Yugoslav Journal of Operations Research
32 (2022), Number 4, 515–528
DOI: https://doi.org/10.2298/YJOR220318025S

FLEXIBLE INVENTORY SYSTEM OF IMPERFECT PRODUCTION UNDER DETERIORATION AND INFLATION

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Received: March 2022 / Accepted: July 2022

Abstract: This study emphasizes the development of a flexible inventory system considering rework requirements on imperfect and defective items. This work has considered defective items could be sold at a lower price in the market as compared to the perfect items. The developed model has considered Weibull deterioration and inflation to balance the same amount in the future due to its potential earning capacity. And demand's function depends on price as well as inventory level because a large pile of goods and their price strategy attracts more customers to generate higher demand. The work also supports managerial decision-making by focusing on the volume flexibility system for smooth production runs. The mathematical formulation of the developed inventory system tries to optimize the inventory cost function under a realistic scenario. A solution procedure has been illustrated and assisted with a numerical example. Later, a validation test is also performed to check the robustness of the proposed mathematical model. The findings of the study will support policymakers, strategists, and firms to implement flexible inventory systems under realistic conditions.

Keywords: Weibull deterioration rate, stock and price-dependent demand rate, flexible production, inflation.

MSC: 90B05, 90B06.

1. INTRODUCTION

Inventory is overall contemplated an obligatory wrongful due to the absence of accompany in the mechanism to follow the rules as system responds in the production to holding inventory indispensable. Although hold a high level of inventory could be expensive due to their deteriorating nature in course of time. With the storing facility some of food products or pharmaceuticals in which deterioration depend upon the atmospheric conditions. Due to these factors, deterioration function depends upon the parameters related to these conditions i.e., Weibull distributed function is more realistic in the real situations. Al-Khedhairi and Tadj [1] investigated continuous-review and periodic-review policies to control the production rate for Weibull deteriorated items. Tripathy and Mishra [2] considered an inventory framework for Weibull deteriorated items in two different cases i.e., with and without shortage. Deterioration is the natural phenomenon of perishable items kept as stock. This Physical decay in items depends upon the nature of the products and the atmospheric conditions. For some perishable items this decay is too less as well as high sometimes. In the regard of perishable products such as foods, pharmaceuticals, fruits and blood etc. deterioration problem is big concern area for business managers [3]. Many researchers have developed inventory systems considering different deterioration figure for instance, sustained type or measure turn on outcomes ([4], [5]). There are the cases in which deterioration function depends upon the parameters related to machine breakdown related conditions [6].

For instancein pharmaceuticals, dry batteries, and food products; deterioration rate increases as time passes i.e., more items remain unused plus the rate of deterioration of the item [7]. They developed manufacturing model with constant demand and variation in production level for Weibull deteriorated products. Therefore, in such real situations deterioration rate can be better expressed by Weibull distribution function [8]. Many authors have investigated the inventory system using Weibull distributions to represents real decay in the items. Profit model for Weibull-distributed items was considered by Sharma et al. [9]. It is essential to maintain the stock during production and keep up the ideal and low-valued items. A portion of the blemished quality items can be sold at a marked down selling cost without renovate them. Phenomenon of volume flexibility relaxed the manufacturing division to adjust production upwards or downwards. Rosenblatt and Lee [10] developed a profitable mass produce sum replica with defective standard articles .A huge study is being done on EPQ/EOQ models to moderate the quality of items by several researchers. Khouja and Mehrez [11] developed an inventory model in which production rate is considered to be variable. Including other researchers,

Salameh and Jaber [12] developed EPQ formulae based model which represented for flawed or inadequate feature items. Sana and Chaudhari [3] developed a flexibility-oriented manufacture replica which is to be accounted for declined units with merchandise-displayed demand. Cardenas-Barron [13] explained production with forethought and temporarily out of stock to firmly, the production size and unavailability quantity, where all flawed articles assumed to altered in an exact revolution. Singh et al. [14] advocated flexibility model under imperfect production process with shortages. Wee et al. [15] explored mathematical model to compute the profit for flawed items with shortage and screening control under Reward Theorem. Malik et al. [16], [17] and Mathur et al. [18] acclaimed that analysis of time varying parameters used in inventory model with different parameters.

Pal et al. [19] evaluated a mathematical concept for distributor to pick up an ideal objects, or by directed a protection process where the production firm tried to gathered or fixed somewhat blemished or faulted units. Recently, Chen [20] looked over the mass production-detailer's strategy to improve flawed production structure. Rout et al. [8] framed a rework policy for final screening of two types of error items. Although, shortages were fully backlogged during the cycle. This circumstance pushes the makers to diminish selling cost with the goal that request of that thing may increment dubious factors. Likewise numerous clients like to purchase from different dealers when stock-out happens. Remembering every one of these realities, a stock model is created for blemished things with diminishing selling value, holding up time subordinate multiplying and selling value subordinate time shifting interest subject to consistent crumbling rate receiving the systems from unsure parameters.

This present circumstance pushes the producers to diminish selling cost so that interest of that thing might increment. Likewise numerous clients like to purchase from different dealers when stock-out happens. Remembering this multitude of realities, a stock model is created for flawed things with diminishing selling cost, holding up time subordinate accumulating and selling cost subordinate time shifting interest subject to consistent crumbling rate embracing the strategies from unsure programming, the above model is adjusted to a questionable model with every one of the expense boundaries as dubious factors.

From last few decades, many researchers and marketing practitioners analysed that if the demand is stock- dependent as well as the function of selling price of the product then it would be a big and important factor to raise the demand. Therefore, price and stock sensitive dependent demand is very realistic in behaviour now-a-days. Datta and Paul [21] focused on cost and stock level-subordinate interest rate and proposed stock model for that condition. Pal et al. [22] considered the issue of deciding the inventory (lot) size of a decaying product with the affecting factors i.e. Stock-dependent, selling cost of a unit and including electronic and print media with thought of deficiencies in terms of shortage of inventory. You and Hsieh [23] analysed multi-period models with stock-ward and cost touchy interest rate. Chang et al. [24] tried to get ideal refill strategies for non-momentary decaying products for cost and stock-dependent kind of demand. Under rework process, two EPQ models for manufactured fresh and defective items considered by Tai [25]. Vashisth et al. ([26], [17]) examined the inventory model with different demand scenario for the non-instantaneous items. Demand rate with stock and time, diminishing with a selling cost of the item to maximize the total profit studied by Singh [27].

Every assembling framework wants to create amazing quality things. Be that as it may, because of genuine issues (work issues, machine breakdown, and so on), a specific level of items are of flawed quality. This kind of study has been considered with inflation by Sarkar and Moon [28]. An effective ordering policy had studied in the reference of deteriorating items under inflation effect [29]. Recently, one inventory based article for decaying products of fix demand and inflation under partial backlogging had proposed with permitted delay in payment to settle the account for Weakening starts following the passage of stock into the distribution centre. Deferred instalments is permissible and level of decay is fixed. Halfway multiplied deficiency is additionally considered and the reason for existing is to limit all out cost of the stock framework in a limited arranging skyline [30]. One of the most attempts to build an inventory model with different realistic scenario is that of Malik and Harish [31].

In the concerned study, a production model for perfect as well as imperfect production with price-stock dependent demand and volume flexibility has been explored. Also, the whole inventory system is considered with inflationary environment. We give the essential presumptions for the developed mathematical model and the notations utilized all through this paper in the section 2. In Section 3, engaged with the numerical part of the model. While section 3 track down the essential and adequate circumstances for an ideal arrangement and propose a calculation for finding the optimize cost and production time. Numerical illustrated in section 4 and studied parameter effect sensitivity analysis on production run time and total inventory cost in the section 5. Conclusion is provided in the last section.

2. ASSUMPTION AND NOTATION

To build the numerical model, the accompanying assumptions and notations are being made:

2.1. Assumptions

1. The model consists of single unit (product) over an infinite planning horizon.
2. Perfect and imperfect items are considered in same cycle length.
3. The function for demand $D(t)$ is given by $D(t) = a - p + bI(t)$, where $a > 0, b > 0$ and $p > 0$
4. Deteriorating item deteriorates with Weibull distribution of time, i.e.,$\theta(t) = \alpha\beta t^{\beta-1}$, where $0 < \alpha < 1, t \geq 0$ and $\beta > 0$. Generally,α is called the scale parameter and β is the shape parameter.

A	Set-up cost
h	Holding cost of perfect items
h'	holding cost of imperfect items
c_d	Deteriorating cost per item
R	Percentage of perfect quality items
r	Inflation rate
p	Selling price for perfect(errorless) unit
q	Selling price for imperfect (errored) unit
I(t)	Inventory stored at any time 't'
μ	Time when the production stops
λ	Time when inventory level gets zero
T	Total cycle length

5. The unit production cost $\kappa(P) = N + \frac{G}{P} + \nu P$ where N,G and ν the raw material cost, energy and labour cost, tool and die cost and all are non-negative real numbers respectively. N is the material cost and $\dfrac{G}{P}$ is cost component which is dependent on P. is a per unit cost component includes tool cost and rework cost.
6. The inflation rate is considered.
7. The shortages of the items have been considered and partially backlogged.

2.2. Notations

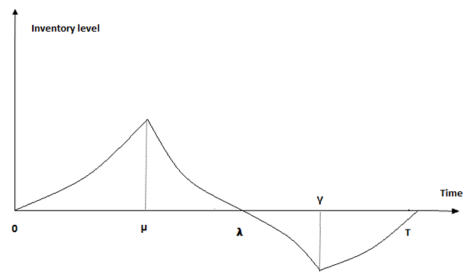

Figure 1: Graph representation of $I(t)$

3. MATHEMATICAL FORMULATION

We have developed the volume flexible inventory system for perfect and imperfect under inflation.Along one complete cycle of replenishment, four phases are included. In the beginning, there was no stock at $t = 0$ and afterward production begins.During the process of production, stock starts to stack up for perfect and imperfect items constantly in the wake of fulfilling demand and decay in terms of

deterioration. Although, manufacturing of products is halted at time $t = \mu$.The accrued stockpile is adequate to represent for demand whereas deterioration is also in process over the interval $[\mu, \lambda]$. After time λ, at time $t = \gamma$, shortage starts partially. Later, at time $t = \lambda$, production starts to consume the unsatisfied demand and the process closes along with no stock at $t = T$.
3.1 In this case, during the production procedure we will discuss about the perfect items.
3.2 This section will covered mathematical analysis of imperfect items.

3.1. Model Formulation for perfect items

Differential equations of $I(t)$ with respect to time 't' described as below:

$$I_1'(t) = -\alpha\beta t^{\beta-1}I(t) + RP - ((a-p) + bI(t); 0 \leq t \leq \mu \tag{1}$$

$$I_2'(t) = -\alpha\beta t^{\beta-1}I(t) - ((a-p) + bI(t))\,; \mu \leq t \leq \lambda \tag{2}$$

$$I_3'(t) = -\delta(a-p); \lambda \leq t \leq \gamma \tag{3}$$

$$I_4'(t) = RP - (a-p); \gamma \leq t \leq T \tag{4}$$

where boundary constraints are $I_1(0) = 0, I_2(\lambda) = 0$ and $I_4(T) = 0$.
Solving above differential equations $(1) - (4)$, we get,

$$I_1(t) = (RP - (a-p))\left(t + \frac{bt^2}{2} + \frac{\alpha t^{\beta+1}}{\beta+1}\right)e^{-\alpha t^{\beta} - bt} \tag{5}$$

$$I_2(t) = -(a-p)\left((t-\lambda) + \frac{b(t^2-\lambda^2)}{2} + \frac{\alpha(t^{\beta+1} - \lambda^{\beta+1})}{\beta+1}\right)e^{-\alpha t^{\beta} - bt} \tag{6}$$

$$I_3(t) = \delta(a-p)(\lambda - t) \tag{7}$$

$$I_4(t) = (RP - (a-p))\,(t - T) \tag{8}$$

Since $I_1(t) = I_2(t)$ at $t = \mu$, we get

$$\mu = -\frac{1}{b+\alpha}\left(1 - \sqrt{1 + \frac{2(a-p)}{RP}\left(\lambda + \frac{b\lambda^2}{2} + \frac{\alpha\lambda^{\beta+1}}{\beta+1}\right)(b+\alpha)}\right) \tag{9}$$

and at $t = \gamma$, $I_3(t) = I_4(t)$ we get,

$$T = \gamma + \frac{(\gamma - \lambda)\delta(a-p)}{RP - (a-p)} \tag{10}$$

Different cost parameters are as follows:
Holding cost

$$
\begin{aligned}
HC = h(RP-(a-p))\Bigg[&\left\{\frac{\mu^2}{2}+\frac{b\mu^3}{6}+\frac{\alpha\mu^{\beta+2}}{(\beta+1)(\beta+2)}\right\}\\
&-(b+r)\left\{\frac{\mu^3}{3}+\frac{b\mu^4}{8}+\frac{\alpha\mu^{\beta+3}}{(\beta+1)(\beta+3)}\right\}-\alpha\left\{\frac{\mu^{\beta+2}}{(\beta+2)}+\frac{b\mu^{\beta+3}}{2(\beta+3)}+\frac{\alpha\mu^{2\beta+2}}{(\beta+1)(2\beta+2)}\right\}\Bigg]\\
&-h(a-p)\Bigg[\Bigg\{\left(-\frac{\lambda^2}{2}-\frac{\mu^2}{2}+\lambda\mu\right)+\frac{b}{2}\left(-\frac{2\lambda^3}{3}-\frac{\mu^3}{3}+\lambda^2\mu\right)\\
&+\frac{\alpha}{\beta+1}\left(\frac{\lambda^{\beta+2}}{\beta+2}-\lambda^{\beta+2}-\frac{\mu^{\beta+2}}{\beta+2}+\lambda^{\beta+2}\mu\right)\Bigg\}-(b+r)\Bigg\{\left(-\frac{\lambda^3}{6}-\frac{\mu^3}{3}+\frac{\lambda\mu^2}{2}\right)\\
&+\frac{b}{2}\left(\frac{\lambda^4}{4}-\frac{\mu^4}{4}+\frac{\lambda^2\mu^2}{2}\right)+\frac{\alpha}{\beta+1}\left(\frac{\lambda^{\beta+3}}{\beta+3}-\frac{\lambda^{\beta+3}}{2}-\frac{\mu^{\beta+3}}{\beta+3}+\frac{\lambda^{\beta+1}\mu^2}{2}\right)\Bigg\}\\
&-\alpha\Bigg\{\left(\frac{\lambda^{\beta+2}}{\beta+2}-\frac{\lambda^{\beta+2}}{\beta+1}-\frac{\mu^{\beta+2}}{\beta+2}+\frac{\lambda\mu^{\beta+1}}{\beta+1}\right)+\frac{b}{2}\left(\frac{\lambda^{\beta+3}}{\beta+3}-\frac{\lambda^{\beta+3}}{\beta+1}-\frac{\mu^{\beta+3}}{\beta+3}+\frac{\lambda^2\mu^{\beta+1}}{\beta+1}\right)\\
&+\frac{\alpha}{\beta+1}\left(\frac{\lambda^{2\beta+2}}{2\beta+2}-\frac{\lambda^{2\beta+2}}{\beta+1}-\frac{\mu^{2\beta+2}}{2\beta+2}+\frac{\lambda^{\beta+1}\mu^{\beta+1}}{\beta+1}\right)\Bigg\}\Bigg]
\end{aligned}
\tag{11}
$$

Shortage Cost

$$
\begin{aligned}
SC = C_2\Bigg[&\delta(a-p)\left\{\frac{(\lambda-\gamma)e^{-r\gamma}}{-r}+\frac{e^{-r\gamma}}{r^2}-\frac{e^{-r\gamma}}{r^2}\right\}\\
&+(RP-(a-p))\left\{-\frac{e^{-r\gamma}}{r^2}+\frac{(\gamma-T)e^{-r\gamma}}{r}+\frac{e^{-r\gamma}}{r^2}\right\}\Bigg]
\end{aligned}
\tag{12}
$$

Lost Sales Cost

$$
LS = C_3(1-\delta)(a-p)\left\{\frac{e^{-r\lambda}-e^{-r\gamma}}{r}\right\} \tag{13}
$$

Production Cost

$$
PC = \left(N+\frac{G}{P}+\nu P\right)\left[RP\left\{\frac{1-e^{-r\mu}}{r}+\frac{e^{-r\gamma}-e^{-rT}}{r}\right\}\right] \tag{14}
$$

Deteriorating cost

$$DC = c_d\alpha\beta\Big(RP - \Big(a-p\Big)\Big)\Big[\Big\{\frac{\mu^{\beta+1}}{\beta+1} + \frac{b\mu^{\beta+2}}{2(\beta+2)} + \frac{\mu^{2\beta+1}}{(\beta+1)(2\beta+1)}\Big\}$$
$$- (b+r)\Big\{\frac{\mu^{\beta+2}}{\beta+2} + \frac{b\mu^{\beta+3}}{2(\beta+3)} + \frac{\alpha\mu^{2\beta+2}}{(\beta+1)(2\beta+2)}\Big\}$$
$$- \alpha\Big\{\frac{\mu^{2\beta+1}}{2\beta+1} + \frac{b\mu^{2\beta+2}}{2(2\beta+2)} + \frac{\alpha\mu^{3\beta+1}}{(\beta+1)(3\beta+1)}\Big\}\Big]$$
$$- c_d\alpha\beta\Big(a-p\Big)\Big[\Big\{\Big(\frac{\lambda^{\beta+1}}{\beta+1} - \frac{\lambda^{\beta+1}}{\beta} - \frac{\mu^{\beta+1}}{\beta+1} + \frac{\lambda\mu^{\beta}}{\beta}\Big) + \frac{b}{2}\Big(\frac{\lambda^{\beta+2}}{\beta+2} - \frac{\lambda^{\beta+2}}{\beta} - \frac{\mu^{\beta+2}}{\beta+2} + \frac{\lambda^2\mu^{\beta}}{\beta}\Big)$$
$$+ \frac{\alpha}{\beta+1}\Big(\frac{\lambda^{2\beta+1}}{2\beta+1} - \frac{\lambda^{2\beta+1}}{\beta} - -\frac{\mu^{2\beta+1}}{2\beta+1} + \frac{\lambda^{\beta+1}\mu^{\beta}}{\beta}\Big)\Big\}$$
$$- (b+r)\Big\{\Big(\frac{\lambda^{\beta+2}}{\beta+2} - \frac{\lambda^{\beta+2}}{\beta+1} - \frac{\mu^{\beta+2}}{\beta+2} + \frac{\lambda\mu^{\beta+1}}{\beta+1}\Big) + \frac{b}{2}\Big(\frac{\lambda^{\beta+3}}{\beta+3} - \frac{\lambda^{\beta+3}}{\beta+1} - \frac{\mu^{\beta+3}}{\beta+3} + \frac{\lambda^2\mu^{\beta+1}}{\beta+1}\Big)$$
$$+ \frac{\alpha}{\beta+1}\Big(\frac{\lambda^{2\beta+2}}{2\beta+2} - \frac{\lambda^{2\beta+2}}{\beta+1} - \frac{\mu^{2\beta+2}}{2\beta+2} + \frac{\lambda^{\beta+1}\mu^{\beta+1}}{\beta+1}\Big)\Big\} - \alpha\Big\{\Big(\frac{\lambda^{2\beta+1}}{2\beta+1} - \frac{\lambda^{2\beta+1}}{2\beta} - \frac{\mu^{2\beta+1}}{2\beta+1} + \frac{\lambda\mu^{2\beta}}{2\beta}\Big)$$
$$+ \frac{b}{2}\Big(\frac{\lambda^{2\beta+2}}{2\beta+2} - \frac{\lambda^{2\beta+2}}{2\beta} - \frac{\mu^{2\beta+2}}{2\beta+2} + \frac{\lambda^2\mu^{2\beta}}{2\beta}\Big) + \frac{\alpha}{\beta+1}\Big(\frac{\lambda^{3\beta+1}}{3\beta+1} - \frac{\lambda^{3\beta+1}}{2\beta} - \frac{\mu^{3\beta+1}}{3\beta+1} + \frac{\lambda^{\beta+1}\mu^{2\beta}}{2\beta}\Big)\Big\}\Big] \tag{15}$$

Hence, the total average cost per unit time for perfect items is

$$TC_p = \frac{1}{T}[HC + DC + LS + SC + PC] \tag{16}$$

3.2. Model Formulation for imperfect items

For average cost of the system per unit time in case of imperfect items are managed by subsequent differential equations with some boundary constraints mentioned below:

$$I'(t) = -\alpha\beta t^{\beta-1}I(t) + (1-R)P - (a-q); 0 \le t \le \mu \tag{17}$$

$$I'(t) = -\alpha\beta t^{\beta-1}I(t) - (a-q); \mu \le t \le \lambda \tag{18}$$

$$I(t) = ((1-R)P - (a-q))\ (t + \frac{\alpha t^{\beta+1}}{\beta+1} + \frac{\alpha^2 t^{2\beta+1}}{2(2\beta+1)}\Big)\ e^{-\alpha t^{\beta}}; 0 \le t \le \mu \tag{19}$$

$$I(t) = ((1-R)P - (a-q))\ (\mu + \frac{\alpha\mu^{\beta+1}}{\beta+1} + \frac{\alpha^2\mu^{2\beta+1}}{2(2\beta+1)}\Big)\ e^{-\alpha t^{\beta}} - (a-q)$$
$$\Big((t-\mu) + \frac{\alpha(t^{\beta+1} - \mu^{\beta+1})}{\beta+1} + \frac{\alpha^2(t^{2\beta+1} - \mu^{2\beta+1})}{2(2\beta+1)}\Big)\ e^{-\alpha t^{\beta}}; \mu \le t \le \lambda \tag{20}$$

And the total quantity of stock at $t = 0$ is given by

$$Q = ((1-R)P - (a-q))\left(\mu + \frac{\alpha\mu^{\beta+1}}{\beta+1} + \frac{\alpha^2\mu^{2\beta+1}}{2(2\beta+1)}\right) e^{-\alpha\mu^{\beta}} \tag{21}$$

Holding cost is

$$\begin{aligned} HC = h'\Bigg[&\frac{\mu^2}{2} + \frac{\alpha\mu^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{\alpha^2\mu^{2\beta+2}}{2(2\beta+1)(2\beta+2)} \\ &- r\left\{\frac{\mu^3}{3} + \frac{\alpha\mu^{\beta+3}}{(\beta+1)(\beta+3)} + \frac{\alpha^2\mu^{2\beta+3}}{2(2\beta+1)(2\beta+3)}\right\} \\ &- \alpha\left\{\frac{\mu^{\beta+2}}{(\beta+2)} + \frac{\alpha\mu^{2\beta+2}}{(\beta+1)(2\beta+2)} + \frac{\alpha^2\mu^{3\beta+3}}{2(2\beta+1)(3\beta+3)}\right\} \\ &+ \left\{(\lambda-\mu) - \frac{r(\lambda^2-\mu^2)}{2} - \frac{\alpha(\lambda^{\beta+1}-\mu^{\beta+1}}{(\beta+1)}\right\} - \left(\frac{\lambda^2}{2} - \mu T\right) + \frac{\mu^2}{2} \\ &+ \frac{\alpha}{\beta+1}\left(\frac{\lambda^{\beta+2}-\mu^{\beta+2}}{\beta+2} - \mu^{\beta+1}(\lambda-\mu)\right) \\ &+ \frac{\alpha^2}{2(2\beta+1)}\left(\frac{\lambda^{2\beta+2}-\mu^{2\beta+2}}{2\beta+2} - \mu^{2\beta+1}(\lambda-\mu)\right) \\ &- r\left\{\left(\frac{\lambda^3}{3} - \frac{\mu\lambda^2}{2} - \frac{\mu^3}{6}\right) + \frac{\alpha}{\beta+1}\left(\frac{\lambda^{\beta+3}-\mu^{\beta+3}}{\beta+3} - \frac{\mu^{\beta+1}(\lambda^2-\mu^2)}{2}\right)\right. \\ &+ \frac{\alpha^2}{2(2\beta+1)}\left(\frac{\lambda^{2\beta+3}-\mu^{2\beta+3}}{2\beta+3} - \frac{\mu^{2\beta+1}(\lambda^2-\mu^2)}{2}\right) \\ &- \alpha\left\{\left(\frac{\lambda^{\beta+2}-\mu^{\beta+2}}{\beta+2} - \frac{\mu(\lambda^{\beta+1}-\mu^{\beta+1})}{\beta+1}\right)\right. \\ &+ \frac{\alpha}{\beta+1}\left(\frac{\lambda^{2\beta+2}-\mu^{\beta+3}}{2\beta+2} - \frac{\mu^{\beta+1}(\lambda^{\beta+1}-\mu^{\beta+1})}{\beta+1}\right) + \frac{\alpha^2}{2(2\beta+1)} \\ &\left.\left(\frac{\lambda^{3\beta+2}-\mu^{3\beta+2}}{3\beta+2} - \frac{\mu^{2\beta+1}(\lambda^{\beta+1}-\mu^{\beta+1})}{\beta+1}\right)\right\}\Bigg] \end{aligned} \tag{22}$$

Deterioration cost is

$$DC = c_{d1}\left[Q - (1-R)P - (a-q)\mu + (a-q)(\lambda-\mu)\right]\frac{1-e^{-r\lambda}}{r} \tag{23}$$

Production Cost is

$$PC = \left(N + \frac{G}{P} + \nu P\right)\left[(1-R)P\left\{\frac{1-e^{-r\mu}}{r} + \frac{e^{-r\gamma}-e^{-r\lambda}}{r}\right\}\right] \tag{24}$$

Therefore, the average total cost per unit time for imperfect items is
$TC_I = \frac{1}{T}$[Holding cost + Deteriorating cost + Production cost]

$$TCI = \frac{1}{T}[HC + DC + PC] \tag{25}$$

The goal is to optimize the total cost function $TC\,(\lambda,\ \ \gamma)$, which is given in equation (26) by given algorithm below. For the minimization of total cost:

- Find $\frac{\partial TC}{\partial \lambda} = 0$ and $\frac{\partial TC}{\partial \gamma} = 0$,
- Again, find $\frac{\partial^2 TC}{\partial \gamma^2}$ and $\frac{\partial^2 TC}{\partial \lambda \partial \gamma}$
- For the convexity, $\frac{\partial^2 TC}{\partial \lambda^2} > 0$, $\frac{\partial^2 TC}{\partial \lambda^2}\frac{\partial^2 TC}{\partial \gamma^2} - \left(\frac{\partial^2 TC}{\partial \lambda \partial \gamma}\right)^2 > 0$ and optimized values of λ^* and γ^*,$TC*$ were determined with the help of MATHEMATICA 11.0 software.

4. NUMERICAL ILLUSTRATION

For the numerical solution, we have taken following data in proper units and solved by mathematical computation tool to get optimized results. A=300, R=85%, P=156, a=90, b=0.3,h=4, α=0.5, β=0.2, h'=2, c_d=0.4,r=0.05 p=40, q=32, C_2=1.5, C_3=1.2, N=15, G=250, ν=0.001, δ=0.2, C_d1=0.8 We get optimum values of μ^*=0.0127, λ^*= 0.0281, γ^*=4.7519, T* = 5.3237 and TC* = 336.94.

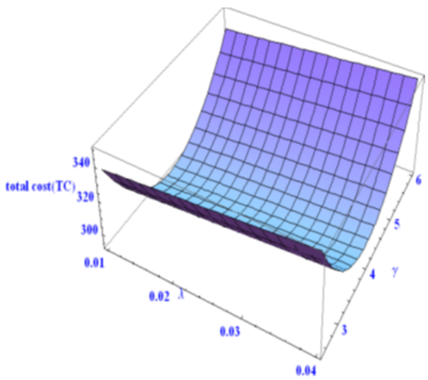

Figure 2: Graph of Convexity of the total cost function verses cycle time

5. SENSITIVITY ANALYSIS

Considering the above numerical results, sensitivity analysis is accommodated for which the all-out cost has accomplished. It is achieved by altering the parameters from −30% to +30%, taking one parameter in turn while all other parameters being constant (See, Table 1).

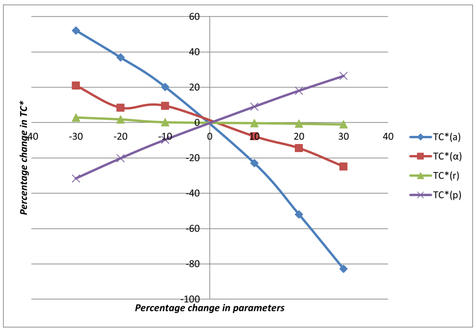

Figure 3: Graph of Variation in different parameters in the reference of total cost

A few interesting outcomes pulled from above table are shown as:

1. Change in increasing way is noticed in the demand parameter as expanded the value of total cost.
2. On the observation, as increment in deterioration coefficient α, total cost of the system also increases.
3. Increment in inflation rate r, the total cost is also raised.
4. The higher selling cost per item increases the total selling price per lot.

6. MANAGERIAL IMPLICATIONS

Optimization of total cost is very crucial if both kind of items (perfect and imperfect) are associated with the production process whereas deterioration takes place Weibull in nature and demand depends on price and stock-level. Meanwhile, a good impact on total cost is noticed while demand is selling price dependent. Both actors i.e., wholesalers and retailers in all actuality do demand to their production network accomplices to give adaptable credit period to settle their records [9]. During production process, store of perfect produced items is made although some of our imperfect products due to some environmental as well as technical error. The concerned assumptions like demand and production process touches the real situation of festive market and gives the accurate idea for present sale as well as future sale. Furthermore, proposed model reflected inflationary situation and shortages which tended to the certified and relevant circumstances to the present era. Moreover, results got through the analysis that the embraced strategy is useful for the arrangement producers.

7. CONCLUSION

This framework concluded a production model for Weibull deteriorated items under the inflation consideration. Factor of flexibility balanced the production

Table 1: Sensitivity analysis of different parameters

Parameter	% changes	Change in λ^*	Change in γ^*	Change in T^*	Change in TC^*
a	-30	0.0525	6.5142	6.7854	161.24
a	-20	0.0342	5.1925	5.5206	212.68
a	-10	0.0105	5.9927	6.5282	268.87
a	10	0.0281	4.7519	5.5092	414.35
a	20	0.0098	3.4301	4.1501	512.06
a	30	0.0098	3.4301	4.3774	616.01
α	-30	0.0342	5.1925	5.8169	266.31
α	-20	0.0885	6.5952	7.3829	308.51
α	-10	0.0105	5.9927	6.7169	304.92
α	10	0.0281	4.7519	5.3256	363.13
α	20	0.0098	3.4301	3.8441	385.69
α	30	0.0532	4.1282	4.6215	420.70
r	30	0.0532	4.1282	4.6215	420.70
r	-30	0.0105	5.9927	6.7169	327.38
r	-20	0.0105	5.9927	6.7169	330.90
r	-10	0.0281	4.7519	5.3237	336.16
r	10	0.0281	4.7519	5.3237	337.90
r	20	0.0281	4.7519	5.3237	339.03
r	30	0.0281	4.7519	5.3237	340.33
p	-30	0.0281	4.7519	5.5815	443.37
p	-20	0.0281	4.7519	5.4864	405.07
p	-10	0.0281	4.7519	5.4009	369.71
p	20	0.0105	5.9927	6.5473	276.10
p	30	0.0105	5.9927	6.4733	247.91

rate. Practically, truth is that during the production process which articles are produced, they all are mixture of error free items i.e., perfect, and somewhat is imperfect. Therefore, defectiveness cannot be ignored for the smooth conduction of business for both supplier and customers. Therefore, to optimize the cost of the total inventory framework we have discussed two stages, first when model explored the total costing of perfectly produced items and second one, calculates the total costing of imperfect quality items. In the case of pottery items, crockery items and fashionable products etc. This model can be applied. Practically, in market this demand rate is very logical because it also consists of selling price function. At last, to emphasize the model there is a numerical and sensitivity assessment has been presented. Furthermore, consequently, we reason a practical for such stock model. Finally, to accentuate the model mathematical model and sensitivity analysis has performed.

Acknowledgements. The authors would like to express their sincere gratitude to all editors and anonymous reviewers for their valuable comments that helped to improve the article.

Funding. This research received no external funding.

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