Yugoslav Journal of Operations Research
33 (2023), Number 3, 467-480
DOI: https://doi.org/10.2298/YJOR220915005D

UNRAVELLING THE ASSIGNMENT PROBLEM UNDER INTUITIONISTIC TRIANGULAR FUZZY ENVIRONMENT BY THE NOVEL HEURISTIC DHOUIB-MATRIX-AP1

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Received: September 2022 / Accepted: February 2023

Abstract: The Assignment Problem (AP) can be stated as *n* activities to be assigned to *n* resources in such a way that the overall cost of assignment is minimized and each activity is assigned to one and only one resource. In real-life, the parameters of the AP are presented as uncertain numbers due to the lack of knowledge, experiences or any other (internal or external) factor. In this paper, the AP is considered under intuitionistic triangular fuzzy number and solved by the novel constructive heuristic Dhouib-Matrix-AP1 (DM-AP1) with a time complexity of *O(n)*. Actually, this paper presents the first enhancement of the novel heuristic DM-AP1 to solve the AP under intuitionistic triangular fuzzy environment. DM-AP1 is composed of three simple steps: computing the total cost, selecting the highest value and finding the minimal element. These steps are repeated in *n* iterations with the use of a standard deviation statistical metric. Two case studies of AP under intuitionistic triangular fuzzy set are taken from the literature and a step-by-step application of the novel DM-AP1 heuristic is presented for more clarification.

Keywords: Operations research, combinatorial optimization, assignment problem, intuitionistic fuzzy set, Dhouib-Matrix, soft computing.

MSC: 90B80, 90C59, 03F55

1. INTRODUCTION

A simplified variant of the well-known Transportation Problem is the Assignment Problem (AP) where all supply and demand quantities are unique (only one) and the main of the objective function is to minimize the total cost of assigning objects (Tasks, Jobs) to other objects (Machines, Workers) so that each object is allocated to only one resource. Various optimization methods have been proposed to solve the AP: The Hungarian method [1], the Linear Programming [2], the Kurtzberg’s method [3], the Auction Algorithm [4], the Efficient Cost Scaling Algorithm [5], the Neural Network [6], the Genetic Algorithm

[7], the Simulated Annealing [8], the Tabu Search Algorithm [9], the Artificial Bee Colony [10], the Ant Colony Optimization [11], the Water Drops Algorithm [12].

In this field, we designed a new constructive heuristic in [13] named Dhouib-Matrix-AP1 (DM-AP1) composed of three simple steps (see Figure 1) repeated n iterations (where n is the number of objects to be assigned). Actually, DM-AP1 requires a polynomial time complexity of O(n) whereas the Hungarian method needs a time complexity of O(n3). Moreover, DM-AP1 is designed regardless the objective function (not as the Hungarian method which is dedicated for the linear sum objective function). Furthermore, DM-AP1 is helpful for the unbalanced Assignment Problem (with constraint on the agents) which outperforms the Hungarian method [14]. Another main advantage of DM-AP1 is its flexibility to use different statistical metrics (Min, Max, Average, Standard Deviation, etc.) in order to generate several initial basic feasible solutions (in this paper the standard deviation metric is used).

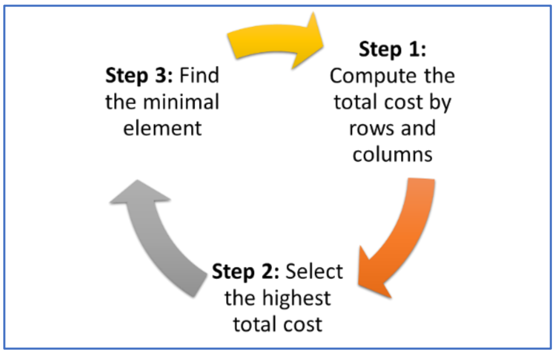

Figure 1: The three simple steps of the DM-AP1 heuristic

In this paper, the novel DM-AP1 method is enhanced to solve the AP under intuitionistic triangular fuzzy environment where the cost parameters are presented as intuitionistic triangular fuzzy numbers (depending on external or internal factors: knowledge, experience and education, etc.). In fact, the intuitionistic fuzzy logic was introduced by [15] in order to wider the concept of fuzzy logic proposed by [16]. The intuitionistic concept was used in several fields: the Travelling Salesman Problem [17], the Classical linear sum Assignment Problem [18, 19], the Bottleneck Assignment Problem [20], the Transportation Problem [21], the Knapsack Problem [22], the Forecasting [23], the Cancer Mediating Biomarkers Recognition [24], the Cloud Vendor Selection [25, 26] and the Station Location Selection [27]. Also, the intuitionistic concept was used for the medical diagnostic [28, 29], the pattern recognition [30], the soft graphs [31], the cycles and trees [32], the education [33] and the answer set programming [34].

Indeed, the main contribution of this paper is the introduction of a new methodology to solve the Assignment Problem under intuitionistic triangular fuzzy environment through the adaptation of the novel DM-AP1 heuristic. This paper is structured as follows: In Section 2, some basic definitions of the intuitionistic triangular fuzzy set are introduced. In section 3, the proposed method DM-AP1 is described and in section 4 two case studies are step-by-step presented for more clarification. Finally, the conclusion and further research are presented.

2. BASIC DEFINITIONS

2.1. Definition 1

Let X be a nonempty set. A fuzzy set $\tilde{E}$ in X is defined as $\tilde{E} = \{\langle x, \mu_{\tilde{E}}(x)\rangle; x \in X\}$. Where $\mu_{\tilde{E}}(x)$ is called membership function which maps each element of X to a value between 0 and 1.

2.2. Definition 2

A fuzzy number $E = (e_1, e_2, e_3)$ is defined to be a triangular fuzzy number (where $e_1 \le e_2 \le e_3$) if its membership functions $\mu_E(x) : \mathbb{R} \to [0,1]$ is equal to

$$\mu_{\tilde{E}}(x) = \begin{cases} 0 & \text{for } x < e_1 \\ \dfrac{x - e_1}{e_2 - e_1} & \text{if } e_1 \le x \le e_2 \\ 1 & \text{if } x = e_2 \\ \dfrac{e_3 - x}{e_3 - e_2} & \text{if } e_2 \le x \le e_3 \\ 0 & \text{for } x > e3 \end{cases}$$

2.3. Definition 3

An intuitionistic fuzzy set, $\tilde{E}^I = \{\langle x, \mu_{\tilde{E}^I}(x), \vartheta_{\tilde{E}^I}(x)\rangle; x \in X\}$ is characterized by its two membership function: the degree of membership function $\mu_{\tilde{E}^I}(x)$ and the degree of non-membership function $\vartheta_{\tilde{E}^I}(x)$ of the element x (where $\mu_{\tilde{E}^I}(x)$, $\vartheta_{\tilde{E}^I}(x)$ is a function from X to [0,1] such that $0 \le \mu_{\tilde{E}^I}(x) + \vartheta_{\tilde{E}^I}(x) < 1, \forall x \in X$).

2.4. Definition 4

An intuitionistic fuzzy numbers $\tilde{E}^I = (e_1, e_2, e_3)(e_1^{'}, e_2, e_3^{'})$ is called to be intuitionistic triangular fuzzy number if its membership function is of the form:

$$\mu_{\tilde{E}^I}(x) = \begin{cases} 0 & \text{for } x < e_1 \\ \dfrac{x - e_1}{e_2 - e_1} & \text{if } e_1 \le x \le e_2 \\ 1 & \text{if } x = e_2 \\ \dfrac{e_3 - x}{e_3 - e_2} & \text{if } e_2 \le x \le e_3 \\ 0 & \text{for } x > e3 \end{cases} \quad \text{and} \quad \vartheta_{\tilde{E}^I}(x) = \begin{cases} 1 & \text{for } x < e_1^{'} \\ \dfrac{e_2 - x}{e_2 - e_1^{'}} & \text{if } e_1^{'} \le x \le e_2 \\ 0 & \text{if } x = e_2 \\ \dfrac{x - e_2}{e_3^{'} - e_2} & \text{if } e_2 \le x \le e_3^{'} \\ 1 & \text{for } x > e_3^{'} \end{cases}$$

2.5. Definition 5

If $\tilde{E}^I = (e_1, e_2, e_3)(e_1^{'}, e_2, e_3^{'})$ is an intuitionistic triangular fuzzy number then the ranking of $\tilde{E}^I$ can be given using the ranking function (Equation 1) developed by Varghese and Kuriakose in [34]:

$$\Re\left(\tilde{E}^I\right) = \frac{1}{3}\left[\frac{\left(e_3^{'} - e_1^{'}\right)\left(e_2 - 2e_3^{'} - 2e_1^{'}\right) + \left(e_3 - e_1\right)\left(e_1 + e_2 + e_3\right) + 3\left(e_3^{'2} - e_1^{'2}\right)}{e_3^{'} - e_1^{'} + e_3 - e_1}\right] \quad (1)$$

For example, the intuitionistic triangular fuzzy number $\tilde{E}^I = (7, 21, 29)(2, 21, 34)$ is converted to a crisp number as follows:

$$\Re\left(\tilde{E}^I\right) = \frac{1}{3}\left[\frac{(34-2)(21-2*34-2*21) + (29-7)(7+21+29) + 3\left(34^2 - 2^2\right)}{34 - 2 + 29 - 7}\right].$$

2.6. Definition 6

The Assignment Problem (AP) can be specified as *n* objects (activities) to be allocated to *n* other objects (resources) where the overall cost of assignment is minimized and each activity is assigned to one and only one resource. The mathematical formulation of the intuitionistic fuzzy assignment problem can be presented as:

$$Min\ \tilde{Z}^I = \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{c}_{ij}^I x_{ij}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = 1, \quad \text{for } i = 1, 2, ..n$$

$$\sum_{i=1}^{n} x_{ij} = 1, \quad \text{for } j = 1, 2, ..n$$

Where

$$\begin{cases} x_{ij} = 1 & \text{if the } i^{th} \text{ job is assigned to the } j^{th} \text{ machine} \\ x_{ij} = 0 & \text{Otherwise} \end{cases}$$

Where *i* and *j* are respectively the index of objects (for examples jobs) to be assigned to other objects (for example machine). Moreover, a binary variable x_{ij} is used and object *i* is allocated to object *j* then $x_{ij} = 1$ otherwise $x_{ij} = 0$. Furthermore, the cost $\tilde{c}_{ij}^I$ is the intuitionistic fuzzy cost of assigning object *i* to object *j*.

3. THE PROPOSED METHOD: DHOUIB-MATRIX-AP1

The Dhouib-Matrix-AP1 (DM-AP1) method was firstly designed and developed under Python programming language in [13] in order to solve the classical Assignment Problem.

Next, an application of the DM-AP1 to solve the trapezoidal fuzzy AP is presented in [36]. Moreover, a new stochastic variant of DM-AP1 entitled DM-AP2 is developed in [14] in order to optimize the unbalanced assignment problem where the constraint is on the agents (more realistic in real world where the number of resources is less than the number of jobs).

DM-AP1 and DM-AP2 are a component of the general concept of Dhouib-Matrix (DM) where several new optimization methods are designed. For the Travelling Salesman Problem, the deterministic Dhouib-Matrix-TSP1 and the stochastic Dhouib-Matrix-TSP2 heuristics are developed [37, 38, 39, 40, 41]. Concerning the Transportation Problem, the Dhouib-Matrix-TP1 technique is designed in [42, 43]. Regarding the Shortest Path Problem, a new exact method entitled Dhouib-Matrix-SPP (DM-SPP) is intended in [44]. DM-SPP presents a complexity of $O(n+m)$ where n is the number of vertices and m is the number of edges. This new method is concurrent to the Dijkstra method for complete graphs and outperforms Dijkstra for incomplete graphs.

Besides, three novel metaheuristics are designed in the field of DM: 1) the iterated stochastic Dhouib-Matrix-3 (DM3) metaheuristic [45, 46, 47], 2) the local search Far-to-Near (FtN) method [48] and 3) the multi-start Dhouib-Matrix-4 (DM4) method [49, 50, 51].

In this paper, DM-AP1 is applied for the classical Assignment Problem where all input data are presented as intuitionistic triangular fuzzy numbers. DM-AP1 is composed of one iterative structure gathering three simple steps repeated in n iterations (see Figure 2).

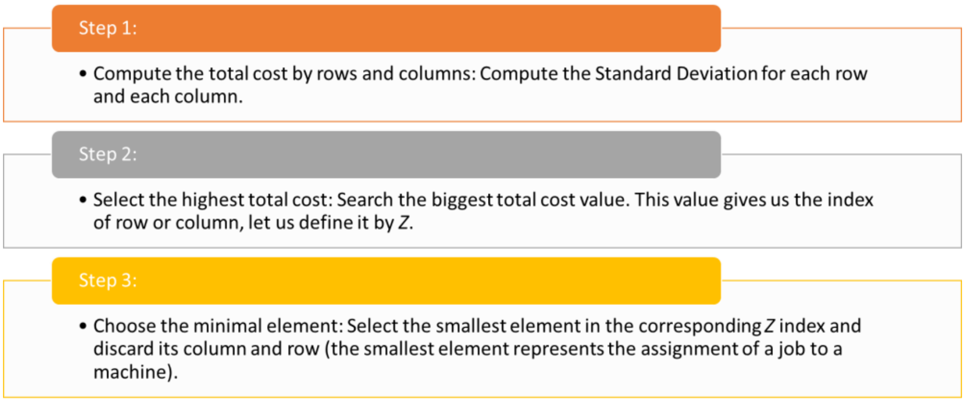

Figure 2: The three steps of the DM-AP1 method

The proposed method DM-AP1 is a constructive heuristic (the algorithm of DM-AP1 is presented in Figure 3), its aim is just to generate an initial basic feasible solution. Thus, for larger instances an iterative structure is required and the DM-AP1 will be used as a good starting solution for any metaheuristic (such as the novels DM3 [44] or DM4 [46] methods). Moreover, different descriptive statistical metrics can be used to generate different initial basic feasible solutions: Thus, DM-AP1 will be nicely used as diversification technique for the DM3 or DM4 metaheuristics.

The advantages of the novel DM-AP1 method are that:

- Only one iterative structure with three simple steps.
- The solution is generated after only n iterations: the computational time is $O(n)$.

- It can be expanded to solve other variants of AP under certain or uncertain environment.

Algorithm of Dhouib-Matrix-AP1 (DM-AP1)

Input: Assignment matrix, *n* = number of tasks
Output: Optimal or near optimal assignment network plan
1. Initialize *S=∞*
2. Set *Assignment-Plan* = {}
3. For *i*=1 to *n*
4. Compute the total cost for each row and each column using the *Standard Deviation* function.
5. Select the highest element (*Z*) from the highest total cost element.
6. Choose the minimal element (at the position d_{xy}) in the corresponding *Z*.
7. Insert *x* and *y* in the list *Assignment-Plan*
8. Discard row *x* and column *y*
9. Compute *S* from *Assignment-Plan*
10. Return *S and Assignment-Plan*

Figure 3: The algorithm of the proposed DM-AP1 method

4. NUMERICAL EXAMPLES

Two numerical case studies are presented in the next subsections in order to give more clarification about the application of DM-AP1: the first one deals about assigning three workers to three jobs and the second case study describes the assignment of four jobs to four machines.

4.1. Illustrative case study 1

Let us consider the problem of TV manufacturing (taken from [19]) with three workers (W_1, W_2 and W_3) and three jobs (J_1: assembling, J_2: testing and J_3: packing). Only one job is assigned to each worker with an intuitionistic triangular fuzzy performing time (in hours) and the deal is to find the assignment plan that minimizes the total working time. Figure 4 depicts the 3×3 assignment matrix.

$$\begin{bmatrix} (7,21,29)(2,21,34) & (7,20,57)(3,20,61) & (12.25.56)(8,25,60) \\ (8,9,16)(2,9,22) & (4,12,35)(1,12,38) & (6,14,28)(3,14,31) \\ (5,9,22)(2,9,25) & (10,15,20)(5,15,25) & (4,16,19)(1,16,22) \end{bmatrix}$$

Figure 4: The intuitionistic triangular fuzzy performance time

Iteration 1: Before starting DM-AP1 the intuitionistic triangular fuzzy numbers are transformed to crisp ones using Equation (1). Next, compute the standard deviation for each row and column and select the highest value (7.8) which is at the third column. Now, select the smallest element in the third column (13) at position d_{33} (assign worker W_3 to job J_3) and discard row 3 and column 3 (see Figure 5).

Iteration 2: Again, compute the standard deviation for each row and column, select the highest value (5.5) which is at the second column and find the smallest element at the second column (17) at position d_{22} (assign worker W_2 to job J_2) and discard row 2 and column 2 (see Figure 6).

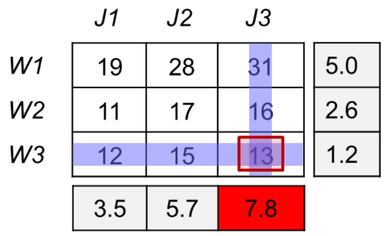

Figure 5: Assign worker (W3) to job (J3)

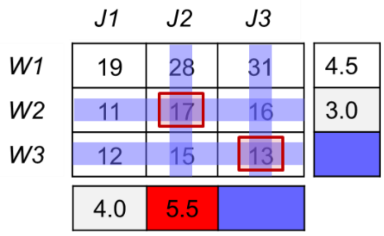

Figure 6: Assign worker (W2) to job (J2)

Iteration 3: Besides, compute the standard deviation for each row and column (0.0), select the highest value (0.0) and choose its corresponding smallest element d_{11} (assign worker W_1 to job J_1) and discard row 1 and column 1 (see Figure 7).

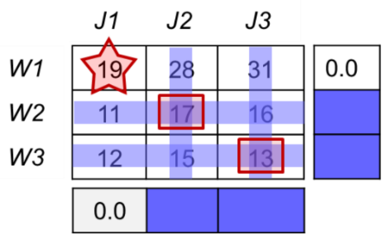

Figure 7: Assign worker (W1) to job (J1)

Thus, the optimal solution is 49 and graphically depicted in Figure 8 for W_1 -> J_1, W_2 - > J_2 and W_3 -> J_3 ($x^*_{11} = x^*_{22} = x^*_{33} = 1$). The corresponding intuitionistic triangular fuzzy value is calculated via the crisp optimal solution:

$$(7, 21, 29)(2, 21, 34)x^*_{11} + (4, 12, 35)(1, 12, 38)x^*_{22} + (4, 16, 19)(1, 16, 22)x^*_{33} = (15, 49, 83)(4, 49, 94)$$

In the above example, the solution generated by DM-AP1 (after n=3 iterations) remains same as that obtained by [19] using the PSK method and by Hungarian method but with a more complicated process and iterations: the Hungarian method requires (n^3=3^3=27) iterations.

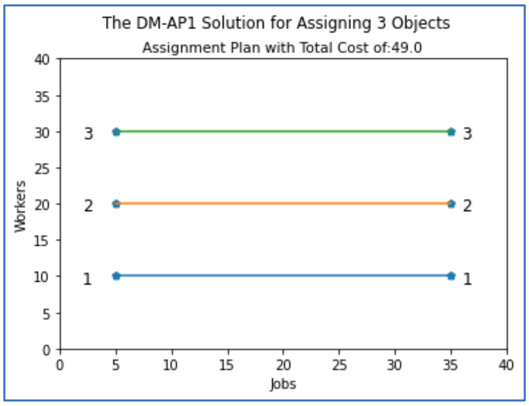

Figure 8: The assignment networks plan for the 3x3 matrix

The optimal intuitionistic triangular fuzzy solution is graphically represented in Figure 9. The degree of acceptance ($\mu_{\tilde{E}^I}(x)$) beyond [15, 83] hours is totally unacceptable for the decision maker and will be totally satisfied with the total assignment time value of 49 hours (the degree of acceptance increases from 14 to 49 hours and then decreases until 83 hours). Similarly, the degree of non-acceptance ($\vartheta_{\tilde{E}^I}(x)$) is totally unacceptable beyond [4, 49], fully acceptable at 49 hours, decreases from 4 to 49 hours and increases from 49 to 94 hours.

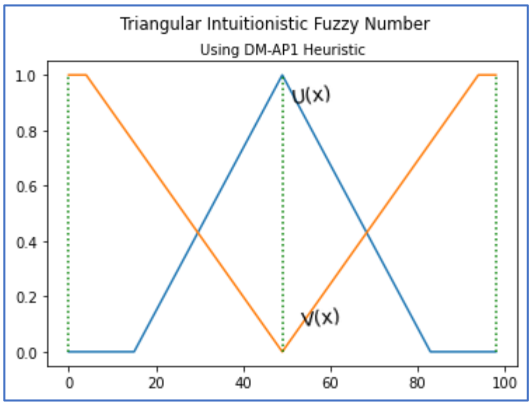

Figure 9: Graphical representation of the minimal total intuitionistic cost using the DM-AP1 heuristic

$$\mu_{\tilde{E}^I}(x) = \begin{cases} 0 & \text{for } x < 15 \\ \dfrac{x-15}{34} & \text{if } 15 \le x \le 49 \\ 1 & \text{if } x = 49 \\ \dfrac{83-x}{34} & \text{if } 49 \le x \le 83 \\ 0 & \text{for } x > 83 \end{cases} \quad \text{and} \quad \vartheta_{\tilde{E}^I}(x) = \begin{cases} 1 & \text{for } x < 4 \\ \dfrac{49-x}{45} & \text{if } 4 \le x \le 49 \\ 0 & \text{if } x = 49 \\ \dfrac{x-49}{45} & \text{if } 49 \le x \le 94 \\ 1 & \text{for } x > 94 \end{cases}$$

4.2. Illustrative case study 2

Let us consider the problem of assigning four jobs (J_1, J_2, J_3 and J_4) to four machines (M_1, M_2, M_3 and M_4) (taken from [19]). Figure 10 depicts the intuitionistic triangular fuzzy number.

$$\begin{bmatrix} (27,50,109)(13,50,123) & (56,67,111)(40,67,127) & (8,22,120)(4,22,124) & (75,100,128)(62,100,141) \\ (52,68,93)(44,68,101) & (43,90,119)(35,90,127) & (34,56,93)(18,56,109) & (60,70,89)(50,70,99) \\ (72,80,109)(58,80,123) & (78,90,108)(65,90,121) & (85,98,150)(76,98,159) & (52,68,93)(44,68,101) \\ (23,40,81)(17,40,87) & (44,58,90)(38,58,96) & (63,89,109)(49,89,123) & (64,72,95)(51,72,108) \end{bmatrix}$$

Figure 10: The intuitionistic triangular fuzzy matrix

Iteration 1: At first, transform the intuitionistic triangular fuzzy numbers to crisp ones using equation (1). Next, compute the standard deviation for each row and column and select the highest value (23.6) which is at the third column. Then, find the smallest element in the third column (50) at position d_{13} (assign job J_3 to machine M_1) and discard row 1 and column 3 (see Figure 11).

	J1	J2	J3	J4	
M1	62	78	50	101	19.0
M2	71	84	61	73	8.1
M3	87	92	111	71	14.2
M4	48	64	87	77	14.6
	14.1	10.2	23.6	12.0	

Figure 11: Assign job (J3) to machine (M1)

Iteration 2: Next, compute the standard deviation for each row and column, select the highest value (16) which is at the first column and find the smallest element at the fourth column (48) at position d_{41} (assign job J_1 to machine M_4) and discard row 4 and column 1 (see Figure 12).

	J1	J2	J3	J4	
M1	62	78	50	101	
M2	71	84	61	73	5.7
M3	87	92	111	71	8.9
M4	48	64	87	77	11.8
	16.0	11.7		2.4	

Figure 12: Assign job (J1) to machine (M4)

Iteration 3: Hence, compute the standard deviation for each row and column, select the highest value (10.5) which is at the third row and find the smallest element at the fourth column (71) at position d_{34} (assign job J_4 to machine M_3) and discard row 3 and column 4 (see Figure 13).

	J1	J2	J3	J4	
M1	62	78	50	101	
M2	71	84	61	73	5.5
M3	87	92	111	71	10.5
M4	48	64	87	77	
		4		1	

Figure 13: Assign job (J4) to machine (M3)

Iteration 4: Moreover, compute the standard deviation for each row and column (0.0), select the highest value (0.0) and select its corresponding smallest element d_{22} (assign job J_2 to worker W_2) and discard row 2 and column 2 (see Figure 14).

	J1	J2	J3	J4	
M1	62	78	50	101	
M2	71	84	61	73	0.0
M3	87	92	111	71	
M4	48	64	87	77	
		0.0			

Figure 14: Assign job (J2) to machine (M2)

Thus, DM-AP1 needs only (n = 4) simple iterations to generate the assignment network (graphically depicted in Figure 15): J_1 -> M_3, J_2 -> M_2, J_3 -> M_4 and J_4 -> M_1 so $(x^*_{13} = x^*_{22} = x^*_{34} = x^*_{41} = 1)$. This solution is the optimal one (253), also found by the Hungarian method but with a more complicated process and iterations (the Hungarian method requires (n^3=4^3=64) iterations). Also, the corresponding intuitionistic triangular fuzzy value can be calculated via the crisp optimal solution:

$$(8,22,120)(4,22,124)x^*_{13} + (43,90,119)(35,90,127)x^*_{22} + (52,68,93)(44,68,101)x^*_{34} +$$
$$(23,40,81)(17,40,87)x^*_{41} = (126,220,413)(100,220,439)$$

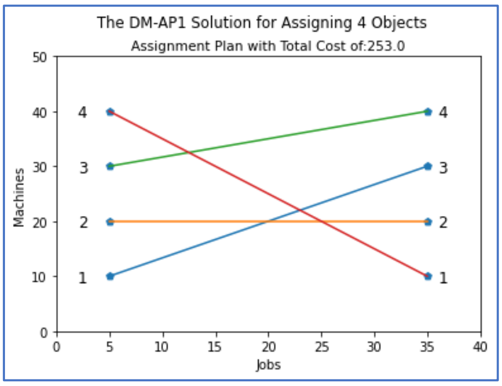

Figure 15: The assignment network plan for the 4x4 matrix

The optimal intuitionistic triangular fuzzy solution is graphically represented in Figure 16. The degree of acceptance $(\mu_{\tilde{E}^I}(x))$ beyond [126, 413] is totally unacceptable for the decision maker and will be totally satisfied with the total assignment time value of 220 (the degree of acceptance increases from 126 to 220 and then decreases until 413 hours). Similarly, the degree of non-acceptance $((\vartheta_{\tilde{E}^I}(x)))$ is totally unacceptable beyond [100, 439], fully acceptable at 220, decreases from 100 to 220 and increases from 220 to 439.

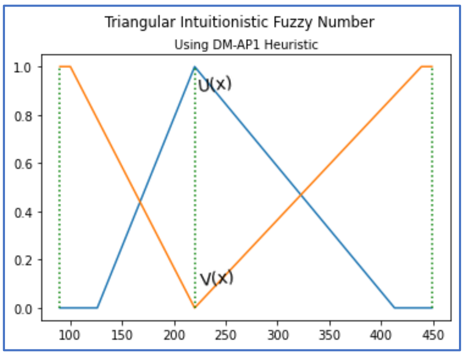

Figure 16: Graphical representation of the minimal total intuitionistic cost using the DM-AP1 heuristic

$$\mu_{\tilde{E}^I}(x) = \begin{cases} 0 & \text{for } x < 126 \\ \dfrac{x-126}{94} & \text{if } 126 \le x \le 220 \\ 1 & \text{if } x = 220 \\ \dfrac{413-x}{193} & \text{if } 220 \le x \le 413 \\ 0 & \text{for } x > 413 \end{cases} \quad \text{and} \quad \vartheta_{\tilde{E}^I}(x) = \begin{cases} 1 & \text{for } x < 100 \\ \dfrac{220-x}{120} & \text{if } 100 \le x \le 220 \\ 0 & \text{if } x = 220 \\ \dfrac{x-220}{219} & \text{if } 220 \le x \le 439 \\ 1 & \text{for } x > 439 \end{cases}$$

3. CONCLUSION

In this paper, the Assignment Problem is considered under intuitionistic triangular fuzzy sets and for the first time the constructive heuristic Dhouib-Matrix-AP1 (DM-AP1) is proposed to solve it with a time complexity of $O(n)$. In fact, DM-AP1 presents only one iterative structure with three simple steps and using the standard deviation statistical metric. The performance of the DM-AP1 heuristic to solve Assignment Problem under intuitionistic triangular fuzzy environment is proved via two case studies with a step-by-step applications.

The major advantage of DM-AP1 is its rapidity to generate an initial basic feasible solution (just n iterations) and its flexibility to use any differential statistical metric (several initial solutions can be generated by the use of different statistical metrics). However, DM-AP1 is a greedy deterministic method without any stochastic perturbation. Thus for further research, DM-AP1 will be combined with the novel multi-start metaheuristic Dhouib-Matrix-4 [46, 47, 48] in order to solve larger variants of the Assignment Problem. Further empirical studies have to be performed to evaluate the performance of the proposed method DM-AP1 for unraveling the Assignment Problem under multi-objective domain and neutrosophic environment. Moreover, other applications of DM-AP1 on different variants of the Assignment Problem need to be performed such as minimizing the bottleneck Assignment Problem or the generalized Assignment Problem.

Funding. This research received no external funding.

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