Yugoslav Journal of Operations Research
33 (2023), Number 3, 449–466
DOI: https://doi.org/10.2298/YJOR220915039F

INVENTORY MODEL FOR INSTANTANEOUS DETERIORATING ITEMS WITH TIME SENSITIVE DEMAND FOR POST COVID-19 RECOVERY

Sadaf FATMA
Department of Mathematics and Scientific Computing, Madan Mohan Malaviya University of Technology, Gorakhpur-273010, U.P., India
sadaff573@gmail.com

Vinod Kumar MISHRA
Department of Mathematics and Scientific Computing, Madan Mohan Malaviya University of Technology, Gorakhpur-273010, U.P., India
vkmmsc@mmmut.ac.in

Ranu SINGH
Department of Mathematics and Scientific Computing, Madan Mohan Malaviya University of Technology, Gorakhpur-273010, U.P., India
ranusinghgkp@gmail.com

Received: September 2022 / Accepted: December 2022

Abstract: The Covid-19 epidemic has caused substantial obstacles to the supply network globally. Hence there is urgency and necessity to build a model for cash flow in the chain of demand and supply system. This research suggests an inventory model to assist retailers in determining the optimal ordering quantity and replenishment cycle to reduce the total cost in different payment cases. The current study looks toward a partial advance and delays in the payment system considering time-sensitive demand, shortage, and partial backlogging for instantaneous deteriorating items. During the financial crisis, the partial advance and delay-in-payment strategy is planned to keep orders flowing from retailers to suppliers and customers to retailers. The impact of advanced and delayed payments on the total cost of a retailer is examined. To exemplify the model's application, numerical examples are used. A sensitivity study of critical parameters has been done to identify more sensitive parameters which reveal the clear depiction of present problems.

Keywords: Deteriorating items, advance and delay-in-payment, time sensitive demand, supply chain, Covid-19.

MSC: 90B05.

1. INTRODUCTION

The supply chain is more critical than ever in today's growing business. Enhancing resilience and responsibility will help businesses respond rapidly to current disturbances and reuse or rebuild supply chains for the future. Businesses need to quickly meet the demands of their employees, clients, and suppliers while navigating the financial and operational obstacles posed by the coronavirus. The worldwide *COVID-19* epidemic has permanently altered our attitudes, behaviours, and experiences as consumers, employees, citizens, and human beings. The crisis is significantly changing how and what consumers buy, accelerating significant structural changes in businesses like consumer goods. Due to the COVID-19 problem, businesses have been thrown off balance by fundamental changes in customer behaviour, supply networks, and routes to market. According to Gupta and Chutani [1], the retailer may help its supplier in a pandemic by making advance purchases or payments. The buyers and sellers often utilise the three payment options: immediate payment, advance payment, and trade credit or payment deferral. These payment arrangements are advantageous for trade promotion since they are beneficial for both the supplier and the consumer. The first policy is an instant pay-out, comparable to the well-known EOQ model popularised by Harris [2]. The second policy requires that the customer pays for the goods before they are delivered. The buyer's purchase price may need to be paid in full or in part in advance, depending on the seller (Zhang et al. [3]; Lashgari et al. [4]). Retailers permit trade credit or payment delays under the third policy. Full payment and partial payment are the two parts of the delayed payment policy. In contrast to a *partial payment delay*, which requires that part of the total purchase price be paid at the time of deliveries and the remaining amount to be paid later i.e., a complete payment delay permits payment of all purchasing costs after the specified date. Due to a financial issue in covid 19, both the retailer and the consumer avoid making full payments at once. Considering this issue, a partial advance and payment delay system with *time dependent demand* has been taken in this study.

Deterioration is the loss of a stock's usability or marginal value as a result of the change, decay, damage, obsolescence, spoilage, and pilferage. Medicine, blood, seafood, liquor, gas, food, and radioactive substances all have a finite shelf life and start to deteriorate soon as they are received in stock. Therefore, the product's deterioration cannot be overlooked. Most inventory models erroneously assume that all demand is lost or backlogged during stockout situations. In reality, some customers wait for replenishment, especially if the delay is short, while others get agitated and walk away. In view of these stockout situations, partial backlogging has been employed in this investigation.

To address the existing challenges for post COVID-19 recovery, our study includes advanced and delayed payment, time-sensitive demand for instantaneous deteriorating items, shortages and partial backlogging as potential solutions. The following are the study's main contributions:

- In the context of the pandemic condition, this model concurrently examined two payment options and their techniques.
- A hybrid payments strategy is taken into consideration to boost the post-Covid-19 recovery.

This section presents a brief literature review on the inventory control. Harris [5] was the first to propose the EOQ model. Raymond [6] wrote the first book based on inventory problems in 1931. This book concentrated on the application of a simple lot size problem. In 1951, Arrow et al. [7] offered a mathematical study of the inventory model, and in 1958, a comprehensive book titled "Studies in mathematical theory of inventory and production" was released. Hadley and Whitin [8] devoted a full-length book to "Analysis of Inventory System". Naddor [9] published a book on the "inventory system." As a result of its widespread application, inventory control has captured the attention of researchers, and it is rapidly evolving.

Whitin [10] first examined the inventory of deteriorating items and focused on the decay of fashion items at the end of the allotted storage time. Ghare [11] created a mathematical model of deteriorating inventory by adding exponential inventory reduction due to deterioration to the traditional EOQ formula. The first model of deteriorating stock with a linear demand trend was created by Dave and Patel [12]. In his opinion, demand was a linear function that envolved through time. Goyal and Giri [13] focused on the most recent modelling trends in deteriorating inventories. They categorised inventory models according to various criteria or restrictions, including variations in demand. Ouyang et al. [14] presented an inventory model for deteriorating goods with an exponentially declining demand and partial backlog. Alamri and Balkhi [15] investigated how memory loss and learning affect the optimum manufacturing lot size for perishable goods with cyclical demand and also deterioration rates. Dye et al. [16] determined the optimal lot size and selling price in terms of a changing rate of deterioration and an exponential partial backlog. They claim that as the interval between replenishments grows shorter, the percentage of customers who backlog their purchases increases rapidly. Dye and Ouyang [17] examined a deteriorating inventory model with fluctuating demand and trade policies.To determine a retailer's optimal replenishment quantity, selling price, and replenishment schedule under two level of trade policies, they created a deterministic EOQ model. A generalised type demand, backorder, and deterioration rate inventory model was introduced by Hung [18]. Mishra and Singh [19] developed a stock model by taking time-dependent demand and holding costs under partial backlog. Lee and Dye [20] provided a model of deteriorating inventories with stock-dependent demand that takes into account the cost of preservation technology as a decision parameter together with a replacement strategy. Mahata [21] suggested a down-stream trade credit from the retailer and an up-stream from the supplier in an EOQ model when the selling items are deteriorating. In order to maximize a retailer's overall profit while maintaining deterioration at a manageable level, Mishra et al. [22] examined trade credit and demand that is influenced by prices. Feng [23] also offered a pricing choice for things that were deteriorating, but this time it was due to a quality investment. It was predicted that demand would rely on both price and quality. Singh and Mishra [24] introduced a green inventory model that uses a combined ordering approach and carbon emissions for non-instantaneous substitutable deteriorating items. Some researchers have created different inventory model by considering different pattern based demand. Pervin et al. [25] developed an inventory model with time based demand and holding cost under stochastic deterioration rate. Under a two-level trade credit policy, Mahato and Mahata [26] addressed a model for non-instantaneous decaying products with price and time dependent demand and deterioration rate. Additionally, they also considered preservation technologies to stop product degradation/decay in order to lower the pace of deterioration. Paul et al. [27] investigated a retailer's model for decaying items with price-based demand in order to optimize overall profit.

It has long been customary in business to prepay or make an advance payment. Researchers have thus considered how this practice's affects on inventory decisions. The research on the use of prepayments in inventory choices was initiated by Zhang [28]. The inventory model with the pre-payment and price discount working simultaneously was solved by Gupta et al. [29] by using a genetic algorithm. Thangam [30] employed a simultaneous prepayment and a delayed payment to draw in more consumers (trade credit). Taleizadeh et al. [31] created inventory models to accommodate multi-advance payments for three distinct shortage and backorder situations. Zia and Taleizadeh [32] introduced an EOQ model that supports backorders, multiple advance and delayed payments. Additional studies on prepayment for degrading goods have been addressed by (Taleizadeh [33], Teng et al. [34]). For instance, Lashgari et al. [35] evaluated the impact on non-instantaneous deterioration by accounting for both advance and delay in payments. Mahata et al. [36] investigated the issue of dynamic decision-making problem with degrading goods under the conditions of price inflation and permissible payment delays. Barman et al.[37] proposed a multi-cycle production inventory model for vendor-buter considering decay products. They assumed that the purchase price along with interest is paid in advance by the customer to the seller before the order amount is replenished.

Goyal [38] discussed how trade policies are frequently applied in an EOQ model with charged interest. Ouyang et al. [39] created an EOQ model under the presumption that suppliers provide both cash discounts and payment delays. Additional research significantly widened this study. Mohanty et al. [40] recently expanded trade credit analyses by accounting for preservation technology investment. Mashud et al. [41] used an EOQ model to take into consideration trade credit and price-based demand with full backlogged and shortages. Mashud et al. [42] suggested a hybrid payment inventory model for non-instantaneously deteriorating products. Based on above literature survey and our knowledge, no study has taken the partial advance and delays in the payment system considering time-sensitive demand, shortage, and partial backlogging for instantaneous deteriorating items. Various techniques for COVID-19 have been formulated by many scholars (Lotfi et al. [43], Khalilpourazari et al. [44]). A hybrid reinforcement learning system and SIDARTHE to forecast the COVID-19 pandemic's trajectory in Quebec, Canada is developed by Khalilpourazari and Doulabi [45]. They demonstrated that the suggested technique offers high-quality answers to the most challenging standards.

Based on above literature review and our analysis, no study has considered advanced and delayed payment with time sensitive demand for deteriorating products under shortages and partial backlogging. To fill the gaps identified in earlier studies in all possible ways, this research motivated us to develop an inventory model for post COVID-19 recovery considering all the above parameters in single model. This study is extension of Mashud et al. [42] by considering time sensitive demand and different objective function.

The following are the paper's outlines: The inventory model is introduced and a quick assessment of the literature is presented in section 1. Section 2 discusses the notations, assumptions. Section 3 describes the problem statment. The model formulation is done in section 4. Section 5 contains the results of the numerical illustration. Sensitivity analysis and practical consequences are discussed in section 6. The study is concluded in the section 7.

2. ASSUMPTIONS AND NOTATIONS

2.1. Assumptions

The assumptions and notations used by this model are listed below:

1. The replenishment rate is treated as infinite.
2. Lead time is constant.
3. Planning horizon is infinite.
4. The demand rate $D = a + bt$, is time sensitive dependent $a > 0, b > 0$.
5. The deterioration occur in the interval $[0, t_1]$ at a constant rate θ.
6. Demands that have not been satisfied are considered to be partially backlogged.The percentage of backorders rises as consumers waiting period $(T-t)$ decreases. The partial backorder rate is $e^{-v(T-t)}$, where v stands for positive backlogging parameter.
7. Prepayment in n installments is requested by the supplier. The retailer next obtains a financial loan from an institution at a specific interest rate. The total capital costs (per cycle) of pre-payments are then calculated using an approach identical to Lashgari et al. [35] throughout the paper.

2.2. Notations

Notation	Description
A	Ordering cost
a	Basic Demand
b	Demand function's slope
C, C_3	Purchase and lost sale cost respectively
C_1, C_2	Holding and shortage cost respectively
D	Annual demand
$I_i(t), i = 1, 2$	Inventory level at time t, $0 \leq t \leq t_1$, and $t_1 \leq t \leq T$ respectively
L	Lead time
n	Number of installments for prepayment
M	Delay in payment period
T	Replenishment Cycle
S, R	Maximum stock and shortage per cycle respectively
t_1	Time at which the stock level drops to zero
v	Backlogging variable, $v > 0$
p	Selling price
θ	Deterioration rate
I_e	Annual rate of interest earned
I_{cc}	Annual rate of interest on loan
I_c	interest rate charged per year
$CCCP$	Cyclic capital cost
PC	Total purchase cost
LSC	Lost sale cost
BC	Backorder cost
σ	fraction of the purchase price that must be paid before delivery
Q	Order quantity
IE_i	Interest earned, $i = 1, 2$
IC_i	Interest charged, $i = 1, 2$
TC_i	Total cost, $i = 1, 2$

3. PROBLEM DESCRIPTION

On the basis of the above presumption, a supplier will accept σ % of a retailer's total purchase cost when the order is placed, and the other remain $(1-\sigma)$% will be paid when the order is received. Customers may also be offered a payment delay by the retailer. The product nature is non-instantaneous deteriorating, and the payment method is hybrid type. The stock level depletes due to requirement and decay during $[0, t_1]$ and after that a part of requirements are backlogged due to shortage. The retailer will determine the replenishment cycle and time at which inventory level drops to zero based on the impact of the epidemic on the market. Figure 1 depicts the proposed EOQ models.

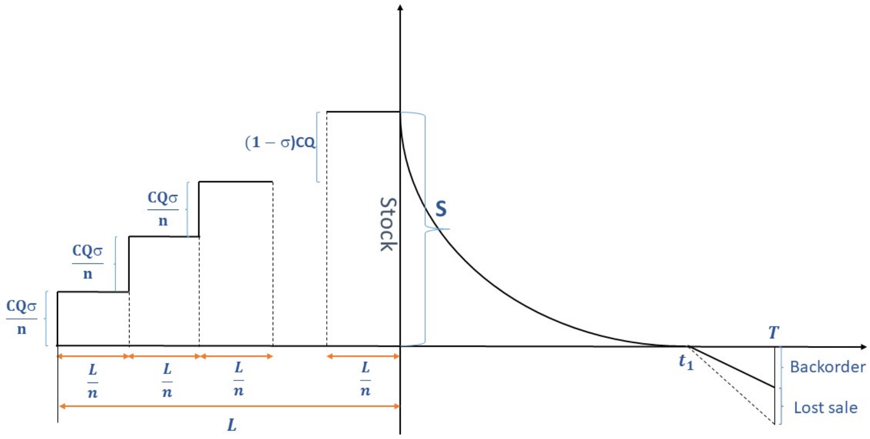

Figure 1: $I(t)$ versus time with shortage

4. MATHEMATICAL FORMULATION

Taking shortages into account, this research provides two models for a hybrid payment system that comprises advance payment and a trade policy. The inventory level $I(t)$ in this situation drops to zero at $t = t_1$ due to both demand and deterioration, during the period $[0, t_1]$, whereas shortages arise during the period $[t_1, T]$ due to demand and a portion of requirments are backlogged. The left part of Figure 1 illustrates that multiple prepayments were made in an equal duration of $\frac{L}{n}$ within the lead-time L. The large section represents payments made in advance, while the small portion represents the amount that must be paid by the retailer when the products are received. The suggested inventory system's differential equation is,

$$\frac{dI_1(t)}{dt} + \theta I_1(t) = -(a + bt), \quad 0 < t \leq t_1, \quad with \;\; I_1(t_1) = 0. \tag{1}$$

Equation (1) gives,

$$I_1(t) = \left(\frac{a + bt_1}{\theta} - \frac{b}{\theta^2}\right) e^{\theta(t_1 - t)} - \left(\frac{a + bt}{\theta} - \frac{b}{\theta^2}\right). \tag{2}$$

Now, applying the condition, $I_1(0) = S$. The initial stock is given by

$$S = \left(\frac{a + bt_1}{\theta} - \frac{b}{\theta^2}\right) e^{\theta t_1} - \left(\frac{a}{\theta} - \frac{b}{\theta^2}\right). \tag{3}$$

After the period $t = t_1$, inventory levels gradually decline below zero, causing shortages. During the period of shortage $[t_1, T]$, the requirments are partially backlogged at a rate for $(T-t)$. For this, the differential equation can be expressed as:

$$\frac{dI_2(t)}{dt} = -(a+bt)\mathrm{e}^{-v(T-t)}, \qquad t_1 < t \le T, \tag{4}$$

Using the boundary condition $I_2(t_2) = 0$. Equation (4) gives,

$$I_2(t) = \mathrm{e}^{-v(T-t_1)}\left(\frac{a+bt_1}{v} - \frac{b}{v^2}\right) - \mathrm{e}^{-v(T-t)}\left(\frac{a+bt}{v} - \frac{b}{v^2}\right), \quad t_1 < t \le T. \tag{5}$$

Therefore, the maximum shortage is given by,

$$R = -I_2(T) = \left(\frac{a+bT}{v} - \frac{b}{v^2}\right) - \mathrm{e}^{-v(T-t_1)}\left(\frac{a+bt_1}{v} - \frac{b}{v^2}\right). \tag{6}$$

The retailer's total order is calculated using the total initial stock and total shortages, and the outcome has the following mathematical form:

$$\begin{aligned} Q = S + R = &\left(\frac{a+bt_1}{\theta} - \frac{b}{\theta^2}\right)\mathrm{e}^{\theta t_1} - \left(\frac{a}{\theta} - \frac{b}{\theta^2}\right) + \left(\frac{a+bT}{v} - \frac{b}{v^2}\right) \\ &- \mathrm{e}^{-v(T-t_1)}\left(\frac{a+bt_1}{v} - \frac{b}{v^2}\right). \end{aligned} \tag{7}$$

The system's ordering cost may be expressed as,

$$OC = A. \tag{8}$$

A retailer pays a supplier for products. He also has to keep the inventory for a specific amount of time. As a result, the holding cost is expressed as:

$$\begin{aligned} HC &= C_1 \int_0^{t_1} I_1(t)dt, \\ &= C_1\left[\left(\frac{a+bt_1}{\theta} - \frac{b}{\theta^2}\right)\left(\frac{\mathrm{e}^{\theta t_1} - 1}{\theta}\right) - \left(\frac{2at_1\theta + bt_1^2\theta - 2bt_1}{2\theta^2}\right)\right]. \end{aligned} \tag{9}$$

$PC = CQ$, represents the overall purchase cost.

$$\begin{aligned} PC = &C\left[\left(\frac{a+bt_1}{\theta} - \frac{b}{\theta^2}\right)\mathrm{e}^{\theta t_1} - \left(\frac{a}{\theta} - \frac{b}{\theta^2}\right) + \left(\frac{a+bT}{v} - \frac{b}{v^2}\right)\right. \\ &\left. - \mathrm{e}^{-v(T-t_1)}\left(\frac{a+bt_1}{v} - \frac{b}{v^2}\right)\right]. \end{aligned} \tag{10}$$

Due to supply challenges, certain customers will have to wait for the next lot. This means that in the interval $t_1 < t \le T$, there is some unsatisfied demand with a value, known as backorder cost. As a result, the backorder cost (BC) takes the following mathematical form:

$$
\begin{aligned}
BC &= C_2 \int_{t_1}^{T} -I_2 dt, \\
&= C_2 \left[\left(\frac{a+bT}{v^2} - \frac{2b}{v^3} \right) - \left[\left(\frac{a+bt_1}{v^2} - \frac{2b}{v^3} \right) + \right. \right. \\
&\left. \left. + (T-t_1) \left(\frac{a+bt_1}{v} - \frac{b}{v^2} \right) \mathrm{e}^{-v(T-t_1)} \right. \right.
\end{aligned} \tag{11}
$$

Stockouts of popular goods can result in lost sales in the form of missed opportunities, which can be stated mathematically as:

$$
\begin{aligned}
LSC &= \int_{t_1}^{T} (a+bt) \left[1 - \mathrm{e}^{-v(T-t)} \right], \\
&= C_3 \left[\left(aT + \frac{bT^2}{2} \right) - \left(at_1 + \frac{bt_1^2}{2} \right) - \left(\frac{a+bT}{v} - \frac{b}{v^2} \right) + \right. \\
&\left. + \left(\frac{a+bt_1}{v} - \frac{b}{v^2} \right) \mathrm{e}^{-v(T-t_1)} \right. .
\end{aligned} \tag{12}
$$

As seen in figure 1, before collecting the items, the retailer's pays a fraction σ of his/her purchase price, and the rest amount $(1-\sigma)$ is anticipated to be paid upon delivery. The cycle capital cost is therefore written as:

$$
CCCP = \frac{C\sigma I_{cc} Q(n+1)L}{2n}. \tag{13}
$$

To increase sales and reverse the unfavourable situation, the retailer extends payment time period (M) to consumers after the receiving time. The various size of M situations i.e., $M < L$, lead to several circumstances that have varying consequences on revenue and expense. The specifics of two distinct inventory systems are covered in the following section.

Case 1: $0 < M < t_1$

The interest rate I_e is related to the amount of interest earned (IE_i). The retailer receive compound interest before the shortage (t_1), but during the shortage, they get simple interest. The expression for IE_1 is,

$$
\begin{aligned}
IE_1 &= (1-\sigma) \left[pI_e \int_0^{t_1} \int_0^{t} (a+bt)\, du dt + \frac{pI_e}{v} \int_0^{t_1} (a+bt) \left[1 - \mathrm{e}^{-v(T-t_1)} \right] dt \right], \\
&= (1-\sigma)\, pI_e \left[\left(\frac{at_1^2}{2} + \frac{bt_1^3}{3} \right) + \left(\frac{1-\mathrm{e}^{-v(T-t_1)}}{v} \right) \left(at_1 + \frac{bt_1^2}{2} \right) \right].
\end{aligned} \tag{14}
$$

This is the interest earned on the sum of the inventories up to $(1-\sigma)$ portion. The consumer is under no obligation to make payment until M due to the reasonable delay. The retailer is subsequently charged after M for the inventory that is on hand. As a result, the interest charged (IC_1) on the entire inventory relative to interest rate I_c is expressed as:

$$\begin{aligned} IC_1 &= CI_c \int_M^{t_1} I_1(t)dt, \\ &= CI_c \left[\left(\frac{a+bt_1}{\theta} - \frac{b}{\theta^2} \right) \left(\frac{e^{\theta(t_1-M)}-1}{\theta} \right) - \right. \\ &\left. - \left\{ \left(\frac{a}{\theta} - \frac{b}{\theta^2} \right)(t_1-M) + \frac{b}{2\theta}\left(t_1^2-M^2\right) \right\} \right]. \end{aligned} \tag{15}$$

The entire cost per cycle is now represented as follows:

$$\begin{aligned} TC(t_1,T) = & \frac{1}{T}\big(\text{Ordering Cost+Holoding Cost+ Backorder Cost+Lost sale Cost} \\ & + \text{Purchase Cost + Cyclic capital Cost - Intesert earned + Interest charged}\big), \\ = & \frac{1}{T}\big(OC+HC+BC+LSC+PC+CCCP-IE_1+IC_1\big). \end{aligned}$$

$$TC_1(t_1,T) = \frac{1}{T}\left(\begin{array}{c} A + C_1\left[\left(\frac{a+bt_1}{\theta}-\frac{b}{\theta^2}\right)\left(\frac{e^{\theta t_1}-1}{\theta}\right) - \left(\frac{2at_1\theta+bt_1^2\theta-2bt_1}{2\theta^2}\right)\right] \\ +C_2\left[\left(\frac{a+bT}{v^2}-\frac{2b}{v^3}\right) - \left[\left(\frac{a+bt_1}{v^2}-\frac{2b}{v^3}\right)+(T-t_1)\left(\frac{a+bt_1}{v}-\frac{b}{v^2}\right)\right]e^{-v(T-t_1)}\right] \\ +C_3\left[\left(aT+\frac{bT^2}{2}\right)-\left(at_1+\frac{bt_1^2}{2}\right)-\left(\frac{a+bT}{v}-\frac{b}{v^2}\right)+\left(\frac{a+bt_1}{v}-\frac{b}{v^2}\right)e^{-v(T-t_1)}\right] \\ +C\left[\left(\frac{a+bt_1}{\theta}-\frac{b}{\theta^2}\right)e^{\theta t_1}-\left(\frac{a}{\theta}-\frac{b}{\theta^2}\right)+\left(\frac{a+bT}{v}-\frac{b}{v^2}\right)-e^{-v(T-t_1)}\left(\frac{a+bt_1}{v}-\frac{b}{v^2}\right)\right] \\ +\frac{C\sigma I_{cc}Q(n+1)L}{2n} - (1-\sigma)pI_e\left[\left(\frac{at_1^2}{2}+\frac{bt_1^3}{3}\right)+\left(\frac{1-e^{-v(T-t_1)}}{v}\right)\left(at_1+\frac{bt_1^2}{2}\right)\right] \\ +CI_c\left[\left(\frac{a+bt_1}{\theta}-\frac{b}{\theta^2}\right)\left(\frac{e^{\theta(t_1-M)}-1}{\theta}\right)-\left\{\left(\frac{a}{\theta}-\frac{b}{\theta^2}\right)(t_1-M)+\frac{b}{2\theta}\left(t_1^2-M^2\right)\right\}\right] \end{array} \right). \tag{16}$$

Case 2: $t_1 < M < T$
In this case,

$$\begin{aligned} IE_2 &= IE_1, \\ &= (1-\sigma)\,pI_e\left[\left(\frac{at_1^2}{2}+\frac{bt_1^3}{3}\right)+\left(\frac{1-e^{-v(T-t_1)}}{v}\right)\left(at_1+\frac{bt_1^2}{2}\right)\right]. \end{aligned} \tag{17}$$

Moreover, there are no positive stocks after M. Hence, the interest charged (IC_2) is, $IC_2 = 0$.

$TC_2(t_1,T)=$

$$\frac{1}{T}\left(\begin{array}{c} A+C_1\left[\left(\frac{a+bt_1}{\theta}-\frac{b}{\theta^2}\right)\left(\frac{e^{\theta t_1}-1}{\theta}\right)-\left(\frac{2at_1\theta+bt_1^2\theta-2bt_1}{2\theta^2}\right)\right] \\ +C_2\left[\left(\frac{a+bT}{v^2}-\frac{2b}{v^3}\right)-\left[\left(\frac{a+bt_1}{v^2}-\frac{2b}{v^3}\right)+(T-t_1)\left(\frac{a+bt_1}{v}-\frac{b}{v^2}\right)\right]e^{-v(T-t_1)}\right] \\ +C_3\left[\left(aT+\frac{bT^2}{2}\right)-\left(at_1+\frac{bt_1^2}{2}\right)-\left(\frac{a+bT}{v}-\frac{b}{v^2}\right)+\left(\frac{a+bt_1}{v}-\frac{b}{v^2}\right)e^{-v(T-t_1)}\right] \\ +C\left[\left(\frac{a+bt_1}{\theta}-\frac{b}{\theta^2}\right)e^{\theta t_1}-\left(\frac{a}{\theta}-\frac{b}{\theta^2}\right)+\left(\frac{a+bT}{v}-\frac{b}{v^2}\right)-e^{-v(T-t_1)}\left(\frac{a+bt_1}{v}-\frac{b}{v^2}\right)\right] \\ +\frac{C\sigma I_{cc}Q(n+1)L}{2n}-(1-\sigma)\,pI_e\left[\left(\frac{at_1^2}{2}+\frac{bt_1^3}{3}\right)+\left(\frac{1-e^{-v(T-t_1)}}{v}\right)\left(at_1+\frac{bt_1^2}{2}\right)\right] \end{array}\right). \quad (18)$$

To acquaire the optimum value for both cases $TC_i(t_1,T), i=1,2$. Differentiate partially $(TC_i(t_1,T), i=1,2)$ with respect to t_1 and T and equating

$$\frac{\partial TC_i(t_1,T)}{\partial t_1}=0 \quad \text{and} \quad \frac{\partial TC_i(t_1,T)}{\partial T}=0.$$

Solving the above for t_1 and T, we obtain the optimal value of t_1 and T. To show the sufficient condition for $(TC_i(t_1,T), i=1,2)$ at t_1 and T

$$\frac{\partial^2 TC_i(t_1,T)}{\partial {t_1}^2}>0\ , \quad \frac{\partial^2 TC_i(t_1,T)}{\partial T^2}>0,$$

and $$\frac{\partial^2 TC_i(t_1,T)}{\partial {t_1}^2}\,\frac{\partial^2 TC_i(t_1,T)}{\partial T^2}-\frac{\partial^2 TC_i(t_1,T)}{\partial t_1\partial T}\,\frac{\partial^2 TC_i(t_1,T)}{\partial T\partial t_1}>0.$$

The total cost function $(TC_i(t_1,T), i=1,2)$ is highly non-linear, so illustrating the optimality condition in analytic form is complicated. Therefore we have shown the nature of the $(TC_i(t_1,T), i=1,2)$ i.e., graphically as shown in Figures 4 and 7.

5. NUMERICAL ILLUSTRATIONS

To demonstrate the application of model, a numerical examples for various trade-credit periods are presented, and the data is shown in Table 1. Using the above approach and with the help of MAPLE-2018, we have obtained the optimal value of t_1, replenishment cycle (T), S, R, total cost (TC) and order quantity (Q) for both cases as shown below:

For case 1: $t_1=0.85$, $T=1.85$, $S^*=147$, $R^*=122$,$TC=\$1475.35$ and $Q^*=269$.

For case 2: $t_1=0.84$, $T=1.80$, $S^*=144$, $R^*=118$, $TC=\$1450.96$ and $Q^*=262$.

After comparing the outcome of both cases, case 2 results a significant reduction in total cost of \$24.39 with respect to decision parameters. For the retailer, case 2 illustration yields the best outcome. Figures 2 and 3 show the convexity of the total cost function (TC) for case 1 and 2 respectively.

Table 1: Data for Numerical Experiment

Parameters	Units	Values
Ordering cost (A)	\$/order	800
Basic demand (a)	−	100
Slope of demand function (b)	−	50
Purchase and lost sale cost (C, C_3)	\$/ unit	5, 6
Holding and shortage cost (C_1, C_2)	\$/unit	0.5, 5
Deterioration rate (θ)	%	0.75
Backlogging parameter (v)	−	0.7
fraction of the purchase price that must be paid before delivery (σ)	−	0.7
Interest rate on loan (I_{cc})	%/year	0.09
Interest charged (I_c)	\$	0.09
Interest earned (I_e)	\$	0.12
Lead time(L)	year	0.2
Number of installments for prepayment (n)	−	3
Permitted delay in payment period (M)	year	0.2,1.2
Selling price (p)	\$/unit	7

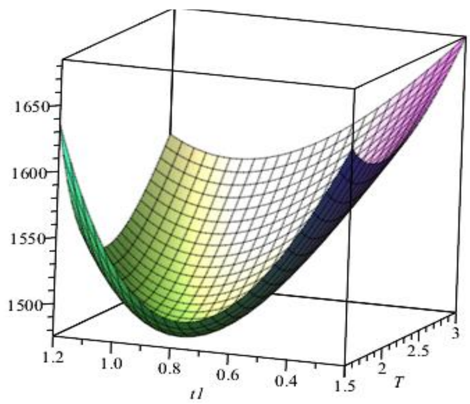

Figure 2: Total cost function vs. t_1 and T for case 1

6. SENSITIVITY ANALYSIS

A sensitivity table is presented to investigate the variational effects of the decision variables by rotating the parameter values from −20% to +20% (See Table 2). The results are obtained by altering each parameter in turn while leaving other values unchanged.

From the sensitivity study, the following results can be drawn:

- Total cost (TC) is more responsive to the parameter C and less responsive to the parameters A, a, b, v, σ, M, and n.
- Increment in A, a, b, n causes minor change in total cost.
- The increment in parameter σ, M causes negligible change in total cost.
- The increase in value of parameter v gives positive result.

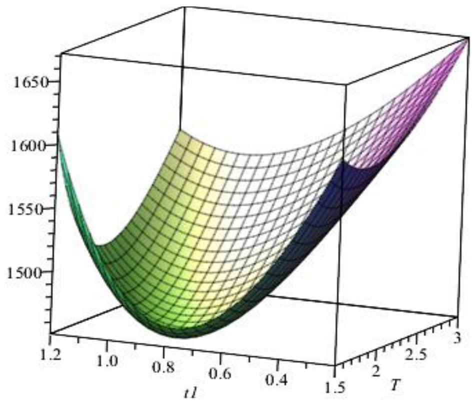

Figure 3: Total cost function vs. t_1 and T for case 2

Table 2: Sensitivity analysis for different parameters

parameters	variation in parameter	variation in TC	variation in t_1	variation in T
	640	1384.39	0.81	1.67
A	720	1431.39	0.83	1.76
	880	1517.56	0.87	1.94
	960	1557.95	0.88	2.02
	80	1384.50	0.92	1.88
a	90	1430.37	0.88	1.87
	110	1519.52	0.82	1.83
	120	1562.96	0.79	1.81
	40	1351.67	0.82	2.00
b	45	1414.56	0.84	1.92
	55	1534.12	0.86	1.78
	60	1590.93	0.87	1.72
	4	1316.97	0.97	1.86
C	4.5	1397.91	0.91	1.85
	5.5	1549.63	0.79	1.86
	6	1621.04	0.74	1.87
	0.56	1601.83	0.99	1.79
v	0.63	1527.49	0.91	1.83
	0.77	1438.47	0.80	1.86
	0.84	1412.00	0.75	1.88
	0.56	1466.39	0.86	1.86
σ	0.63	1470.89	0.86	1.85
	0.77	1479.77	0.84	1.85
	0.84	1484.15	0.85	1.85
	0.16	1474.80	0.85	1.85
M	0.18	1475.07	0.85	1.85
	0.22	1475.63	0.85	1.85
	0.24	1475.92	0.85	1.85
	640	1384.39	0.81	1.67
n	720	1431.06	0.83	1.76
	880	1517.56	0.87	1.94
	960	1557.95	0.88	2.02

It can be concluded that parameter C is more profound than the rest parameters, and hence the manager should pay more attention to it. Figures 4, 5, 6 and 7 exhibit a graphical depiction of sensitivity analysis.

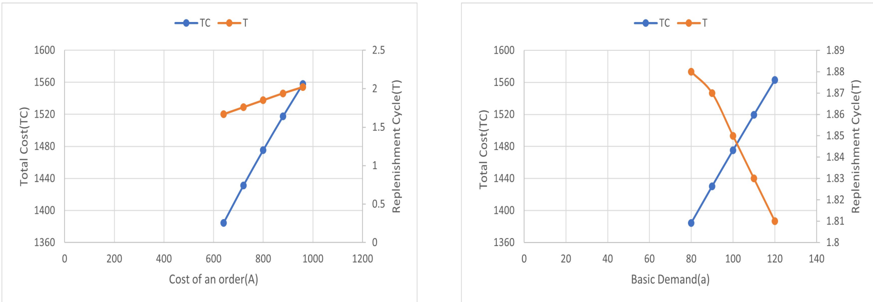

Figure 4: Sensitivity analysis represented graphically with changes to A and a

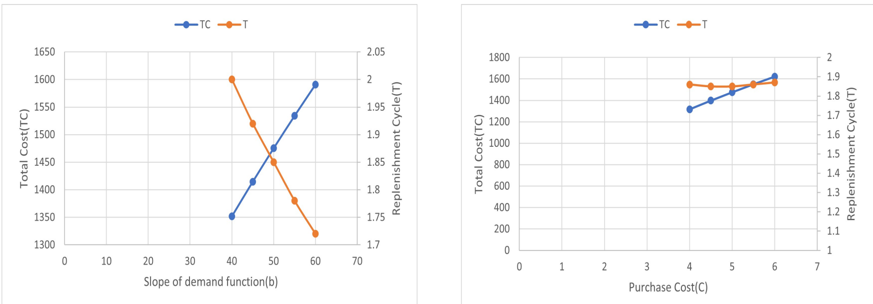

Figure 5: Sensitivity analysis represented graphically with changes to b and C

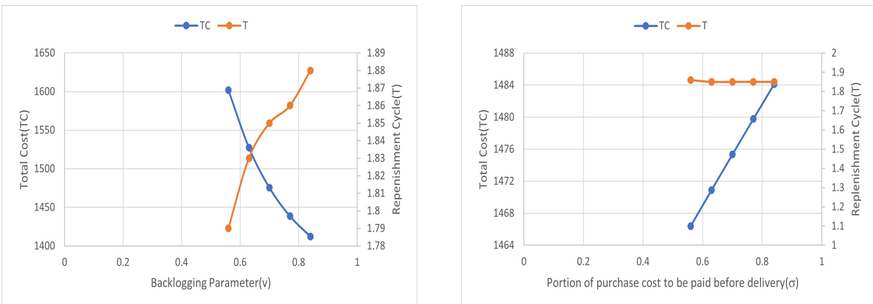

Figure 6: Sensitivity analysis represented graphically with changes to v and σ

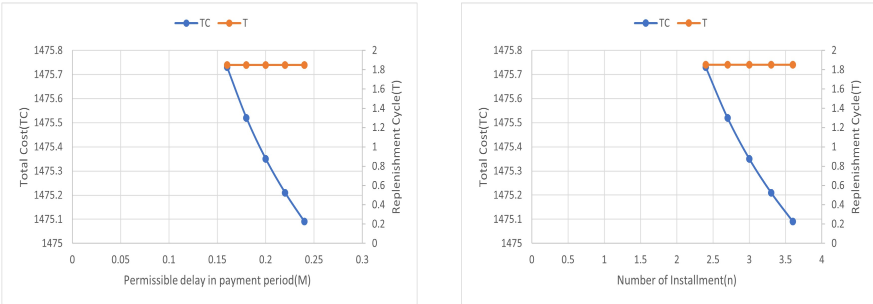

Figure 7: Sensitivity analysis represented graphically with changes to M and n

7. CONCLUSION

The payment strategy between vendors, retailers and consumers is a crucial part of supply management credit policy that influences each party's overall costs and purchasing decisions. This research addresses the business collapse and failure caused by COVID-19 and proposes a strategy to promote supply chain economic recovery. An alternate technique for maintaining business relations via a hybrid payment scheme is proposed. In this study, the supplier requests for prepayment in the number of n instalments. Also, a financial institution provides loans to the retailer at a fixed interest rate. This research work considers instantaneous deteriorating items with time-sensitive demand and partial backlogging. A numerical example with different trade credit periods illustrates the model's applicability. Software Maple-18 is used to find the optimal ordering quantity, replenishment cycle, maximum stock and shortage per cycle to minimize the total cost in different payment cases. The numerical illustration shows that effectively implementing trade credit and delayed payment strategies help to reduce the retailer's overall cost. The convexity of the overall cost function is depicted graphically with respect to decision variables.

This study is limited to single retailer-supplier-consumer, assuming customer demand is time-sensitive, which may be extended by considering multi-retailer-supplier-consumer. This work can be further studied by considering various demand patterns like (price and stock-dependent demand). One can also look into how using preservation technique affects the rate of deterioration. Consideration of the carbon emissions factor (carbon cap policy) under fuzzy environment to handle uncertainty may be one of the interesting extensions of this study (Özmen et al. [46] ; Barman et al. [47]; Pervin and Roy [48]; Singh and Mishra [49]). The suggested model may be analyzed by considering the influence of greening level on selling price and manufacturing cost (as examined in Paul et al. [50]).

Funding. This research received no external funding.

REFERENCES

[1] V. Gupta and A. Chutani, "Supply chain financing with advance selling under disruption," *International Transactions in Operational Research*, vol. 27, no. 5, pp. 2449-2468, 2020.

[2] F. W. Harris, " How many parts to make at once," 1913.

[3] Q. Zhang, Y. C. Tsao, and T. H. Chen, "Economic order quantity under advance payment," *Applied Mathematical Modelling*, vol. 38, no. 24, pp. 5910-5921, 2014.

[4] M. Lashgari, A. A. Taleizadeh, and A. Ahmadi, "Partial up-stream advanced payment and partial down-stream delayed payment in a three-level supply chain," *Annals of Operations Research*, vol. 238, no. 1, pp. 329-354, 2016.

[5] F. W. Harris, "Operations and cost," *Factory management series*, pp. 48-52, 1915.

[6] F. E. Raymond, "Quantity and economy in manufacture," *McGraw-Hill*, 1931.

[7] K. J. Arrow, S. Karlin, and H. E. Scarf, "Studies in the mathematical theory of inventory and production," 1958.

[8] G. Hadley, and T. M. Whitin, "*Analysis of inventory systems*," no. 658.787 H3, 1963.

[9] E. Naddor, "On the equivalence of some inventory systems," *Management Science*, vol. 9, no. 3, pp. 482-489, 1963.

[10] T. M. Whitin, "Theory of inventory management," *Princeton University Press*, 1957.

[11] P. M. Ghare, "A model for an exponentially decaying inventory," *J. ind. Engng*, vol. 14, pp. 238-243, 1963.

[12] U. Dave, and L. K. Patel, "(T, S i) policy inventory model for deteriorating items with time proportional demand," *Journal of the Operational Research Society*, vol. 32, no. 2, pp. 137-142, 1981.

[13] S. K. Goyal, and B. C. Giri, "Recent trends in modeling of deteriorating inventory," *European Journal of operational research*, vol. 134, no. 1, pp. 1-16, 2001.

[14] L. Y. Ouyang, K. S. Wu, and M. C. Cheng, "An Inventory Model for Deteriorating Items With Exponential Declining Demand and Partial Backlogging," *The Yugoslav Journal of Operations Research*, vol. 15, no. 30, pp. 277-288, 2005.

[15] A. A. Alamri, and Z. T. Balkhi, "The effects of learning and forgetting on the optimal production lot size for deteriorating items with time varying demand and deterioration rates," *International Journal of Production Economics*, vol. 107, pp. 125-138, 2007.

[16] C. Y. Dye, L. Y. Ouyang, and T. P. Hsieh, "Deterministic inventory model for deteriorating items with capacity constraint and time-proportional backlogging rate," *European Journal of Operational Research*, vol. 178, no. 3, pp. 789-807, 2007.

[17] C. Y. Dye, and L. Y. Ouyang, "A particle swarm optimization for solving joint pricing and lot-sizing problem with fluctuating demand and trade credit financing," *Computers & Industrial Engineering*, vol. 60, no. 1, pp. 127-137, 2011.

[18] K. C. Hung, "An inventory model with generalized type demand, deterioration and backorder rates," *European Journal of Operational Research*, vol. 208, no. 3, pp. 239-242, 2011.

[19] V. K. Mishra, and L. S. Singh, "Deteriorating inventory model for time dependent demand and holding cost with partial backlogging," *International Journal of Management Science and Engineering Management*, vol. 6, no. 4, pp. 267-271, 2011.

[20] Y.-P. Lee, and C.-Y. Dye, "An inventory model for deteriorating items under stockdependent demand and controllable deterioration rate," *Computers and Industrial Engineering*, vol. 63, no. 2, pp. 474-482, 2012.

[21] G. C. Mahata, "Retailer's optimal credit period and cycle time in a supply chain for deteriorating items with up-stream and down-stream trade credits," *Journal of Industrial Engineering International*, vol. 11, no. 3, pp. 353-366, 2015.

[22] U. Mishra, J. Tijerina-Aguilera, S. Tiwari, and L. E. Cardenas-Barron, "Retailer's joint ordering, pricing, and preservation technology investment policies for a deteriorating item under permissible delay in payments," *Mathematical Problems in Engineering*, vol. 2018, Article ID 6962417, 2018.

[23] L. Feng, "Dynamic pricing, quality investment, and replenishment model for perishable items," *International Transactions in Operational Research*, vol. 26, no. 4, pp. 1558-1575, 2019.

[24] R. Singh, and V. K. Mishra, "An Inventory Model for Non-instantaneous Deteriorating

Items with Substitution and Carbon Emission Under Triangular Type Demand," *International Journal of Applied and Computational Mathematics*, vol. 7, no. 4, pp. 1-21, 2021.

[25] M. Pervin, S. K. Roy, G. W. and Weber, "Analysis of inventory control model with shortage under time-dependent demand and time-varying holding cost including stochastic deterioration," *Annals of Operations Research*, vol. 260, no. 1, pp. 437-460, 2018.

[26] C. Mahato, and G. C. Mahata, "Optimal replenishment, pricing and preservation technology investment policies for non-instantaneous deteriorating items under two-level trade credit policy," *Journal of Industrial and Management Optimization*, vol. 18, no. 5, pp. 3499-3537, 2021.

[27] A. Paul, M. Pervin, S. K. Roy, G. W. Weber, and A. Mirzazadeh, "Effect of price-sensitive demand and default risk on optimal credit period and cycle time for a deteriorating inventory model," *Rairo-Operations Research*, vol. 55, pp. S2575-S2592, 2021.

[28] A. X. Zhang, "Optimal advance payment scheme involving fixed per-payment costs," *Omega*, vol. 24, no. 5, pp. 577-582, 1996.

[29] R. K. Gupta, A. K. Bhunia, and S. K. Goyal, "An application of genetic algorithm in solving an inventory model with advance payment and interval valued inventory costs," *Mathematical and Computer Modelling*, vol. 49, no. 5-6, pp. 893-905, 2009.

[30] A. Thangam, "Dominant retailer's optimal policy in a supply chain under Advance Payment scheme and trade credit," *International Journal of Mathematics in Operational Research*, vol. 3, no. 6, pp. 658-679, 2011.

[31] A. A. Taleizadeh, D. W. Pentico, M. S. Jabalameli, M. and Aryanezhad, "An economic order quantity model with multiple partial prepayments and partial backordering," *Mathematical and Computer Modelling*, vol. 57, no. 3-4, pp. 311-323, 2013.

[32] N. P. Zia, A. A. and Taleizadeh, "A lot-sizing model with backordering under hybrid linked-to-order multiple advance payments and delayed payment," *Transportation Research Part E: Logistics and Transportation Review*, vol. 82, pp. 19-37, 2015.

[33] A. A. Taleizadeh, "An EOQ model with partial backordering and advance payments for an evaporating item," *International Journal of Production Economics*, vol. 155, pp. 185-193, 2014.

[34] J. T. Teng, L. E. Cárdenas-Barrón, H. J. Chang, J. Wu, and Y. Hu, "Inventory lot-size policies for deteriorating items with expiration dates and advance payments," *Applied Mathematical Modelling*, vol. 40, no. 19-20, pp. 8605-8616, 2016.

[35] M. Lashgari, A. A. Taleizadeh, and S. J. Sadjadi, "Ordering policies for non-instantaneous deteriorating items under hybrid partial prepayment, partial trade credit and partial backordering," *Journal of the Operational Research Society*, vol. 69, no. 8, pp. 1167-1196, 2018.

[36] P. Mahata, G. C. Mahata, and A. Mukherjee, "An ordering policy for deteriorating items with price-dependent iso-elastic demand under permissible delay in payments and price inflation," *Mathematical and Computer Modelling of Dynamical Systems*, vol. 25, no. 6, pp. 575-601, 2019.

[37] D. Barman, G. C. Mahata, and B. Das, "Advance payment based vendor–buyer production inventory model with stochastic lead time and continuous review policy," *Opsearch*, vol. 58, no. 4, pp. 1217-1237, 2021.

[38] S. K. Goyal, "Economic order quantity under conditions of permissible delay in payments," *Journal of the operational research society*, pp. 335-338, 1985.

[39] L. Y. Ouyang, C. T. Chang, and J. T. Teng, "An EOQ model for deteriorating items under trade credits," *Journal of the Operational Research Society*, vol. 56, no. 6, pp. 719-726, 2005.

[40] D. J. Mohanty, R. S. Kumar, and A. Goswami, "Trade-credit modeling for deteriorating item inventory system with preservation technology under random planning horizon," *Sādhanā*, vol. 43, no. 3, pp. 1-17, 2018.

[41] A. H. Md Mashud, M. Hasan, H. M. Wee, and Y. Daryanto, "Non-instantaneous deteriorating inventory model under the joined effect of trade-credit, preservation technology and advertisement policy," *Kybernetes: The International Journal of Systems and Cybernetics*, vol. 49, no. 6, pp. 1645-1674, 2019.

[42] A. H. M. Mashud, M. R. Hasan, Y. Daryanto, and H. M. Wee, "A resilient hybrid payment supply chain inventory model for post Covid-19 recovery," *Computers and Industrial*

Engineering, vol. 157, p. 107249, 2021.

[43] R. Lotfi, K. Kheiri, A. Sadeghi, and E. Babaee Tirkolaee, "An extended robust mathematical model to project the course of COVID-19 epidemic in Iran," *Annals of Operations Research*, pp. 1-25, 2022.

[44] S. Khalilpourazari, H. H. Doulabi, A. Ö. Çiftçioğlu, and G. W. Weber, "Gradient-based grey wolf optimizer with Gaussian walk: Application in modelling and prediction of the COVID-19 pandemic," *Expert Systems with Applications*, vol. 177, p. 114920, 2021.

[45] S. Khalilpourazari, and H. Hashemi Doulabi, "Designing a hybrid reinforcement learning based algorithm with application in prediction of the COVID-19 pandemic in Quebec," *Annals of Operations Research*, vol. 312, no. 2, pp. 1261-1305, 2022.

[46] A. Özmen, E. Kropat, and G. W Weber, "Robust optimization in spline regression models for multi-model regulatory networks under polyhedral uncertainty," *Optimization*, vol. 66, no. 12, pp. 2135-2155, 2017.

[47] H. Barman, M. Pervin, S. K. Roy, and G. W. Weber, "Back-ordered inventory model with inflation in a cloudy-fuzzy environment," *Journal of Industrial and Management Optimization*, vol. 17, no. 4, p. 1913, 2021.

[48] M. Pervin, and S. K. Roy, "Fuzzy Inventory Model with Demand, Deterioration and Inflation: A Comparative Study Through NGTFN and CNTFN," *In Advances in Applied Mathematical Analysis and Applications*, River Publishers, pp. 113-137, 2022.

[49] R. Singh, and V. K. Mishra, "Sustainable Integrated Inventory Model for Substitutable Deteriorating Items Considering Both Transport and Industry Carbon Emissions," *Journal of Systems Science and Systems Engineering*, pp. 1-21, 2022.

[50] A. Paul, M. Pervin, S. K. Roy, N. Maculan, and G. W. Weber, "A green inventory model with the effect of carbon taxation," *Annals of Operations Research*, vol. 309, no. 1, pp. 233-248, 2022.