Yugoslav Journal of Operations Research
32 (2022), Number 4, 453–470
DOI: https://doi.org/10.2298/YJOR220115021P

FUZZY OPTIMIZATION MODEL OF TWO PARAMETER WEIBULL DETERIORATING RATE WITH QUADRATIC DEMAND AND VARIABLE HOLDING COST UNDER ALLOWABLE SHORTAGES

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Received: January 2022 / Accepted: July 2022

Abstract: In this paper, a fuzzy inventory model with a Weibull deterioration rate, a quadratic demand rate, and a variable holding cost under permissible shortages has been developed. The deterioration rate is expressed by a two-parameter Weibull distribution. During a shortage, some buyers wait for the actual product, while others do not. This shortfall is considered partially backlogged in this model. Some buyers wait for the actual product during such shortages, but many do not. Therefore, partially backlogged shortages are taken into account in this approach. In a traditional inventory model, all parameters such as purchasing cost, shortage cost, holding cost, etc. are predetermined. However, there will be some variations. As a result, fuzzy factors are more accurate to deal with the real world's problems. This research attempts to cut down the cost in a fuzzy environment by using quadratic demand, shortage, Weibull deterioration rate, and variable holding cost. Costs such as ordering, shortage, and deterioration are addressed as pentagonal fuzzy numbers that are defuzzified using a graded mean representation approach. Finally, sensitivity analysis was carried out to investigate the influence of cost parameters on total inventory cost. A numerical example is used to validate the proposed model in a real-world system.

Keywords: EOQ model, deterioration, Weibull deterioration rate, variable dependent demand, shortages, pentagonal number, graded mean integration method.

MSC: 90B05.

1. INTRODUCTION

The fuzzy set strategy is associated with dealing effectively with uncertainties and imprecise information, and it provides another tool to the decision maker in addition to the basic deterministic and probabilistic mathematical tools used throughout real-world issues. For many real-world problems, the EOQ model is an ideal approach. Mostly, researchers investigated various EOQ models in a fuzzy environment to optimize the objective function. Inventory control policies play a critical role in the manufacturing process by considering many production systems. Various models have been made to achieve a balance between having too much inventory, which results in high holding costs, and having too little inventory, which leads to stock outs and poor customer service. The main objective is to reduce inventory-related average costs over time; reality is not accurate and can only be computed to the extent that fuzzy set theory provides more realistic outcomes for inventory issues. Some fuzzy techniques, on the other hand, are too complex to evaluate, which might lead to incorrect interpretation and, as a result, inaccurate judgments. Zadeh [1] was the first to give various features of fuzzy sets and demonstrate the separation theorem for convex fuzzy sets without necessitating the fuzzy sets to really be disjoint.kaufman et.al [2] introduced fuzzy arithmetic operations, whereas Zimmermann [3] developed the notion of fuzzy set theory and its applications. The traditional economic order quantity (EOQ) model assumes that the manufactured goods are non-perishable and have an indefinite useful life. However, in the current world, most products are at risk of deterioration, dryness, breakage, evaporation, expiration, and depreciation. Health kits, chemicals, medicines, foodstuffs, volatile substances, and so on, degrade rapidly over time. As a result, over a period of time, these goods are no longer able to perform their primary purpose and are quickly discarded. As a result, the significance of deterioration-related loss cannot be avoided. A significant number of researchers have been working on inventory models for things that deteriorate over time. Among them, [4]were the first to create an updated version of the EOQ model for deteriorating objects. Many researchers have contributed significantly in recent years to inventory challenges with variable deterioration and slightly varying demands, which are a crucial component of the inventory system. It might be a constant, random variable with a known probability density, or it could change over time, based on selling price, purchase cost, and customer behavior. Many things, such as mobile phones, televisions, attractive apparel, and so on, have fluctuating demand rates due to various factors. Some items have an increasing demand throughout time. On the other hand, demand for some items has decreased due to the arrival of more attractive commodities

2. LITERATURE REVIEW

One of the practical utilizations of decision science was inventory analysis. In a fuzzy environment, Zadeh [1] was the one who initiated the very first attempt to develop the theory of fuzzy sets, and after that, some researchers started applying fuzzy set theory to inventory management difficulties. When the cost of inventory is a fuzzy number, Vujosevic et al. [5] proposed an EOQ formula based on a trapezoidal fuzzy number. Gen et al. [6] proposed a strategy for addressing an inventory control challenge using input data characterized by triangular fuzzy numbers. Chang et al. [7] presented an issue in inventory with backorders in a fuzzy sense and justified the model by considering the backlogged quantity as a triangular fuzzy number. Lee et al. [8] developed a fuzzy inventory with and without a backorder for fuzzy order quantity.

In a fuzzy environment with backorder Yao et al. [9] developed and modified an inventory model. The authors concentrate on total demand, which is based on a fuzzy set with interval values. The centroid technique is used to estimate demand, which is expressed as triangular fuzzy numbers. Wu et al. [10] establish a solution to the problem of backordered inventory. The overall cost is calculated by fuzzifying the order quantity and shortfall quantity and then using the centroid technique to defuzzify them. Dutta et al. [11] developed a problem involving single inventory and treated demand as a fuzzy random variable to make it more realistic. A triangular number was employed to achieve computational efficiency. The graded mean integration approach is used to defuzzify the values.

Kazemi et al. [12] developed a model containing fuzzy parameters and decision variables as well as backorder. The study employs two kinds of fuzzy numbers, triangular and trapezoidal, to explore an inventory model with backorders in a fuzzy condition. Defuzzification has been accomplished using graded mean integration. Fuzzy environment, Uthayakumar and Valliathal [13] established an economic production model for Weibull deteriorating objects across an indefinite horizon, in which some cost components were represented as triangular fuzzy quantities and the cost function was defuzzified using the signed distance approach. Using two forms of fuzzy numbers, trapezoidal and triangular numbers, Mahata et al. [14] discussed an inventory model for products of imperfect quality and shortfall backordering in fuzzy environments. A defuzzification method, particularly the graded mean integration method, is used to discover the accurate estimate of the profit function for each fuzzy model. The optimal policy for the developed model is then identified. Rajput et al. [15] introduced an economic production quantity model with a finite production rate established for cloudy normalized triangular fuzzy number (CNTFN), [15] developed an economic production quantity model with a trade-credit policy and their classical model presented with a fuzzy demand rate under a cloudy fuzzy environment. Arora et al. [16] develops a traditional economic order quantity model with a fuzzy approach and provide a suitable structure to handle such uncertain parameters, improving the exactness and computational efficiency of the inventory system.

An EOQ model with power demand for perishable goods was formulated by Rajoria et al. [17]. Authors considered two parameters of the Weibull distribution (scale and shape) and backlog rate is assumed to be inversely proportionate to the waiting time for the next renewal. Later, Pal et al. [18] used a fuzzy environment to solve the inventory model to determine the model's optimum solution in various scenarios. To defuzzify the parameters, a total λ-integral value is used with a triangular fuzzy number assumed. An economic order quantity model for perishable items with time and stock dependent demand with allowable shortages under an inflationary environment has been formulated by Rajoria et al. [19], E.Shekharian et al. [20] provide an overview of the fuzzy inventory models based on fuzzy inventory models under different environments and consider many other factors. Garai et al. [21] proposed a fuzzy inventory model based on the uncertainty principle with price dependent demand and time-changing while holding costs. Poswal et.al. [22]investigated ordering quantity model having uncertain demand for medicinal product in healthcare sector and considered expiration date of medicinal product under consideration of effect of price level and partially backordering cost. A fuzzy inventory model was studied by Srinivasan and Vijayan [23]. The authors of this research used the centroid ranking approach to find the best ordering strategy in an inventory model with deteriorating materials in a fuzzy environment. In a fuzzy approach and treating triangular numbers as triangular numbers, Srivastava et al. [24]discussed an optimal inventory management system for perishable goods with linear demand, a variable rate of deterioration, partial backlog, and shortage. The findings were obtained using graded mean integration, signed distance, and centroid methods. Padiyar et al. [25] proposed a framework for such items that have time and pricing constraints to achieve the ideal production run time by applying this fuzzy approach to increases the total turnover of firm Poswal et al. [22] developed a framework for remanufacturing process of sustainable products with trade credit policy under a fuzzy environment.

A model for controlled emissions under fuzzy demand and energy consumption has been developed by Priyan et al. [26] using a sustainable dual-channel vendor-buyer supply chain. The objective is to achieve a balance between cost and emissions as well as a sustainable plan for managing both physical and digital orders. This study discusses inventory models for decaying items with a Weibull deterioration rate, quadratic demand, and variable holding costs in order to fill this research gap. Numerical examples are used for this sensitivity analysis. According to the results of this study, no paper on the relevant topic has been written in a fuzzy environment with two parameters: Weibull deterioration rate and variable holding cost with allowable shortages. Poswal et al. [22] developed a Fuzzy EOQ model for deteriorating products having stock and price dependent demand function under shortages. Authors considered a Fuzzy triangular number to fuzzified costs and signed distance method for defuzzification.

Table 1: Analysis of authors' contribution

Authors	Model	Objective function	Deterioration	Demand function	Shortage
Garai et al. [21]	FEOQ	Profit optimization	NO	Price and time	NO
Srinivasan and Vijayan [23]	FEOQ	Profit optimization	Yes (Constant)	Time dependent	NO
Srivastava et al.[24]	FEOQ	Cost optimization	Yes (Variable)	Linear	Yes (Partially backlogged)
Padiyar et al. [25]	FEOQ	Cost optimization	Yes(Constant)	Price dependent	Yes (Fully backlogged)
Priyan et al. [26]	Carbon emission	Cost optimization	NO	Trapezoidal	NO
Poswal et al. [22]	FEOQ	Cost optimization	Yes(Linear)	Price and Stock	Yes (Fully backlogged)
This paper	FEOQ	Cost optimization	Yes(Weibull)	Quadratic	Yes (Partially backlogged)

On the analysis of authors' contribution Table 1, we found research gap to extend the model by considering two parameters the weibull deterioration rate, to fuzzy model has been developed in this paper. A fuzzy pentagonal number are used to fuzzified the uncertainties like holding costs, deterioration costs, and shortage costs. The total average inventory costs with a Weibull deteriorating rate and quadratic demand with variable holding costs in the fuzzy environment are derived under partially shortages. The fuzzy model is defuzzified by the GMIR method (Graded mean representation method). Some of the parameters in this system may have variable values rather than fixed values, which generally represents the actual situation.

3. DEFINITIONS AND PRELIMINARIES

3.1. Pentagonal Fuzzy number

The membership function μ_A is used to described a pentagonal fuzzy number $A \approxeq (p,\ q,\ r,\ s,\ t)$ as:

$$\mu_{\widetilde{A}} = \begin{cases} L_1(x) = \frac{x-p}{q-p}, & p \leq x \leq q \\ L_2(x) = \frac{x-q}{r-q}, & q \leq x \leq r \\ 1, & x = r \\ R_1(x) = \frac{s-x}{s-r}, & r \leq x \leq s \\ R_2(x) = \frac{t-x}{t-s}, & s \leq x \leq t \\ 0, & otherwise \end{cases}$$

The α–cut of $\widetilde{A} = (p,\, q,\, r,\, s,\, t)$, $0 \leq \alpha \leq 1$ is A_α=$[A_L(\alpha), A_R(\alpha)]$ where

$$A_{L_1}(\alpha) = p + (q-p)\,\alpha = \ {L_1}^{-1}(\alpha)$$

$$A_{L_2}(\alpha) = q + (r-q)\,\alpha = \; {L_2}^{-1}(\alpha)$$
$$A_{R_1}(\alpha) = s - (s-r)\,\alpha = \; {R_1}^{-1}(\alpha)$$
$$A_{R_2}(\alpha) = t - (t-s)\,\alpha = \; {R_2}^{-1}(\alpha)$$

$$L^{-1}(\alpha) = \frac{{L_1}^{-1}(\alpha) + {L_2}^{-1}(\alpha)}{2}$$
$$= \frac{p + (q-p)\,\alpha + q + (r-q)\alpha}{2} = \frac{p + q + (q - p + r - q)\alpha}{2} = \frac{p + q + (r-p)\alpha}{2}$$

$$R^{-1}(\alpha) = \frac{{L_1}^{-1}(\alpha) + {L_2}^{-1}(\alpha)}{2}$$
$$= \frac{s - (s-r)\,\alpha + t - (t-s)\alpha}{2} = \frac{s + t - (s - r + t - s)\alpha}{2} = \frac{s + t - (t-r)\alpha}{2}$$

3.2. Graded Mean Integration (GMI) Representation

If $A \cong (p,\ q,\ r,\ s,\ t)$ is a pentagonal fuzzy number, then $\widetilde{A}$'s GMI representation is described as follows

$$P(\widetilde{A}) = \frac{1}{2}\frac{\int_0^{W_A} h\left[L^{-1}(h) + R^{-1}(h)\right] dh}{\int_0^{W_A} h\ dh}, \quad 0 \le h \le W_A \ \ \& \ \ 0 \le W_A \le 1$$

$$P(\widetilde{A}) = \frac{1}{2}\frac{\int_0^1 h\left[\frac{p+q+(r-p)h}{2} + \frac{s+t-(t-r)h}{2}\right] dh}{\int_0^1 h\ dh} = \frac{p + 3q + 4r + 3s + t}{12}$$

4. ASSUMPTIONS AND NOTATIONS

The following assumptions and notations are used to explain this mathematical model.

4.1. Assumptions

- The quadratic demand rate D(t) is assumed to be $D(t) = u + vt + wt^2$; u, v, w are constants.
- Replenishment takes place in the inventory model.
- Shortages are permitted.
- The distribution of time until the items deteriorate follows a two-parameter model. The Weibull distribution is a type of probability distribution.
- As soon as objects are placed in inventory, they rapidly deteriorate.
  As a result, degradation rate $\theta(t) = \alpha\beta t^{\beta-1}$ where α(scale parameter) $>$ 0,β(shape parameter) > 0.
- Throughout this model, the cost of holding is time based. i.e., $H_c = m + nt$ where $m > 0$, $n > 0$.

4.2. Notations

In the creation of a mathematical model, we employ the following notations:

S_C	Shortage cost per unit time
O_C	Ordering cost/order
D_C	Deterioration cost/orde
S	The maximum amount of inventory
R	The maximum inventory level for each ordering cycle
Q	The order quantity ($Q = S + R$)
$q(t)$	Inventory level at time t.
t_1	Time at which shortage start
T	Total length of each ordering cycle
TIC	Total inventory cost over the period $(0, T)$
m,n	Holding cost parameter
$\widetilde{S_C}$	Fuzzy Shortage cost per unit time
$\widetilde{O_C}$	Fuzzy Ordering cost per order
$\widetilde{D_C}$	Fuzzy Deterioration cost
$Q*$	Fuzzy order quantity
$t1*$	Time at which shortage start in fuzzy environment
$T*$	Total length of each ordering cycle in fuzzy environment
$\widetilde{TIC}$	Fuzzy total inventory cost over the period $(0, T)$
TIC_G	Defuzzification value of $\widetilde{TIC}$

5. MATHEMATICAL FORMULATION

The replenishment of a decaying commodity with shortfalls and variable holding costs is assumed in this research. At $t = 0$, the inventory level reaches its maximum and replenishment begins. Following that, the inventory level decreases during the time range $[0, t]$, finally dropping to zero at $t = t1$. During the period $[t1, T]$ shortage occur at the point $t1$. Finally, the optimum order quantity Q and the overall optimal inventory cost are also noted. The graph below Figure 1 depicts inventory behavior at any given period. Now, till the shortages are permitted at interval $[0, t1]$, The differential equation of Weibull deteriorate goods for quadratic demand is given by

$$\frac{dq_1(t)}{dt} + \alpha\beta t^{\beta-1} q_1(t) = -(u + vt + \mathrm{w}\mathrm{t}^2) \quad 0 \le t \le t_1 \tag{1}$$

The shortfall arises within the interval $[t_1, T]$, hence the differential equation is:

$$\frac{dq_2(t)}{dt} = -(u + vt + \mathrm{w}\mathrm{t}^2); \quad t_1 \le t \le T \tag{2}$$

The boundary conditions are as follows: $t = 0$, $q(0) = R$

$t = t_1$; $q(t_1) = 0$

$t = T$; $q(T) = -S$

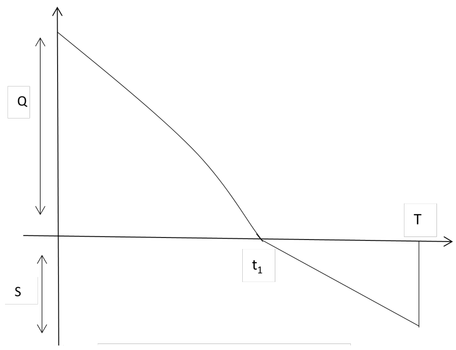

Figure 1: Flow of inventory

Now, by solve the equations (1), we obtain the following:

$$q_1(t) = a(t_1 - t) + \frac{b}{2}(t_1^2 - t^2) + \frac{c}{3}(t_1^3 - t^3) + \frac{a\alpha}{\beta+1}(t_1^{\beta+1} - t^{\beta+1}) + \frac{b\alpha}{\beta+2}(t_1^{\beta+2} - t^{\beta+2}) + \frac{c\alpha}{\beta+3}(t_1^{\beta+3} - t^{\beta+3}) \tag{3}$$

Also, by solve the (2) we obtain,

$$q_2\,(t) = \; u(t_1 - T) - \frac{v}{2}(t_1^2 - T^2) - \frac{w}{3}(t_1^3 - T^3) \tag{4}$$

Now, for each cycle, at t = 0, the maximum allowable inventory level is given by

$$q_1\,(0) = R$$

$$t = 0$$

$$R = q_1\,(0) = ut_1 + \frac{vt_1^2}{2} + \frac{wt_1^3}{3} + \frac{u\alpha}{\beta+1}{t_1}^{\beta+1} + \frac{v\alpha}{\beta+2}{t_1}^{\beta+2} + \frac{w\alpha}{\beta+3}{t_1}^{\beta+3}$$

The maximum level of quadratic demands each cycle is determined by when t = T.

$$T = T$$

$$q_2\,(t) = -S$$

$$S = -u(t_1 - T) - \frac{v}{2}(t_1^2 - T^2) - \frac{w}{3}(t_1^3 - T^3)$$

The order quantity Q = R + S for each cycle is now determined by

$$Q\;ut_1 + \frac{vt_1^2}{2} + \frac{wt_1^3}{3} + \frac{u\alpha}{\beta+1}t_1^{\beta+1} + \frac{v\alpha}{\beta+2}t_1^{\beta+1} + \frac{w\alpha}{\beta+3}t_1^{\beta+1} - u(t_1 - T) - \frac{bw}{2}(t_1^2 - T^2) - \frac{c}{3}(t_1^3 - T^3) \tag{5}$$

6. COST ASSOCIATED

The following costs are included in the developed model:

6.1. Holding cost

The total of processing costs, compensation costs, insurance coverage, and taxation, and also obsolescence and deformation cost, is assessed as the holding cost (also defined as the carrying inventory cost) that an organization undertakes for managing and storing its whole unsold stock.

$$\begin{aligned}\text{Holding Cost} &= \int_0^{t_1} (m+nt)\, q_1(t)\, dt \\ &= \frac{um}{2}t_1^2 + (\frac{mv}{3}+\frac{un}{6})t_1^3 + (\frac{mw}{4}+\frac{vn}{8})t_1^4 + (\frac{um\alpha}{\beta+2})t_1^{\beta+2} + \\ &\alpha(\frac{v}{\beta+3}+\frac{u}{2(\beta+3)})t_1^{\beta+3} + \alpha(\frac{mw}{\beta+4}+\frac{vn}{\beta+4})t_1^{\beta+4} + \frac{w\alpha}{2(\beta+5)}t_1^{\beta+5}\end{aligned} \tag{6}$$

6.2. Shortage cost

Shortage costs are the expenses experienced by a company when it does not have enough inventory on hand. This is really a crucial phenomenon while assessing how much inventory to keep on hand, specifically for businesses that compete on customer service and support.

$$\begin{aligned}\text{Shortage cost} &= -S_C \int_{t_1}^{T} q_2(t) \\ &= -S_C \left\{ u(t_1 T - \frac{T^2}{2} - \frac{t_1^2}{2}) + v(\frac{{t_1}^2 T}{2} - \frac{T^3}{6} - \frac{t_1^3}{3}) + w(\frac{{t_1}^3 T}{3} - \frac{T^4}{12} - \frac{t_1^4}{4}) \right\}\end{aligned} \tag{7}$$

6.3. Ordering cost

The fees spent on making and execute an order to a seller are known as ordering costs. These expenses are used into the assessment of the economic order quantity for an inventory object.

$$\text{Ordering cost} = O_c \tag{8}$$

6.4. Deteriorating cost

The expense of deterioration is the cost of the system's deteriorating goods. Since 'D_c' signifies the per-unit cost of degradation. Deterioration costs can be calculated as follows:

$$\begin{aligned}\text{Deterioration cost} &= D_C[R - \int_0^{t_1} D(t)\, dt] \\ &= D_C(\frac{u\alpha}{\beta+1}{t_1}^{\beta+1} + \frac{v\alpha}{\beta+2}{t_1}^{\beta+2} + \frac{w\alpha}{\beta+3}{t_1}^{\beta+3})\end{aligned} \tag{9}$$

6.5. Total Cost

As a result, the total cost per unit time per unit cycle is calculated as follows:

$$
\begin{aligned}
TSC(t_1,T) &= \frac{1}{T}(\text{Holding cost + shortage cost + ordering cost + deteriorating cost})\\
&= \frac{1}{T}(\frac{um}{2}t_1^2 + \left(\frac{mv}{3}+\frac{un}{6}\right)t_1^3 + \left(\frac{mw}{4}+\frac{vn}{8}\right)t_1^4 + \left(\frac{um\alpha}{\beta+2}t_1^{\beta+2}\right) + \\
&\alpha\left(\frac{v}{\beta+3}+\frac{u}{2(\beta+3)}\right){t_1}^{\beta+3} + \alpha\left(\frac{mw}{\beta+4}+\frac{vn}{2(\beta+4)}\right){t_1}^{\beta+4} + \frac{w\alpha}{2(\beta+5)}{t_1}^{\beta+5}\\
&- S_C\left(u\left(t_1T-\frac{T^2}{2}-\frac{t_1^2}{2}\right)+v\left(\frac{{t_1}^2T}{2}-\frac{T^3}{6}-\frac{t_1^3}{3}\right)+w\left(\frac{{t_1}^3T}{3}-\frac{T^4}{12}-\frac{t_1^4}{4}\right)\right)\\
&+ O_C + D_C\left(\frac{u\alpha}{\beta+1}{t_1}^{\beta+1}+\frac{v\alpha}{\beta+2}{t_1}^{\beta+2}+\frac{w\alpha}{\beta+3}{t_1}^{\beta+3}\right))
\end{aligned} \tag{10}
$$

Our purpose is to minimize the average total system cost (TSC). As a result, we've found the minimal solution. The requirements for a satisfactory minimal average total cost are as follows:

$$\frac{\mathbf{d(TSC)}}{\mathbf{dT}} = 0, \frac{\mathbf{d(TSC)}}{\mathbf{dt_1}} = \mathbf{0} \tag{11}$$

7. FUZZY INVENTORY MODEL

Since it is difficult to describe all of the defined parameters precisely due to the uncertainty in the surroundings, therefore we suppose that some of them such as S_C, O_C, D_C will modify within certain limits.

Let $\widetilde{S_C}=(\widetilde{S_{C1}},\widetilde{S_{C2}},\widetilde{S_{C3}},\ \widetilde{S_{C4}},\ \widetilde{S_{C5}})$,

$\widetilde{O_C}=(\widetilde{O_{C1}},\widetilde{O_{C2}},\widetilde{O_{C3}},\widetilde{O_{C4}},\widetilde{O_{C5}})$

$\widetilde{D_C}=(\widetilde{D_{C1}},\ \widetilde{D_{C2}},\ \widetilde{D_{C3}},\widetilde{D_{C4}},\widetilde{D_{C5}})$ be the pentagonal fuzzy number

In a fuzzy sense, the overall cost of the system per unit time is calculated by

$$
\begin{aligned}
\widetilde{TSC}\,(t_1,T) &= \frac{1}{T}(\frac{um}{2}t_1^2 + \left(\frac{mv}{3}+\frac{un}{6}\right)t_1^3 + \left(\frac{mw}{4}+\frac{vn}{8}\right)t_1^4 + \left(\frac{um\alpha}{\beta+2}t_1^{\beta+2}\right)\\
&+ \alpha\left(\frac{v}{\beta+3}+\frac{u}{2(\beta+3)}\right){t_1}^{\beta+3}\\
&+ \alpha\left(\frac{mw}{\beta+4}+\frac{vn}{2(\beta+4)}\right){t_1}^{\beta+4} + \frac{w\alpha}{2(\beta+5)}{t_1}^{\beta+5}\\
&- \widetilde{S_C}\left(u\left(t_1T-\frac{T^2}{2}-\frac{t_1^2}{2}\right)+v\left(\frac{{t_1}^2T}{2}-\frac{T^3}{6}-\frac{t_1^3}{3}\right)+w\left(\frac{{t_1}^3T}{3}-\frac{T^4}{12}-\frac{t_1^4}{4}\right)\right)\\
&+ \widetilde{O_C} + \widetilde{D_C}\left(\frac{u\alpha}{\beta+1}{t_1}^{\beta+1}+\frac{v\alpha}{\beta+2}{t_1}^{\beta+2}+\frac{w\alpha}{\beta+3}{t_1}^{\beta+3}\right))
\end{aligned} \tag{12}
$$

Using a graded mean representation methodology, we defuzzify the fuzzy total cost

$$TSCG(t_1,T)=\frac{1}{12}(TICG1(t_1,T),TICG2(t_1,T),\\ TICG3(t_1,T),TICG4(t_1,T),TICG5(t_1,T)) \tag{13}$$

Where

$$\begin{aligned}TSCG_1(t_1,T)=&\frac{1}{T}(\frac{um}{2}t_1^2+\left(\frac{mv}{3}+\frac{un}{6}\right)t_1^3+\left(\frac{mw}{4}+\frac{vn}{8}\right)t_1^4+\left(\frac{um\alpha}{\beta+2}t_1^{\beta+2}\right)\\ &+\alpha\left(\frac{v}{\beta+3}+\frac{u}{2\left(\beta+3\right)}\right){t_1}^{\beta+3}\\ &+\alpha\left(\frac{mw}{\beta+4}+\frac{vn}{2\left(\beta+4\right)}\right){t_1}^{\beta+4}+\frac{w\alpha}{2\left(\beta+5\right)}{t_1}^{\beta+5}\\ &-S_{C1}\left(u\left(t_1T-\frac{T^2}{2}-\frac{t_1^2}{2}\right)+v\left(\frac{{t_1}^2T}{2}-\frac{T^3}{6}-\frac{t_1^3}{3}\right)+w\left(\frac{{t_1}^3T}{3}-\frac{T^4}{12}-\frac{t_1^4}{4}\right)\right)\\ &+O_{C1}+D_{C1}\left(\frac{u\alpha}{\beta+1}{t_1}^{\beta+1}+\frac{v\alpha}{\beta+2}{t_1}^{\beta+2}+\frac{w\alpha}{\beta+3}{t_1}^{\beta+3}\right))\end{aligned}$$

$$\begin{aligned}TSCG_2(t_1,T)=&\frac{1}{T}(\frac{um}{2}t_1^2+\left(\frac{mv}{3}+\frac{un}{6}\right)t_1^3+\left(\frac{mw}{4}+\frac{vn}{8}\right)t_1^4+\left(\frac{um\alpha}{\beta+2}t_1^{\beta+2}\right)\\ &+\alpha\left(\frac{v}{\beta+3}+\frac{u}{2\left(\beta+3\right)}\right){t_1}^{\beta+3}\\ &+\alpha\left(\frac{mw}{\beta+4}+\frac{vn}{2\left(\beta+4\right)}\right){t_1}^{\beta+4}+\frac{w\alpha}{2\left(\beta+5\right)}{t_1}^{\beta+5}\\ &-S_{C2}\left(u\left(t_1T-\frac{T^2}{2}-\frac{t_1^2}{2}\right)+v\left(\frac{{t_1}^2T}{2}-\frac{T^3}{6}-\frac{t_1^3}{3}\right)+w\left(\frac{{t_1}^3T}{3}-\frac{T^4}{12}-\frac{t_1^4}{4}\right)\right)\\ &+O_{C2}+D_{C2}\left(\frac{u\alpha}{\beta+1}{t_1}^{\beta+1}+\frac{v\alpha}{\beta+2}{t_1}^{\beta+2}+\frac{w\alpha}{\beta+3}{t_1}^{\beta+3}\right))\end{aligned}$$

$$\begin{aligned}TSCG_3(t_1,T)=&\frac{1}{T}(\frac{um}{2}t_1^2+\left(\frac{mv}{3}+\frac{un}{6}\right)t_1^3+\left(\frac{mw}{4}+\frac{vn}{8}\right)t_1^4+\left(\frac{um\alpha}{\beta+2}t_1^{\beta+2}\right)\\ &+\alpha\left(\frac{v}{\beta+3}+\frac{u}{2\left(\beta+3\right)}\right){t_1}^{\beta+3}\\ &+\alpha\left(\frac{mw}{\beta+4}+\frac{vn}{2\left(\beta+4\right)}\right){t_1}^{\beta+4}+\frac{w\alpha}{2\left(\beta+5\right)}{t_1}^{\beta+5}\\ &-S_{C3}\left(u\left(t_1T-\frac{T^2}{2}-\frac{t_1^2}{2}\right)+v\left(\frac{{t_1}^2T}{2}-\frac{T^3}{6}-\frac{t_1^3}{3}\right)+w\left(\frac{{t_1}^3T}{3}-\frac{T^4}{12}-\frac{t_1^4}{4}\right)\right)\\ &+O_{C3}+D_{C3}\left(\frac{u\alpha}{\beta+1}{t_1}^{\beta+1}+\frac{v\alpha}{\beta+2}{t_1}^{\beta+2}+\frac{w\alpha}{\beta+3}{t_1}^{\beta+3}\right))\end{aligned}$$

$$\begin{aligned}
TSCG_4(t_1,T) = & \frac{1}{T}(\frac{um}{2}t_1^2 + \left(\frac{mv}{3}+\frac{un}{6}\right)t_1^3 + \left(\frac{mw}{4}+\frac{vn}{8}\right)t_1^4 + \left(\frac{um\alpha}{\beta+2}t_1^{\beta+2}\right) \\
& + \alpha\left(\frac{v}{\beta+3}+\frac{u}{2\left(\beta+3\right)}\right){t_1}^{\beta+3} \\
& + \alpha\left(\frac{mw}{\beta+4}+\frac{vn}{2\left(\beta+4\right)}\right){t_1}^{\beta+4} + \frac{w\alpha}{2\left(\beta+5\right)}{t_1}^{\beta+5} \\
& - S_{C4}\left(u\left(t_1T-\frac{T^2}{2}-\frac{t_1^2}{2}\right)+v\left(\frac{{t_1}^2T}{2}-\frac{T^3}{6}-\frac{t_1^3}{3}\right)+w\left(\frac{{t_1}^3T}{3}-\frac{T^4}{12}-\frac{t_1^4}{4}\right)\right) \\
& + O_{C4} + D_{C4}\left(\frac{u\alpha}{\beta+1}{t_1}^{\beta+1}+\frac{v\alpha}{\beta+2}{t_1}^{\beta+2}+\frac{w\alpha}{\beta+3}{t_1}^{\beta+3}\right))
\end{aligned}$$

$$\begin{aligned}
TSCG_5(t_1,T) = & \frac{1}{T}(\frac{um}{2}t_1^2 + \left(\frac{mv}{3}+\frac{un}{6}\right)t_1^3 + \left(\frac{mw}{4}+\frac{vn}{8}\right)t_1^4 + \left(\frac{um\alpha}{\beta+2}t_1^{\beta+2}\right) \\
& + \alpha\left(\frac{v}{\beta+3}+\frac{u}{2\left(\beta+3\right)}\right){t_1}^{\beta+3} \\
& + \alpha\left(\frac{mw}{\beta+4}+\frac{vn}{2\left(\beta+4\right)}\right){t_1}^{\beta+4} + \frac{w\alpha}{2\left(\beta+5\right)}{t_1}^{\beta+5} \\
& - S_{C5}\left(u\left(t_1T-\frac{T^2}{2}-\frac{t_1^2}{2}\right)+v\left(\frac{{t_1}^2T}{2}-\frac{T^3}{6}-\frac{t_1^3}{3}\right)+w\left(\frac{{t_1}^3T}{3}-\frac{T^4}{12}-\frac{t_1^4}{4}\right)\right) \\
& + O_{C5} + D_{C5}\left(\frac{u\alpha}{\beta+1}{t_1}^{\beta+1}+\frac{v\alpha}{\beta+2}{t_1}^{\beta+2}+\frac{w\alpha}{\beta+3}{t_1}^{\beta+3}\right)
\end{aligned}$$

Therefore have

$$\begin{aligned}
& TSCG(t_1,T) = \frac{1}{12}(TSCG_1(t_1,T) + 3TSCG_2(t_1,T) \\
& + 4TSCG_3(t_1,T) + 3TSCG_4(t_1,T) + TSCG_5(t_1,T))
\end{aligned} \tag{14}$$

8. OPTIMALITY CONDITION

To minimize total cost function $TSC_G(t_1,T)$ per unit time, the optimal value of t_1 and T may be achieved by solving the following equations.

$$\frac{\partial TSC_G(t_1,T)}{\partial t_1} = 0 \;\; and \;\; \frac{\partial TSC_G(t_1,T)}{\partial T} = 0 \tag{15}$$

Also,

$$\frac{\partial^2 TSC_G(t_1,T)}{\partial {t_1}^2}\frac{\partial^2 TSC_G(t_1,T)}{\partial T^2} - \left(\frac{\partial^2 TSC_G(t_1,T)}{\partial t_1 \partial T}\right) > 0 \tag{16}$$

9. METHOD FOR FINDING THE FUZZY TOTAL COST

Step 1: For the specified crisp values, we first compute total cost (TSC) in the crisp environment, as shown in eqn. (11).

Step 2: Using fuzzy arithmetic operations on cost and demand, and pentagonal fuzzy numbers, calculate the fuzzy total cost (provided in eqn. (15), (16).

Step 3: Using a graded mean representation technique, defuzzify $\widetilde{TSC}$ and determine T^* and $t1*$

10. EMPIRICAL VERIFICATION WITH NUMERICAL EXAMPLE

Now, by using suitable numerical example to show how the model have performed and validate. the inventory system's parameters are consider for seasonal products in which measure and describe the model having two parameter Weibull Deteriorating rate with Quadratic Demand and Variable Holding Cost under allowable Shortages. Use the Mathematica and data from previous study as a source to understand the phenomenon.

10.1. Under crisp environment

The system's parameters are as follows to measure and illustrate the model: Sc = \$6/unit time, OC = \$100 per order, DC = \$12 per order, u = 18, v = 14, w = 12, α= 0.05, m = 0.05, n = 20, β=2.

The use of optimality Criteria defined in section 9, we obtain optimal cycle time and length of ordering cycle as follow.

Consequently, Minimum total inventory cost, TSC= \$132.272 per order, the behavior of the unit time profit function is convex with respect to critical time t1=0.59671 per unit time and optimal length of ordering cycle is T = 1.15175 unit time, which can be easily seen in Figure 2.

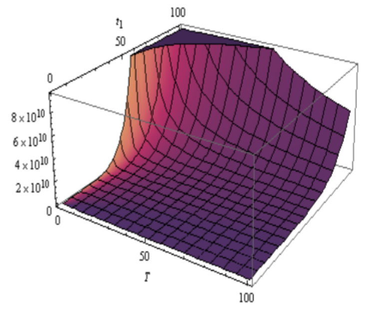

Figure 2: Behavior of TSC with respect to cycle time and ordering cycle under crips

10.2. In fuzzy environment

The system's parameters are as follows to measure and illustrate the model $\widetilde{S_c}$=($\widetilde{S_{c1}}$,$\widetilde{S_{c2}}$, $\widetilde{S_{c3}}$, $\widetilde{S_{c4}}$, $\widetilde{S_{c5}}$) = (1,3,6,9,12), $\widetilde{O_c}$=($\widetilde{O_{c1}}, \widetilde{O_{c2}}, \widetilde{O_{c3}}, \widetilde{O_{c4}}, \widetilde{O_{c5}}$) = (90,95,100,105,110), $\widetilde{D_c}$=($\widetilde{D_{c1}}$, $\widetilde{D_{c2}}$, $\widetilde{D_{c3}}, \widetilde{D_{c4}}, \widetilde{D_{c5}}$) = (8,10,12,14,16)

The use of optimality Criteria in section 9, use the graded mean integration method we obtain Total inventory cost in uncertain environment $\widetilde{TSC(t_1, T)}$ = \$96.4161 per order, the behavior of the unit time profit function is convex with respect to critical time $t1*$=1.18722 per unit time and optimal length of ordering cycle is $T*$=1.46202 unit time. which can be easily seen in Figure 3.

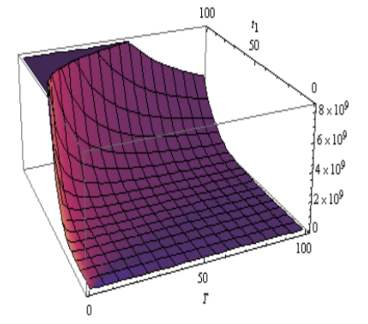

Figure 3: Behavior of $\widetilde{TSC}$ with respect to cycle time and ordering cycle under crips

11. COMPARATIVE STUDY

An evaluation of inventory model, Items having two parameter Weibull Deteriorating rate with Quadratic Demand and Variable Holding Cost under allowable Shortages has been done in this research. we observe that the total cost in fuzzy is more significant than the crisp. The result shows the following optimal values of total system cost is decrease from crisp model (\$132.271per order) to fuzzy model (\$96.4161per order).

12. SENSITIVITY ANALYSIS

The purpose of this section is to describe the sensitivity of the model in presented Scenario and discuss the effect on total system cost by changing the percentage of the important parameters.

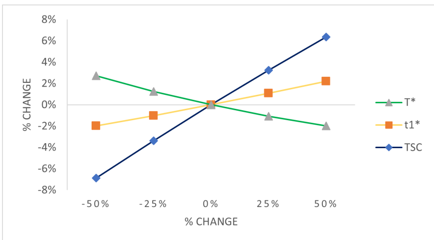

Figure 4: Effect of Percentage changes in initial demand parameter u

From the Table 2 following examination obtained

- Total system cost (TSC) is highly sensitive to change with the initial demand parameter u, the percentage increases and decreases in demand parameter by 25%, 50%, the system cost will increase and decreases, but time $t1*$ at which shortages start will increases and decreases, and vice versa. But

Table 2: Effect of shape parameters of demand and deterioration on system

Parameters	Percentage changes in parameters	Changes in total cost (TSC^*)	$t_1{}^*$	$T*$
u	-50%	89.7892	1.24515	1.53100
	-25%	93.1713	1.21492	1.49548
	0%	96.4161	1.18722	1.46202
	+25%	99.5351	1.16169	1.43047
	+50%	102.538	1.13805	1.40070
v	-50%	91.0746	1.26191	1.55161
	-25%	93.8555	1.22146	1.50369
	0%	96.4161	1.18722	1.46202
	+25%	98.7348	1.15762	1.42524
	+50%	101.020	1.13164	1.39240
w	-50%	93.4089	1.2172	1.53285
	-25%	94.9906	1.20126	1.49432
	0%	96.4161	1.1872	1.46202
	+25%	97.7198	1.17456	1.43421
	+50%	98.9250	1.16299	1.40980
α	-50%	90.8513	1.30831	1.54732
	-25%	93.8561	1.24115	1.49924
	0%	96.4161	1.18722	1.46202
	+25%	98.6486	1.14220	1.43194
	+50%	100.629	1.10384	1.40692
β	-50%	97.8172	1.24334	1.51085
	-25%	96.9839	1.21204	1.48345
	0%	96.4161	1.18722	1.46202
	+25%	96.003	1.16716	1.44486
	+50%	95.6879	1.15072	1.43088

Total length of ordering cycle time will reduce on increasing the demand parameter, shown in Figure 5.

- The percentage of demand parameter v and w increases and decreases by 25%, 50%, system cost and increases and decreases, but total ordering cycle time will have reversal phenomenon while time at which shortages start will slightly effected by these parameters shown in Figure 5. the model is influenced by demand, graphically shown in Figure 5 to Figure 6 respectively.
- The percentage of deterioration parameter α increases and decreases by 25%, 50%, system cost will also increase and decreases, But ordering cycle time

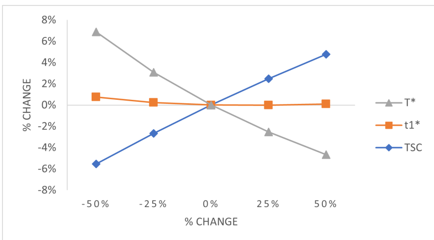

Figure 5: Effect of Percentage changes in demand shape parameter v

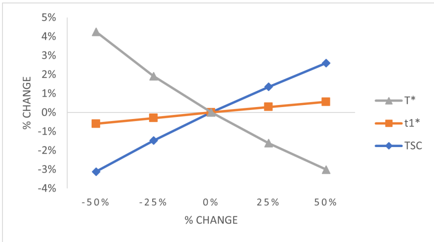

Figure 6: Effect of Percentage changes in demand shape parameter w

and time at which shortages start will decreases and increases. Shown in Figure 7.

- The percentage of deterioration parameter β increases and decreases by 25%, 50%, system cost, ordering cycle time and time at which shortages start will decreases and increases. Shown in Figure 8. We can say β is highly sensitive.

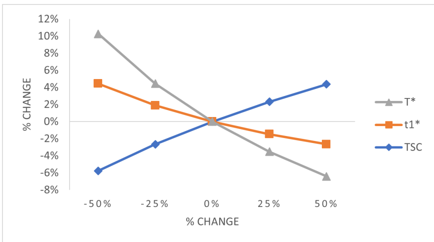

Figure 7: Effect of Percentage changes in deterioration parameter α

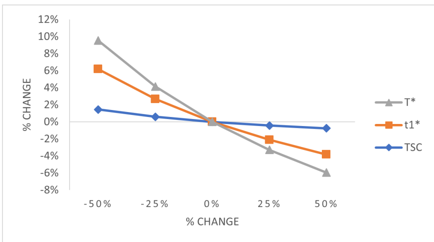

Figure 8: Effect of Percentage changes in deterioration parameter β

13. CONCLUSION

This research has been done for inventory models with quadratic demand, changing deterioration rates, and varying holding costs over time. This model has been demonstrated by the numerical and graphical analysis. The acquire results specify the stability of the model. By examining the presented model, it was discovered that the minimal overall inventory system cost in a fuzzy environment is lower than in a crisp environment and observed that minimum total inventory system cost under fuzzy environment has been reducing compared to the crisp environment.

The provided model is very helpful in case of time dependent demand and holding cost under uncertain scenario. In the present situation, with the uncertainties in the various cost involve, the fuzziness occurs in the inventory management, thus for the future scope, there is wide applicability of this model of the inventory in numerous domains as it helps in the reduction of the total cost of the inventory system.

Funding. This research received no external funding.

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