INVENTORY AND TRANSPORTATION COST MINIMIZATION IN THE DELIVERY LOGISTICS OF SWINE FLU VACCINE

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Abstract: In this paper,we present a mathematical model of delivery logistics of swine flu vaccine in a rural area. The objective is to make the medicine available at the Primary health centres of that area in such a way that the total cost (purchasing and cartage) of obtaining the flu vaccination is minimized and the demand of the demand centres is met. One more constraint is taken into consideration: once the bottle of the medicine is opened, it has to be given to a fixed number of patients requiring swine flu vaccine simultaneously, failing which, the proportion of unused medicine becomes obsolete and can not be administered to the patients. But if the bottle of medicine could not be used fully at a time then, the unused medicine can be sold back at a discounted price. Under such conditions, our next objective is to determine the size of order that must be placed so that the expected total profit per year is maximized.The methodology to achieve this objective is: first, formulate the problem as a fixed charge capacitated transportation model with bounds on rim conditions to determine the number of units that should be purchased from various distribution centres such that the total cost (purchasing cost + cartage) of obtaining the medicine is minimized. Further,model the problem as an inventory model to determine the size of order.

Keywords: Logistics, Capacitated, Transportation Problem, Inventory, Fixed Charge.

MSC: 90C08, 90B06.

1. INTRODUCTION

Logistics management is defined as a part of supply chain management that plans, implements and controls the efficient, effective forward and reverse flow

and storage of goods, services and related information between the point of origin and the point of consumption in order to meet customer's requirements. It includes transportation network design, warehouse location, materials handling, inventory control, order management and fulfilment, procurement and customer service, and spans the entire supply chain. Leuthesser et. al.[13] suggested various methodologies to improve customer service and increase profit while decreasing total costs. In 2005, Qi, X.[14] refers to logistics scheduling as providing the job scheduling and transportation with a single framework. A large portion of academic articles are based on either transportation scheduling or scheduling of resources such as people or machines. Tracey [15] examined transportation through the use of structural equation modeling to improve delivery schedules and to provide a source of competitive advantage to the supply chain. Many researchers such as Derouich et.al. [24], Devine et.al. [25], Rodrigues et.al.[23] have presented mathematical models to control disease transmission.

In this paper, we discuss a problem which provides greater insight into the situation of the logistics scheduling of order and delivery of bio-perishable materials in the medical field and across the supply chain involved. The case of swine flu vaccination has been used to bring better understanding and knowledge to a situation that effects a large number of people and continues to spread throughout the world. Influenza epidemics occur when novel strains of the influenza virus emerge in human populations and spread throughout.

In 2015, swine flu emerged in Delhi, Haryana, and many other neighboring states of North India. During that time, we visited several hospitals, such as Ram Manohar Lohia Hospital, Delhi, City Hospital in Sonepat district, Haryana, Primary Health Centre at Gohana, and many others. We also met various doctors at their own private clinics and nursing homes. In hospitals, we contacted swine-flu patients and asked them about their problems. Their major problem is that they are not admitted to the hospitals easily as there are too many patients suffering from this disease and not enough beds are available in hospitals. Moreover, even if admitted, the hospitals are short of medicine, so the patients do not get medicine in time. Doctors have suggested that the primary control measures for swine flu epidemic are antiviral medications and vaccines, as well as non-pharmaceutical interventions such as social distancing measures, school closures, and hygienic precautions.Antiviral drugs are believed to reduce disease severity and duration of infectiousness in individual patients, if taken sufficiently early.Another problem addressed by the hospital staff is the problem in obtaining the medicine (antiviral drugs and vaccination)as the medicine is to be transported from a long distance. Moreover, the hospitals also have the problem of storing the medicine. During that period, these problems were also addressed extensively in newspapers [4, 9, 8]. All these problems motivated us to develop a model which can ensure the availability of medicine at all health centres at minimum cost taking into consideration the problem of storing the medicine. Keeping this perspective in mind, we proposed a mathematical model which at its first level

minimizes the total cost of obtaining swine flu medicine such that the demand of health centres is met. At the second level, we determine the size of order at which the expected total profit per year is maximized. Hypothetical data is taken to exemplify the proposed mathematical model.Our objective is to optimize the distribution of swine flu vaccination in Haryana. An attempt has been made to model such a situation as a capacitated transportation problem. Further the problem is modeled as an economic order quantity model for items with deteriorating quality.

An extensive literature is available in the field of capacitated transportation problem. Many researchers such as Dahiya et.al. [3], Pandian et.al.[11], Xie et.al. [17] have contributed a lot in the field of capacitated transportation problem. Basu et.al. [2] developed an algorithm for the optimum cost-time trade off pairs in a fixed charge linear transportation problem giving same priority to cost as well as time. Arora et. al.[1] developed an algorithm for solving a capacitated fixed charge bi-criterion indefinite quadratic transportation problem with restricted flow. Gupta et.al.[5, 6, 7] studied optimum time-cost trade off pairs in a capacitated transportation problem. Xie et.al. [16] developed a technique for duration and cost minimization for transportation problem. Jain and Arya [18]studied an inverse version of capacitated transportation problem. Jaggi et.al. [10, 12] studied economic order quantity model for deteriorating items with imperfect quality and permissible delay on payment. Eroglu et.al. [19] studied an economic order quantity model with defective items and shortages. Many researchers such as Lin [21], Chen et.al. [20] and Khan et.al. [22] have contributed a lot in the field of inventory.

This paper is organized as following- In section 2, a mathematical model of a capacitated fixed charge transportation problem is developed. A related transportation problem is then formulated to solve it. Optimality criterion for solving capacitated fixed charge transportation problem is also established. In section 3, an algorithm is developed, which describes the procedure of solving capacitated transportation problem and finding the improved feasible solution at each iteration until the optimal solution is reached. In section 4, another mathematical model of economic order quantity with deteriorating quality is proposed. In section 5, the developed models are illustrated by using the problem of swine flu vaccination delivery.

2. MATHEMATICAL MODEL OF A CAPACITATED FIXED CHARGE TRANSPORTATION PROBLEM

Let $I = \{1, 2, 3, ..., m\}$ be the index set of m origins where the medicine is available.
$J = \{1, 2, 3, 4, ..., n\}$ is the index set of n destinations (hospitals).
x_{ij} = decision variable which denotes the number of units transported from i^{th} origin to j^{th} destination .
c_{ij} = cost of transporting one unit of commodity from i^{th} origin to j^{th} destination

(hospital).

l_{ij} and u_{ij} are respectively the lower and upper bounds on number of units to be transported from i^{th} origin to j^{th} destination.

a_i and A_i are the bounds on the availability at the ith origin, $i \in I$

b_j and B_j are the bounds on the demand at the jth destination, $j \in J$

F_i is the fixed cost incurred when the goods are shipped from the ith origin.

Then a capacitated fixed charge transportation problem with bounds on rim conditions can be formulated as

$$(P1) : min\{\sum_{i \in I} \sum_{j \in J} c_{ij}x_{ij} + \sum_{i \in I} F_i\}$$

subject to

$$a_i \leq \sum_{j \in J} x_{ij} \leq A_i, \forall i \in I$$

$$b_j \leq \sum_{i \in I} x_{ij} \leq B_j, \forall j \in J$$

$$l_{ij} \leq x_{ij} \leq u_{ij} \text{ and integers}, \forall i \in I, \forall j \in J$$

For the formulation of $F_i, (i = 1, 2........, m)$, we assume that $F_i, (i = 1, 2.......m)$ depends on the number of units shipped from the ith origin in a lot.

$$F_i = \sum_{l=1}^{q} F_{il}\delta_{il}, i = 1, 2, ..., m$$

where

$$\delta_{i1} = \begin{cases} 1 & 0 < \text{number of units supplied by the i}^{th}\text{origin} \leq a_{i1} \\ 0 & \text{otherwise} \end{cases}$$

$$\delta_{i2} = \begin{cases} 1 & a_{i1} < \text{number of units supplied by the i}^{th}\text{origin} \leq a_{i2} \\ 0 & \text{otherwise} \end{cases}$$

$$\vdots$$
$$\vdots$$
$$\vdots$$
$$\vdots$$

$$\delta_{iq} = \begin{cases} 1 & \text{number of units supplied by the i}^{th}\text{origin} > a_{iq} \\ 0 & \text{otherwise} \end{cases}$$

for $l = 1, 2, 3......q$ and $i = 1, 2, 3.........m$.

Here, $0 = a_{i1} < a_{i2} < .... < a_{iq}$.

Also, $a_{i1}, a_{i2}, \cdots, a_{iq}, (i = 1, 2, \cdots, m)$ are constants and F_{il} are the fixed costs $\forall i = 1, 2, \cdots, m$ and $l = 1, 2, 3, \cdots, q$.

In order to solve problem ($P1$), a related transportation problem ($P2$) is constructed which is as follows.

$$(P2) : min\{\sum_{i \in I'} \sum_{j \in J'} c'_{ij} y'_{ij} + \sum_{i \in I} F'_i\}$$

subject to

$$\sum_{j \in J'} y'_{ij} = A'_i, \forall i \in I'$$

$$\sum_{i \in I'} y'_{ij} = B'_j, \forall j \in J'$$

$l_{ij} \leq y_{ij} \leq u_{ij}$ and integers, $\forall i \in I, \forall j \in J$

$0 \leq y_{m+1,j} \leq B_j - b_j, \forall j \in J$

$0 \leq y_{i,n+1} \leq A_i - a_i, \forall i \in I$

$y_{m+1,n+1} \geq 0$

$l_{m+1,j} = 0, l_{i,n+1} = 0, l_{m+1,n+1} = 0, \forall i \in I, \forall j \in J$

$$u_{i,n+1} = A_i - a_i, \ u_{m+1,j} = B_j - b_j, \ u_{m+1,n+1} = M \ > \ \sum_{i \in I} \sum_{j \in J} x_{ij}$$

where M is a positive real number.

$$A_i = A'_i, \forall i \in I, \ A'_{m+1} = \sum_{j \in J} B_j, B'_j = B_j, \ \forall j \in J, \ B'_{n+1} = \sum_{i \in I} A_i$$

$c'_{ij} = c_{ij}, \forall i \in I, \forall j \in J, \ c'_{m+1,j} = c'_{i,n+1} = c'_{m+1,n+1} = 0 \ \forall i \in I, \forall j \in J$

$F'_i = F_i, \forall i \in I, F'_{m+1} = 0$

$I = \{1, 2, \cdots, m, m + 1\}, J = \{1, 2, \cdots, n, n + 1\}$

It can be shown that problems ($P1$) and ($P2$) are equivalent with the help of the following theorems.

Theorem 2.1. *There is one to one correspondence between a feasible solution of problem* ($P2$) *and a feasible solution of problem* ($P1$).

Theorem 2.2. *The value of objective function of problem* ($P1$) *at a feasible solution is equal to the value of objective function of problem* ($P2$) *at its corresponding feasible solution and conversely.*

Theorem 2.3. *There is one to one correspondence between the optimal solution to problem* (*P*1) *and the optimal solution to problem* (*P*2).

For the proof of the above theorems, refer to [12]

Theorem 2.4. *(Optimality Criterion)Let* $X = \{X_{ij}\}$ *be a basic feasible solution of problem* (*P*2) *with basis matrix B.*
Let ΔF_{ij} *be the change in fixed cost* $\sum\limits_{i \in I} F_i$ *when some non- basic variable* x_{ij} *undergoes change by an amount of* θ_{ij}
$\theta_{ij} = $ *level at which a non- basic cell* (*i*, *j*) *enters the basis by replacing some basic cell of B.*
N_1 *and* N_2 *denote the set of non- basic cells* (*i*, *j*) *that are at their lower bounds and upper bounds, respectively.*
Let u_i *and* v_j *be the dual variables which are determined by using the following equations and taking one of the* u_i*'s or* v_j*'s as zero.*

$$u_i + v_j = c_{ij}, \forall (i, j) \in B$$
$$u_i + v_j = z_{ij}, \forall (i, j) \in N_1 \text{ and } N_2$$

Then $X = \{X_{ij}\}$ *will be an optimal basic feasible solution if*

$$R^1_{ij} = \theta_{ij}(c_{ij} - z_{ij}) + \Delta F_{ij} \geq 0; \ \forall (i, j) \in N_1$$

and

$$R^2_{ij} = -\theta_{ij}(c_{ij} - z_{ij}) + \Delta F_{ij} \geq 0; \ \forall (i, j) \in N_2$$

Proof. Let z^0 be the objective function value of the problem (*P*2).
Let $z^0 = z_1 + F^0$ where $F^0 = \sum\limits_{i \in I} F_i$ and $z_1 = \sum\limits_{i \in I} \sum\limits_{j \in J} c_{ij} x_{ij}$

Let $\hat{z}$ be the objective function value at the current basic feasible solution $\hat{X} = \{x_{ij}\}$, corresponding to the basis B obtained on entering the non- basic cell $x_{ij} \in N_1$ into the basis which undergoes change by an amount θ_{ij} and is given by min$\{u_{ij} - l_{ij}; x_{ij} - l_{ij}$ for all basic cells (i,j) with a $(-\theta)$entry in the $\theta-$ loop; $u_{ij} - x_{ij}$ for all basic cells (i,j) with a $(+\theta)$ entry in the $\theta-$loop}.
Then

$$\hat{z} = [z_1 + \theta_{ij}(c_{ij} - z_{ij})] + F^0 + \Delta F_{ij}$$
$$\hat{z} - z^0 = \theta_{ij}(c_{ij} - z_{ij}) + \Delta F_{ij}$$

This basic feasible solution will give an improved value of z if
$\hat{z} < z^0$ or if $\theta_{ij}(c_{ij} - z_{ij}) + \Delta F_{ij} < 0$
Therefore, one can move from one basic feasible solution to another basic feasible solution on entering the cell $(i, j) \in N_1$ into the basis for which the above condition is satisfied. It will be an optimal basic feasible solution if

$$R^1_{ij} = \theta_{ij}(c_{ij} - z_{ij}) + \Delta F_{ij} \geq 0; \ \forall (i, j) \in N_1$$

Similarly, when non- basic variable $x_{ij} \in N_2$ undergoes change by an amount θ_{ij}, then

$$\hat{z} - z^0 = -\theta_{ij}(c_{ij} - z_{ij}) + \Delta F_{ij} < 0$$

It will be an optimal basic feasible solution if

$$R_{ij}^2 = -\theta_{ij}(c_{ij} - z_{ij}) + \Delta F_{ij} \geq 0; \ \forall (i, j) \in N_2$$

□

3. ALGORITHM

Step 1: Starting from the given problem ($P1$), form a problem ($P2$) by introducing an additional row, and an additional column, and assigning the cost.

$$c'_{m+1,j} = c'_{i,n+1} = c'_{m+1,n+1} = 0 \ \forall i \in I, \forall j \in J$$

Step 2: Find an initial basic feasible solution to the problem ($P2$) with respect to the variable costs by upper bound simplex technique . Let B be the current basis.
Step3: Calculate the fixed cost of the current basic feasible solution and denote it by $F(current)$, where $F(current) = \sum_{i \in I} F_i$
Step 4(a): Find $\Delta F_{ij} = F(NB) - F(current)$, where $F(NB)$ is the total fixed cost obtained when some non- basic cell (i, j) undergoes change.
Step 4(b): Calculate $\theta_{ij}, (c_{ij} - z_{ij})$ for all non- basic cells such that

$$u_i + v_j = c_{ij}, \forall (i, j) \in B$$
$$u_i + v_j = z_{ij}, \forall (i, j) \in N_1 \ and \ N_2$$

θ_{ij}= level at which a non- basic cell (i, j) enters the basis replacing some basic cell of B.
N_1 and N_2 denote the set of non- basic cells (i, j) that are at their lower bounds and upper bounds, respectively.
Note: u_i, v_j are the dual variables which are determined by using above equations and taking one of the $u'_i s$ or $v'_j s$ as zero.
Step 4(c): Find $R_{ij}^1, \forall (i, j) \in N_1$ and $R_{ij}^2, \forall (i, j) \in N_2$ where

$$R_{ij}^1 = \theta_{ij}(c_{ij} - z_{ij}) + \Delta F_{ij} \geq 0; \ \forall (i, j) \in N_1$$
$$R_{ij}^2 = -\theta_{ij}(c_{ij} - z_{ij}) + \Delta F_{ij} \geq 0; \ \forall (i, j) \in N_2$$

Step5: If $R_{ij}^1 \geq 0 \forall (i, j) \in N_1$ and $R_{ij}^2 \geq 0, \forall (i, j) \in N_2$, then the current solution so obtained is the optimal solution to ($P2$).Go to step 6.Otherwise, some $(i, j) \in N_1$

for which $R^1_{ij} < 0$, or some $(i, j) \in N_2$ for which $R^2_{ij} < 0$ will undergo change. Go to step 3.

Step 6: Find the optimal cost of the problem $(P2)$ yielded by the basic feasible solution y_{ij}. This is the optimal solution of the problem $(P1)$.

4. MATHEMATICAL MODEL OF ECONOMIC ORDER QUANTITY OF ITEMS WITH DETERIORATING QUALITY

Let y be the order size that is delivered instantaneously with a purchasing price of c per unit and an ordering cost of K. It is assumed that each lot received contains percentage defectives, p, with a known probability density function, $f(p)$. The selling price of good quality item is s per unit. The defective items are sold as a single batch at a discounted price. A 100% screening process of the lot is conducted at a rate of x units per unit time; items of poor quality are kept in stock and sold prior to receiving the next shipment as a single batch at a discounted price of v per unit. Let D be the demand per year and T be the cycle length. Let d be the unit screening cost. Let h be the holding cost per unit per unit time. The number of good items in each order denoted by $N(p, y)$ is obtained by subtracting the defective items from the lot size. i.e.,

$$N(p, y) = y - py = y(1 - p) \tag{4.1}$$

In order to avoid shortages, the number of good items in each order is at least equal to the demand during screening time t. Then,

$$N(p, y) \geq Dt \tag{4.2}$$

Substitute (4.1) in equation (4.2) and put $t = \frac{y}{x}$ we get $(1 - p)y \geq D\frac{y}{x}$

Simplifying, we get $p \leq 1 - \frac{D}{x}$

Let $TR(y)$ denote the total revenue earned by selling the good quality items at s per unit and imperfect quality items at a price of v per unit. Then,

$$TR(y) = sy(1 - p) + vyp$$

Moreover, the total cost = procurement cost per cycle + screening cost per cycle+ holding cost per cycle.

Procurement cost $= K + cy$

Screening cost $= dy$

Holding cost $= h(\frac{y(1-p)T}{2} + \frac{py^2}{x})$

Therefore, Total cost $= K + cy + dy + h(\frac{y(1-p)T}{2} + \frac{py^2}{x})$

We know that Profit per cycle = Revenue per cycle - Cost per cycle. That is,

$$TP(y) = TR(y) - TC(y)$$

$$\Rightarrow TP(y) = sy(1-p) + vyp - (K + cy + dy + h(\frac{y(1-p)T}{2} + \frac{py^2}{x}))$$

$Total\ profit\ per\ unit\ time = TPU(y) = \dfrac{TP(y)}{T}$

$$\Rightarrow TPU(y) = D(s - v + \frac{hy}{x}) + D(v - \frac{hy}{x} - c - d - \frac{K}{y}) \times \frac{1}{1-p} - hy.\frac{1-p}{2}$$

$Expected\ value\ of\ total\ profit\ per\ unit\ time\ is\ :$

$$\Rightarrow ETPU(y) = D(s - v + \frac{hy}{x}) + D(v - \frac{hy}{x} - c - d - \frac{K}{y}) \times E[\frac{1}{1-p}] - hy.\frac{1-E[p]}{2}$$

$$\frac{d(ETPU)}{dy} = \frac{hD}{x} - \frac{hD}{x} \times E[\frac{1}{1-p}] + \frac{KD}{y^2} \times E[\frac{1}{1-p}] - \frac{h}{2} + \frac{hE[p]}{2}$$

Put $\frac{d(ETPU)}{dy} = 0$, we get

$$y = \sqrt{\frac{KDE[\frac{1}{1-p}]}{h[\frac{1-E[p]}{2} - \frac{D}{x}(1 - E[\frac{1}{1-p}])]}} \qquad (4.3)$$

$$\frac{d^2(ETPU)}{dy^2} = \frac{-2KDE[\frac{1}{1-p}]}{y^3} \leq 0\ for\ all\ values\ of\ y$$

This implies that $ETPU$ is maximum at y given by (4.3).

5. PROBLEM OF SWINE FLU VACCINATION DELIVERY

In this paper, we consider the situation when swine flu becomes an epidemic in a rural area of Haryana. In order to protect the population in that area, local government establishes three centres, namely centre 1 , centre 2, and centre 3 to make necessary arrangements for the availability of swine flu vaccination in that area. Purchase managers at these centres purchase this medicine from a pharmaceutical company which has its distribution centres at Delhi and Chandigarh. Depending upon the location of these centres, the distribution centres at Delhi and Chandigarh have decided to supply a minimum and a maximum number of units to each centre.The figures (in thousand units) are shown in the Table 1 .

Table 1: Bounds on number of units (in thousands)

centres→	centre1	centre2	centre3
Delhi	10	10	5
	1	2	0
Chandigarh	15	15	20
	0	3	1

Note: The entries in the upper left corner and the lower left corner of each cell denote, respectively the maximum and minimum number of units (in thousands). In addition, a minimum of 5000 units and a maximum of 30,000 units of flu vaccine are available at Delhi all the time, whereas the availability at Chandigarh varies from 10000 to 40000 units. The demand of centre 1 varies from 10000 units to 30000 units, whereas the demand of centre 2 varies between 7000 units to 20000 units. The figures of demand for centre 3 are 5000 units and 30000 units. The selling price (in '00 rupees) per one thousand unit charged by distribution centres at Delhi and Chandigarh from the three centres of Haryana are given in Table 2

Table 2: Cost table (in hundreds of rupees)

centres	centre 1	centre2	centre3
Delhi	5	9	9
Chandigarh	4	6	2

The cartage F_1 (in hundreds of rupees) of delivery vans used for transporting cartons of medicine from Delhi to three centres are as follows:
$F_1 = F_{11}\delta_{11} + F_{12}\delta_{12} + F_{13}\delta_{13}$ where $F_{11} = 150, F_{12} = 50, F_{13} = 50$
The cartage F_2 (in hundreds of rupees)of delivery vans used for transporting cartons of medicine from Chandigarh to three centres are as follows:
$F_2 = F_{21}\delta_{21} + F_{22}\delta_{22} + F_{23}\delta_{23}$ where $F_{21} = 200, F_{22} = 100, F_{23} = 50$
where for $i = 1, 2$

$$\delta_{i1} = \begin{cases} 1 & 0 < \text{number of units (in thousands) supplied} \leq 10 \\ 0 & \text{otherwise} \end{cases}$$

$$\delta_{i2} = \begin{cases} 1 & 10 < \text{number of units (in thousands) supplied by the i}^{\text{th}}\text{origin} \leq 20 \\ 0 & \text{otherwise} \end{cases}$$

$$\delta_{i3} = \begin{cases} 1 & \text{number of units (in thousands) supplied by the i}^{\text{th}}\text{origin} > 20 \\ 0 & \text{otherwise} \end{cases}$$

Now, the problem of purchase managers at the three centres is to determine the order size that should be placed at Delhi and Chandigarh so that the total cost (purchasing cost + cartage) of obtaining the goods is minimized and their demand is met. Once the medicine is received, it is sent by all centres to cold storage, otherwise the medicine may get spoiled. The medicine is now sold from the office of cold storage to Primary Health Centre (P.H.C) in Haryana where the medicine is given to the patients. One unit of medicine contains 5 doses. Once the bottle of the medicine is opened, it has to be given to 5 patients simultaneously, failing which, the proportion of unused medicine is of no use. The price per unit charged by the cold storage from P.H.C is 25 per unit. If one bottle of medicine could not be used fully at a time, then the unused medicine will be taken back by the cold storage at a discounted price of $Rs.20$ per unit. The P.H.C has an annual demand of 5000 units. Ordering cost is $Rs.10$ per order. Holding cost is $Rs.0.5$ per unit per year. It also cost $Rs.0.5$ per unit when the medicine is delivered to the patient. The P.H.C charges 50 per unit from the patient. It is assumed that the percentage of the unused medicine is a random variable, p, which is uniformly distributed with its p.d.f as

$$f(p) = \begin{cases} 25 & 0 \leq p \leq 0.04 \\ 0 & otherwise \end{cases}$$

It was found that , on an average, 17520 units per year are delivered to the patients. The problem is to determine the size of order that must be placed so that the expected total profit per year is maximized. Shortages are avoided.

The problem under consideration is solved in two parts. In the first part, we solve the problem of the purchase manager of centres 1, 2 and 3 i.e., we determine the number of units that they should purchase from Delhi and Chandigarh such that the total cost (purchasing cost + cartage) of obtaining the medicine is minimized. And in the second part, we determine the order size that should be placed in order to maximize the total expected profit per year. First part of the problem can be modeled as a capacitated transportation problem.

Let $i = 1$ denote the distribution centre at Delhi, and $i = 2$ is the distribution centre at Chandigarh.

$j = 1, 2$, and 3 denote, respectively centres 1, 2, and 3 at Haryana.

x_{ij} = number of units(in thousands) ordered by j^{th} centre from i^{th} distribution centre.

c_{ij} = purchasing cost(in hundreds of rupees) per one thousand unit of medicine ordered by j^{th} centre from i^{th} distribution centres .

l_{ij} and u_{ij} are the bounds on number of units to be supplied by i^{th} distribution centre to j^{th} centre.

a_i and A_i are the bounds on the availability of medicine at the ith distribution centre.

b_j and B_j are the bounds on the demand at the jth centre.

Then, the given problem becomes

$$(P1) : min\{\sum_{i=1}^{2} \sum_{j=1}^{3} c_{ij}x_{ij} + \sum_{i=1}^{2} F_i\}$$

subject to

$$5 \leq \sum_{j=1}^{3} x_{1j} \leq 30; \quad 10 \leq \sum_{j=1}^{3} x_{2j} \leq 40; \quad 10 \leq \sum_{i=1}^{2} x_{i1} \leq 30; \quad 7 \leq \sum_{i=1}^{2} x_{i2} \leq 20;$$

$$5 \leq \sum_{i=1}^{2} x_{i3} \leq 30$$

$$1 \leq x_{11} \leq 10; \ 2 \leq x_{12} \leq 10; \ 0 \leq x_{13} \leq 5; \ 0 \leq x_{21} \leq 15; \ 3 \leq x_{22} \leq 15; \ 1 \leq x_{23} \leq 20$$

$$c_{11} = 5; \ c_{12} = 9; \ c_{13} = 9; \ c_{21} = 4; \ c_{22} = 6; \ c_{23} = 2$$

F_i is the cartage associated with i^{th} distribution centre.

$F_1 = F_{11}\delta_{11} + F_{12}\delta_{12} + F_{13}\delta_{13}$ where $F_{11} = 150, F_{12} = 50, F_{13} = 50$

$F_2 = F_{21}\delta_{21} + F_{22}\delta_{22} + F_{23}\delta_{23}$ where $F_{21} = 200, F_{22} = 100, F_{23} = 50$

where for $i = 1, 2$

$$\delta_{i1} = \begin{cases} 1 & 0 < \sum\limits_{j=1}^{3} x_{ij} \leq 10 \\ 0 & otherwise \end{cases}$$

$$\delta_{i2} = \begin{cases} 1 & 10 < \sum\limits_{j=1}^{3} x_{ij} \leq 20 \\ 0 & otherwise \end{cases}$$

$$\delta_{i3} = \begin{cases} 1 & \sum\limits_{j=1}^{3} x_{ij} > 20 \\ 0 & otherwise \end{cases}$$

Set up the transportation problem as shown in Table 3

Table 3: Cost Matrix of problem ($P1$)

$i \cdots j$	D_1		D_2		D_3		A_i
O_1	5	∟10	9	∟10	9	∟5	30
		⌐1		⌐2		⌐0	
O_2	4	∟15	6	∟15	2	∟20	40
		⌐0		⌐3		⌐1	
B_j	30		20		30		

Here, O_1 and O_2 are distribution centres at Delhi and Chandigarh, respectively. D_1, D_2, D_3 are centres 1, 2, and 3, respectively. Values in upper left corner of each cell shows $c'_{ij}s$. Values in the ∟ and ⌐ of each cell denotes the upper bounds u_{ij} and lower bounds l_{ij} of each decision variable, respectively. A_i shows the maximum number of units available at i^{th} distribution centre. B_j denotes the maximum number of units demanded by j^{th} centre.
Convert the above problem ($P1$) into related transportation problem ($P2$) as discussed in the algorithm by introducing a dummy origin and a dummy destination with $c_{i4} = 0$ for all $i = 1, 2$ and $c_{3j} = 0$ for all $j = 1, 2, 3, 4$. Also, we have $0 \leq x_{14} \leq 25, 0 \leq x_{24} \leq 30, 0 \leq x_{31} \leq 20, 0 \leq x_{32} \leq 13, 0 \leq x_{33} \leq 25, 0 \leq x_{34} \leq M$, and $F_{3j} = 0$ for $j = 1, 2, 3, 4$. In this way , we form the problem ($P2$).
Using Computing software MATHEMATICA, we find an initial basic feasible solution of problem ($P2$), which is given in Table 4 below.

Table 4: Cost Matrix of Related Transportation Problem ($P2$) with initial basic feasible solution.

$i \cdots j$	D_1		D_2		D_3		D_4		u_i
O_1	5	∟10	9	∟10	9	∟5	0	∟25	1
	3	⌐1	$\underline{2}$	⌐2	$\underline{0}$	⌐0	$\overline{25}$	⌐0	
O_2	4	∟15	6	∟15	2	∟20	0	∟30	0
	7	⌐0	**5**	⌐3	**5**	⌐1	**23**	⌐0	
O_3	0	∟20	0	∟13	0	∟25	0	∟M	0
	$\underline{20}$	∟0	$\underline{13}$	⌐0	$\underline{25}$	⌐0	**22**	⌐0	
v_j	4		6		2		0		

Note: In Table 4, entries in bold represent basic cells, and entries of the form $\underline{a}$ and $\overline{b}$ represent non- basic cells which are at their lower bounds and upper bounds,

respectively. Entries in ⌞ represent upper bounds in each cell, and entries in ⌜ represents lower bounds in each cell.

$F(CurrentSolution) = 150 + 300 + 0 = 450$

Applying steps 3 to 6 of the algorithm , we get Table 5.

Table 5: Optimality condition

NB	O_1D_2	O_1D_3	O_1D_4	O_3D_1	O_3D_2	O_3D_3
$c_{ij} - z_{ij}$	2	6	-1	-4	-6	-2
θ_{ij}	2	2	7	8	10	15
$\theta_{ij}(c_{ij} - z_{ij})$	4	12	-7	-32	-60	-30
$F(NB)$	450	450	500	500	500	500
ΔF_{ij}	0	0	-50	-50	-50	-50
R_{ij}^1 and R_{ij}^2	4	12	-43	-18	10	-20

Since all R_{ij}^1 and R_{ij}^2 are not greater than or equal to zero in Table 5, so we enter O_1D_4 into basis and proceed as before. We then get the solution shown in Table 6.

$F(CurrentSolution) = 200 + 200 + 0 = 400$

Applying steps 3 to 6 of the algorithm , we get Table 7. Since all R_{ij}^1 and $R_{ij}^2 \geq 0$, solution given in Table 6 is an optimal solution.

    Minimum $Z = 50 + 18 + 30 + 10 + 400 = 508$

Since the cost figures are given in hundreds of rupees, therefore, the minimum total cost is $Rs.50800$ and centre 1 will place an order of 10000 units at Delhi centre, while centre 2 will place an order of 2000 units at Delhi centre and 5000 units at

Table 6: Iterated solution

$i \cdots j$	D_1		D_2		D_3		D_4		u_i
O_1	5	⌞10	9	⌞10	9	⌞5	0	⌞25	0
	10	⌜1	2	⌜2	0	⌜0	**18**	⌜0	
O_2	4	⌞15	6	⌞15	2	⌞20	0	⌞30	-1
	0	⌜0	**5**	⌜3	**5**	⌜1	30	⌜0	
O_3	0	⌞20	0	⌞13	0	⌞25	0	⌞M	0
	20	⌞0	13	⌜0	25	⌜0	**22**	⌜0	
v_j	5		7		3		0		

Table 7: Optimality condition

NB	O_1D_2	O_1D_3	O_2D_4	O_3D_1	O_3D_2	O_3D_3
$c_{ij} - z_{ij}$	2	6	1	-5	-7	-3
θ_{ij}	2	4	7	0	0	0
$\theta_{ij}(c_{ij} - z_{ij})$	4	24	7	0	0	0
$F(NB)$	400	400	450	400	400	400
ΔF_{ij}	0	0	50	0	0	0
R_{ij}^1 and R_{ij}^2	4	24	43	0	0	0

Chandigarh centre. Centre 3 will place an order of 5000 units at Chandigarh. With this planning strategy, the three centres can meet their demands at minimum cost. For the second part, we do the problem formulation of the following inventory situation.

Now, Let y be the order size placed by P.H.C that is delivered instantaneously with a purchasing price c of $Rs.25$ per unit. A fixed cost$K$ of $Rs.10$ per order is incurred each time an order is placed . It is assumed that in each lot received, percentage of medicine that remain unused follow uniform distribution with p.d.f , $f(p)$ given by

$$f(p) = \begin{cases} 25 & 0 \leq p \leq 0.04 \\ 0 & otherwise \end{cases}$$

The selling price of medicine delivered to the patient is $s = Rs.50$ per unit. Let the rate at which the medicine is delivered to the patients per year is $x = 17520$ units per year ; portion of medicine which is not used are kept in stock and sent back to cold storage prior to receiving the next shipment as a single batch at a discounted price of $v = Rs.20$ per unit. Annual Demand of medicine = D = 5000 units per year. The cost of administering the medicine each time is $d = Rs.0.5$ per unit. Let $h = Rs.0.5$ be the holding cost per unit per unit time. In order to avoid shortage, we must ensure that $p \leq 1 - \frac{D}{x} = 1 - \frac{5000}{17520} = 0.7146$

$$E(p) = \int_a^b pf(p)dp = \int_0^{0.04} 25p\,dp = 0.02$$

$$E[\frac{1}{1-p}] = \int_a^b \frac{1}{1-p} f(p)dp = \int_0^{0.04} \frac{25}{1-p}dp = 1.02055$$

$$\text{optimum order size} = y = \sqrt{\frac{10 \times 5000 \times 1.02055}{0.5[\frac{1-0.02}{2} - \frac{5000}{17520}(1 - 1.02055)]}} = 453.66 \sim 454$$

Expected total profit per unit =

$$ETPU = 5000(50 - 20 + \frac{0.5 \times 454}{17520})$$
$$+ 5000(20 - \frac{0.5 \times 454}{17520} - 25 - 0.5 - \frac{10}{454}) \times 1.02055$$
$$= 121709.9183 \text{ per year}$$

The purchase manager at centre 1 will place an order of 10000 units at Delhi centre, while centre 2 will place an order of 2000 units of swine flu vaccination at Delhi centre and 5000 units at Chandigarh centre. Centre 3 will place an order of 5000 units at Chandigarh. With this planning strategy, the three centres can meet their demands at minimum of *Rs*.50800. On the other hand, P.H.C will place an order of 454 units of medicine at a time. By ordering this much , a maximum of expected total profit of *Rs*.121709.9183 per year is earned.

6. CONCLUSION

Due to the difficulties in treating swine flu disease, controlling and preventing its outbreak is essential for keeping people healthy, particularly in the regions of Asia and Africa. We discussed a mathematical model for the availability of medicine at minimum cost. This problem is formulated as a fixed charge capacitated transportation problem with bounds of rim conditions. Once the medicine is made available, the next objective is to determine the size of order to be placed such that the expected total profit per year is maximized. The objective is formulated as an economic order quantity model for items with deteriorating quality.

As a future work, we intend to extend the model for swine flu transmission by including different decision variables that represent distinct measures for controlling the disease. Multi-objective programming can be used to consider the problems of availability of vaccine, medicine for the treatment of swine flu, availability of beds and doctors in hospitals, storage of medicine, etc. Investigating impacts of different values of model parameters is also the subject of the future work. During the last decades, the global prevalence of dengue progressed dramatically. The present model or extended model such as multi-objective approach can be used for optimal control of dengue transmission as this disease has become endemic in more than one hundred countries of Africa, Asia, America, and the western Pacific.

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