bar{A} + \tfrac{1}{12}(\hat{A}^e - \bar{A})\big)\mathbf{x}_1 + \big(\tfrac{5}{12}\check{B}^e + \tfrac{7}{12}\bar{B} + \tfrac{1}{12}(\hat{B}^e - \bar{B})\big)\mathbf{x}_2 \\
& \quad \leq \big(\tfrac{7}{12}\bar{\mathbf{b}} + \tfrac{5}{12}\hat{\mathbf{b}}^e + \tfrac{1}{12}(\check{\mathbf{b}}^e - \bar{\mathbf{b}})\big) \\
& \quad \mathbf{x}_1, \mathbf{x}_2 \geq 0.
\end{aligned}
\tag{7}
$$

Again by using the proposed method in [53] for (6), we have:
the worst-worst model:

$$
\begin{aligned}
\max_{\mathbf{x}_1 \in X} \quad & \big(\tfrac{5}{12}\check{\mathbf{c}}_1^u + \tfrac{7}{12}\bar{\mathbf{c}}_1 + \tfrac{1}{12}(\hat{\mathbf{c}}_1^l - \bar{\mathbf{c}}_1)\big)\mathbf{x}_1 + \big(\tfrac{5}{12}\check{\mathbf{d}}_1^u + \tfrac{7}{12}\bar{\mathbf{d}}_1 + \tfrac{1}{12}(\hat{\mathbf{d}}_1^l - \bar{\mathbf{d}}_1)\big)\mathbf{x}_2 \\
\text{s.t.} \quad & \max_{\mathbf{x}_2 \in X_2} \quad \big(\tfrac{5}{12}\check{\mathbf{c}}_2^u + \tfrac{7}{12}\bar{\mathbf{c}}_2 + \tfrac{1}{12}(\hat{\mathbf{c}}_2^l - \bar{\mathbf{c}}_2)\big)\mathbf{x}_1 + \big(\tfrac{5}{12}\check{\mathbf{d}}_2^u + \tfrac{7}{12}\bar{\mathbf{d}}_2 + \tfrac{1}{12}(\hat{\mathbf{d}}_2^l - \bar{\mathbf{d}}_2)\big)\mathbf{x}_2 \\
& \quad \text{s.t.} \quad \big(\tfrac{7}{12}\bar{A} + \tfrac{5}{12}\hat{A}^u + \tfrac{1}{12}(\check{A}^l - \bar{A})\big)\mathbf{x}_1 + \big(\tfrac{7}{12}\bar{B} + \tfrac{5}{12}\hat{B}^u + \tfrac{1}{12}(\check{B}^l - \bar{B})\big)\mathbf{x}_2 \\
& \quad \leq \tfrac{5}{12}\check{\mathbf{b}}^u + \tfrac{7}{12}\bar{\mathbf{b}} + \tfrac{1}{12}(\hat{\mathbf{b}}^l - \bar{\mathbf{b}}) \\
& \quad \mathbf{x}_1, \mathbf{x}_2 \geq 0,
\end{aligned}
\tag{8}
$$

By solving BLPP (8) $\check{\mathbf{x}}_1^u$, $\check{\mathbf{x}}_2^u$, $\check{Z}^u$ and $\check{z}^u$ are achieved.
the worst-best model:

$$
\begin{aligned}
\max_{\mathbf{x}_1\in X_1} \quad & \big(\frac{5}{12}\check{\mathbf{c}}_1^u+\frac{7}{12}\bar{\mathbf{c}}_1+\frac{1}{12}(\hat{\mathbf{c}}_1^u-\bar{\mathbf{c}}_1)\big)\mathbf{x}_1+\big(\frac{5}{12}\check{\mathbf{d}}_1^u+\frac{7}{12}\bar{d}\mathbf{d}_1+\frac{1}{12}(\hat{\mathbf{d}}_1^u-\bar{\mathbf{d}}_1)\big)\mathbf{x}_2\\
\text{s.t.} \quad \max_{\mathbf{x}_2\in X_2} \quad & \big(\frac{5}{12}\check{\mathbf{c}}_2^u+\frac{7}{12}\bar{\mathbf{c}}_2+\frac{1}{12}(\hat{\mathbf{c}}_2^u-\bar{\mathbf{c}}_2)\big)\mathbf{x}_1+\big(\frac{5}{12}\check{\mathbf{d}}_2^u+\frac{7}{12}\bar{\mathbf{d}}_2+\frac{1}{12}(\hat{\mathbf{d}}_2^u-\bar{\mathbf{d}}_2)\big)\mathbf{x}_2\\
\text{s.t.} \quad & \big(\frac{7}{12}\bar{A}+\frac{5}{12}\hat{A}^u+\frac{1}{12}(\check{A}^u-\bar{A})\big)\mathbf{x}_1+\big(\frac{7}{12}\bar{B}+\frac{5}{12}\hat{B}^u+\frac{1}{12}(\check{B}^u-\bar{B})\big)\mathbf{x}_2\\
& \le \frac{5}{12}\check{\mathbf{b}}^u+\frac{7}{12}\bar{\mathbf{b}}+\frac{1}{12}(\hat{\mathbf{b}}^u-\bar{\mathbf{b}})\\
& \mathbf{x}_1,\mathbf{x}_2\ge 0,
\end{aligned}
\tag{9}
$$

which is a BLPP that $\check{\mathbf{x}}_1^l$, $\check{\mathbf{x}}_2^l$, $\check{Z}^l$ and $\check{z}^l$ get by solving it.

Now, by using the method [53], the best-worst and the best-best models are obtained from (7) as follows:
the best-worst model:

$$
\begin{aligned}
\max_{\mathbf{x}_1\in X_1} \quad & \big(\frac{7}{12}\bar{\mathbf{c}}_1+\frac{5}{12}\hat{\mathbf{c}}_1^u+\frac{1}{12}(\check{\mathbf{c}}_1^u-\bar{\mathbf{c}}_1)\big)\mathbf{x}_1+\big(\frac{7}{12}\bar{\mathbf{d}}_1+\frac{5}{12}\hat{\mathbf{d}}_1^u+\frac{1}{12}(\check{\mathbf{d}}_1^u-\bar{\mathbf{d}}_1)\big)\mathbf{x}_2\\
\text{s.t.} \quad \max_{\mathbf{x}_2\in X_2} \quad & \big(\frac{7}{12}\bar{\mathbf{c}}_2+\frac{5}{12}\hat{\mathbf{c}}_2^u+\frac{1}{12}(\check{\mathbf{c}}_2^u-\bar{\mathbf{c}}_2)\big)\mathbf{x}_1+\big(\frac{7}{12}\bar{\mathbf{d}}_2+\frac{5}{12}\hat{\mathbf{d}}_2^u+\frac{1}{12}(\check{\mathbf{d}}_2^u-\bar{\mathbf{d}}_2)\big)\mathbf{x}_2\\
\text{s.t.} \quad & \big(\frac{5}{12}\check{A}^u+\frac{7}{12}\bar{A}+\frac{1}{12}(\hat{A}^u-\bar{A})\big)\mathbf{x}_1+\big(\frac{5}{12}\check{B}^u+\frac{7}{12}\bar{B}+\frac{1}{12}(\hat{B}^u-\bar{B})\big)\mathbf{x}_2\\
& \le \frac{7}{12}\bar{\mathbf{b}}+\frac{5}{12}\hat{\mathbf{b}}^u+\frac{1}{12}(\check{\mathbf{b}}^u-\bar{\mathbf{b}})\\
& \mathbf{x}_1,\mathbf{x}_2\ge 0,
\end{aligned}
\tag{10}
$$

By solving the BLPP (10), $\hat{\mathbf{x}}_1^l$, $\hat{\mathbf{x}}_2^l$, $\hat{Z}^l$ and $\hat{z}^l$ are obtained, and
the best-best model:

$$
\begin{aligned}
\max_{\mathbf{x}_1\in X_1} \quad & \big(\frac{7}{12}\bar{\mathbf{c}}_1+\frac{5}{12}\hat{\mathbf{c}}_1^u+\frac{1}{12}(\check{\mathbf{c}}_1^l-\bar{\mathbf{c}}c_1)\big)\mathbf{x}_1+\big(\frac{7}{12}\bar{\mathbf{d}}_1+\frac{5}{12}\hat{\mathbf{d}}_1^u+\frac{1}{12}(\check{\mathbf{d}}_1^l-\bar{\mathbf{d}}_1)\big)\mathbf{x}_2\\
\text{s.t.} \quad \max_{x_2\in X_2} \quad & \big(\frac{7}{12}\bar{\mathbf{c}}_2+\frac{5}{12}\hat{\mathbf{c}}_2^u+\frac{1}{12}(\check{\mathbf{c}}_2^l-\bar{\mathbf{c}}_2)\big)\mathbf{x}_1+\big(\frac{7}{12}\bar{\mathbf{d}}_2+\frac{5}{12}\hat{\mathbf{d}}_2^u+\frac{1}{12}(\check{\mathbf{d}}_2^l-\bar{\mathbf{d}}_2)\big)\mathbf{x}_2\\
\text{s.t.} \quad & \big(\frac{5}{12}\check{A}^u+\frac{7}{12}\bar{A}+\frac{1}{12}(\hat{A}^l-\bar{A})\big)\mathbf{x}_1+\big(\frac{5}{12}\check{B}^u+\frac{7}{12}\bar{B}+\frac{1}{12}(\hat{B}^l-\bar{B})\big)\mathbf{x}_2\\
& \le \frac{7}{12}\bar{\mathbf{b}}+\frac{5}{12}\hat{\mathbf{b}}^u+\frac{1}{12}(\check{\mathbf{b}}^l-\bar{\mathbf{b}})\\
& \mathbf{x}_1,\mathbf{x}_2\ge 0.
\end{aligned}
\tag{11}
$$

By solving (11), $\hat{\mathbf{x}}_1^u$, $\hat{\mathbf{x}}_2^u$, $\hat{Z}^u$ and $\hat{z}^u$ are achieved.

Therefore, by solving (3), (8), (9), (10) and (11), we obtain the optimal solution $\tilde{\tilde{\mathbf{x}}}_{1opt} = ((\check{x}^l_{1opt}, \bar{\mathbf{x}}_{1opt}, \hat{\mathbf{x}}^l_{1opt}), (\check{\mathbf{x}}^u_{1opt}, \bar{\mathbf{x}}_{1opt}, \hat{\mathbf{x}}^u_{1opt}))$ and $\tilde{\tilde{\mathbf{x}}}_{2opt} = ((\check{\mathbf{x}}^l_{2opt}, \bar{\mathbf{x}}_{2opt}, \hat{\mathbf{x}}^l_{2opt})$ $, (\check{\mathbf{x}}^u_{2opt}, \bar{\mathbf{x}}_{2opt}, \hat{\mathbf{x}}^u_{2opt}))$ and optimal values $\tilde{\tilde{Z}}_{opt} = ((\check{Z}^l_{opt}, \bar{Z}_{opt}, \hat{Z}^l_{opt}), (\check{Z}^u_{opt}, \bar{Z}_{opt}, \hat{Z}^u_{opt}))$ and $\tilde{\tilde{z}}_{opt} = ((\check{z}^l_{opt}, \bar{z}_{opt}, \hat{z}^l_{opt}), (\check{z}^u_{opt}, \bar{z}_{opt}, \hat{z}^u_{opt}))$ for (2).

4. COMPARATIVE ANALYSIS AND DISCUSSION

Considering that BLPP with IT2TFN parameters has not been studied so far, in this section, we discuss and examine the advantages of the proposed method by reviewing similar problems.

One of the common methods to solve optimization problems in uncertainty environment is to use ranking functions. Ranking methods assign a crisp number to each fuzzy number or IT2FNs and turn the problem into a crisp problem that is easy to solve.

Chen in [40] expressed an ELECTRE-based outranking method for multi-criteria decision making within the environment of IT2FS. Chen used a hybrid averaging approach with signed distances to construct a collective decision matrix and proposes using ELECTRE-based outranking methods to analyze the collective interval type-2 fuzzy data. Hu et al. [44] proposed a new approach based on possibility degree as well as deviation degree, to construct a key optimal model for using interval type-2 fuzzy number in multi-criteria decision making problems in which the weights were partially known. Then, they calculated the overall value of each alternative by the defined aggregation operators. Furthermore, they introduced a new possibility degree, which is proposed to overcome some drawbacks of the existing methods, for comparisons between the overall values of alternatives to construct a possibility degree matrix. Based on the constructed matrix, Hu et al. ranked all of the alternatives according to the ranking vector derived from the matrix, and selected the best one. Javanmard and Mishmast Nehi [46] proposed a new ranking function method for IT2FN. Then they considered a linear programming problem in which the parameters were IT2FN. Using the proposed ranking method, they assigned a crisp number to each IT2FN and turned the linear programming problem with IT2FN parameters into a linear programming problem and solved it easily.

Also, the ranking method has been used to solve BLPPs with fuzzy parameters. Ren [54] converted BLPP with fuzzy parameters into a BLPP using the deviation degree measures and a ranking function method of fuzzy numbers. Taking into account the overall balance between improving the objective function value and decreasing the allowed deviation degree, they presented a computational method to obtain a fuzzy optimal solution. Ren et al. [55] transformed the BLPP with fuzzy parameters to crisp form under different constraint feasibility degrees based on a fuzzy relation for ranking fuzzy numbers. Considering the overall balance between better objective function values and higher feasibility degrees of the constraints, they developed an interactive programming approach to find a balance solution for the BLPP with fuzzy parameters. Tayebnasab et al. [56] solved the BLPP with triangular fuzzy parameters using the ranking fuzzy numbers with integral value [57], and converted the BLPP with triangular fuzzy parameters into BLPP. Then they used the Kth-best method to solve the BLPP and obtained the optimal solution and the optimal value as triangular fuzzy numbers.

According to the above, although it is common to use the ranking method and replace a fuzzy number or an IT2FN with a crisp number to solve the optimization problem in uncertainty environment, there is a general defect of losing problem information due to ranking in all ranking methods. Using a well-defined approximate closed interval to tackling uncertainties is more acceptable and logical than replacing it with a crisp number. In this article, we assigned an interval to each IT2TFN using the Ban and Coroianu's nearest interval approximation [52] and solved the problem. So far, no approximation method for solving the BLPP in which each IT2TFN in the problem is assigned to a closed interval, has been presented; so, the proposed technique has a new and different nature. Here, using an approximation, we convert the BLPP with IT2TFN parameters into an interval BLPP. Then we achieve five BLPPs that by solving them, the optimal value and the optimal solution obtain as IT2TFNs.

5. NUMERICAL EXAMPLES

In this section, we solve two examples. In the first example, we express a FBLPP with interval type-2 symmetric triangular fuzzy numbers and in the second example, we express the FBLPP with IT2TFNs which are not necessarily symmetric.

Example 15. *Consider the BLPP with IT2TFNs as:*

$$\begin{aligned} &\max_{x_1} \tilde{\tilde{c}}_1 x_1 + \tilde{\tilde{d}}_1 x_2 \\ &\qquad s.t. \ \max_{x_2} \ \tilde{\tilde{c}}_2 x_1 + \tilde{\tilde{d}}_2 x_2 \\ &\qquad s.t. \ \tilde{\tilde{a}}_{i1} x_1 + \tilde{\tilde{a}}_{i2} x_2 \le \tilde{\tilde{b}}_i \quad i = 1, 2 \\ &\qquad x_1, x_2 \ge 0, \end{aligned} \tag{12}$$

where,

$\tilde{\tilde{c}}_1 = \big((\check{c}^l_1, \bar{c}_1, \hat{c}^l_1), (\check{c}^u_1, \bar{c}_1, \hat{c}^u_1)\big) = \big((1.5, 2, 2.5), (1, 2, 3)\big)$,
$\tilde{\tilde{d}}_1 = \big((\check{d}^l_1, \bar{d}_1, \hat{d}^l_1), (\check{d}^u_1, \bar{d}_1, \hat{d}^u_1)\big) = ((2.5, 3, 3.5), (2, 3, 4)\big)$,
$\tilde{\tilde{c}}_2 = \big((\check{c}^l_2, \bar{c}_2, \hat{c}^l_2), (\check{c}^u_2, \bar{c}_2, \hat{c}^u_2)\big) = \big((1, 1.5, 2), (0, 1.5, 3)\big)$,
$\tilde{\tilde{d}}_2 = \big((\check{d}^l_2, \bar{d}_2, \hat{d}^l_2), (\check{d}^u_2, \bar{d}_2, \hat{d}^u_2)\big) = \big((2, 3, 4), (1, 3, 5)\big)$,
$\tilde{\tilde{a}}_{11} = \big((\check{a}^l_{11}, \bar{a}_{11}, \hat{a}^l_{11}), (\check{a}^u_{11}, \bar{a}_{11}, \hat{a}^u_{11})\big) = \big((0.5, 1, 1.5), (0, 1, 2)\big)$,
$\tilde{\tilde{a}}_{12} = \big((\check{a}^l_{12}, \bar{a}_{12}, \hat{a}^l_{12}), (\check{a}^u_{12}, \bar{a}_{12}, \hat{a}^u_{12})\big) = \big((3, 4, 5), (2, 4, 6)\big)$,
$\tilde{\tilde{b}}_1 = \big((\check{b}^l_1, \bar{b}_1, \hat{b}^l_1), (\check{b}^u_1, \bar{b}_1, \hat{b}^u_1)\big) = \big((9, 11, 13), (7, 11, 15)\big)$,
$\tilde{\tilde{a}}_{21} = \big((\check{a}^l_{21}, \bar{a}_{21}, \hat{a}^l_{21}), (\check{a}^u_{21}, \bar{a}_{21}, \hat{a}^u_{21})\big) = \big((3, 4, 5), (2, 4, 6)\big)$,

$\tilde{\tilde{a}}_{22} = \big((\check{a}^l_{22}, \bar{a}_{22}, \hat{a}^l_{22}), (\check{a}^u_{22}, \bar{a}_{22}, \hat{a}^u_{22})\big) = \big((1.5, 2.5, 3.5), (0.5, 2.5, 4.5)\big)$ *and* $\tilde{\tilde{b}}_2 = \big((\check{b}^l_2, \bar{b}_2, \hat{b}^l_2)\big), (\check{b}^u_2, \bar{b}_2, \hat{b}^u_2) = \big((8, 12, 16), (4, 12, 20)\big)$ *are IT2TFN.*
According to the problem data, the middle problem gets as follows:

$$\begin{aligned} &\max_{x_1} 2x_1 + 3x_2 \\ &\quad s.t.\ \max_{x_2}\ 1.5x_1 + 3x_2 \\ &\qquad s.t.\ x_1 + 4x_2 \le 11 \\ &\qquad 4x_1 + 2.5x_2 \le 12 \\ &\qquad x_1, x_2 \ge 0. \end{aligned} \tag{13}$$

By solving (13)*, we obtain* $\bar{x}_{1opt} = 1.51851$, $\bar{x}_{2opt} = 2.37037$ *and optimal values are* $\bar{Z}_{opt} = 10.14815$ *and* $\bar{z}_{opt} = 9.38888$.
By using Ban and Coroianu's nearest interval approximation, we have:
1. The worst-worst model:

$$\begin{aligned} &\max_{x_1} \frac{39}{24}x_1 + \frac{63}{24}x_2 \\ &\quad s.t.\ \max_{x_2}\ \frac{22}{24}x_1 + \frac{54}{24}x_2 \\ &\qquad s.t.\ 33x_1 + 114x_2 \le 228 \\ &\qquad 114x_1 + 78x_2 \le 216 \\ &\qquad x_1, x_2 \ge 0, \end{aligned} \tag{14}$$

$\check{x}^u_{1opt} = 0.65630$, $\check{x}^u_{2opt} = 1.81001$, $\check{Z}^u_{opt} = 5.81778$ *and* $\check{z}^u_{opt} = 4.67415$ *are the optimal solution of worst-worst problem that by solving* (14) *are obtained.*
2. The worst-best model:

$$\begin{aligned} &\max_{x_1} \frac{40}{24}x_1 + \frac{64}{24}x_2 \\ &\quad s.t.\ \max_{x_2}\ \frac{24}{24}x_1 + \frac{56}{24}x_2 \\ &\qquad s.t.\ 32x_1 + 112x_2 \le 232 \\ &\qquad 112x_1 + 76x_2 \le 224 \\ &\qquad x_1, x_2 \ge 0. \end{aligned} \tag{15}$$

By solving (15)*, we obtain* $\check{x}^l_{1opt} = 0.73734$, $\check{x}^l_{2opt} = 1.86075$, $\check{Z}^l_{opt} = 6.19092$ *and* $\check{z}^l_{opt} = 5.07911$.

3. The best-worst model:

$$\begin{aligned}
&\max_{x_1} \frac{56}{24}x_1 + \frac{80}{24}x_2 \\
&\quad s.t.\ \max_{x_2}\ \frac{48}{24}x_1 + \frac{88}{24}x_2 \\
&\qquad s.t.\ 16x_1 + 80x_2 \le 296 \\
&\qquad 80x_1 + 44x_2 \le 352 \\
&\qquad x_1, x_2 \ge 0,
\end{aligned} \tag{16}$$

which $\hat{x}^l_{1opt} = 2.65730$, $\hat{x}^l_{2opt} = 3.16853$, $\hat{Z}^l_{opt} = 16.76217$ *and* $\hat{z}^l_{opt} = 16.93258$ *are achieved from solving* (16).

4.The best-best model:

$$\begin{aligned}
&\max_{x_1} \frac{57}{24}x_1 + \frac{81}{24}x_2 \\
&\quad s.t.\ \max_{x_2}\ \frac{50}{24}x_1 + \frac{90}{24}x_2 \\
&\qquad s.t.\ 15x_1 + 78x_2 \le 300 \\
&\qquad 78x_1 + 42x_2 \le 360 \\
&\qquad x_1, x_2 \ge 0.
\end{aligned} \tag{17}$$

By solving (17), $\hat{x}^u_{1opt} = 2.83828$, $\hat{x}^u_{2opt} = 3.30033$, $\hat{Z}^u_{opt} = 17.87954$ *and* $\hat{z}^u_{opt} = 18.28933$ *are obtained.*

Then, for problem (12), *by using Ban and Coroianu's nearest interval approximation, we have:*

$$\tilde{\tilde{x}}_{1opt} = \big((0.73734, 1.51851, 2.65730), (0.65630, 1.51851, 2.83828)\big),$$

$$\tilde{\tilde{x}}_{2opt} = \big((1.86075, 2.37037, 3.16853), (1.81001, 2.37037, 3.30033)\big),$$

$$\tilde{\tilde{Z}}_{opt} = \big((6.19092, 10.14815, 16.76217), (5.81778, 10.14815, 17.87954)\big)$$

and

$$\tilde{\tilde{z}}_{opt} = \big((5.07911, 9.38888, 16.93258), (4.67415, 9.38888, 18.28933)\big).$$

Figures 3 and 4 show the optimal solution and the optimal value of example 15, respectively. As we see in figures 3 and 4, the optimal solution and the optimal value for leader and follower are obtained as an IT2TFNs, but unlike the parameters that were interval type-2 symmetric triangular fuzzy numbers, the optimal solutions and the optimal value were not obtained interval type-2 symmetric triangular fuzzy numbers.

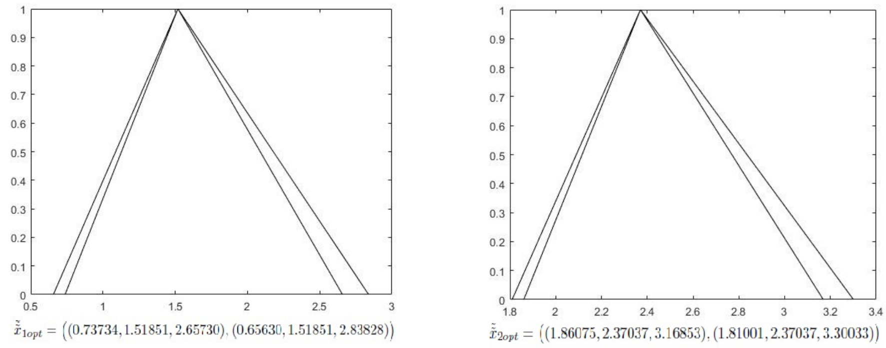

Figure 3: The optimal solution of example 15

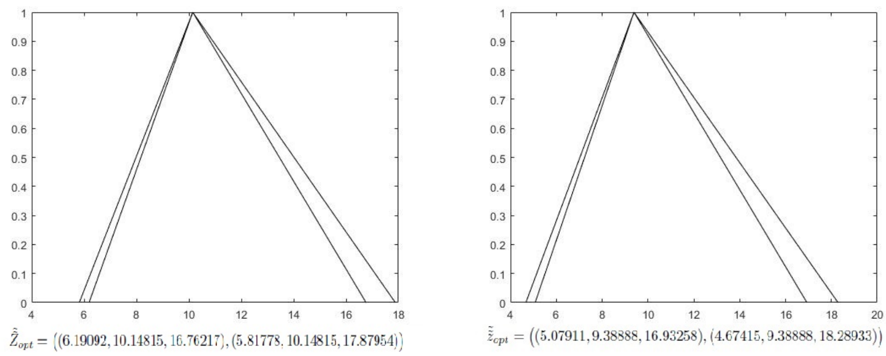

Figure 4: The optimal value of example 15

Example 16. *Consider the BLPP with IT2TFNs as:*

$$\begin{aligned}
&\max_{x_1} \tilde{\tilde{c}}_1 x + \tilde{\tilde{d}}_1 x_2 \\
&\qquad s.t. \ \max_{x_2} \ \tilde{\tilde{c}}_2 x_1 + \tilde{\tilde{d}}_2 x_2 \\
&\qquad s.t. \ \tilde{\tilde{a}}_{i1} x_1 + \tilde{\tilde{a}}_{i2} x_2 \leq \tilde{\tilde{b}}_i \quad i = 1, 2 \\
&\qquad x_1, x_2 \geq 0,
\end{aligned} \tag{18}$$

where,
$\tilde{\tilde{c}}_1 = \big((\check{c}_1^l, \bar{c}_1, \hat{c}_1^l), (\check{c}_1^u, \bar{c}_1, \hat{c}_1^u)\big) = \big((2.5, 3, 4), (2, 3, 5)\big)$,
$\tilde{\tilde{d}}_1 = \big((\check{d}_1^l, \bar{d}_1, \hat{d}_1^l), (\check{d}_1^u, \bar{d}_1, \hat{d}_1^u)\big) = \big((2, 4, 5), (1, 4, 6)\big)$,
$\tilde{\tilde{c}}_2 = \big((\check{c}_2^l, \bar{c}_2, \hat{c}_2^l), (\check{c}_2^u, \bar{c}_2, \hat{c}_2^u)\big) = \big((0.5, 1, 2), (0.25, 1, 3)\big)$,
$\tilde{\tilde{d}}_2 = \big((\check{d}_2^l, \bar{d}_2, \hat{d}_2^l), (\check{d}_2^u, \bar{d}_2, \hat{d}_2^u)\big) = \big((3, 5, 6), (2, 5, 7)\big)$,
$\tilde{\tilde{a}}_{11} = \big((\check{a}_{11}^l, \bar{a}_{11}, \hat{a}_{11}^l), (\check{a}_{11}^u, \bar{a}_{11}, \hat{a}_{11}^u)\big) = \big((1, 2, 3), (0.5, 2, 4)\big)$,

$\tilde{\tilde{a}}_{12} = \big((\check{a}^l_{12}, \bar{a}_{12}, \hat{a}^l_{12}), (\check{a}^u_{12}, \bar{a}_{12}, \hat{a}^u_{12})\big) = \big((5,6,7),(4,6,8)\big)$,
$\tilde{\tilde{b}}_1 = \big((\check{b}^l_1, \bar{b}_1, \hat{b}^l_1), (\check{b}^u_1, \bar{b}_1, \hat{b}^u_1)\big) = \big((6,8,9),(5,8,10)\big)$,
$\tilde{\tilde{a}}_{21} = \big((\check{a}^l_{21}, \bar{a}_{21}, \hat{a}^l_{21}), (\check{a}^u_{21}, \bar{a}_{21}, \hat{a}^u_{21})\big) = \big((3,5,6),(2,5,7)\big)$,
$\tilde{\tilde{a}}_{22} = \big((\check{a}^l_{22}, \bar{a}_{22}, \hat{a}^l_{22}), (\check{a}^u_{22}, \bar{a}_{22}, \hat{a}^u_{22})\big) = \big((3,4,6),(2,4,8)\big)$ *and*
$\tilde{\tilde{b}}_2 = \big((\check{b}^l_2, \bar{b}_2, \hat{b}^l_2)\big), (\check{b}^u_2, \bar{b}_2, \hat{b}^u_2) = \big((5,6,8),(4,6,9)\big)$ *are IT2TFNs.*

According to the problem data, the middle problem gets as follows:

$$\begin{aligned} &\max_{x_1} 3x_1 + 4x_2 \\ &\qquad s.t.\ \max_{x_2}\ x_1 + 5x_2 \\ &\qquad\qquad s.t.\ 2x_1 + 6x_2 \leq 8 \\ &\qquad\qquad 5x_1 + 4x_2 \leq 6 \\ &\qquad\qquad x_1, x_2 \geq 0. \end{aligned} \tag{19}$$

By solving (19)*, we obtain* $\bar{x}_{1opt} = 0.1818$, $\bar{x}_{2opt} = 1.2727$ *and optimal values are* $\bar{Z}_{opt} = 5.6363$ *and* $\bar{z}_{opt} = 6.5454$.
By using Ban and Coroianu's nearest interval approximation, we have:
1. The worst-worst model:

$$\begin{aligned} &\max_{x_1} \frac{64}{24}x_1 + \frac{68}{24}x_2 \\ &\qquad s.t.\ \max_{x_2}\ \frac{19}{24}x_1 + \frac{92}{24}x_2 \\ &\qquad\qquad s.t.\ 66x_1 + 162x_2 \leq 164 \\ &\qquad\qquad 136x_1 + 134x_2 \leq 128 \\ &\qquad\qquad x_1, x_2 \geq 0, \end{aligned} \tag{20}$$

$\check{x}^u_{1opt} = 0$, $\check{x}^u_{2opt} = 0.9552$, $\check{Z}^u_{opt} = 2.7064$ *and* $\check{z}^u_{opt} = 3.6616$ *are the optimal solution of worst-worst problem that by solving* (20) *are obtained.*
2. The worst-best model:

$$\begin{aligned} &\max_{x_1} \frac{66}{24}x_1 + \frac{70}{24}x_2 \\ &\qquad s.t.\ \max_{x_2}\ \frac{21}{24}x_1 + \frac{94}{24}x_2 \\ &\qquad\qquad s.t.\ 65x_1 + 160x_2 \leq 166 \\ &\qquad\qquad 134x_1 + 132x_2 \leq 130 \\ &\qquad\qquad x_1, x_2 \geq 0. \end{aligned} \tag{21}$$

By solving (21)*, we obtain* $\check{x}^l_{1opt} = 0$, $\check{x}^l_{2opt} = 0.9848$, $\check{Z}^l_{opt} = 2.8724$ *and* $\check{z}^l_{opt} = 3.8573$.

3. The best-worst model:

$$\begin{aligned}
&\max_{x_1} \frac{90}{24}x_1 + \frac{110}{24}x_2 \\
&\qquad s.t.\ \max_{x_2}\ \frac{43}{24}x_1 + \frac{134}{24}x_2 \\
&\qquad\qquad s.t.\ 37x_1 + 128x_2 \leq 206 \\
&\qquad\qquad 94x_1 + 84x_2 \leq 170 \\
&\qquad\qquad x_1, x_2 \geq 0,
\end{aligned} \tag{22}$$

which $\hat{x}^l_{1opt} = 0.4993$, $\hat{x}^l_{2opt} = 1.4650$, $\hat{Z}^l_{opt} = 8.5872$ *and* $\hat{z}^l_{opt} = 9.0744$ *are achieved from solving* (22).

4.The best-best model:

$$\begin{aligned}
&\max_{x_1} \frac{91}{24}x_1 + \frac{112}{24}x_2 \\
&\qquad s.t.\ \max_{x_2}\ \frac{43}{24}x_1 + \frac{136}{24}x_2 \\
&\qquad\qquad s.t.\ 35x_1 + 126x_2 \leq 208 \\
&\qquad\qquad 92x_1 + 80x_2 \leq 172 \\
&\qquad\qquad x_1, x_2 \geq 0.
\end{aligned} \tag{23}$$

By solving (23), $\hat{x}^u_{1opt} = 0.5723$, $\hat{x}^u_{2opt} = 1.4918$, $\hat{Z}^u_{opt} = 9.1319$ *and* $\hat{z}^u_{opt} = 9.4790$ *are obtained.*

Then, for problem (18), *by using Ban and Coroianu's nearest interval approximation, we have:*

$$\tilde{\tilde{x}}_{1opt} = \big((0, 0.1818, 0.4993), (0, 0.1818, 0.5723)\big),$$

$$\tilde{\tilde{x}}_{2opt} = \big((0.9848, 1.2727, 1.4650), (0.9552, 1.2727, 1.4918)\big),$$

$$\tilde{\tilde{Z}}_{opt} = \big((2.8724, 5.6363, 8.5872), (2.7064, 5.6363, 9.1319)\big)$$

and

$$\tilde{\tilde{z}}_{opt} = \big((3.8573, 6.5454, 9.0744), (3.6616, 6.5454, 9.4790)\big).$$

Figures 5 and 6 show the optimal solution and the optimal value of example 16, respectively. As figures 5 and 6 show, using the proposed method, the optimal solution and the optimal value have been obtained as IT2TFNs. Also, the optimal solution and the optimal value were not interval type-2 symmetric triangular fuzzy numbers.

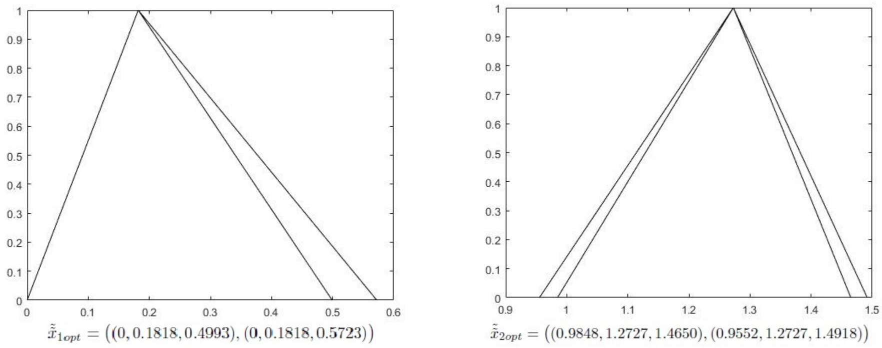

Figure 5: The optimal solution of example 16

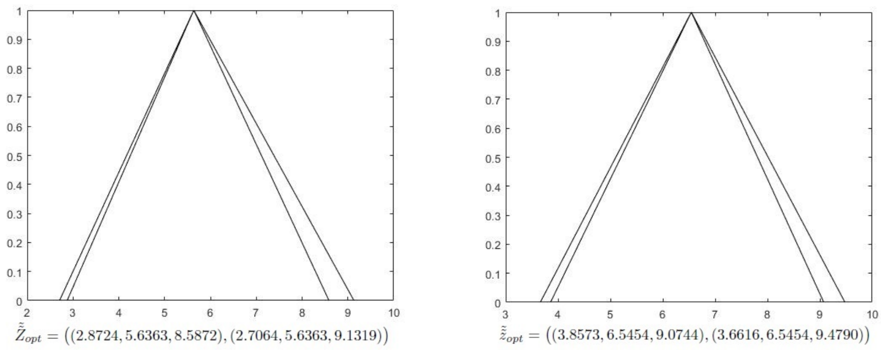

Figure 6: The optimal value of example 16

6. CONCLUSIONS

In this paper, we considered a BLPP where all the parameters were IT2TFNs. Then it is converted to an interval BLPP using Ban and Coroianu's approximation. In the next step, we attained five BLPP, by solving which, the optimal value of problem was achieved as IT2TFN.

We recommend working on solving the BLPP with parameters of the type-2 fuzzy numbers or uncertainty in constraints and objective functions of type-2 fuzzy numbers and even both of them.

Also, we recommend carrying out the study on interval type-2 non-triangular fuzzy BLPP.

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