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PERFORMANCE APPROXIMATIONS OF MARKOV MODULATED POISSON PROCESS ARRIVAL QUEUE WITH MARKOVIAN SERVICE USING MATRIX GEOMETRIC APPROACH

Raghavendran RAMANATHAN
Department of Mathematics, School of Advanced Sciences, VIT-Chennai, India
raghavendran.r2017@vitstudent.ac.in

Vidhya VARADHARAJAN
Department of Mathematics, School of Advanced Sciences, VIT-Chennai, India
vidhya.v@vit.ac.in

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Abstract: Queue analysis of correlated work of the arrival and the service process are not available much in literature. This research paper applies matrix geometric approach to study the $MMPP/MSP/1$ queuing model under the quasi-birth death(QBD) process. Customers admitted in the system are having two different intensity of arrival followed by Markov Modulated Poisson Process($MMPP$). $MMPP$ is a versatile class of MAP and it is necessary to adapt the first characterization of the queuing model. In service mechanism Markov Service Process(MSP) is adopted. As like $MMPP$, MSP is also a versatile and can capture the correlation of the modulated arrivals. The methodology presented in this research work calculates the metric performances not depending on the series of infinite state of the system. The assumed model is formulated as QBD process to obtain the steady state probability of the system. The complexity model formulated involves structures matrices with appropriate dimension. Neut's pioneered the matrix geometric approach to such complex models which involves the rate computation of rate matrix R. The rate matrix R is approximated to derive the performance metrics of the system and then compute the cost for the underlying Markov chain. To attain this approximation the sensitivity analysis is done with numerical results.

Keywords: ($MMPP$), (MSP), performance analysis, quasi-birth-death process, structured Markov chain, optimal cost.

MSC: 37A50, 37L55, 60G05, 60G20.

1. INTRODUCTION

Queues embedded in Network related models are applied largely in analyzing the evaluation metrics of data transfer in computer networks and communication systems. The fundamental role of stochastic modeling aims to find a versatile process to describe the data transfer mechanism in accurate way. Traffic models based on Markovian arrival process (MAP) assured the development of traffic related queuing network study [1, 2]. The MAP expansion plays key role in the extension of traffic related analysis of queuing networks in order to manage the inter dependent arrival and services[3, 4]. The extension of multi-class MAP, referred to as $MMPP$ is a stochastic model that allow to capture the inter level dependency of the modulated arrivals of traffic [5, 6, 7]. The author [8] developed an approximation algorithm to fit two state $M3PP$ and introduced the interposition operations which enables composing many $M3PPs$ by preserving the corresponding marginal counting process.
In modeling a various phenomenon like queuing process, the Markov modulated type Poisson arrival process, a special case of MAP can be used. The authors [9] analyzed $MMPP$ queue behavior in long range dependence is exhibited. The $MMPP$ has various application in bursty traffic model. The discrete time batch Markovian Arrival process model is considered with variable available bit rate and their superposition are studied in [10] and obtained that the output process belong to the same process. The essential characteristic of $MMPP's$ is the unique feature of arrivals that are not available in other Markov models [11]. The modulated characteristic enables the use of non-renewal process in models which is important because of temporal dependency and self-correlation can adverse system performance which are important in maths modeling/ The stimulative work behind the wide application of $MMPP$ is that $MMPP$ keeps the tractability of the path traced out by Poisson process. The classical Markov model cannot be sufficiently adapted to many computer traffic models such as internet browsing, teleconferencing and VoIP, at the same time it is important to approach a suitable to model traffic system to elaborate the various multimedia level applications over network models. Like MAP, MSP is a versatile type process with correlated service times.
The MSP is correlated and analyzed for two types of customer rejection like partial and total rejection policy[12] which has potential applications in telecommunication system. The authors [13] obtained the closed analysis of discrete time with MSP based on roots obtained from the characteristic equation corresponding to the system length vector generating function. The authors [14, 15, 16] considered a single server queue with renewal input to measure some block probability, mean length of queue and waiting times using MSP. Okamura et al.[17] proposed a numerical procedure for fitting a MAP to $MMPP$ and estimate the parameters of MAP and $MMPP$ using expectation-maximization(EM) algorithm. In the literature, only limited number of works on Markov service process are studied. An

explicit solution for the sojourn time distribution queue was derived by [18] along with correlated coefficient of lagged inter departure interval. The Markov service process and its stationary characteristics are studied by [19, 20]. The study of packet loss pattern in wireless communication was carried by [21, 22] that includes loss period and loss distance. The author [23] considered single server involving differentiated service that correspond to the modulated arrival and refers service process of two states as MSP to represent the server behavior as a $CTMC$ indicating the particular type of service engaged in. Few other contributions of various authors on the queuing model with MSP can be referred in [2, 24, 25]. Meier-Hellstern et al.[26] proposed a simple $MMPP$ intrinsic traffic model that approximate more accurately the long range dependence(LRD) characteristic of internet traffic observed in the measured traces over the time scale of interest. Also, they fit an algorithm for $MMPP$ with two arrival rate of inhomogeneity to yield analytic results, but the performance values are not computed numerically. Neut [27] discussed the concept of pioneered the queuing approach using matrix geometric method that applies for the models of complexity. The arrival and serv0ice rate are used in matrix entries to form a structured Markov chain. Around three decades a large amount of work done by many authors justifies that this methodology played a significant role in queuing works.
In this work, we present an numerical approximations of the performance metrics of the single serve with modulated arrival. As the arrivals are inhomogeneous in nature Markov Service Process is necessary to admit in order to correlate the modulated arrival and functioning of the queuing mechanism. We construct the model in such a way that the input and the inter traffic are assumed to be $MMPP's$, in order to allow the input traffic permitting inter class dependency. The remaining content of the paper is organized as follows: A brief introduction to $MMPP$ and its evolution from MAP is provided in section 2. The analytic part of the performance metrics and the service process of single server $MMPP/MSP/1$ is described in the section 3. Section 4 presents some numerical results in which the proposed analysis and compared with the numerical data.

2. ARRIVAL, SERVICE CHARACTERIZATION AND MODEL DESCRIPTION

In this section, the $MMPP$ information is provided which has the modulated arrival of the customers considered under model study.

A discrete time QBD of homogeneous type is a Markov process described by a 2 dimensional process with its state space of the form $\{(i,j,),0 \leq i, 1 \leq j \leq r\}$ in which the first term is called as level and the second one referred as phase of the state respectively. The transition value from (i,j) to (i',k) is allowed to take the possible values with $|i'-i| < 2$ and (i,j) to (i',k) may depend on j,k and $|i'-i|$, but at the same time not on the partial value of i and i'.

$$Q = \begin{pmatrix} L_0 & \bar{L}_1 & & & \\ \bar{L}_{-1} & L_0 & L_1 & & \\ & L_{-1} & L_0 & L_1 & \\ & & \ddots & \ddots & \ddots \end{pmatrix}$$

2.1. MSPP

The $MMPP$ is a two state generalized version of Poisson process with switched state arrivals. The two possibilities can occur finally at the exit of an exponentially distributed time of sojourn in state i with average value as $\frac{1}{\lambda_i}$. If $i = 1$ the probability α (for $i = 2$, the probability β), no occurrences of event and the system enters to the state $i \neq j$. Otherwise with the assumption of probability $1 - \alpha$ if $i = 1$ (for $i = 2$, the probability $1 - \beta$) an event occurs and the system continue to be in the same state. An $MMPP$ is a simple discrete case of arrival process that takes the value of finite count of Poisson process modulated under a discrete time Markov chain. In state in a different way, $MMPP$ is a process of double measure stochastic whose Poisson arrival is assumed to be modulated type by an irreducible finite state $CTMc$, also called as phase of the process. The $MMPP$ can be characterized in the form rate matrices $\{D_0, D_1\}$ where

$$D_0 = \begin{pmatrix} -\lambda_1 & \lambda_1\alpha \\ \lambda_2\beta & -\lambda_2 \end{pmatrix}, D_1 = \begin{pmatrix} \lambda_1(1-\alpha) & 0 \\ 0 & \lambda_2(1-\beta) \end{pmatrix} \tag{1}$$

with the infinitesimal irreducible generator

$$Q = D_0 + D_1 = \begin{pmatrix} D_0 & D_1 & & & \\ & D_0 & D_1 & & \\ & & D_0 & D_1 & \\ & & & \ddots & \ddots \end{pmatrix}$$

2.2. MSP

A Markov service process can be expressed by a Markov chain. Let the Markov chain $\{(N(t), J(t))\}_{t\geq 0}$ along with the state space $\{(i,j), i \geq 0, 1 \leq j \leq r\}$ in two dimension and the transition probability matrix assumes the block matrix form

$$\begin{pmatrix} M_0 & M_1 & & & \\ & M_0 & M_1 & & \\ & & M_0 & M_1 & \\ & & & \ddots & \ddots \end{pmatrix}$$

where $N(t)$ denotes the counting variable, $J(t)$ represents the phase variable or axillary state and $M_i's$ are non-negative $m \times m$ block matrices with entries between 0 and 1. The probability entry in the transition matrix from the state value (i,j) to state $(i+1,j)$ and denoting the service time departure from $(j,j')\,th$

entry $(M_1)_{j,j'}$ of the $m \times m$ matrix M_1. The expression $(M_1)_{j,j'}$ are based on the phases j and j'. The sum $M = M_0 + M_1$ is a $m \times m$ block stochastic matrix with the matrix of transition probability value of the underlying Markov chain $\{J(t)\}$ corresponding to MSP.

The matrix $(I - M_0)$ which is non-singular and found to be at any state $\{(i, j), i \geq 0, 1 \leq j \leq r\}$ the sojourn time is finite with probability 1. The MSP has the fundamental service rate μ given by $\mu = \pi M_1 e$. The invariant probability vector of $M = M_0 + M_1$ is $\pi M = \pi,\ Me = 1$ and e, where π is the vector of column type with all entries 1 and the dimension is designated as required.

2.3. $MMPP/MSP/1$ QUEUE

A server of single with infinite capacity queuing system accommodating the customers are served according to Markov service process. The $MMPP/MSP/1$ is a queue model of special type based on the general single server $MAP/MSP/1$ where the arrival pattern and service mechanism are allowed to follow $MMPP$ and MSP respectively. In this work we frame a quasi-birth-death(QBD) processes an efficient form to analyze the many general complex model using matrix geometric method [28]. An embedded three dimensional Markov chain $\{(N(t), J(t), K(t))\}_{t \geq 0}$ of the $MMPP/MSP/1$ queue can be applied to model the queue size behavior. The state space of the corresponding Markov chain is

$$\{0, 1, 2, \ldots, N, N_{loss}\} \times \{1, 2, \ldots m\} \times \{1, 2, \ldots, m\}$$

by denoting the buffer size, the state of $MMPP$, the phase of the MSP as $N(t), J(t)$and $K(t)$ respectively. The state of the system can defined as $X(t) = (N(t), J(t), K(t))$. The embedded Markov chain generator of $\{X(t)\}$ which has an irreducible transition probability matrix whose block structure form denoted by

$$Q^{MMPP} = \begin{pmatrix} D_0 \otimes M_0 & D_0 \otimes M_1 + D_1 \otimes M_0 & 0 & 0 & 0 \\ \bar{D}_0 \otimes M_0 & D_0 \otimes M_1 + D_1 \otimes M_0 & D_1 \otimes M_1 & 0 & 0 \\ 0 & D_0 \otimes M_0 & D_0 \otimes M_1 + D_1 \otimes M_0 & D_1 \otimes M_1 & 0 \\ 0 & \ddots & \ddots & \ddots & \ddots \end{pmatrix}$$

The Kronecker type product $\otimes$ operation is used as defined in [29, 30]. For $1 \leq l$, the level space $\mathcal{L}(l)$ defined by $\mathcal{L}(l) = \{(l, j, k); j \in \mathcal{J}, k \in \mathcal{K}\}$ which refers to level 0. Let $\pi(l, j, k)$ denote the stationary probability of the process at (l, j, k) state. The vector $\pi(l) = (\pi(l, j, k), j \in \mathcal{J}, k \in \mathcal{K})$ denote the probabilities of row vector such that the process at level l and the stationary probability distribution of the process represented by $\pi = (\pi(l), l \geq 0)$. It is assumed that π the stationary distribution always exists. Consider the time length over a period of time of the process that starts in the state (l, j, k) for some $l \geq 0$ and finish as soon as it reaches level $l - 1$ for the first time.

Theorem 1. *Suppose that the Markov chain Q^{MMPP} of $MMPP/MSP/1$ type defined by [28] is stochastic and irreducible then Q^{MMPP} is positive recurrent*

$$\pi (D_1 \otimes M_1) e < \pi (D_0 \otimes M_1) e \tag{2}$$

Under the stability condition [28] of the system, there exists the stationary state probability vector π satisfying $\pi Q^{MMPP} = 0$ and $\pi e = 1$. The stationary probability vector π can be partitioned as $\pi = (\pi(0), \pi(1), \pi(2), \dots)$ and is given by

$$\pi(0)\left(D_0 \otimes M_0\right) + \pi(1)\left(\bar{D}_0 \otimes M_0\right) = 0$$

$$\pi(0)\left(D_0 \otimes M_1 + \bar{D}_1 \otimes M_0\right) + \pi(1)\left(D_0 \otimes M_1 + D_1 \otimes M_0\right) + \pi(2)\left(D_1 \otimes M_1\right) = 0$$

$$\pi(l) = \pi(2) R^{l-2}, l = 3, 4, 5, \dots$$

and the normalized equation is

$$\pi(0)e + \pi(1)e + \pi(2)\left(I - R\right)^{-1} e = 1$$

Let n_{st}, where $s = (j, k), t = (j', k')$ be the average sojourn time during the period of the process in the state (l, j', k') and let N be $m \times m$ matrix given by $N = (n_{st})$. The role of N plays important in analysis of the model assumed. As $\{X(t)\}$ is a quasi-birth-death process, the vector π is calculated by the matrix geometric method [**?**]

$$\pi(l) = \pi(0)\left(D_0 \otimes M_1 + D_1 \otimes M_0\right) N R^{l-1}, l \geq 1$$

where the matrix R has the elements with rate entries of $\{X(t)\}$ given by

$$R = \left(D_1 \otimes M_1\right) N$$

The non-negative row vector $\pi(0)$ satisfies

$$\pi(0)\left\{D_0 \otimes M_0 + \left(D_0 \otimes M_1 + D_1 \otimes M_0\right) N \left(\bar{D}_0 \otimes M_0\right)\right\} = 0^t$$

and the condition of normalizing

$$\pi(0)\left[e' + \left(D_0 \otimes M_1 + D_1 \otimes M_0\right) N \left(I - R\right)^{-1} e\right] = 1$$

where 0 is the column vector of $0's$, e is that of $1's$ and 0^t is the null column vector of required dimension. The rate matrix R satisfies the quadratic matrix equation

$$0 = D_0 \otimes M_1 + R\left(D_0 \otimes M_1 + D_1 \otimes M_0\right) + R^2\left(D_0 \otimes M_0\right)$$

The square matrix wit transition rate values represented as R will converge as $L_{-1} \otimes M_1$ and $\left(L_0 \otimes M_1 + L_1 \otimes M_0\right)^{-1}$ are positive. The rate value of the matrix R will tend to increase monotonically by nature and iterations will be carried out until the condition $max\left|R^{l+1} - R^l\right| < \epsilon$ reaches, where R^l denotes the value of R in the lth iteration and ϵ is the required degree of accuracy.

3. PERFORMANCE MEASURES

This section consists of some important measures that are listed out to bring out the qualitative analysis of the assumed model considered in the study. The average count of customers and second moment in the model considered are calculated by

$$E(n) = \pi(1)e + \pi(2)\left[2(I-R)^{-1} + R(I-R)^{-2}\right]e$$

$$E(n^2) = -\pi(2)\left[(I-R)^{-1} + R(I-R)^{-2} + 2(I-R)^{-3} - 4I\right]e + \pi(1)e + 4\pi(2)e$$

where the term n denotes the size of the system at an arbitrary time. By using the values the variance can also be found. The probability of zero customers in the system is given by

$$P_0 = \pi(0)e$$

The expected count of customers in the model

$$L_s = \sum_{l=0}^{\infty} l\pi(l)e \tag{3}$$

The expected count of customers in the line

$$L_q = \sum_{l=0}^{\infty} l\pi(l+1)e \tag{4}$$

The average time of wait of the model is

$$W_s = \frac{1}{\lambda}L_s \tag{5}$$

The average time of wait of the line is

$$W_q = \frac{1}{\lambda}L_q \tag{6}$$

4. WIRELESS CONTROL COMMUNICATIONS

This particular section considers a single server communication system in which the signals tend to fade, noise and interference so that the occurrence of error chances are high. Fitting a model analytically to study the data simulation have shown some idea that the Markov hidden model can suitably fits to the basic behavior of the error that occur even in some complex problems [21]. Assume the data transmission through a wireless link with bursty process relative to a high rate data transmission. A single server provide the access of arrival process through the transmitted with full bandwidth. The data packets on prescribed at the transmitted in which the entering packets are queued, wait in order to be transmitted. The exhibited memory of fading Rayleigh channel is examined by using error process of hidden Markov model. Consider the service provided by transmission of channel that can be elaborated by service process. The service process can be considered to model an embedded hidden model of Markov type by defining matrices M_0 and M_1. The matrices M_0 and M_1 are the transmission probability of packet transmission happened during non occurrence of higher priority packet and priority packet respectively. The state sequence is a Markov chain with $M = M_0 + M_1$ as the transition state probability matrix.The process considered is assumed to have stationary and the initial state probability vector satisfying the condition $\pi M = \pi,\ \pi e = 1$.

5. SENSITIVITY ANALYSIS

In this section, we focus on sensitivity analysis of the model applied on the optimal value R and its expected cost $C(r)$ whose values are based parameter α and β. For the numerical calculation, we consider the input data defined by $MMPP$ as

$$D_0 = \begin{pmatrix} -0.3 & 0.3\alpha \\ 0.5\beta & -0.5 \end{pmatrix}, D_1 = \begin{pmatrix} 0.3(1-\alpha) & 0 \\ 0 & 0.5(1-\beta) \end{pmatrix} \tag{7}$$

Figure 1 plots the increasing trend of the average length L_s and L_q increases as the parameter α decreases with $\alpha = 1/5$ to $\alpha = 1/10$ for fixed value of $\beta = 1/3$. The increasing trend of the average waiting time W_s and W_q of the system and queue along with L_s and L_q are illustrated in the same Figure 1.

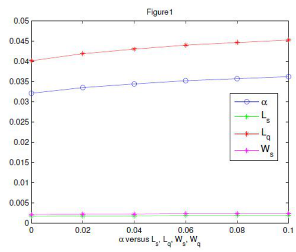

Figure 1: α versus L_s, L_q, W_s, W_q

In Figure 2, we plot the average length of the system and queue along with waiting time W_s and W_q of the system and queue respectively. It is observed that when $\alpha = 1/5$ and β varies from $\beta = 1/4$ to $\beta = 1/10$, the trend of L_s, L_q, W_s, W_q are increasing.

In Figure 3, the value of λ_1 ranging from $\lambda_1 = 1$ to $\lambda_1 = 5$ with fixed value of $\lambda_2 = 0.5$ along with $\alpha = 1/5, \beta = 1/4$ the average mean length of the system L_s and L_q and the waiting time of the system W_s and queue W_q are also increasing.

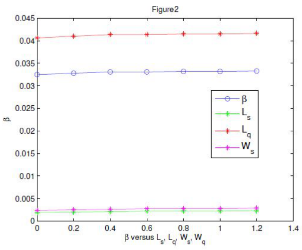

Figure 2: β versus L_s, L_q, W_s, W_q

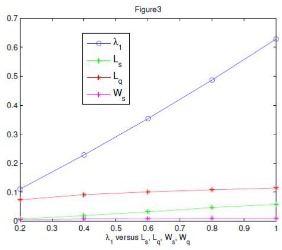

Figure 3: λ_1 versus L_s, L_q, W_s, W_q

6. COST STUDY

In this section, we study the steady state expected cost function of the decision variable r. The objective is to obtain the critical value r to minimize the expected total cost/unit time. Let C_1 denote the cost/unit time when the customers are in buffer for service, C_2 denote the cost/unit time of busy server, C_3 denote the cost/unit time of idle server. The cost function parameter listed above are used to determine the expected total cost/unit time which is given by $C(r) = C_1 L_q + C_2 \pi(l) + C_3 P_0$. The performance of sensitivity work on the optimal value r are based on the variable of the system parameter values such as the rate of arrival λ_1 the probability α, β and cost parameter. Let the cost parameters be $C_1 = 100, C_2 = 200, C_3 = 300$. The calculated numeric value of optimal critical r and the expected minimum cost $C(r)$ are illustrated in Figure 4 and Figure 5.

In Figure 4, the parameter β is varied from $\beta = 1/5$ to $\lambda = 1/10$ and the corresponding expected cost increases its trend with respect to critical value r. In Figure 5, the parameter β is varied from $\lambda = 1/5$ to $\lambda = 1/10$ and the corresponding expected cost trend line with respect to critical value r is presented.

7. CONCLUSION

In this work we applied the matrix geometric methodology to study the $MMPP$ queuing model under the QBD process. As the arrival and service process are correlated, the Markovian service process is considered for serving the two types of

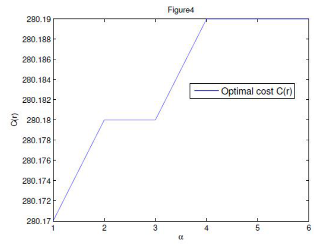

Figure 4: α versus Cost function

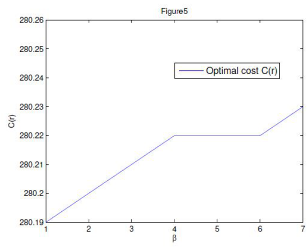

Figure 5: β versus Cost function

customers that arrive at two different rates. The rate matrix R computed is applied to obtain some performance metrics of the system. The results established for $MMPP/M/1$ can also be extended to batch type $MMPP$ queue. The current work of this paper can be extended to study the multi-server queuing model.

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Code availability. There is no code required in this paper.

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