Yugoslav Journal of Operations Research
32 (2022), Number 4, 425–437
DOI: https://doi.org/10.2298/YJOR220315019S

MATHEMATICAL MODEL FOR ANALYSING AVAILABILITY OF THRESHING COMBINE MACHINE UNDER REDUCED CAPACITY

Shakuntla SINGLA
1Department of Mathematics and Humanities, Maharishi Markandeshwar (Deemed to be University), Mullana, Ambala, India
shakus25@gmail.com

Umar Muhammad MODIBBO
Department of Statistics and Operations Research, Modibbo Adama University, Yola, P.M.B. 2076, Yola, Nigeria
umarmodibbo@mautech.edu.ng

Mohammed MIJINYAWA
Department of Statistics and Operations Research, Modibbo Adama University, Yola, P.M.B. 2076, Yola, Nigeria
m.mijinyawa@mautech.edu.ng

Subhash MALIK
Department of Mechanical Engineering, Maharishi Markandeshwar (Deemed to be University), Mullana, Ambala, India
subhasmalik604@gmail.com

Shubham VERMA
Department of Mechanical Engineering, Maharishi Markandeshwar (Deemed to be University), Mullana, Ambala, India
shubham6140005@gmail.com

Pooja KHURANA
Department of Applied Sciences, Manav Rachna International Institute of Research and Studies, Faridabad, India
pooja.fet@mriu.edu.in

Received: March 2022 / Accepted: August 2022

Abstract: Obtaining system availability in an engineering design is trickish and challenging, especially when there is a reduction in capacity; however, it supports system maintainability. In this paper, a mathematical model for finding the availability under the reduced capacity has been proposed using the Chapman Kolmogorov approach with the help of transition diagrams associated with various possible combinations of probabilities. The paper observes the most critical subsystem by selecting variable failure and repair rates from different subsystems. It deals with the sensitivity analysis of a complex repairable threshing combined machine comprising subsystems in a series configuration and the threshing machine consisting of 21 subsystems. The device works in total capacity when the threshing drum and feeding Hooper work in the complete state, and the concave subsystem and blower work with reduced power. This study dealt with uncertain data and was analyzed analytically using a complex repairable system. The availability of the entire machine has been investigated analytically, and various availability indices such as subsystems extruder have been computed and reported. The study discovered that subsystem extruder has the most impact on some subsystems' overall system availability.

Keywords: Availability, supplementary variable technique, mean time to failure, mean time between failure, maintainability, mathematical modeling, reliability.

MSC: 90B85, 90C26.

1. INTRODUCTION

Every farmer in the agriculture field wants to maximize profit by making the machine without failure by adopting proper maintenance and planning. Nowadays, sole operating machines and multi-operating machines exist, e.g., harvesting, threshing and reaping, combined in one machine, i.e., combined harvester. This agricultural implement comprises approximately 21 parts, including header, belts, layers, sieves, rotating blades, reel, cutter bar, grain tank, augers, conveyors, unloading pipe, etc. It could extract, winnow, and swain crops such as rice, sunflower, pulses, corn, wheat, barley, flax, soybeans, etc. Reliability techniques are used to judge the availability and maintainability of a system. This paper aims to evaluate faults in subsystems and suggest ways to improve the tests to minimize the weakness. Firstly, Cox [1] established systematic solutions of reliability analysis using the supplementary variable technique.

Shakuntla et al.[2] discussed poly tube availability analysis using the supplementary variable technique. Also, reducing the risk of hazards for optimum reliability was studied by Yang et al. [3]. The availability of banking services with a standby system has been evaluated in [4]. Aggarwal et al.[5] used a genetic algorithm (GA) to offer mathematical modelling for availability optimization within butter oil manufacturing. Ram and Nagiya [6] investigated the total system dependability of such a gas power plant using mathematical models and the Laplace transform. Kumar et al. [7] used Regenerative Point Graphical Technique (RPGT) to explore the edible oil refinery business. The authors used RPGT to understand the performance of a paper mill's cleaning unit. Similarly, RPGT has been used to analyze the behaviour and profit of a thresher plant in agriculture under steady-state assuming a single server [8].

The sensitivity analysis of the 3:4:G system was analyzed by [9]. Later, Kumar et al. [10] examined the behaviour of a bread-making system composed of five unique subsystems and assessed system characteristics beneficial to management using RPGT within steady-state conditions. Savolainen et al. [11] studied a system configuration with three demand-dependent unit functionality. A non-markovian mechanism has been investigated using extra variables (Shakuntla et al., 2019). Sharma and Ailawalia [12] studied the reliability, availability and maintainability (RAM) of a threshing machine in agriculture and considered good, reduced and failed as states of the device. The problem is modelled using the supplementary variable technique and solved with the Lagrangian method. Shelare et al. [13] formulated a mathematical model for Deshelled Nut quantity in the Charoli Nut Deshelling machine for improving processing efficiency and waste reductions.

Malik and Tewari [14] studied the formability of a thermal power soil's coal system application. Modibbo et al. [15] proposed two different approaches to reliability function estimation for both system and subsystem components. The study emplored the maximum likelihood estimator (MLE) and uniformly minimum unbiased estimator (UMUVE) approaches to derive the new methodology for estimating systems' parts. It further hybridized an optimization procedure to minimize the cost of purchasing failed components and applied it to a real-life engineering firm. Several types of research have been conducted in different fields using different approaches to optimize the process. Several applications of optimization and reliability calculations can be found in [16, 17, 14]. However, this paper proposes a model that can select various subsystems' variable failure and repair rates in the most critical subsystem. The authors further studied system availability comparing four different approaches and concluded POS outperformed others [18]. Khan et al. [19] have studied a selective maintenace allocation problem in the reliability theory, and formulated the problem as bilevel nonlinear optimization.

2. SYSTEM DESCRIPTION

The system under consideration comprises four prominent parts-threshing drums, feeding hopper, concave and blower. As defined, explained and shown in Figure 1.

1. **The threshing Drum (A)** rotates at high speed (500rpm). Its shape is cylindrical.
2. **The feeding hopper (B)** is located upon that threshing pneumatic cylinder tip.
3. **Concave (C):** It can work in a reduced state. It separates the grain from the crop
4. **Blower (D):** Blower can also work in a reduced state. It cleans and separates the straw from the grain.

Figure 1: The Vidhataa multi-functional thresher

2.1. Terminology

Every machine has specific terms used to describe its parts and functionality for clear understanding and to avoid unnecessary errors from the operators. Table 1 presents and explains the terminology of the Vidhataa multi-functional thresher machine briefly under study.

Table 1: Machine Terminology and explanation.

Terminology	Explanation
A, B, C, D:	These terms depict the whole working state of the subsystems
$\bar{C}$, $\bar{D}$:	This shows that C and D work in a reduced capacity.
a, b, c, d:	This shows the complete failed states of A, B, C, and D.
$\lambda_1(v)$, $\lambda_2(v)$:	Failure Rate of subsystem C and D.
$\alpha_1(v)$, $\alpha_2(v)$:	The transition rate of subsystem Aand B.
$\Phi(u)$, $\psi(u)$, $\mu(u)$, $\sigma(u)$:	Depicts the repair rate of the subsystems A,B, C,
$P_{Ti}(u, v, t)$:	The likelihood that the system seems to be in State I during time t must have an elapsed breakdown ($i = 1, \ldots, 20$) time v and then an elapsed recovery time u.

2.2. System Assumptions

The following assumptions are expected to hold regarding the system.

i. The subsystem of the machine can work with reduced capacity.

ii. The repair process starts as soon as possible.

iii. The failure rate is constant, and the repair rate of the subsystems is variable.

iv. Failure and repair events are all statistically independent.

v. The repair subsystem is as good as a new one

2.3. Conceptual Model

For evaluating the system performance under actual operating conditions, it is essential to draw a conceptional model for the system/subsystem known as the transition diagram.

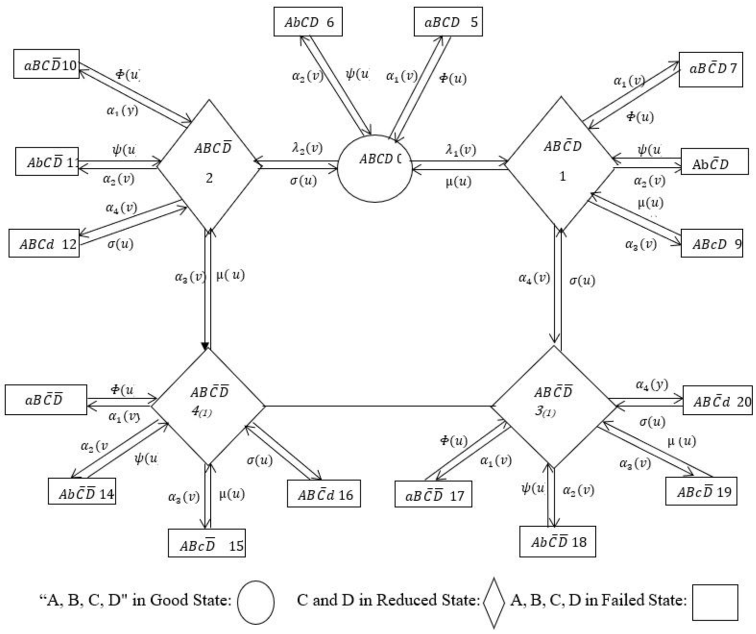

Figure 2: Transition diagram of Combine Threshing Machine

2.4. Mathematical Model

The equations of the combined machine when two subsystems concave and blower fail simultaneously by using the Chapman Kolgmrogrov rule are given as follows:

2.4.1. Transitory state with variable failure and repair rates

$$[d/dt + S_{T0}]P_{T0}(t) = M_{T0}(t) \tag{1}$$

$$[\delta/\delta t + \delta/\delta u + \delta/\delta v + S_{T1}(u, v)]P_{T1}(u, v, t) = M_{T1}(u, v, t) \tag{2}$$

$$[\delta/\delta t + \delta/\delta u + \delta/\delta v + S_{T2}(u, v)]P_{T2}(u, v, t) = M_{T2}(u, v, t) \tag{3}$$

$$[\delta/\delta t + \delta/\delta u + \delta/\delta v + S_{T3(1)}(u, v)]P_{T3(1)}(u, v, t) = M_{T3}(u, v, t) \tag{4}$$

$$[\delta/\delta t + \delta/\delta u + \delta/\delta v + S_{T3(2)}(u, v)]P_{T4(2)}(u, v, t) = M_{T4}(u, v, t) \tag{5}$$

$$[\delta/\delta t + \delta/\delta u + \delta/\delta v + \Phi(u)]P_{T5}(u, v, t) = \alpha_1(v)P_{T0}(t) \tag{6}$$

$$[\delta/\delta t + \delta/\delta u + \delta/\delta v + \psi(u)]P_{T6}(u,v,t) = \alpha_2(v)P_{T0}(t) \tag{7}$$

$$[\delta/\delta t + \delta/\delta u + \delta/\delta v + \Phi(u)]P_{T7}(u,v,t) = \alpha_1(v)P_{T1}(u,v,t) \tag{8}$$

$$[\delta/\delta t + \delta/\delta u + \delta/\delta v + \psi(u)]P_{T8}(u,v,t) = \alpha_2(v)P_{T1}(u,v,t) \tag{9}$$

$$[\delta/\delta t + \delta/\delta u + \delta/\delta v + \mu(u)]P_{T9}(u,v,t) = \alpha_3(v)P_{T1}(u,v,t) \tag{10}$$

$$[\delta/\delta t + \delta/\delta u + \delta/\delta v + \Phi(u)]P_{T10}(u,v,t) = \alpha_1(v)P_{T2}(u,v,t) \tag{11}$$

$$[\delta/\delta t + \delta/\delta u + \delta/\delta v + \psi(u)]P_{T11}(u,v,t) = \alpha_2(v)P_{T2}(u,v,t) \tag{12}$$

$$[\delta/\delta t + \delta/\delta u + \delta/\delta v + \sigma(u)]P_{T12}(u,v,t) = \alpha_4(v)P_{T2}(u,v,t) \tag{13}$$

$$[\delta/\delta t + \delta/\delta u + \delta/\delta v + \Phi(u)]P_{T13}(u,v,t) = \alpha_1(v)P_{T4(2)}(u,v,t) \tag{14}$$

$$[\delta/\delta t + \delta/\delta u + \delta/\delta v + \psi(u)]P_{T14}(u,v,t) = \alpha_2(v)P_{T4(2)}(u,v,t) \tag{15}$$

$$[\delta/\delta t + \delta/\delta u + \delta/\delta v + \mu(u)]P_{T15}(u,v,t) = \alpha_3(v)P_{T4(2)}(u,v,t) \tag{16}$$

$$[\delta/\delta t + \delta/\delta u + \delta/\delta v + \sigma(u)]P_{T16}(u,v,t) = \alpha_4(v)P_{T4(2)}(u,v,t) \tag{17}$$

$$[\delta/\delta t + \delta/\delta u + \delta/\delta v + \Phi(u)]P_{T17}(u,v,t) = \alpha_1(v)P_{T3(1)}(u,v,t) \tag{18}$$

$$[\delta/\delta t + \delta/\delta u + \delta/\delta v + \psi(u)]P_{T18}(u,v,t) = \alpha_1(v)P_{T3(1)}(u,v,t) \tag{19}$$

$$[\delta/\delta t + \delta/\delta u + \delta/\delta v + \mu(u)]P_{T19}(u,v,t) = \alpha_1(v)P_{T3(1)}(u,v,t) \tag{20}$$

$$[\delta/\delta t + \delta/\delta u + \delta/\delta v + \sigma(u)]P_{T20}(u,v,t) = \alpha_1(v)P_{T3(1)}(u,v,t) \tag{21}$$

Where

$S_{T0} = \sum_{i=1}^{2} \alpha_i(u) + \sum_{i=1}^{2} \lambda_i$
$S_{T1}(u,v) = \sum_{i=1}^{4} \alpha_i(v) + \mu(u)$
$S_{T2}(u,v) = \sum_{i=1}^{4} \alpha_i + \sigma(u)$
$S_{T3(1)}(u,v) = \sum_{i=1}^{4} \alpha_i(v) + \sigma(u)$
$S_{T4(2)}(u,v) = \sum_{i=1}^{4} \alpha_i(v) + \mu(u)$
$M_{T0}(t) = \int P_{T5}(u,v,t)\Phi(u)dudv + \int P_{T6}(u,v,t)\psi(u)dudv + \int P_{T2}(u,v,t)\sigma(u)dudv$
$+ \int P_{T1}(u,v,t)\mu(u)dudv$
$M_{T1}(u,v,t) = \lambda_1(v)P_{T0}(t) + P_{T7}(u,v,t)\Phi(u) + P_{T8}(u,v,t)\psi(u) + P_{T3(1)}(u,v,t)\sigma(u) + P_{T9}(u,v,t)\mu(u)$
$M_{T2}(u,v,t) = \lambda_2(v)P_{T0}(t) + P_{T10}(u,v,t)\Phi(u) + P_{T11}(u,v,t)\psi(u) + P_{T12}(u,v,t)\sigma(u) + P_{T4(2)}(u,v,t)\mu(u)$
$M_{T3}(u,v,t) = \alpha_4(v)P_{T1}(t) + P_{T17}(u,v,t)\Phi(u) + P_{T18}(u,v,t)\psi(u) + P_{T19}(u,v,t)\mu(u) + P_{T20}(u,v,t)\sigma(u)$
$M_{T4}(u,v,t) = \alpha_3(v)P_{T2}(t) + P_{T13}(u,v,t)\Phi(u) + P_{T14}(u,v,t)\psi(u) + P_{T15}(u,v,t)\mu(u) + P_{T16}(u,v,t)\sigma(u)$

2.4.2. The Extremity Conditions

$$P_{T1}(0,v,t) = \lambda_1(v)P_{T0}(t) \tag{22}$$

$$P_{T2}(0,v,t) = \lambda_2(v)P_{T0}(t) \tag{23}$$

$$P_{T3(1)}(0,v,t) = \int \alpha_4(v)P_{T1}(u,v,t)du \tag{24}$$

$$P_{T4(2)}(0,v,t) = \int \alpha_2(v)P_{T2}(u,v,t)du \tag{25}$$

$$P_{T5}(0,v,t) = \alpha_1(v)P_{T0}(t) \tag{26}$$

$$P_{T6}(0,v,t) = \alpha_2(v)P_{T0}(t) \tag{27}$$

$$P_{T7}(0,v,t) = \int \alpha_1(v) P_{T1}(u,v,t)du \tag{28}$$

$$P_{T8}(0,v,t) = \int \alpha_2(v) P_{T1}(u,v,t)du \tag{29}$$

$$P_{T9}(0,v,t) = \int \alpha_3(v) P_{T1}(u,v,t)du \tag{30}$$

$$P_{T10}(0,v,t) = \int \alpha_1(v) P_{T2}(u,v,t)du \tag{31}$$

$$P_{T11}(0,v,t) = \int \alpha_2(v) P_{T2}(u,v,t)du \tag{32}$$

$$P_{T12}(0,v,t) = \int \alpha_1(v) P_{T2}(u,v,t)du \tag{33}$$

$$P_{T13}(0,v,t) = \int \alpha_1(v) P_{T4(2)}(u,v,t)du \tag{34}$$

$$P_{T14}(0,v,t) = \int \alpha_2(v) P_{T4(2)}(u,v,t)du \tag{35}$$

$$P_{T15}(0,v,t) = \int \alpha_3(v) P_{T4(2)}(u,v,t)du \tag{36}$$

$$P_{T16}(0,y,t) = \int \alpha_4(v) P_{T4(2)}(u,v,t)du \tag{37}$$

$$P_{T17}(0,y,t) = \int \alpha_1(v) P_{T3(1)}(u,v,t)du \tag{38}$$

$$P_{T18}(0,y,t) = \int \alpha_2(v) P_{T3(1)}(u,v,t)du \tag{39}$$

$$P_{T19}(0,y,t) = \int \alpha_3(v) P_{T3(1)}(u,v,t)du \tag{40}$$

$$P_{T20}(0,y,t) = \int \alpha_4(v) P_{T3(1)}(s,t)du \tag{41}$$

2.4.3. Baseline Conditions

$$P_{Ti}(u,v,0) = 0, \quad (i = 1, 2, \ldots, 20) \tag{42}$$

$$P_{T0}(0) = 0 \tag{43}$$

2.4.4. For Long Run Availability

A decision-maker wants a machine to operate with optimal performance (less failure rate) in the agricultural field. Therefore, $t \longrightarrow \infty$ is taken, consequently $d/dt \longrightarrow 0,\ \delta/\delta t \longrightarrow 0,\ \delta/\delta u \longrightarrow 0,\ and\ \delta/\delta v \longrightarrow 0$. Then, the ordinary differential equation with constant transition rates moderates the system under defined linear algebraic equations (Eqn. 44-64).

$$[\alpha_1 + \alpha_2 + \lambda_1 + \lambda_2]\, P_{T0} = \Phi P_{T5} + \psi P_{T6} + \sigma P_{T1} + \mu P_{T2} \tag{44}$$

$$[\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \mu]\, P_{T1} = \lambda_1 P_{T0} + \sigma P_{T3(1)} + \mu P_{T9} + \psi P_{T8} + \Phi P_{T7} \tag{45}$$

$$[\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \sigma]\, P_{T2} = \lambda_2 P_{T0} + \Phi P_{T10} + \psi P_{T11} + \sigma P_{T12} + \mu P_{T4(2)} \tag{46}$$

$$[\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \sigma]\, P_{T3(1)} = \alpha_4 P_{T1} + \sigma P_{T20} + \mu P_{T19} + \psi P_{T18} + \Phi P_{T17} \tag{47}$$

$$[\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \mu]\, P_{T4(2)} = \alpha_3 P_{T2} + \Phi P_{T13} + \psi P_{T14} + \sigma P_{T16} + \mu P_{T15} \tag{48}$$

$$\Phi P_{T5} = \alpha_1 P_{T0} \tag{49}$$

$$\psi P_{T6} = \alpha_2 P_{T0} \tag{50}$$

$$\phi P_{T7} = \alpha_1 P_{T1} \tag{51}$$

$$\psi P_{T8} = \alpha_2 P_{T1} \tag{52}$$

$$\mu P_{T9} = \alpha_3 P_{T1} \tag{53}$$

$$\Phi P_{T10} = \alpha_1 P_{T1} \tag{54}$$

$$\psi P_{T11} = \alpha_2 P_{T2} \tag{55}$$

$$\sigma P_{T12} = \alpha_4 P_{T2} \tag{56}$$

$$\Phi P_{T13} = \alpha_1 P_{T4(2)} \tag{57}$$

$$\psi P_{T14} = \alpha_2 P_{T4(2)} \tag{58}$$

$$\mu P_{T15} = \alpha_3 P_{T4(2)} \tag{59}$$

$$\sigma P_{T16} = \alpha_4 P_{T4(2)} \tag{60}$$

$$\phi P_{T17} = \alpha_1 P_{T3(1)} \tag{61}$$

$$\psi P_{T18} = \alpha_2 P_{T3(1)} \tag{62}$$

$$\mu P_{T19} = \alpha_3 P_{T3(1)} \tag{63}$$

$$\sigma P_{T20} = \alpha_4 P_{T3(1)} \tag{64}$$

2.4.5. Elucidation

The linear differential equation (1) is a first-order ordinary differential equation, and the remaining Eqns. (2-21) are defined as partial differential equations (PDE) of the first order. Calculating the machine availability, Eqns. (1-21) along with the extremities Eqns. (22-41) and baseline conditions Eqns. (42-43) are evaluated by applying Lagrange's method as well as probabilities $P_{Ti}(t)(i = 1, \ldots, 20)$ for each state are as follows:

$$P_{T0}(t) = e^{-\int S_{T0}(t)dt}\left[1 + \int M_{T0}(t)e^{\int S_{T0}(t)dt}dt\right] \tag{65}$$

$$P_{T1}(u,v,t) = e^{-\int S_{T1}(u,v)dx}\left[\lambda_1(v-u)P_{T0}(t-u) + \int M_{T1}(u,v,t)e^{\int S_{T1}(u,v)du}du\right] \tag{66}$$

$$P_{T2}(u,v,t) = e^{-\int S_{T2}(u,v)dx}\left[\lambda_2(v-u)P_{T0}(t-u) + \int M_{T2}(u,v,t)e^{\int S_{T2}(u,v)du}du\right] \tag{67}$$

$$P_{T3(1)}(u,v,t) = e^{-\int S_{T3(1)}(u,v)dx}\left[\int \alpha_4(v-u)P_{T1}(u,v-u,t-u)dv + \int M_{T3}(u,v,t)e^{\int S_{T3(1)}(u,v)du}du\right] \tag{68}$$

$$P_{T4(2)}(u,v,t) = e^{-\int S_{T4(2)}(u,v)dx}\left[\int \alpha_3(v-u)P_{T2}(u,v-u,t-u)dv + \int M_{T4}(u,v,t)e^{\int S_{T4(2)}(u,v)du}du\right] \tag{69}$$

$$P_{T5}(u,v,t) = e^{-\int \Phi(u)du}\left[\alpha_1(v-u)P_{T0}(t-u) + \int \alpha_1(v)P_{T0}(t)e^{\int \Phi(u)du}du\right] \tag{70}$$

$$P_{T6}(u,v,t) = e^{-\int \psi(u)du}\left[\alpha_2(v-u)P_{T0}(t-u) + \int \alpha_2(v)P_{T0}(t)e^{\int \psi(u)du}du\right] \tag{71}$$

$$P_{T7}(u,v,t) = e^{-\int \Phi(u)du}\left[\int \alpha_1(v-u)P_{T1}(u,v-u,t-u)du + \int \alpha_1(v)P_{T1}(u,v,t)e^{\int \Phi(u)du}du\right] \tag{72}$$

$$P_{T8}(u,v,t) = e^{-\int \psi(u)du}\left[\int \alpha_2(v-u)P_{T1}(u,v-u,t-u)du + \int \alpha_2(v)P_{T1}(u,v,t)e^{\int \psi(u)du}du\right] \tag{73}$$

$$P_{T9}(u,v,t) = e^{-\int \mu(u)du}\left[\int \alpha_3(v-u)P_{T1}(u,v-u,t-u)du + \int \alpha_3(v)P_{T1}(u,v,t)e^{\int \mu(u)du}du\right] \tag{74}$$

$$P_{T10}(u,v,t) = e^{-\int \Phi(u)du}\left[\int \alpha_1(v-u)P_{T2}(u,v-u,t-u)du + \int \alpha_1(v)P_{T2}(u,v,t)e^{\int \Phi(u)du}du\right] \tag{75}$$

$$P_{T11}(u,v,t) = e^{-\int \psi(u)du}\left[\int \alpha_2(v-u)P_{T2}(u,v-u,t-u)du + \int \alpha_2(v)P_{T2}(u,v,t)e^{\int \psi(u)du}du\right] \tag{76}$$

$$P_{T12}(u,v,t) = e^{-\int \sigma(u)du}\left[\int \alpha_4(v-u)P_{T2}(u,v-u,t-u)du + \int \alpha_4(v)P_{T2}(u,v,t)e^{\int \sigma(u)du}du\right] \tag{77}$$

$$P_{T13}(u,v,t) = e^{-\int \Phi(u)du}\left[\int \alpha_1(v-u)P_{T4(2)}(u,v-u,t-u)du + \int \alpha_1(v)P_{T4(2)}(u,v,t)e^{\int \Phi(u)du}du\right] \tag{78}$$

$$P_{T14}(u,v,t) = e^{-\int \psi(u)du}\left[\int \alpha_2(v-u)P_{T4(2)}(u,v-u,t-u)du + \int \alpha_2(v)P_{T4(2)}(u,v,t)e^{\int \psi(u)du}du\right] \tag{79}$$

$$P_{T15}(u,v,t) = e^{-\int \mu(u)du}\left[\int \alpha_3(v-u)P_{T4(2)}(u,v-u,t-u)du + \int \alpha_3(v)P_{T4(2)}(u,v,t)e^{\int \mu(u)du}du\right] \tag{80}$$

$$P_{T16}(u,v,t) = e^{-\int \sigma(u)du}\left[\int \alpha_4(v-u)P_{T4(2)}(u,v-u,t-u)du + \int \alpha_4(v)P_{T4(2)}(u,v,t)e^{\int \sigma(u)du}du\right] \tag{81}$$

$$P_{T17}(u,v,t) = e^{-\int \Phi(u)du}\left[\int \alpha_1(v-u)P_{T3(1)}(u,v-u,t-u)du + \int \alpha_1(v)P_{T3(1)}(u,v,t)e^{\int \Phi(u)du}du\right] \tag{82}$$

$$P_{T18}(u,v,t) = e^{-\int \psi(u)du}\left[\int \alpha_2(v)P_{T3(1)}(u,v-u,t-u)du + \int \alpha_2(v)P_{T3(1)}(u,v,t)e^{\int \psi(u)du}du\right] \tag{83}$$

$$P_{T19}(u,v,t) = e^{-\int \mu(u)du}\left[\int \alpha_3(v)P_{T3(1)}(u,v-u,t-u)du + \int \alpha_3(v)P_{T3(1)}(u,v,t)e^{\int \mu(u)du}du\right] \tag{84}$$

$$P_{T20}(u,v,t) = e^{-\int \sigma(u)du}\left[\int \alpha_2(v)P_{T3(1)}(u,v-u,t-u)du + \int \alpha_2(v)P_{T3(1)}(u,v,t)e^{\int \sigma(u)du}du\right] \tag{85}$$

Thus, probabilities for all the states have been obtained in terms of $P_{T0}(t)$ using Eqn. (1). Therefore time-dependent availability (A_{Tv}) is given by

$$A_{Tv} = P_{T0}(t) + \int \sum_{i=1}^{2} P_{Ti}(u,v,t)dudv + \int P_{T4(2)}(u,v,t)dudv + \int P_{T3(1)}(u,v,t)dudv \tag{86}$$

Further, the linear Eqns. (44-64) have been solved recursively to determine the steady-state availability. All the probabilities of the different states are obtained in terms of P_{T0} and are given below:

$$P_{T1} = R_1 P_{T0} \tag{87}$$

$$P_{T2} = R_2 P_{T0} \tag{88}$$

$$P_{T3(1)} = M_1 P_{T0} \tag{89}$$

$$P_{T4(2)} = M_2 P_{T0} \tag{90}$$

Where

$$M_1 = \frac{\lambda_1 \alpha_4}{\sigma\mu}, \quad M_2 = \frac{\lambda_2 \alpha_3}{\sigma\mu}, \quad R_1 = \frac{\lambda_1}{\mu}, \quad R_2 = \frac{\lambda_2}{\sigma}$$

Now using normalizing conditions $\sum_{i=1}^{20} P_{Ti} = 1$, we get

$$P_{T0} = \left[1 + \tfrac{\alpha_1}{\Phi} + \tfrac{\alpha_2}{\psi} + \left(1 + \tfrac{\alpha_1}{\Phi} + \tfrac{\alpha_2}{\psi} + \tfrac{\alpha_3}{\mu} + \tfrac{\alpha_4}{\sigma}\right) M_1 + \left(1 + \tfrac{\alpha_1}{\Phi} + \tfrac{\alpha_2}{\psi} + \tfrac{\alpha_4}{\sigma} + \tfrac{\alpha_3}{\mu}\right) M_2\right]^{-1}$$
$$+ \left[\left(1 + \tfrac{\alpha_1}{\Phi} + \tfrac{\alpha_4}{\sigma} + \tfrac{\alpha_3}{\mu}\right) R_1 + \left(1 + \tfrac{\alpha_1}{\Phi} + \tfrac{\alpha_2}{\psi} + \tfrac{\alpha_4}{\sigma}\right) R_2\right]^{-1}$$

Consequently, the long-run availability A_{Tv} will be calculated as:

$$A_{Tv} = [1 + M_1 + M_2 + T_1 + T_2]\, P_0 \tag{91}$$

Table 2: Effect of failure α_1) and repair (Φ) rate of the subsystem Threshing Drum (A) on availability.

$\alpha_1 \longrightarrow$ $\Phi \downarrow$	0.0057	0.0059	0.0061	0.0063
0.5	0.9769	0.9732	0.9694	0.9656
0.7	0.9802	0.9775	0.9748	0.9721
0.9	0.9821	0.9796	0.9770	0.9757
1.1	0.9832	0.9815	0.9797	0.9780

Table 3: Effect of failure α_2) and repair (ψ) rates of the subsystem Feeding Hooper (B) on availability.

$\alpha_2 \longrightarrow$ $\psi \downarrow$	0.007	0.009	0.011	0.013
2	0.9769	0.9660	0.9651	0.9541
4	0.9886	0.9881	0.9877	0.9872
6	0.9892	0.9889	0.9885	0.9882
8	0.9895	0.9892	0.9890	0.9886

Table 4: Effect of failure α_3) and repair (μ) rates of the subsystem concave (C) on availability.

$\alpha_1 \longrightarrow$ $\mu \downarrow$	0.01	0.02	0.03	0.04
0.33	0.9769	0.9762	0.9760	0.9757
0.53	0.9770	0.9767	0.9464	0.9763
0.73	0.9789	0.9775	0.9772	0.9770
0.93	0.9790	0.9789	0.9780	0.9779

3. RESULT AND DISCUSSION

The decision-maker (farmer) always prefers a combined threshing machine to operate as long as possible in an optimal fashion without unnecessary failures. Machine availability has to be determined to achieve the long-time operating goal. Therefore, the proposed model (Eqn. 91) can help calculate the availability of threshing machines with different values of failure and repair rates. Machines' availability increases with the repair rates and decreases with the failure rates. We then evaluate the impact of various characteristics on the system's long-term availability. Using Eqn. (91), several subsystem repair rates have been calculated with varying configurations. However, the study did not analyze all possible pairings of subsystems, but only the unique subsystem combinations were evaluated. Tables 2-5 presents the long-run availability of threshing combine machine. The analysis was carried out by altering failure as well as repair rates of such mixtures $\alpha_1 = 0.0057, 0.0059, 0.0061, 0.0063, \varPhi = 0.5, 0.7, 0.9, 1.1$ and by taking other parameters as fixed: $\alpha_2 = 0.007, \alpha_3 = 0.01, \alpha_4 = 0.015, \psi = 2, \mu = 0.33, \sigma = 0.02$. As given in Table 2, the calculated result implies that a rise in the mixture's failure rate (α_1) reduces long-run availability by roughly 0.12 per cent. In contrast, a rise in the repair rate ($\varPhi$) increases availability by approximately 0.26 per cent. Table 3 demonstrates that the rise in the failure rate (α_2) of extruders reduces the system's long-run availability from 0.3 per cent to 0.07 per cent. When we raise the repair rate ($\varPhi$) of the extruder from 2.0 to 8.0, availability improves by 0.6 per cent to 0.85 per cent. It is then estimated the system's long-term availability by altering failure and repair rates. The results obtained as employed in this study are: $\alpha_1 = 0.0057, \alpha_2 = 0.007, \alpha_4 = 0.015, \varPhi = 0.5, \psi = 2.0, = 2$. Four values of $\alpha_3 (= 0.01, 0.02, 0.03, 0.04) and \mu (= 0.33, 0.53, 0.73, 0.93)$ have been taken into account while studying the influence of failure and repair rates of subsystem die. Table 4 displays the outcomes acquired as a consequence of this procedure. Failure as well as repair rates for all subsystems except that of cutter are as follows: $\alpha_1 = 0.0057, \alpha_2 = 0.007, \alpha_3 = 0.33, \Phi = 0.5, \psi = 0.04, \mu = 0.02$. The failure as well as repair rates of cutter were calculated as follows: $\alpha_4 = 0.015, 0.030, 0.045, 0.060, \sigma = 2.0, 4.0, 6.0, 8.0$. Table 5 displays the results of the long-run availability calculation.

4. CONCLUSION

Reliability, maintainability, and replacement of system components are important aspects of industrial advancement. Agriculture requires complex farm implements for its viability, productivity and sustainability. This study investigates the sensitivity analysis of complex repairable threshing combined machine comprising subsystems in a series configuration and the threshing machine consisting of 21 subsystems. A mathematical model under different conditions is proposed to analyze the system's availability. The device works in total capacity when the threshing drum and feeding Hooper work in the entire state, and the concave subsystem and blower work with reduced power. The analysis results are compared under different variables and presented in Tables 2–5. It demonstrates that subsystem extruder has the most impact on some subsystems' overall system availability. Those specific subsystems have almost no effect on the availability of polytube manufacturing facilities. As a result, we conclude that management should prioritize subsystem extruder maintenance to increase overall system availability. This study is helpful for researchers and policymakers in the agricultural sector and other related engineering fields. In future, the analysis can explore the variables under fuzzy, intuitionistic and Neutrosophic environments.

Acknowledgements. The authors are thankful to the editor-in-chief and his team, the special issue guest editors Prof. A.K. Malik and his team and the anonymous refrees for finding this article worthy of publication in this esteemed journal.

Funding. This research received no external funding.

REFERENCES

[1] D. R. Cox, "The analysis of non-markovian stochastic processes by the inclusion of supplementary variables," in *Mathematical Proceedings of the Cambridge Philosophical Society*, vol. 51, no. 3. Cambridge University Press, 1955, pp. 433–441.

[2] S. Shakuntla, A. Lal, S. Bhatia, and J. Singh, "Reliability analysis of polytube industry using supplementary variable technique," *Applied Mathematics and Computation*, vol. 218, no. 8, pp. 3981–3992, 2011.

[3] T. Yang, C. Cui, Y. Shen, and Y. Lv, "A novel denitration cost optimization system for power unit boilers," *Applied Thermal Engineering*, vol. 96, pp. 400–410, 2016.

[4] R. Kumar and P. Goel, "Availability analysis of banking server with warm standby," 2016.

[5] A. K. Aggarwal, V. Singh, and S. Kumar, "Availability analysis and performance optimization of a butter oil production system: a case study," *International Journal of System Assurance Engineering and Management*, vol. 8, no. 1, pp. 538–554, 2017.
[6] M. Ram and K. Nagiya, "Gas turbine power plant performance evaluation under key failures," *Journal of Engineering science and Technology*, vol. 12, no. 7, pp. 1871–1886, 2017.
[7] A. Kumar, D. Garg, and P. Goel, "Mathematical modeling and profit analysis of an edible oil refinery industry," *Airo Int Res J*, vol. 12, pp. 1–8, 2017.
[8] S. Kumari, P. Khurana, and S. Singla, "Behavior and profit analysis of a thresher plant under steady state," *International Journal of System Assurance Engineering and Management*, vol. 13, no. 1, pp. 166–171, 2022.
[9] A. Kumar, D. Garg, P. Goel, and O. Ozer, "Sensitivity analysis of 3: 4:: good system," *International Journal of Advance Research in Science and Engineering*, vol. 7, no. 2, pp. 851–862, 2018.
[10] A. Kumar, P. Goel, and D. Garg, "Behaviour analysis of a bread making system," *International Journal of Statistics and Applied Mathematics*, vol. 3, no. 6, pp. 56–61, 2018.
[11] J. Savolainen and M. Urbani, "Maintenance optimization for a multi-unit system with digital twin simulation," *Journal of Intelligent Manufacturing*, vol. 32, no. 7, pp. 1953–1973, 2021.
[12] A. Sharma, P. Ailawalia *et al.*, "Ram (reliability, availability and maintainability) of threshing machine in agriculture," *Agriculture and Natural Resources*, vol. 55, no. 6, pp. 1057–1061, 2021.
[13] S. D. Shelare, R. Kumar, and P. B. Khope, "Formulation of a mathematical model for quantity of deshelled nut in charoli nut deshelling machine," in *Advances in Metrology and Measurement of Engineering Surfaces.* Springer, 2021, pp. 89–97.
[14] S. Malik and P. Tewari, "Optimization of coal handling system performability for a thermal power plant using pso algorithm," *Grey Systems: Theory and Application*, vol. 10, no. 3, pp. 359–376, 2020.
[15] U. M. Modibbo, M. Arshad, O. Abdalghani, and I. Ali, "Optimization and estimation in system reliability allocation problem," *Reliability Engineering & System Safety*, vol. 212, p. 107620, 2021.
[16] A. K. Malik, S. K. Yadav, and S. R. Yadav, "Optimization techniques, i. k international pub," *Pvt. Ltd., New Delhi*, 2012.
[17] A. Malik, V. Kumar, and A. Malik, "Importance of operations research in higher education," *International Journal of Operations Research and Optimization*, vol. 7, no. 1-2, pp. 35–40, 2016.
[18] Y. S. Raghav, M. Mradula, R. Varshney, U. M. Modibbo, A. A. H. Ahmadini, and I. Ali, "Estimation and optimization for system availability under preventive maintenance," *IEEE Access*, 2022.
[19] M. F. Khan, U. M. Modibbo, N. Ahmad, and I. Ali, "Nonlinear optimization in bi-level selective maintenance allocation problem," *Journal of King Saud University-Science*, vol. 34, no. 4, p. 101933, 2022.