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A LITERATURE REVIEW ON RETRIAL QUEUEING SYSTEM WITH BERNOULLI VACATION

Nicholas Gnana Sekar MICHEAL MATHAVAVISAKAN
Department of Mathematics, School of Advanced Sciences,
Vellore Institute of Technology, Vellore 632 014, Tamil Nadu, India
michealmathava.visakan2020@vitstudent.ac.in

Kandaiyan INDHIRA
Department of Mathematics, School of Advanced Sciences,
Vellore Institute of Technology, Vellore 632 014, Tamil Nadu, India
kindhira@vit.ac.in

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Abstract: The retrial phenomenon occurs inherently in a wide range of queueing systems. The majority of retrial queueing models do not account for vacation. However, in practice, retrial queueing systems undergo vacations for maintenance or other reasons. In this study, we provide an in-depth analysis of the many possible retrial queueing systems when Bernoulli vacations are in effect. Moreover, this study outlines the key principles and reviews the relevant literature. The framework of a retrial queue with Bernoulli vacation has numerous applications in computer networking systems, manufacturing and production mechanisms, inventory systems, including network service, mail service and file transfer service, etc. Several retrial queueing systems have been investigated, notably $M/M/1$, $M/M/C$, $M/G/1$, $M^{[X]}/G/1$, and $Geo/G/1$. Many other important situations, such as server interruption, feedback, G-queue, impatient customers, priority customers, etc., have been explored in relation to retrial queues with Bernoulli vacation and the results of these investigations are also highlighted. The foremost objective of this study is to help researchers, administrators and technical workers who want to use queuing theory to simulate congestion and need to know where to find details on the right models. Finally, some open problems and potential future lines of survey are also covered.

Keywords: $M/M/1$ and $M/M/C$ retrial queue model, $M/G/1$ and $M^X/G/1$ retrial queue model, $Geo/G/1$ retrial queue model, Bernoulli vacation.

MSC: 60K25, 90B22, 60K30.

1. INTRODUCTION

The mathematical model for analyzing line congestion and delays is known as the queueing theory. Customers and servers form the basis of a queueing system. We often witness queues in our daily lives because the outcomes are used to determine the resources needed to provide service. A.K. Erlang (1878–1929), who is regarded as the "Father of Queuing Theory," was a pioneer in this field. His articles were mainly on the investigation of telephone traffic congestion, which paved the way for the introduction of efficient network theory and telephone network analysis.

For a long time, there seemed to be a substantial contribution to the retrial queueing systems. Consumer perception and the retrial idea may have a substantial influence on computer network performance, resulting in increased network complexity. The phenomenon of retrials is well known; the server can enter the orbital interaction of blocked subscribers and retry after an arbitrary duration of time if the server is occupied. The queueing system that includes such a retrial effect is called a retrial queue (RQ), which is shown in Figure 1.

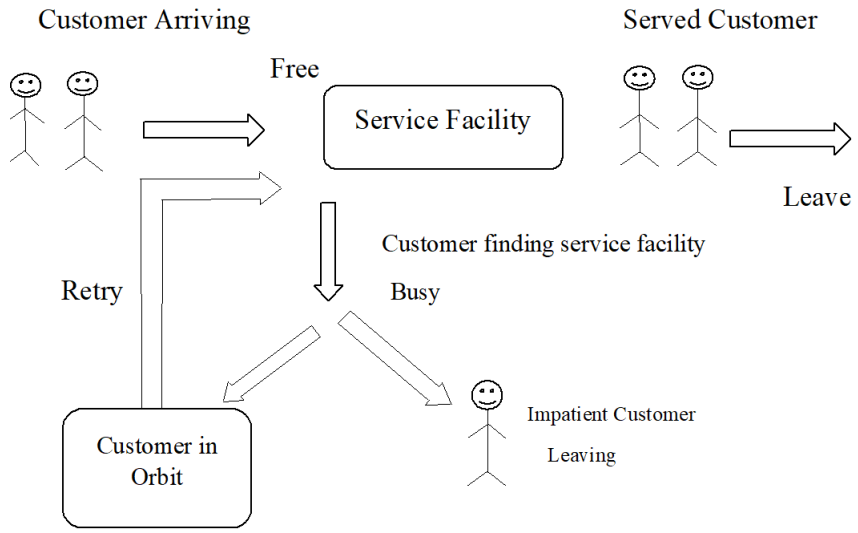

Figure 1: Retrial Queue

Many computer network structures, telephone switching structures and wireless network structures employ RQs as mathematical models. Peripheral devices in a computer networking structure may performs retrials in order to get service from a central processing unit (CPU). Hosts on local area networks (LANs) might attempt many retries to get connection to the network channel, which is expressly stated in the Carrier Sense Multiple Access (CSMA) protocol, which oversees this access.

Recently, vacation queues have been a major topic in queueing theory. In the RQ, there are several sorts of vacation models. In this study, we utilized the retrial queue with Bernoulli vacation, which is shown in Figure 2. A Bernoulli vacation (BV) is a service discipline that determines whether a server will take a vacation or serve the next consumer after completing a service. Here, we briefly review diverse sorts of arrival and service operations with various categories of Bernoulli vacation models that have been analyzed in the last few decades. This will be helpful for the readers who are working in the area of Bernoulli vacation queues.

The manufacturing industry also makes use of RQ with BV, namely in the production of Verteiler Ensprintz Pumps, when an order of VE pumps reaches the calibration bench. When one pump is being calibrated, the others can be assembled and the calibration bench can be used for periodic service. The retry structure also plays a significant role in manufacturing lines, like those seen in cookie and biscuit factories. In an average operation, raw materials arrive in batches, each of which is subjected to a primary inspection before being processed. Batches that are being sent to the server will have to wait if it is currently experiencing high demand and the server itself may also undergo maintenance as well.

Our article is outlined as follows: In section 2, we covered the queueing models with Bernoulli vacation. Section 3 reviews the $M/M/1$ and $M/M/C$ retrial queueing models with Bernoulli vacation. Furthermore, $M/G/1$ and $M^{[X]}/G/1$ retrial queueing models with Bernoulli vacation are discussed in section 4. Section 5 presents the recent work on $Geo/G/1$ retrial queueing models with Bernoulli vacation. Section 6 delves more into recent developments in Bernoulli vacation models. Section 7 indicates the recent advancements and future scope of the retrial queueing model. Finally, section 8 presents the conclusion and summary of the work.

1.1. Motivation and Scope

Due to their significance in numerous environments, retrial queues have been considered the subject of a great deal of discussion. There is a plethora of literature summarizing queueing systems in a variety of contexts. However, very few were on retrial queueing systems. Moreover, to maximize the server's potential and the system's efficiency while keeping costs to a minimum, the vacationing server function is an additional vital component. The server has the option of performing additional duties, like maintenance, during its vacation. Similarly, plenty of literature reviews investigating various queueing vacation frameworks have been presented. Still, a study on a retrial queueing system with vacation has not been published. In light of the relevance of the aforementioned occurrences, we present a literature review on retrial queueing system (QS) within a BV framework so as to help fill the research gap.

The retrial QS with BV, which is then evaluated in a wide variety of queueing scenarios, is the primary topic of this research review. Queueing models with BV, $M/M/1$ and $M/M/C$ RQ with BV, $M/G/1$ and $M^{[X]}/G/1$ RQ models with

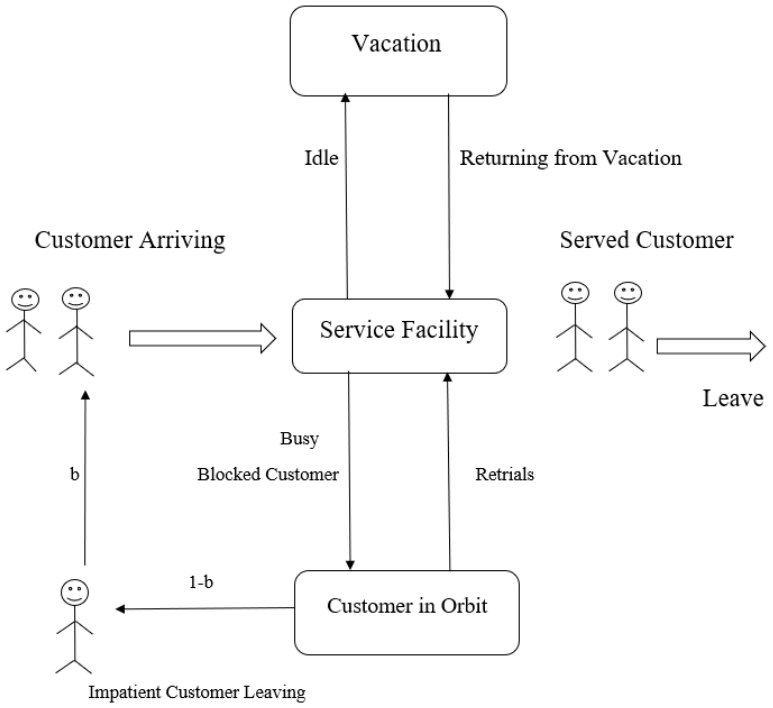

Figure 2: Retrial Queue with Bernoulli Vacation

BV, $Geo/G/1$ RQ models with BV and other BV models have been analyzed. Additionally, the most current development in retrial queues has been provided. In addition, the significant contributions of certain papers have been highlighted. Several suggestions for future research have also been made. Finally, we provide a discussion of several unsolved issues and intriguing research paths.

2. QUEUEING MODELS WITH BERNOULLI VACATION

Due to the broad variety of applications in real-world queueing systems, such as manufacturing and production systems, distribution and service systems, transportation systems, the telecommunications industry and computer networking systems, QS with vacation policies have received a lot of attention. Tsuyoshi Katayama [1] studied a novel class of multi-class priority queues with three controllable parameters (P_1, P_2, P_N) and they functions as follows: Once a message of class n has been served, the system will either serve the next message in queue n with prob., P_n $(1 \geq P_n \geq 0)$ or the highest class message present in the system with prob., $1 - P_n$. In addition, he explored many class prime queues with

general parameters and server vacations using the probability generating function (PGF) technique. Finally, he presented essential recursive equations for obtaining the moments of the expected waiting distribution for every class in a sequential manner. Atencia and Moreno [2] explored a single-server retrial queue (SSRQ) system with an unlimited loading, Poisson arrivals, generic service time distribution and a linear retrial strategy. In the steady state distribution (SSD), they discovered the ergodicity of the embedding Markov chain (EMC), its stablility and the combined generating procedure of the group of consumers in each categories. Furthermore, they found the system size distribution function, which extrapolates the Pollaczek-Khinchin (PK) formula. A single server (SS) bulk queueing system with an unstable server, formative assessments, N-policy, Bernoulli scheduling and a stand-by server was analyzed by Ayyappan and Karpagam [3]. Working breakdowns and multi-stage service may be added to this system to make it even more useful. Based on their numerical analysis, they discovered that when the service rate increases, the behaviour metrics decreases. As a result, they culminated that having a standby machine in a production system has been extremely beneficial in reducing huge losses. Under a Bernoulli-schedule (BS) controlled strategy, Tao Jiang and Baogui Xin [4] examined a SS queueing system with functioning breakdowns and postponed repair. They obtained the SSD for easy process of behaviour metrics and the Laplace-Stieltjes transform (LST) of an arbitrary customer's stationary sojourn time using the matrix analysis technique and the spectral expansion method, respectively. Ayyappan and Gowthami [5] discussed a two-server queuing system with a Markovian arrival process (MAP) arrival rate and a phase type (PH)-distribution service rate for each servers, as well as BV, Bernoulli feedback (BF) and customer reneging. Additionally, they use the matrix-analytic approach and analyzed the resulting (quasi-birth-and-death) QBD process when it is stable. Moreover, they examined the system's busy times and waiting times.

Madhu Jain and Sandeep Kaur [6] explored a queueing model with a (p, N)-policy to estimate various behaviour of the QS by taking into account a variety of genuine aspects such as balking, BF, vacation, bulk input, optional phase service, unconvincing server and so on. Amina Angelika Bouchentouf et al. [7] studied a SS Markovian queueing model with BF, K-variant working vacation policy, balking, reneging and customer retention for reneged customers in which the server must wait until the end of the ordinary busy period before going on vacation. In addition, the theorems relating to stochastic decomposition are proved. Moreover, a quadratic fit search method (QFSM) is used to optimize costs. The proposed model is inspired by its widespread applicability in various real-time systems, such as computer and communication networks. Chandra Shekhara et al. [8] worked on a multi-server queueing (MSQ) model with a finite capacity, a reneged customer retention policy and a Bernoulli planned modified vacation policy. This study may be expanded by including other aspects such as bulk arrival and synchronized vacation and it can also be modeled for the problem of machine interference. Sijia Liu et al. [9] developed a $K - out - of - N$ retrial model with mixed standby elements and BV of a single repairman. The Vector Markov process and Laplace transform theory were used to derive formulas for SSD, reliability function and

average period to system initial failure. Moreover, the sensitivity analysis and a relative sensitivity analysis were also presented. Ayyappan and Gowthami [10] described a traditional queueing system with a MAP arrival rate, SS, a PH-service rate, vacation, setup time, failure, repair, feedback, balking and reneging. The slowdown in server performance was taken into account by Choudhary et al. [11]. After a certain number of services have been completed, vacation time is allocated for maintenance purposes. When the maintenance is complete, the server will resume operations in its original service rate. Their model is more realistic because it takes into account consumer impatience and server unreliability. Ayyappan and Karpagam [12] studied a bulk queue with an unreliable server, instantaneous feedback, N-policy, stand-by server and multiple instances of the Bernoulli schedule. The bulk arrival, fixed batch service queue with unreliable server, Bernoulli vacation and delay time was studied by Kalyanaraman and Nagarajan [13]. In addition, some measurements of the model's performance in different server states have been obtained using the PGF method. The model has been compared to previously established ones by making assumptions about the distributions of the random variables. Under Bernoulli vacation schedule and N-policy, Begum and Choudhury [14] addressed a situation in which a single server queue handles a large influx of customers and provides two types of heterogeneous optional repeating service with delayed repair. Additionally, by designing an appropriate linear cost structure of the underlying model and employing a difference operator, they obtained a locally optimal N-policy at a lower cost. Rajendiran and Kandaiyan [15] investigated the effects of Bernoulli vacation on a batch arrival feedback queueing system featuring balking and two stages of variable service with distinct service levels.

3. AN $M/M/1$ AND $M/M/C$ RETRIAL QUEUEING MODELS WITH BERNOULLI VACATION

The phenomena of an incoming consumers finding the service busy upon arriving, joining the virtual area known as orbit and retrying for service at a later arbitrary duration of time is described as the retrial phenomena. Because of the possible real-time applications in telephone services, computers and network of information, the analysis of RQs has become more important. Keilson and Servi [16] were the first to use the Bernoulli approach to create a vacation queue. They worked on a system wherein a server either takes a vacation right after a service has been done if the system is free or if there are clients in the system, it keep working on. If the queue is empty after the vacation finishes, the server switches to busy mode and waits for new clients. In real life, a server may be needed to undertake extra duties after service, for example system administration, troubleshooting or simply being out of commission on a regular basis.

Lakshmi and Kasturi Ramanath [17] used matrix geometric approaches (MGM) to model the conduct of an Interactive Voice Response System (IVRS). Using the truncation method they investigated the system's stability and further investigated the level-dependent QBD model in order to derive expressions for the system's per-

formance metrics. In a SSRQ with BV, arrivals with a large number of consumers inside the orbit was analyzed by Ke Sun and Jinting Wang [18] with blocked customers' equilibrium joining behaviour. Using a vacation interruption policy the varied balking possibilities of customers in busy regular and working vacation states was produced by Poonam Gupta and Naveen Kumar [19]. Furthermore, they determined the greatest value for a reduced service charge during the working vacation period by making use of the parabolic method. This concept may be used in a diverse network and telecommunications systems.

Multi-server retrial queues (MSRQ) (shown in Figure 3) are way more complex to investigate than SSRQ. A level-dependent QBD process can be used to describe most MSRQ. The spatial variability induced by transitions owing to repeated attempts is the main feature of its infinitesimal generator. The analytical complexity of retrial models is increased by this lack of homogeneity.

A SSD Markov decision MSRQ system with BV was probed by Krishna Kumar et al. [20]. Using the MGM, they were able to extract interesting and relevant behaviour computations for the system under discussion. Jau-Chuan Ke et al. [21] studied the MGM and analysed an $M/M/c$ RQ with BV ($M/M/c/BSV$ RQ). A QBD process was used to model the queueing system. An $M/M/c$ RQ with BF and startup failures was discovered by Dong-Yuh Yanga et al. [22] and the sufficient and required conditions for the system's stability and stationary probability vectors were found using the QBD and MGM. In k-stages, the Bernoulli vacation in group arrival RQs with inconsistent delivery service was rigorously explored by Radha et al. [23]. By utilizing supplementary variable methodologies (SVT), Rajam and Uma [24] handled ordinary and conditional re-service for the server RQ's service while the system was occupied, on vacation or during maintenance with the integrated presence of consumers. Moreover, all customers need and expect regular service and suitable re-service, but only some customers need additional service and re-service. A few re-service options are available at the end of each service. Finally, several numerical parameters were used to analyze the required measure of reliability. Jain and Kaur [25] investigated the Bernoulli vacation model for the $M^{[X]}/G/1$ unstable server retrial queue. In order to be computationally manageable, the many performance measurements that can be used are made plain via the SVT. Furthermore, the maximum entropy principle (MEP) method has also been used to show its far-reaching potential when it comes to assessing the functional characteristics of complex systems for which explicit analytical results are unavailable. Additionally, situations such as an admissions policy, a vacation policy, etc. might be added to the current model. The unstable server $M^{[X]}/G/1$ retrial feedback queue with balking, working vacation and vacation interruption was investigated by Jain and Kumar [26]. Results from the investigation of many performance indicators using the SVT reveal useful information for enhancing the quality of service (QOS) of such queueing systems across a variety of dimensions, including system size, server availability and more. System organizers and decision makers can benefit from the cost optimization performed to identify the best service rates, in order to support QOS at low cost.

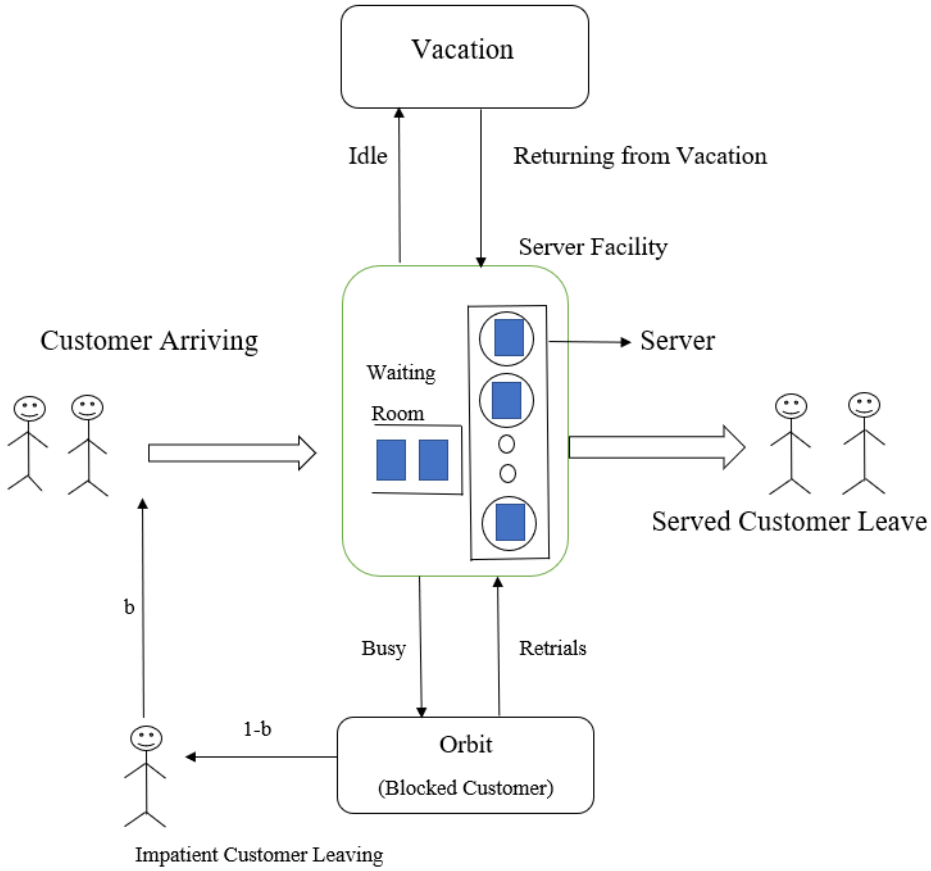

Figure 3: Multi Server Retrial Queue with Bernoulli Vacation

4. THE $M/G/1$ AND $M^{[X]}/G/1$ RETRIAL QUEUEING MODELS WITH BERNOULLI VACATION

A modified $M/G/1$ RQ with an infinite waiting position was investigated by Bong Dae choi and Kwang kyu park [27] in which arriving consumers find the service busy, either join the retrial area with wait to infinite space or leave the system. In the retry and priority groups, they discovered the optimal value of p for various costs per user per unit time. Otherwise the endless waiting room where they will be served with probability $q(= 1-p)$. Using the SVT, they construct the joint generating function of the two groups customer counts. Krishna Kumar and Arivudainambi [28] studied the $M/G/1$ RQ. Here the incoming customers, after finding the service busy they enter into the orbit according to FCFS and they also derived the general decomposition law (GDL) for this model. Zhou Wen-

hui [29] analyzed the $M/G/1$ retrial queue with generic retrial, vacation, setup, service times and Bernoulli vacation. In this instance, consumers who encounter a busy server are assumed to form a queue in the orbit, with priority given to the customer at the front of the queue. Finally, he examined the ergodic condition of this model to derive the PGF. Atencia and Moreno [30] extended the work of Zhou Wenhui [29] model and also obtained the stationary distribution of the priority queue size. They established the system size distribution's generating function, which generalizes the well-known PK formula. Senthil Kumar and Arumuganathan [31] described a SSRQ model (which is shown in Figure 4) with bulk arrivals and two stages of essential service and common vacation time with BS under the condition of stability. This study was inspired by real-world implementations of LANs, CSMA/CD protocols, client-server interactions, telephony infrastructures and internet-based email services.

Gautam Choudhury [32] developed the $M/G/1$ RQ model with linear strat-

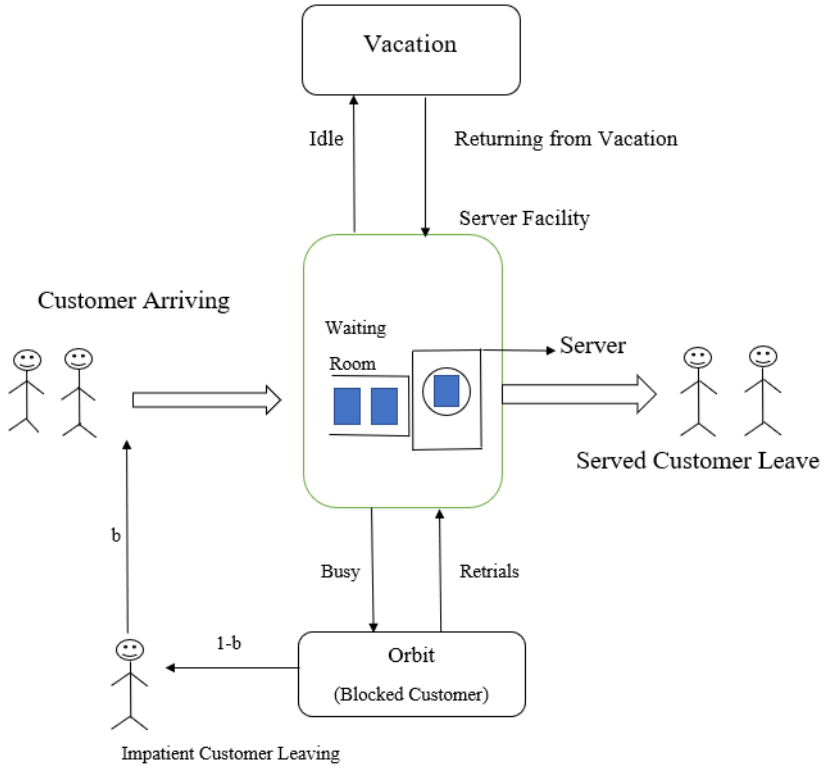

Figure 4: Single Server Retrial Queue with Bernoulli Vacation

egy and two stage service along with the BS. Moreover, they included recursive limiting probabilities for this model. Yong Wan Lee and Young Ho Jang [33]

dealt with $M/G/1$ feedback RQ which has been served with BS. Moreover, in this model the clients can either join the retrial area or leave the system and they also obtained the combined generating procedure of numbers in the two groups using SVT. Under the BS, Senthil Kumar and Arumuganathan [34] examined a SSRQ model with impatient consumers at two stages of services and common vacation time. Nathan P. Sherman and Jeffrey P. Kharouufeh [35] analyzed the system where the server is full and it fails under a Bernoulli routing policy (BRP) which sends a percentage of incoming clients straight to the orbit. They also established enough circumstances to guarantee the existence of a unique, optimum BRP that reduce the total projected prices per unit time of keeping consumers in the initial queue and orbit. An $M/G/1$ RQ system with BF, non-permanent clients and two stages of vital service was examined by Kasturi Ramanath and Lakshmi [36]. The steady-state PGFs of the orbit size and the system size were produced using SVT. They've found a condition that is both necessary and sufficient for the SSD to exist. The inspiration for this system comes from the IVRS which is used at banks, train stations, telecommunications centres, contact centres and other places.

Kalidass and Kasturi Ramanath [37] probed a two-stage $M/G/1$ RQ with a limited amount of instantaneous BF. To provide a sufficient requirement for the model to reach a SSD, they used the EMC procedure. The system size PGF, including the orbit length and queue size marginal PGFs, was obtained. Moreover, asymptotic behaviour in the case of a large rate of retrials was also investigated. Santhakumaran and Shanmugasundaram [38] analyzed a SS system which may serve both retrial and non-retrial consumers with a broad service time distribution. They have derived analytical formulas for a wide range of performance metrics that matter and stability of the system is established by deriving the necessary conditions. Finally, the optimal policies are implemented. In order to determine the optimum settings for the controller, numerical examples have also been performed. The state-dependent retrial queueing model with group entry and server vacation was explored by Ebenesar and Udaya [39]. An $M/G/1$ RQ with unfavourable customers and priority service which has been subjected to server failure and maintenance under a Bernoulli vacation schedule (BVS) was analyzed by Jinbiao Wu and Zhaotong Lian [40]. They developed the general condition for the ergodicity of an EMC using Lyapunov functions. They established SSD solutions for both the queueing measurements and reliability values using SVT. This system generated the famous model discussed by Choi and Park [27]. Krishnakumar et al. [41] have studied a SSRQ with unfavourable customers and two kinds of BF. A SSRQ with Poisson input, k stages of heterogeneous service and feedback was discovered by Ebenesar and Udaya [42]. Radha et al. [43] dealt with a group arrival feedback RQ with multi-stage service and a single vacation where the system has been prone to failures and repairs. In addition, various system probabilities and performance indicators were derived, including the mean number of consumers in the system/orbit and the average waiting time of customers in the system. Moreover, it is demonstrated that the general decomposition law applies to this model.

Using the approach of SVT, Krishna Kumar et al. [44] studied a non-Markovian

feedback SSRQ with collisions and general retrial times. Lakshmi and Kasturi Ramanath [45] extended their work [36] with BF, consistent customers and failure with repair. Gautam Choudhury and Jau-Chuan Ke [46] studied the SSD behaviour of an $M/G/1$ queue with retrial time and a BV with an unstable server, which also includes a phase of failure and a term of delay. For a fixable $M/G/1$ RQ with BS and common retrial strategy, under working vacations (WV) and vacation interruptions, Rajadurai et al. [47] presented the performance of the $M/G/1$ pre-emptive preference RQ with instantaneous BF. The SVT was used to calculate the PGFs for the no. of consumers in the system when they are free, busy or on vacation. This paradigm is used in the Simple Mail Transfer Protocol (SMTP) mail system to send information to e-mail systems and medical service systems' telephone consultation systems. This study has been unique in a way, since it generalizes both single and multiple working vacation models in the condition of RQs. Shan Gao et al. [48] performed both queueing analysis and reliability evaluation. The ergodicity of two linked EMC had been investigated, including the accompanying stationary distributions. Other essential system behaviour computations and reliability value were derived from the SSD combined generating procedure for the no. of clients in each queues. Jinting Wang and Jianghua Li [49] addressed an unconvincing $M/G/1$ RQ with BV and second multi-optional service that allows balking of new arrivals and reneging on users. Rajadurai et al. [50] studied a RQ system with Bernoulli WV and vacation interruption. This model has been used in the SMTP mail system to send information to mail systems and medical service systems as well. Varalakshmi et al. [51] analyzed a SSRQ model with two service stages, immediate BF and single vacation and beginning failures. This concept is used in stop-and-wait ARQ (Automatic Repeat Request), which is a telecommunications mechanism for sending information between two linked devices. This work may be expanded upon by including topics such as postponing repairs, working vacation rules and impatient clients. Gautam Choudhury et al. [52] investigated the SSD performance of an $M/G/1$ RQ with two service stages and normal retrial times for an unstable server under a BVS.

Rajadurai et al. [53] looked into a SSRQ model with negative customers during single and many WVs, as well as during vacation interruptions, where the server was broken down and repaired. This proposed model might be used in a formation to order system to increase the presentation of the manufacturing serviceability and prevent it from getting overloaded, as well as in a computer processing system and telephone consultation system for medical care purposes. This research will be very useful to system managers who will be able to build a system with efficient management and make well-informed judgments about the system's size and other elements. Under the condition of stability, Elakiya Ravitha and Revathi [54] discussed a SSRQ with intolerant subscribers, broad retrial time, two stages of essential service and general vacation time under the BS. Nawel Arrar et al. [55] analyzed the asymptotic performance of the no. of consumers in the retrial area during busy time and found that a minimum retrial rate constitutes a contribution. Under the specific premise when the server can only be occupied by the consumer at the front of the orbit queue, the asymptotic performance of the

SSD of the no. of users in orbit, which did not reveal the nature of the variable was analyzed and moreover it enabled us to estimate it with known distributions. Boualem et al. [56] derived insensitive limits for numerous behaviour computations of a SSRQ with typically distributed inter-retrial periods and a BS. They established that the EMC transition operator is monotonic in the case of high excessive stochastic ordering and growing convex ordering. For the distribution of the no. of clients in the system, they obtained the comparability conditions. The stationary distribution has boundaries as well as some basic constraints for the system's mean features. The verification of various disparities for few continuous probabilities related with each state (m, n) of the system was used to prove these findings. Varalakshmi et al. [57] examined a SSRQ model with two service stages and immediate BF where the server might fail and can be repaired. This technique is useful in packet switching systems that use stop-and-wait ARQ. Using the SVT, a SSRQ with a second possible service, balking, BV and feedback was discussed by Pavai Madheswari and Josephine [58]. In this case, the consumer has the right to object if he is not immediately attended upon arrival. Additionally, stochastic decomposition law (SDL) has been proven without any balking. This system is most commonly used in cloud computing. Tao Li et al. [59] studied how working vacation was managed using a BS in an $M/G/1$ RQ with common retrial times. In a future study, a comparable model with bulk arrival may be dealt in with the same procedure. Under non-pre-emptive prime groups, Geetha Priya and Francis Raj [60] observed that the server offers two sorts of service, namely priority and non-priority. They projected that after serving the system's final essential unit, the server would go on a random-duration vacation. After concluding the service, the server will check if there were any clients in the retrial performance measure. For a Bernoulli schedule in a steady-state $M/G/1$ retrial queue, Liu and Zhao [61] investigated the asymptotic behaviour of the tail probability of the no. of customers under the premise with the service time distribution having a regularly fluctuating tail. Furthermore they get detailed tail asymptotic properties for the conditional probability of the no. of customers in the (priority) queue and orbit, using the newly proposed exhaustive stochastic decomposition approach. An $M/G/1$ retrial queue with balking customers and Bernoulli working vacation interruption was looked at by Li et al. [62]. In addition, a discussion of various numerical examples and an analysis of cost optimization were provided. Malik et al. [63] conducted research on retrial G-queues under various scenarios and wrote a review of their findings. The $M/G/1$ retrial queue with delayed repair and feedback under working vacation policy with impatient customers was researched by GnanaSekar and Kandaiyan [64]. The retrial queues with generally distributed retrial times was addressed by Fiems [65]. Priya and Raj [66] considered a Bernoulli vacation-based single-server retrial G-queue with pre-emptive resume priority. The effects of a modified Bernoulli server vacation, starting failure with balking and reneging was analyzed in a study conducted by Raj et al. [67].

An $M^{[X]}/G/1$ queue is a queueing model in which unit arrivals are in groups and adhere to the Poisson process (PP) with state-dependent arrival rates, service times with a general distribution. An $M^{[X]}/G/1$ queueing model in which the no.

of consumers arriving at the system follows a compound PP with an arrival rate λ. Consumers are admitted to the system in X-sized groups. With a probability mass function (pmf) $a_n = P\{X = n\}; n \geq 1$, the batches has been identically and independently distributed (iid) random variables. Gautam Choudhury [68] analyzed a $M^{[X]}/G/1$ queueing system that operates under a classical retrial strategy which features two stages of mixed service with a BVS. The concept of classical retrial strategy was also introduced, as it is the control of admittance to the retrial area via the Bernoulli admission(BA) mechanism. Gautam Choudhury and Kandarpa Deka [69] proposed to investigate an unconvincing $M^{[X]}/G/1$ RQ with a second possible service channel that has been susceptible to server failure and delayed server repair. The model also includes the concept of BA process. It generalizes the $M^{[X]}/G/1$ RQ with random failure and BA process and also the $M^{[X]}/G/1$ queue with second alternative service and unstable server. Gautam Choudhury and Jau-Chuan Ke [70] investigated an $M^{[X]}/G/1$ RQ with common retrial times and a BVS for an unstable server, which includes a server's breakdown time with time intervals. In addition, a cost effectiveness maximization model was established in order to show the level of efficiency achieved for each dollar spent. This research presents an extension of the vacation model theory and the analysis of the model will provide a useful performance evaluation tool for more general situations that arise in practical applications. For instance, an access point is used to connect between wireless and wired networks in wireless networks. Madhu Jain et al. [71] analyzed at the performance of a bulk entry RQ with random service interruption and BV and concluded that this approach is more adaptable in real-world congestion conditions since it includes BV and SOS. The vacation models examined in this study may be useful in queuing systems when the server intends to use idle time for relaxation or other duties. Gautam Choudhury and Kandarpa Deka [72] explored an $M^{[X]}/G/1$ queueing system that operates under a linear retrial strategy and features two stages of mixed service with a BVS. In addition, upon arrival, each consumer is susceptible to a control admittance strategy. By treating the entry, service and vacation rates as fuzzy numbers, Upadhyaya [73] examined the system features of a batch entry retrial queuing model with a BVS. In addition, the purpose of describing the family of crisp queues with a vacationing server is to create a pair of parametric non-linear programmes, sometimes known as PNLPs. Also, the numerical examples are made easier to understand for a variety of service time and vacation time distributions so that the validity of the suggested technique may be demonstrated.

Rajadurai et al. [74] introduced a batch arrival feedback RQ model with two stages of service, orbit search and delayed repair, under BV. The SVT approach was used to calculate the PGF of no. of consumers in the system and orbit. The mean no. of clients in the system/orbit, as well as under unusual circumstances, were investigated. Under BV, breakdown and repair, Rajadurai et al. [75] probed a bulk arrival RQ model with two stage service. The inspiration for this model derives from a variety of real-world applications, such as computer and network of information, where information is processed in two stages by a SS under the BV policy. This model identifies theoretical interrelationships in medical pathology,

where medicine is used to manage a group of disorders. Rajadurai et al. [76] discovered a group arrival feedback RQ with intolerant consumers as well as orbital search under the BVS, in which the overload server has been prone to failure owing to the arrival of unfavourable customers. In particular for wired networks, the model that they have developed has the potential to find application in a packet switched network. Charan Jeet Singh [77] analyzed that the customers can wait in a virtual group in the RQ model with an unstable server under Bernoulli's vacation policy. This research can be expanded to find the best N-strategy or admission control-based queueing system.

Francis Raj [78] discussed a group arrival RQ with a second possible service and modified extended BV that was subjected to server failures. The proposed approach might be useful in the email system's transfer model. The messages has been sent between mail servers using the SMTP. Using the SVT methodology, Yuvarani and Saravanarajan [79] explored a SSRQ with customized BV under delayed repair. Revathi et al. [80] investigated a SS bulk arrival RQ with extra re-service under a modified BV that includes breakdown time with time intervals. The primary premise of this study is that recovery does not occur quickly after a failure and that there is a time period until restore may begin. Gautam Choudhury and Lotfi Tadj [81] analyzed the unification of multiple classes of related bulk arrival queueing systems by extending the major $M^{[X]}/G/1$ RQ under the BVS with a linear retrial strategy and BA method for an unstable server. Rajam and Uma [82] examined BV of the $M^{[X]}/G/1$ queue with a two-tier service-based unstable server in depth. These techniques used $M^{[X]}/G/1$ queues as a two-stage service and demonstrated that BV adheres to the threshold requirements for unconvincing servers, downtime and latency. It also has a system of numerous queues connected for batch delivery based on the addition of extra variables and a complicated PP with a degree of arrival. The proactive renewal of service hours policy, which was applied to current clients in the line, was extensively examined. The batch arrival RQ with impatient consumers, BV, feedback and a server prone to startup failure was examined by Nila and Sumitha [83]. Madhu Jain and Sandeep Kaur [84] investigated the unstable $M^{[X]}/G/1$ RQ, which includes a variety of realistic elements such as probable feedback, vacation, many-probable service, balking, server failure and delay repair. The SVT was used to evaluate the queue length distribution and other system behaviour measures. The MEP approach was used to offer approximate solutions for the SSD and waiting time. Many real-time systems uses the retrial bulk model described, including manufacturing and assembly organizations, communications networks, health-care systems, banking and computer networks and so on.

5. AN $Geo/G/1$ RETRIAL QUEUEING MODELS WITH BERNOULLI VACATION

One of the major motivations for studying discrete time (DT) queues is that they are more appropriate for computer and telecommunications systems modeling than their continuous-time counterparts because they work on a DT basis,

with events occurring only at equally spaced epochs. Researchers are interested in the use of DT queues for slotted device applications like ALOHA and ATM (asynchronous transfer mode) throughout B-ISDN (broadband integrated services digital network), where a device supply chain among a controller as well as the transfer rate of one compartment in ATM are basic components of duration as an example bits, bytes, inflexible size packets and so on. In a variety of contexts, scholars have put their efforts into analyzing DT queues.

Choi and Park [27] studied a $M/G/1$ RQ model with BS using a continuous-time model. From both theoretical and practical aspects, it will be beneficial if their research has to be extended to the equivalent DT model. Hui Li and Tao Yang [85] analyzed regarding the blocked customers'options to either enter the unlimited venue or to leave the system with the aid of DT $Geo/G/1$ RQ with a BS. The PGF of the combined distribution of the consumers in the waiting area and the retrial circle in SSD was deduced using an analytic formula and was also proven that a SDL holds. Moreover, the established recursive techniques may also be utilized to evaluate the minimal SSD of consumers in the prime and non-prime groups in this case. Jinting Wang [86] extended the retrial queueing theory to the DT scenario in the case of server vacations. The ergodicity requirement of the Markov chain that underlies the queueing system was determined in this study. He presented two SDL, as well as constraints for the closeness between the model's system size distributions and the equivalent model without retrials as an application. With BF, Atencia and Moreno [87] investigated a DT $Geo^{[X]}/G_H/1$ RQ. They constructed a SDL and used it to put limitations on the distance between the SSD of their queueing system and the standard system. They also established recursive equations for determining the SSD of the orbit size in the particular situation of individual arrivals. Shan Gao and Zai-ming Liu [88] worked with a DT group arrival RQ on a server that's prone to errors at the start. At any moment, they may calculate the SSD of the service state and the no. of consumers in the system or orbit. A SDL was also derived. They also established recursive equations for determining the SSD of the orbit size in the particular situation of individual arrivals and also looked at the relationship between the DT and continuous-time systems. Shweta Upadhyaya [89] discussed the ideas of favoured and intolerant customers under BF and estimated the performance of a DT RQ. Moreover, recent study examines the modeling of a variety of digital systems in a discrete context, including CCNs (computer and network of information), ALOHA and ATM.

6. OTHER BERNOULLI VACATION MODELS

Another recent development in BV model research was retrial queueing model with basic recurring tries and initial losses, where the server gives two stages of composite service to arriving clients, one after the other, according to a BVS which was introduced by Jau-Chuan Ke and Fu-Min Chang's [90]. The PGFs of the service state and the no. of consumers in the system or orbit were generated analytically with clear formulations. Madhu Jain et al. [91] studied how the server delivers two stages of essential service to all entering clients in a group

arrival RQ model with BF and balking including a modified vacation strategy. In this research, the server experiences a starting failure. They analyzed the transient solution of the queueing system. The distribution of repair, service and vacation times was believed to be random. In regards to their Laplace transforms, the period-dependent PGF have been constructed. Madhu Jain et al. [92] discussed the SSD performance of the large arrival retrial queuing model using the MEP. They have evaluated important performance indicators such as system length, orbit length, waiting time, etc. Further, they offered numerical simulation, both of which may be helpful to the system designers and decision makers for the quantitative assessment of the concerned queuing system. Chen et al. [93] studied a distinct-time feedback RQ with a general distribution, a BV policy on the server and pre-emptive resume for the customer. Moreover, two different stochastic decomposition laws for this system have also been established.

Under the BVS with negative customers, breakdown and repair, Rajadurai et al. [94] investigated a group arrival feedback RQ model with two-stage service. This model has actual real-life applications in a packet switching network, where it is used to forward packets within a network so they can be transmitted. Sasikala et al. [95] discussed a SSD of the $M^{[X]}/G^B/1$ RQ model with BV. Using SVT, some quantitative computations for the proposed model had been obtained along with few numerical examples. And finally, they showed that their future scope of work includes bulk service queueing systems with WVs, bulk service queueing systems with prime customers and bulk service queueing systems with transient solutions. The work of Yuvarani and Saravanarajan [96] presented in terms of a SS pre-emptive priority RQ with common retrial times, orbital search under BV and service disruption. Arivudainambi and Gowsalya [97] implemented a RQ model with two kinds of service under BVS, where the server gives two different kinds of service. Krishna Kumar et al. [98] studied a Markovian RQ with finite orbit capacity for a multi-programming and multiprocessor network system wherein the service channels of the CPU queue take vacations according to the BS. Ayyappan and Supraja [99] analyzed the transient performance of a non-Markovian queueing model in which clients arrive in batches based on a compound PP and bulk service was given by a single site with a lowest and largest batch length. Moreover, the SSD results for time reliant PGFs in ideas of Laplace transforms have also been found.

Ayyappan and Gowthami [100] worked on the retrial queuing system, during which arrival time has been determined by the MAP and service time follows the PH-distribution, subjected under BS vacation, Bernoulli's feedback and breakdown and repair. They have also looked at their model's busy times. Their findings may be used in queueing models in which arrival is determined by batch Markovian arrival process ($BMAP$). A SS batch arrival priority based retrial queueing system with orbital search, required short vacation and optional long vacation that consists of working breakdown and repair under Bernoulli schedule controlled policy is analyzed by Ayyappan et al. [101]. The system also includes Bernoulli schedule controlled policy. Ayyappan and Udayageetha [102] explored a $M^{[X_1]}, M^{[X_2]}/G_1, G_2/1$ RQ with preference service under BV, working disaster

and shut down or begin time. Moreover, the benefit of a customer's impatience on a service system was also investigated. A SS which serves two classes of customers under non-preemptive prime service rule was scrutinized by Ayyappan et al. [103] in addition to working breakdown, admission control, balking and BV. Here the admission control mechanism acts as the filtration process of allowing customers. Moreover, the efficient functioning of the service system was also verified since the customer's service would not be interrupted in the event of a breakdown. Revathi et al. [104] discovered the SSD behaviour of a SSRQ with customized BV, in which each kind of service is a private service susceptible to breakdowns and repairs. The equilibrium balking techniques were identified by Gao et al. [105] in the repairable $M/M/1$ G-retrial queue with complete removals. Also, the system can be modeled as a symmetric non-cooperative game among cooperative customers and the central problem is to find the Nash equilibrium balking strategy. This strategy is stable if and only if all cooperative customers agree to adhere to it, as no one can gain from deviating from it. Moreover, they applied the concepts from queueing theory and game theory to analyze the Nash equilibrium pure strategy in the observable case and the Nash equilibrium mixed strategy in the unobservable situation. Finally, they provide some numerical examples to show that changing these factors can alter the equilibrium behaviours under consideration. Ayyappan and Gowthami [106] utilized the matrix analytic technique to examine a classical model with a single server, a phase-type service distribution, inter-arrival times according to the Markovian arrival process and other random variables with exponential distributions. However, they fail to account for the QBD process under the invariant circumstance. The Markovian arrival process (MAP) was studied by Ayyappan and Gowthami [107] for a conventional queueing model in which clients arrive at the system from a variety of different servers. Two deterministic batch arrival queues with instant feedback, modified Bernoulli vacation and server failure were added in the SS study by Ayyappan et al. [108]. Kannadasan and Padmavathi [109] use a fuzzy method based on a BS and an action based on hexagonal fuzzy numbers to analyze an $Fm/Fg/1$ retrial queue. Using Zadeh's extension approach, they examine the orbital system and waiting-time mean customer counts as well as other system performances of relevance. In order to provide a performance study of various systems, including telephone switching systems and computers contending for access to a central processing unit, fuzzy retrial queueing systems were implemented. Begum and Choudhury [110] analyzed the steady-state repairability of an $M/(G_1, G_2)/1$ queue with service interruption. The BS allows the server to take a single vacation once each service is completed.

7. ADVANCEMENT IN RETRIAL QUEUEING MODEL

When a consumer sees that a server they want to access is already in use, they may elect to join the retrial orbit and wait for their turn to get service. Retry efforts are crucial and cannot be neglected when designing queueing models due to their widespread use and rapid pace of development. Several progresses are listed

here. The evolutionary channel fuzzy inference system serves as the foundation for a new type of artificial neural network called an adaptive neuro-fuzzy inference system (ANFIS). ANFIS is an artificial neural network that takes its cues from the evolutionary channel fuzzy inference system. It is a universal estimator since it can use the advantages of both neural networks and fuzzy logic concepts inside a single framework. When applied to real-world congestion, ANFIS's soft computing approach can yield useful results. Recently, many researchers [111, 112, 113] analyzed retrial QS and compared their numerical outcomes with the neuro-fuzzy results.

Cost optimization is a business-centric, iterative process that seeks to lessen outlays and boost profitability. It comprises negotiating the most favourable terms and conditions for business deals. The running cost of a system has direct bearing on the bottom line in a practical setting. In order to increase the system's profitability, developers and administrators focus on decreasing the operating expenditure per unit of time. This has made cost optimization [112, 113, 114, 115, 116] a crucial part of many retry queueing systems. Rather than giving precise probability distributions, linguistic values like "fast," "slow," and "moderate" are often used to represent the arrival pattern of clients, the service pattern and the vacation pattern of the server. The created queuing models are more widely applicable to real-world problems since they take into account various system characteristics like fuzzy values. While recent years have seen a lot of important work on fuzzy retrial queues [117, 73], the corresponding literature remains sparse.

The use of retrial queue with BV is prevalent in the real world. Particularly important in wireless networks, it bridges the gap between the wireless and wired networks [70]. Additionally, it helps in the transmission of packets in a packed switch network [94] and is used in the basic mail transfer protocol to transport messages [90]. It may find use in a variety of traffic circumstances [77], such as the time lag experienced during electronic equipment quality assurance in a manufacturing setting as well.

7.1. Future scope

There are various cases where $M/M/1$, $M^{[X]}/M/1$ and discrete time retrial QS can occur. So far, they have received little attention because most of the contributions made to date only concentrate with $M/G/1$ retrial QS under BV. In addition, while SVT has been extensively examined, other methods, such as the matrix analytical approach, the use of fuzzy variables, etc., have yet to be explored. Another potential snag in the retrial queue is the lack of works on retrial QS under Bernoulli working vacation. Cost optimization in a variety of queuing systems [114, 115, 116], including those involving vacation time, has been studied extensively using PSO and ABC algorithms. Yet, there are still many unanswered concerns about these models. Some examples include cost optimization on retrial QS with vacation time, optimization using other algorithms like GWO, DE, BAT, etc.

8. CONCLUSION

As surveyed by Doshi [118] many significant projects have been done in the vacation model region over the last few decades. A brief literature review of retrial queueing systems with BV has been done to show how they evolved from the beginning to the present. The pioneers of some retrial systems with BV that have been used in various instances like server interruption, feedback, G-queue, impatient customers, priority customers, etc. have been presented. Practitioners and queue theorists equally will benefit from this literature review on Bernoulli vacation for various retrial models because it clarifies the new dimensions and applicability of queueing models for system upgrades and efficiency enhancements at the optimal cost, all within the context of techno-economic constraints. Moreover, it can assist statisticians, operations analysts, researchers, engineers and managers in the implementation of retrial queueing models with BV. Thus, the present paper on retrial queueing models with BV is believed to supply useful insights and important performance assessments for future design and development of service and manufacturing organizations by providing various realistic and noble aspects. Moreover, our hope is that these aspects may be integrated into future retrial QS to help address the research gap and facilitate the development of different retrial systems.

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REFERENCES

[1] T. Katayama, "Priority queues with bernoulli schedules," in *Teletraffic Science and Engineering*. Elsevier, 1997, vol. 2, pp. 309–318.

[2] I. Atencia and P. Moreno, "A queueing system with linear repeated attempts, bernoulli schedule and feedback," *Top*, vol. 11, no. 2, pp. 285–310, 2003. doi: https://doi.org/10.1007/BF02579046

[3] G. Ayyappan and S. Karpagam, "Analysis of a bulk queue with unreliable server, immediate feedback, n-policy, bernoulli schedule multiple vacation and stand-by server," *Ain Shams Engineering Journal*, vol. 10, no. 4, pp. 873–880, 2019. doi: https://doi.org/10.1016/j.asej.2019.03.008

[4] T. Jiang and B. Xin, "Computational analysis of the queue with working breakdowns and delaying repair under a bernoulli-schedule-controlled policy," *Communications in Statistics-Theory and Methods*, vol. 48, no. 4, pp. 926–941, 2019. doi: https://doi.org/10.1080/03610926.2017.1422756

[5] G. Ayyappan and R. Gowthami, "Analysis of $map/ph(1), ph(2)/2$ queue with bernoulli schedule vacation, bernoulli feedback and renege of customers," *International Journal of Applied and Computational Mathematics*, vol. 5, pp. 1–47, 2019. doi: https://doi.org/10.1007/s40819-019-0744-6

[6] M. Jain and S. Kaur, "(p, n)-policy for unreliable server bulk queue with bernoulli feedback," *International Journal of Applied and Computational Mathematics*, vol. 6, no. 6, p. 170, 2020. doi: https://doi.org/10.1007/s40819-020-00912-4

[7] A. A. Bouchentouf, L. Yahiaoui, M. Kadi, and S. Majid, "Impatient customers in markovian queue with bernoulli feedback and waiting server under variant working vacation policy," *Operations Research and Decisions*, vol. 30, 2020. doi: https://doi.org/10.37190/ord200401

[8] C. Shekhar, S. Varshney, and A. Kumar, "Matrix-geometric solution of multi-server queueing systems with bernoulli scheduled modified vacation and retention of reneged customers: A meta-heuristic approach," *Quality Technology & Quantitative Management*, vol. 18, no. 1, pp. 39–66, 2021. doi: https://doi.org/10.1080/16843703.2020.1755088

[9] S. Liu, L. Hu, Z. Liu, and Y. Wang, "Reliability of a retrial system with mixed standby components and bernoulli vacation," *Quality Technology & Quantitative Management*, vol. 18, no. 2, pp. 248–265, 2021. doi: https://doi.org/10.1080/16843703.2020.1853320

[10] G. Ayyappan and R. Gowthami, "A map/ph/1 queue with setup time, bernoulli vacation, reneging, balking, bernoulli feedback, breakdown and repair," *Reliability: Theory & Applications*, vol. 16, no. 2 (62), pp. 191–221, 2021. doi: https://doi.org/10.24412/1932-2321-2021-262-191-221

[11] A. Choudhary, N. Mahla, and D. C. Sharma, "Cost optimization of the queueing system with degrading service rate, bernoulli vacation, and a regular vacation after fixed services," *International Journal of Applied and Computational Mathematics*, vol. 8, no. 3, p. 124, 2022. doi: https://doi.org/10.1007/s40819-022-01319-z

[12] G. Ayyappan and S. Karpagam, "Analysis of a bulk queue with unreliable server, immediate feedback, n-policy, bernoulli schedule multiple vacation and stand-by server," *Ain Shams Engineering Journal*, vol. 10, no. 4, pp. 873–880, 2019. doi: https://doi.org/10.1016/j.asej.2019.03.008

[13] R. Kalyanaraman and P. Nagarajan, "Bulk arrival, fixed batch service queue with unreliable server, bernoulli vacation and with delay time," in *AIP Conference Proceedings*, vol. 2177, no. 1. AIP Publishing LLC, 2019. doi: https://doi.org/10.1063/1.5135209 p. 020034.

[14] A. Begum and G. Choudhury, "Analysis of a bulk arrival n-policy queue with two-service genre, breakdown, delayed repair under bernoulli vacation and repeated service policy," *RAIRO-Operations Research*, vol. 56, no. 2, pp. 979–1012, 2022. doi: https://doi.org/10.1051/ro/2021167

[15] R. Rajendiran and I. Kandaiyan, "Transient scrutiny of $m^X/g(a,b)/1$ queueing system with feedback, balking and two phase of service subject to server failure under bernoulli vacation," *AIMS Mathematics*, vol. 8, no. 3, pp. 5391–5412, 2023. doi: https://doi.org/10.3934/math.2023271

[16] J. Keilson and L. Servi, "Oscillating random walk models for gi/g/1 vacation systems with bernoulli schedules," *Journal of applied Probability*, vol. 23, no. 3, pp. 790–802, 1986. doi: https://doi.org/10.2307/3214016

[17] K. Lakshmi and K. Ramanath, "A matrix geometric approach to the $m/m/1$ two-phase multi optional retrial queue with bernoulli feedback, impatient customers and a server subject to breakdown and repair," *Opsearch*, vol. 51, pp. 36–49, 2014. doi: https://doi.org/10.1007/s12597-013-0131-8

[18] K. Sun and J. Wang, "Equilibrium joining strategies in the single-server constant retrial queues with bernoulli vacations," *RAIRO - Operations Research*, vol. 55, pp. S481 – S502, 2021. doi: https://doi.org/10.1051/ro/2019087

[19] P. Gupta and N. Kumar, "Cost optimization of single server retrial queueing model with bernoulli schedule working vacation, vacation interruption and balking," *Journal of Mathematics and Computer Science*, vol. 11, no. 3, pp. 2508–2523, 2021.

[20] B. K. Kumar, R. Rukmani, and V. Thangaraj, "An $m/m/c$ retrial queueing system with bernoulli vacations," *Journal of Systems Science and Systems Engineering*, vol. 18, pp. 222–242, 2009. doi: https://doi.org/10.1007/s11518-009-5106-1

[21] J.-C. Ke, C.-H. Wu, and W. L. Pearn, "A heuristic algorithm for the optimization of a retrial system with bernoulli vacation," *Optimization*, vol. 62, no. 3, pp. 299–321, 2013. doi: https://doi.org/10.1080/02331934.2011.579966

[22] D.-Y. Yang, J.-C. Ke, and C.-H. Wu, "The multi-server retrial system with bernoulli feedback and starting failures," *International Journal of Computer Mathematics*, vol. 92,

no. 5, pp. 954–969, 2015. doi: https://doi.org/10.1080/00207160.2014.932908
[23] J. Radha, K. Indhira, and V. Chandrasekaran, "Multi stage unreliable retrial queueing system with bernoulli vacation," in *IOP Conference Series: Materials Science and Engineering*, vol. 263, no. 4. IOP Publishing, 2017. doi: https://doi.org/10.1088/1757-899X/263/4/042148 p. 042148.
[24] V. Rajam and S. Uma, "Retrial queue $m^X/g/c$ with bernoulli's vacation, repair, service and re-service in orbit," *Turkish Journal of Computer and Mathematics Education (TURCOMAT)*, vol. 12, no. 12, pp. 4535–4540, 2021.
[25] M. Jain and S. Kaur, "Bernoulli vacation model for $mx/g/1$ unreliable server retrial queue with bernoulli feedback, balking and optional service," *RAIRO-Operations Research*, vol. 55, pp. S2027–S2053, 2021. doi: https://doi.org/10.1051/ro/2020074
[26] M. Jain and A. Kumar, "Unreliable server $m[x]/g/1$ retrial feedback queue with balking, working vacation and vacation interruption," *Proceedings of the National Academy of Sciences, India Section A: Physical Sciences*, vol. 93, no. 1, pp. 57–73, 2023. doi: https://doi.org/10.1007/s40010-022-00777-w
[27] B. D. Choi and K. K. Park, "The m/g/1 retrial queue with bernoulli schedule," *Queueing systems*, vol. 7, pp. 219–227, 1990. doi: https://doi.org/10.1007/BF01158476
[28] B. K. Kumar and D. Arivudainambi, "The $m/g/1$ retrial queue with bernoulli schedules and general retrial times," *Computers & Mathematics with Applications*, vol. 43, no. 1-2, pp. 15–30, 2002. doi: https://doi.org/10.1016/S0898-1221(01)00267-X
[29] Z. Wenhui, "Analysis of a single-server retrial queue with fcfs orbit and bernoulli vacation," *Applied Mathematics and Computation*, vol. 161, no. 2, pp. 353–364, 2005. doi: https://doi.org/10.1016/j.amc.2003.12.032
[30] I. Atencia and P. Moreno, "A single-server retrial queue with general retrial times and bernoulli schedule," *Applied Mathematics and Computation*, vol. 162, no. 2, pp. 855–880, 2005. doi: https://doi.org/10.1016/j.amc.2003.12.128
[31] M. S. Kumar and R. Arumuganathan, "On the single server batch arrival retrial queue with general vacation time under bernoulli schedule and two phases of heterogeneous service," *Quality Technology & Quantitative Management*, vol. 5, no. 2, pp. 145–160, 2008. doi: https://doi.org/10.1080/16843703.2008.11673393
[32] G. Choudhury, "Steady state analysis of an $m/g/1$ queue with linear retrial policy and two phase service under bernoulli vacation schedule," *Applied Mathematical Modelling*, vol. 32, no. 12, pp. 2480–2489, 2008. doi: https://doi.org/10.1016/j.apm.2007.09.020
[33] Y.-W. Lee and Y.-H. Jang, "The $m/g/1$ feedback retrial queue with bernoulli schedule," *Journal of applied mathematics & informatics*, vol. 27, no. 1, pp. 259–266, 2009.
[34] M. S. Kumar, R. Arumuganathan *et al.*, "Performance analysis of single server retrial queue with general retrial time, impatient subscribers, two phases of service and bernoulli schedule," *Journal of Applied Science and Engineering*, vol. 13, no. 2, pp. 135–143, 2010. doi: https://doi.org/10.6180/jase.2010.13.2.03
[35] N. P. Sherman and J. P. Kharoufeh, "Optimal bernoulli routing in an unreliable $m/g/1$ retrial queue," *Probability in the Engineering and Informational Sciences*, vol. 25, no. 1, pp. 1–20, 2011. doi: https://doi.org/10.1017/S0269964810000215
[36] K. Ramanath and K. Lakshmi, "An $m/g/1$ two phase multi-optional retrial queue with bernoulli feedback and non-persistent customers," *International Journal of Latest Trends in Software Engineering*, vol. 1, no. 1, pp. 1–9, 2011.
[37] K. Kalidass and K. Ramanath, "A priority retrial queue with second multi optional service and m immediate bernoulli feedbacks," in *Proceedings of the 6th International Conference on Queueing Theory and Network Applications*, 2011. doi: https://doi.org/10.1145/2021216.2021226 pp. 67–76.
[38] A. Santhakumaran and S. Shanmugasundaram, "A single server retrial queue in bernoulli schedule with feedback on non-retrial customers," *Southeast Asian Bulletin of Mathematics*, vol. 35, no. 2, 2011.
[39] J. E. A. Bagyam and K. U. Chandrika, "Batch arrival retrial queuing system with state dependent admission and bernoulli vacation," *International Journal of Research in Engineering and Technology*, vol. 2, no. 10, 2013.
[40] J. Wu and Z. Lian, "A single-server retrial g-queue with priority and unreliable server

under bernoulli vacation schedule," *Computers & Industrial Engineering*, vol. 64, no. 1, pp. 84–93, 2013. doi: https://doi.org/10.1016/j.cie.2012.08.015

[41] B. Krishna Kumar, S. Pavai Madheswari, and S. Anantha Lakshmi, "An $m/g/1$ bernoulli feedback retrial queueing system with negative customers," *Operational Research*, vol. 13, pp. 187–210, 2013. doi: https://doi.org/10.1007/s12351-011-0107-5

[42] J. E. A. Bagyam and K. U. Chandrika, "Multi-stage retrial queueing system with bernoulli feedback," *International Journal of Scientific and Engineering Research*, vol. 4, no. 9, pp. 496–499, 2013.

[43] J. Radha, P. Rajadurai, K. Indhira, and V. Chandrasekaran, "A batch arrival retrial queue with k-optional stages of services, bernoulli feedback, single vacation and random breakdown," *Global Journal of Pure and Applied Mathematics*, vol. 10, no. 2, pp. 265–283, 2014.

[44] B. K. Kumar, R. Rukmani, V. Thangaraj, and U. R. Krieger, "A single server retrial queue with bernoulli feedback and collisions," *Journal of statistical theory and practice*, vol. 4, pp. 243–260, 2010. doi: https://doi.org/10.1080/15598608.2010.10411984

[45] K. Lakshmi and K. Ramanath, "An m/g/1 two phase multi-optional retrial queue with bernoulli feedback, non-persistent customers and breakdown and repair," *International journal of operational research*, vol. 19, no. 1, pp. 78–95, 2014. doi: https://doi.org/10.1504/IJOR.2014.057844

[46] G. Choudhury and J.-C. Ke, "An unreliable retrial queue with delaying repair and general retrial times under bernoulli vacation schedule," *Applied Mathematics and Computation*, vol. 230, pp. 436–450, 2014. doi: https://doi.org/10.1016/j.amc.2013.12.108

[47] P. Rajadurai, S. Yuvarani, and M. C. Saravanarajan, "Performance analysis of preemptive priority retrial queue with immediate bernoulli feedback under working vacations and vacation interruption," *Songklanakarin Journal of Science & Technology*, vol. 38, no. 5, 2016. doi: https://doi.org/10.14456/sjst-psu.2016.67

[48] S. Gao, J. Wang, and T. Van Do, "A repairable retrial queue under bernoulli schedule and general retrial policy," *Annals of Operations Research*, vol. 247, no. 1, pp. 169–192, 2016. doi: https://doi.org/10.1007/s10479-015-1885-6

[49] J. Wang and J. Li, "A repairable $m/g/1$ retrial queue with bernoulli vacation and two-phase service," *Quality Technology & Quantitative Management*, vol. 5, no. 2, pp. 179–192, 2008. doi: https://doi.org/10.1080/16843703.2008.11673395

[50] P. Rajadurai, M. Saravanarajan, and V. Chandrasekaran, "A single server retrial queue with bernoulli working vacation and vacation interruption," *International Journal of Applied Engineering Research*, vol. 11, no. 1, p. 2016.

[51] M. Varalakshmi, P. Rajadurai, M. Saravanarajan, and V. Chandrasekaran, "An $m/g/1$ retrial queueing system with two phases of service, immediate bernoulli feedbacks, single vacation and starting failures," *International Journal of Mathematics in Operational Research*, vol. 9, no. 3, pp. 302–328, 2016. doi: https://doi.org/10.1504/IJMOR.2016.078823

[52] G. Choudhury, L. Tadj, and M. Deka, "An unreliable server retrial queue with two phases of service and general retrial times under bernoulli vacation schedule," *Quality Technology & Quantitative Management*, vol. 12, no. 4, pp. 437–464, 2015. doi: https://doi.org/10.1080/16843703.2015.11673430

[53] P. Rajadurai, V. Chandrasekaran, and M. Saravanarajan, "Analysis of an unreliable retrial g-queue with working vacations and vacation interruption under bernoulli schedule," *Ain Shams Engineering Journal*, vol. 9, no. 4, pp. 567–580, 2018. doi: https://doi.org/10.1016/j.asej.2016.03.008

[54] S. Elakiya ravitha and S. Revathi, "Analysis of single server retrial queue with general retrial time, impatient subscribers, two phases of service and bernoulli schedule," *International Journal of Creative Research Thoughts*, vol. 6, no. 2, pp. 82–95, 2018.

[55] N. Arrar, L. Derrouiche, and N. Djellab, "On the asymptotic behaviour of the $m/g/1$ retrial queue with priority customers, bernoulli schedule and general retrial times," *IAENG International Journal of Applied Mathematics*, vol. 48, no. 2, pp. 1–8, 2018.

[56] M. Boualem, M. Cherfaoui, N. Djellab, and D. Aissani, "A stochastic version analysis of an $m/g/1$ retrial queue with bernoulli schedule," *Bulletin of the Iranian Mathematical Society*, vol. 43, no. 5, pp. 1377–1397, 2017.

[57] M. Varalakshmi, V. Chandrasekaran, and M. Saravanarajan, "An unreliable two phase retrial queue with immediate bernoulli feedbacks," *International Journal of Pure and Applied Mathematics*, vol. 115, no. 9, pp. 447–457, 2017.

[58] S. P. Madheswari and S. Josephine, "Analysis of an $m/g/1$ retrial queue with second optional service and customer feedback, under bernoulli vacation schedule," *Malaya Journal of Matematik (MJM)*, vol. 7, no. 4, p. 2019, 2019. doi: https://doi.org/10.26637/MJM0704/0027

[59] T. Li, L. Zhang, and S. Gao, "An $m/g/1$ retrial queue with single working vacation under bernoulli schedule," *RAIRO-Operations Research*, vol. 54, no. 2, pp. 471–488, 2020. doi: https://doi.org/10.1051/ro/2019008

[60] K. GeethaPriya and L. F. Raj, "An $m/g/1$ retrial queueing model, with non-preemptive resume priority,multi optional service, feedback, bernoulli vacation and orbital search," *Journal of Xi'an University of Architecture & Technology*, vol. 12, 2020.

[61] B. Liu and Y. Q. Zhao, "Tail asymptotics for a retrial queue with bernoulli schedule," *Mathematics*, vol. 10, no. 15, p. 2799, 2022. doi: https://doi.org/10.3390/math10152799

[62] T. Li, L. Zhang, and S. Gao, "An $m/g/1$ retrial queue with balking customers and bernoulli working vacation interruption," *Quality Technology & Quantitative Management*, vol. 16, no. 5, pp. 511–530, 2019. doi: https://doi.org/10.1080/16843703.2018.1480264

[63] G. Malik, S. Upadhyaya, and R. Sharma, "A study on retrial g-queues under different scenarios: A review," in *Proceedings of International Conference on Scientific and Natural Computing: Proceedings of SNC 2021*. Springer, 2021. doi: https://doi.org/10.1007/978-981-16-1528-$3_1$8 pp. 211–220.

[64] M. M. N. GnanaSekar and I. Kandaiyan, "Analysis of an $m/g/1$ retrial queue with delayed repair and feedback under working vacation policy with impatient customers," *Symmetry*, vol. 14, no. 10, p. 2024, 2022. doi: https://doi.org/10.3390/sym14102024

[65] D. Fiems, "Retrial queues with generally distributed retrial times," *Queueing Systems*, vol. 100, no. 3-4, pp. 189–191, 2022. doi: https://doi.org/10.1007/s11134-022-09793-4

[66] K. G. Priya and L. F. Raj, "A single server retrial g-line along preemptive resume priority & non-continuous server as per bernoulli vacation," in *AIP Conference Proceedings*, vol. 2385, no. 1. AIP Publishing LLC, 2022. doi: https://doi.org/10.1063/5.0070868 p. 130052.

[67] L. F. Raj, C. Revathi, and M. Saravanarajan, "An m/g/1 retrial g-queue for balking, reneging, subject to modified bernoulli vacation, starting failure," in *AIP Conference Proceedings*, vol. 2385, no. 1. AIP Publishing, 2022. doi: https://doi.org/10.1063/5.0070792

[68] G. Choudhury, "A two phase batch arrival retrial queueing system with bernoulli vacation schedule," *Applied Mathematics and Computation*, vol. 188, no. 2, pp. 1455–1466, 2007. doi: https://doi.org/10.1016/j.amc.2006.11.011

[69] G. Choudhury and K. Deka, "An $mx/g/1$ unreliable retrial queue with two phases of service and bernoulli admission mechanism," *Applied Mathematics and Computation*, vol. 215, no. 3, pp. 936–949, 2009. doi: https://doi.org/10.1016/j.amc.2009.06.015

[70] G. Choudhury and J.-C. Ke, "A batch arrival retrial queue with general retrial times under bernoulli vacation schedule for unreliable server and delaying repair," *Applied Mathematical Modelling*, vol. 36, no. 1, pp. 255–269, 2012. doi: https://doi.org/10.1016/j.apm.2011.05.047

[71] M. Jain, G. Sharma, and R. Sharma, "A batch arrival retrial queuing system for essential and optional services with server breakdown and bernoulli vacation," *International Journal of Internet and Enterprise Management*, vol. 8, no. 1, pp. 16–45, 2012. doi: https://doi.org/10.1504/IJIEM.2012.049868

[72] G. Choudhury and K. Deka, "A batch arrival retrial queue with two phases of service and bernoulli vacation schedule," *Acta Mathematicae Applicatae Sinica, English Series*, vol. 29, no. 1, pp. 15–34, 2013. doi: https://doi.org/10.1007/s10255-007-7083-9

[73] S. Upadhyaya, "Bernoulli vacation policy for a bulk retrial queue with fuzzy parameters," vol. 3, pp. 1–14, 2013.

[74] P. Rajadurai, K. Indhira, M. Saravanarajan, and V. Chandrasekaran, "Analysis of an $m[x]/g/1$ feedback retrial queue with two phase service, bernoulli vacation, delaying repair and orbit search," *Advances in Physics Theories and Applications*, p. 40, 2015.

[75] P. Rajadurai, M. Varalakshmi, M. Saravanarajan, and V. Chandrasekaran, "Analysis of m

[x]/g/1 retrial queue with two phase service under bernoulli vacation schedule and random breakdown," *International journal of mathematics in operational research*, vol. 7, no. 1, pp. 19–41, 2015. doi: https://doi.org/10.1504/IJMOR.2015.065946

[76] P. Rajadurai, V. Chandrasekaran, and M. Saravanarajan, "Analysis of an $m[x]/g/1$ unreliable retrial g-queue with orbital search and feedback under bernoulli vacation schedule," *Opsearch*, vol. 53, pp. 197–223, 2016. doi: https://doi.org/10.1007/s12597-015-0226-5

[77] C. J. Singh, M. Jain, and B. Kumar, "$mx/g/1$ unreliable retrial queue with option of additional service and bernoulli vacation," *Ain Shams Engineering Journal*, vol. 7, no. 1, pp. 415–429, 2016. doi: https://doi.org/10.1016/j.asej.2015.05.006

[78] L. Francis Raj, "A non markovian retrial queueing system with modified extended vacations under bernoulli schedule and server breakdowns," *International Journal of Pure and Applied Mathematics*, vol. 10, pp. 123–138, 2017.

[79] S. Yuvarani and M. Saravanarajan, "Analysis of $m[x]/g/1$ preemptive priority retrial queue with delayed repairs under modified bernoulli vacation schedule," *International Journal of Pure and Applied Mathematics*, vol. 117, pp. 73–82, 2017.

[80] C. Revathi, L. Francis Raj, and M. Saravanarajan, "On the $m[x]/g/1$ retrial queue with regular and optional re-service under modified bernoulli server vacation and delaying repair," *Journal of Xi'an University of Architecture & Technology*, vol. 3, pp. 5247–5260, 2020.

[81] G. Choudhury and L. Tadj, "An unreliable batch arrival retrial queueing system with bernoulli vacation schedule and linear repeated attempts: Unreliable retrial system with bernoulli schedule," *International Journal of Operations Research and Information Systems (IJORIS)*, vol. 11, no. 1, pp. 83–109, 2020. doi: https://doi.org/10.4018/IJORIS.2020010104

[82] V. Rajam and S. Uma, "Bernoulli's vacation of $m^X/g/1$ queue with two-tier service based volatile server," *Journal of Mathematics and Computer Science*, vol. 12, pp. 1358–1362, 2021.

[83] M. Nila and D. Sumitha, "Two phase retrial queue with starting failures, customer impatience, feedback and bernoulli vacation," *Advances and Applications in Mathematical Sciences*, vol. 21, no. 4, pp. 1773–1786, 2022.

[84] M. Jain and S. Kaur, "Bernoulli vacation model for $mx/g/1$ unreliable server retrial queue with bernoulli feedback, balking and optional service," *RAIRO-Operations Research*, vol. 55, pp. S2027–S2053, 2021. doi: https://doi.org/10.1051/ro/2020074

[85] H. Li and T. Yang, "Geo/g/1 discrete time retrial queue with bernoulli schedule," *European Journal of Operational Research*, vol. 111, no. 3, pp. 629–649, 1998. doi: https://doi.org/10.1016/S0377-2217(97)90357-X

[86] J. Wang, "Discrete-time $geo/g/1$ retrial queues with general retrial time and bernoulli vacation," *Journal of Systems Science and Complexity*, vol. 25, pp. 504–513, 2012. doi: https://doi.org/10.1007/s11424-012-0254-7

[87] I. Atencia and P. Moreno, "Discrete-time $geo[x]/gh/1$ retrial queue with bernoulli feedback," *Computers & Mathematics with Applications*, vol. 47, no. 8-9, pp. 1273–1294, 2004. doi: https://doi.org/10.1016/S0898-1221(04)90122-8

[88] S. Gao and Z.-m. Liu, "A repairable $geox/g/1$ retrial queue with bernoulli feedback and impatient customers," *Acta Mathematicae Applicatae Sinica, English Series*, vol. 30, no. 1, pp. 205–222, 2014. doi: https://doi.org/10.1007/s10255-014-0278-y

[89] S. Upadhyaya, "Performance prediction of a discrete-time batch arrival retrial queue with bernoulli feedback," *Applied Mathematics and Computation*, vol. 283, pp. 108–119, 2016. doi: https://doi.org/10.1016/j.amc.2016.02.026

[90] J.-C. Ke and F.-M. Chang, "$m[x]/(g1, g2)/1$ retrial queue under bernoulli vacation schedules with general repeated attempts and starting failures," *Applied Mathematical Modelling*, vol. 33, no. 7, pp. 3186–3196, 2009. doi: https://doi.org/10.1016/j.apm.2008.10.017

[91] M. Jain, G. Sharma, and R. Sharma, "Unreliable $mx/(g1, g2)/1$ retrial queue with bernoulli feedback under modified vacation policy," *International Journal of Information Management Sciences*, vol. 23, pp. 425–448, 2012.

[92] M. Jain, R. Sharma, and G. Sharma, "Maximum entropy analysis of bulk arrival retrial queue with second optional service and bernoulli vacation," *International Jour-*

nal of Industrial and Systems Engineering, vol. 20, no. 3, pp. 369–396, 2015. doi: https://doi.org/10.1504/IJISE.2015.069918

[93] P. Chen, Y. Zhou, and C. Li, "Discrete-time retrial queue with bernoulli vacation, preemptive resume and feedback customers," *Journal of Industrial Engineering and Management (JIEM)*, vol. 8, no. 4, pp. 1236–1250, 2015. doi: https://doi.org/10.3926/jiem.1487

[94] P. Rajadurai, V. Chandrasekaran, and M. Saravanarajan, "Steady state analysis of batch arrival feedback retrial queue with two phases of service, negative customers, bernoulli vacation and server breakdown," *International journal of mathematics in operational research*, vol. 7, no. 5, pp. 519–546, 2015. doi: https://doi.org/10.1504/IJMOR.2015.071276

[95] S. Sasikala, K. Indhira, and V. Chandrasekaran, "A study on $mx/gb/1$ retrial queueing system with bernoulli vacation schedule and variable server capacity," *International Journal of Knowledge Management in Tourism and Hospitality*, vol. 1, no. 3, pp. 263–277, 2017. doi: https://doi.org/10.1504/IJKMTH.2017.086812

[96] S. Yuvarani and M. Saravanarajan, "Analysis of preemptive priority retrial queue with batch arrival and orbital search under bernoulli vacation," *International Journal of Pure and Applied Mathematics*, vol. 113, pp. 139–147, 2017.

[97] D. Arivudainambi and M. Gowsalya, "A single server non-markovian retrial queue with two types of service and bernoulli vacation," *International Journal of Operational Research*, vol. 33, no. 1, pp. 55–81, 2018. doi: https://doi.org/10.1504/IJOR.2018.09423

[98] B. K. Kumar, R. N. Krishnan, R. Sankar, and R. Rukmani, "Analysis of multiprogramming-multiprocessor retrial queueing systems with bernoulli vacation schedule," *Queueing Models and Service Management*, vol. 1, no. 1, pp. 31–74, 2018.

[99] G. Ayyappan and R. Supraja, "Transient analysis of $m^{[X]}/g(a,b)/1$ queueing system with balking under bernoulli schedule vacation and random breakdown," *Journal of Computer and Mathematical Sciences*, vol. 9, pp. 455–473, 2018.

[100] G. Ayyappan and R. Gowthami, "Analysis of $map/ph/1$ retrial queue with constant retrial rate, bernoulli schedule vacation, bernoulli feedback, breakdown and repair," *Reliability: Theory & Applications*, vol. 14, no. 2, pp. 86–103, 2019.

[101] G. Ayyappan, B. Somasundaram, and P. Thamizhselvi, "Analysis of $m[x1], m[x2]/g1, g2/1$ retrial queueing system with priority services, orbital search, compulsory short vacation, optional long vacation, working breakdown and repair under bernoulli schedule controlled policy," *Malaya Journal of Matematik (MJM)*, vol. 7, no. 2, 2019, pp. 230–244, 2019. doi: https://doi.org/10.26637/MJM0702/0014

[102] G. Ayyappan and J. Udayageetha, "Transient analysis of $m[x1], m[x2]/g1, g2/1$ retrial queueing system with priority services, working breakdown, start up/close down time, bernoulli vacation, reneging and balking," *Pakistan Journal of Statistics and Operation Research*, pp. 203–216, 2020. doi: https://doi.org/10.18187/pjsor.v16i1.2181

[103] G. Ayyappan, P. Thamizhselvi, B. Somasundaram, and J. Udayageetha, "Analysis of an retrial queueing system with priority services, working breakdown, bernoulli vacation, admission control and balking," *Journal of Statistics and Management Systems*, vol. 24, no. 4, pp. 685–702, 2021. doi: https://doi.org/10.1080/09720529.2020.1744812

[104] C. Revathi, L. F. Raj, and M. Saravanarajan, "Single server repairable retrial queueing system with modified bernoulli vacation, optional re-service," *Advances in Mathematics: Scientific Journal*, vol. 9, pp. 5997–6007, 2020. doi: https://doi.org/10.37418/amsj.9.8.68

[105] S. Gao, D. Zhang, H. Dong, and X. Wang, "Equilibrium balking strategies in the repairable $m/m/1$ g-retrial queue with complete removals," *Probability in the Engineering and Informational Sciences*, vol. 35, no. 1, pp. 138–157, 2021. doi: https://doi.org/10.1017/S026996481900010X

[106] G. Ayyappan and R. Gowthami, "A map/ph/1 queue with setup time, bernoulli schedule vacation, balking and bernoulli feedback," *International Journal of Applied and Computational Mathematics*, vol. 8, no. 2, p. 62, 2022. doi: https://doi.org/10.1007/s40819-022-01260-1

[107] ——, "Analysis of $map/ph1, ph2/2$ queue with working breakdown, starting failure, and bernoulli vacation," in *Fuzzy Mathematical Analysis and Advances in Computational Mathematics*. Springer, 2022, pp. 189–208.

[108] G. Ayyappan, S. Nithya, and B. Somasundaram, "Analysis of $m[x1], m[x2]/g1, g(a,b)2/1$

queue with priority services, server breakdown, repair, modified bernoulli vacation, immediate feedback." *Applications & Applied Mathematics*, vol. 17, no. 2, 2022.

[109] G. Kannadasan and V. Padmavathi, "Analysis of $fm/fg/1$ retrial queue with bernoulli schedule and vacation using hexagonal fuzzy numbers," *London Journal of Research in Science: Natural and Formal*, vol. 22, no. 5, 2022.

[110] A. Begum and G. Choudhury, "Analysis of an $m/g1g2/1$ queue with bernoulli vacation and server breakdown," *International Journal of Applied and Computational Mathematics*, vol. 9, no. 1, p. 9, 2023. doi: https://doi.org/10.1007/s40819-022-01481-4

[111] R. Sethi, M. Jain, R. Meena, and D. Garg, "Cost optimization and anfis computing of an unreliable $m/m/1$ queueing system with customers' impatience under n-policy," *International Journal of Applied and Computational Mathematics*, vol. 6, pp. 1–14, 2020. doi: https://doi.org/10.1007/s40819-020-0802-0

[112] A. Ahuja, A. Jain, and M. Jain, "Transient analysis and anfis computing of unreliable single server queueing model with multiple stage service and functioning vacation," *Mathematics and Computers in Simulation*, vol. 192, pp. 464–490, 2022. doi: https://doi.org/10.1016/j.matcom.2021.09.011

[113] M. Vaishnawi, S. Upadhyaya, and R. Kulshrestha, "Optimal cost analysis for discrete-time recurrent queue with bernoulli feedback and emergency vacation," *International Journal of Applied and Computational Mathematics*, vol. 8, no. 5, p. 254, 2022. doi: https://doi.org/10.1007/s40819-022-01445-8

[114] P. Vijaya Laxmi and K. Jyothsna, "Cost and revenue analysis of an impatient customer queue with second optional service and working vacations," *Communications in Statistics-Simulation and Computation*, vol. 51, no. 8, pp. 4799–4814, 2022. doi: https://doi.org/10.1080/03610918.2020.1752378

[115] M. Jain and A. Kumar, "Unreliable server $m[x]/g/1$ queue with working vacation and multi-phase repair with delay in verification," *International Journal of Applied and Computational Mathematics*, vol. 8, no. 4, p. 210, 2022. doi: https://doi.org/10.1007/s40819-022-01383-5

[116] A. Kumar and M. Jain, "Cost optimization of an unreliable server queue with two stage service process under hybrid vacation policy," *Mathematics and Computers in Simulation*, vol. 204, pp. 259–281, 2023. doi: https://doi.org/10.1016/j.matcom.2022.08.007

[117] S. B. Khodadadi and F. Jolai, "A fuzzy based threshold policy for a single server retrial queue with vacations," *Central European Journal of Operations Research*, vol. 20, pp. 281–297, 2012. doi: https://doi.org/10.1007/s10100-010-0169-0

[118] B. T. Doshi, "Queueing systems with vacations-a survey," *Queueing systems*, vol. 1, pp. 29–66, 1986. doi: https://doi.org/10.1007/BF01149327