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INTEGRATED DECISION-MAKING FRAMEWORK FOR HOSPITAL DEVELOPMENT: A SINGLE-VALUED NEUTROSOPHIC PROBABILISTIC HESITANT FUZZY APPROACH WITH INNOVATIVE AGGREGATION OPERATORS

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Abstract: This research article proposes an innovative algorithm for analyzing parallelism in the evolution of hospital building features, with the goal of advancing decision-making processes in both urban and rural hospitals. As an additional generalization of the concepts of fuzzy sets, intuitionistic fuzzy sets, single-valued neutrosophic sets, hesitant fuzzy sets, and probabilistic fuzzy sets this paper proposes a single-valued neutrosophic probabilistic hesitant fuzzy set (SV-NPHFS). It is derived from the combination of single-valued neutrosophic sets, probabilistic fuzzy sets, and hesitant fuzzy sets. The novel algebraic structure and cosine evaluation function of SV-NPHFSs are then introduced. In addition, we introduce novel operators: the single-valued neutrosophic probabilistic hesitant fuzzy weighted geometric (SV-NPHFWG), the single-valued neutrosophic probabilistic hesitant fuzzy ordered weighted geometric (SV-NPHFOWG), the single-valued neutrosophic probabilistic hesitant fuzzy weighted average (SV-NPHFWA), and the single-valued neutrosophic probabilistic hesitant fuzzy ordered weighted average (SV-NPHFOWA). More complex links between features and alternatives can be made with the multi-attribute decision-making procedures outlined in this work. This characteristic highlights their superior practicality and accuracy over existing methods, which often fail to capture the intricate interplay of elements in real-world scenarios.This demonstrates that applying the decision-making strategies covered in this article can lead to the discovery of even additional trait correlations. Finally, we evaluate the performance of our proposed method on a real choice problem and an experimental comparison. The results demonstrate that the new method will be more advantageous in a range of applications where decision-making is uncertain. Figure 1 illustrates all of the manuscript's results in a graphical abstract.

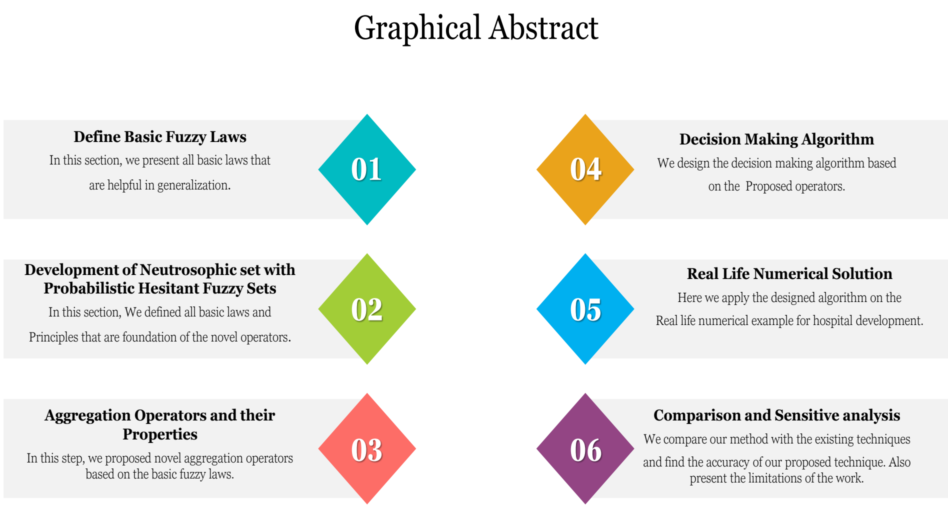

Figure 1: Graphical Abstract

Keywords: Neutrosophic information, probabilistic hesitant fuzzy sets, aggregation operators, decision making.

MSC: 03E72.

1. INTRODUCTION

There are significant differences between the economic development of rural and urban areas, especially in the medical sector. With better equipped hospitals, sophisticated medical facilities, and a greater concentration of healthcare specialists, urban locations typically have a more robust healthcare infrastructure. In metropolitan environments, access to specialist healthcare services and cutting-edge medical technologies is frequently greater than in rural ones. Furthermore, healthcare investments tend to be higher in urban areas, which stimulates more innovation and research [1, 2]. On the other hand, poor infrastructure, a lack of medical experts, and restricted availability of healthcare services are common problems in rural areas. Differences in health outcomes are made worse by the economic gap because rural populations may find it more difficult to access timely and high-quality medical care. To close this disparity, specific funding for rural healthcare infrastructure must be allocated, healthcare workers must be encouraged to work in rural regions, and technology must be used to support telemedicine and remote patient care. It will need a coordinated effort to tackle these discrepancies and make sure that healthcare facilities are available to everyone, everywhere, in order to achieve fair economic growth in the medical sector [3].
Efficient decision-making for growth in the economy necessitates a sophisticated strategy that takes into account the various opportunities and constraints that each location brings, especially in the medical field and in urban as well as rural areas. Prioritizing community health initiatives, telemedicine projects, and healthcare infrastructure can improve accessibility to medical services in remote locations. The logistical challenges of accessing remote communities can also be addressed by making investments in the training of local healthcare personnel and the deployment of mobile medical units. Prioritizing cutting-edge hospitals, research institutes, and specific medical facilities can boost the economy in metropolitan areas. A strong medical ecosystem can be promoted by public-private collaborations and incentives for the development of innovative healthcare technologies. Adapting plans to the unique requirements of every area promotes a more just and long-lasting growth of the health care industry, which advances national economic growth [4]. In the medical fields of both urban and rural areas, where uncertainty and imprecise evidence are prevalent, fuzzy set theory is a useful tool for decision-making related to economic development. Fuzzy set theory takes degrees of membership into account and allows for a more nuanced portrayal of real-world complications than classical set theory, that is based on binary classification (an element either belonging to a set or it does not). Fuzzy set theory helps decision-makers deal with the ambiguity and vagueness that are inherent in the data when it comes to medical resource distribution and planning, where elements like population dynamics, changing healthcare requirements, and resource limits are common.

Because of its adaptability, economic development techniques can be tailored to fit a variety of dynamic healthcare contexts, leading to more responsive and flexible approaches in both urban and rural settings. In the end, the use of fuzzy set theory makes decision-making processes more efficient, which leads to better results while pursuing improved healthcare services and local economic growth. Classical or crisp techniques of dealing with ambiguity and uncertainty in decision-making settings may not always be the most successful. Zadeh [5] pioneered the use of fuzzy sets (FS) to deal with such uncertainty in 1965. In FSs, Zadeh assigns membership grades to elements of a set in the interval [0,1]. Many set theoretic elements of crisp circumstances were established for fuzzy sets, making Zadeh's work in this area notable. Atanassov [6] investigated the intuitionistic fuzzy set (IFS), a better variation of FS that includes membership and non-membership degrees. Over the last few decades, IFSs have been shown to be beneficial and regularly used by scientists to analyst ambiguity and unreliability in data. IFS is supported by many researchers, including: Xu [7] discussed the algebraic arithmetic aggregation information (AgIn) under IFSs and how it can be used to difficult real-world applications. Jiang et al. [8] developed the entropy-based power AgIn under IF information. The Pythagorean Fuzzy Set (PyFS) is a recent active tool for determining uncertainty in MADM problems [9, 10]. The PyFS is defined as the total of membership and non-membership degrees that is less than or equal to one. The PyFS has more users than the IFS. PyFS is the ideal strategy for dealing with ambiguous problems because all PyF degrees contain IF degrees. Xu and Zhang [11] mathematically developed the PyFS and introduced the concept of PyFNs, as well as the theory of PyF TOPSIS for using PyFNs to solve MADM problems. Peng and Yang [12] created the PyF superiority and inferiority grading process for MAGDM using PyFNs. James and Beliakov [13] concentrated on the concept of "average" and how to create aggregate functions that produce output similar to typical fuzzy numbers [14]. PyFN is used by Reformat and Yager to deal with interactive decision support systems [15]. Gou et al. investigated the properties of continuous PyF information [16]. Torra [17] devised the hesitant fuzzy set (HFS), which necessitates that the membership have a set of possible values, to more forcefully characterize the HFS than the earlier classical fuzzy set extensions. A novel model based on HFSs has recently been implemented to deal with situations where experts are divided between many options for an indication, alternative, element, etc. [18, 19]. When experts waver between a number of prospective memberships for an element of a sequence of decisions, HFSs are especially helpful at addressing the challenges with group decision making [20]. The hesitant linguistic word set [21], the hesitant fuzzy uncertain linguistic set [22], the dual hesitant fuzzy set [23, 24], and the generalized hesitant fuzzy set [25] are just a few of the extensions to HFS that have been implemented to handle more complex environments. A number of academics have applied the HFS idea to group decision-making contexts by using aggregation operators [26, 27, 28]. Smarandache first proposed the neutrosophic set (NS), an analytical framework and quantitative tool for comprehending the origin, nature, and extent of neutralities [29]. It is a spiritual practice that focuses on neutralities' origins, nature, and scope as well as how they interact

with other ideational spectrums. The NS generalizes the ideas behind the classical set [30], fuzzy set, interval valued fuzzy set, interval-valued IFS [31], paraconsistent set, dialetheist set, paradoxist set, and tautological set [32]. A NS is characterized by truth membership function $\beta_F(\varkappa)$, indeterminacy membership function $\alpha_F(\varkappa)$ and falsity membership function $\gamma_F(\varkappa)$, where $\beta_F(\varkappa)$, $\alpha_F(\varkappa)$ and $\gamma_F(\varkappa)$ are real standard or nonstandard elements from $]0^{-,}1^{+}[$. It will be difficult to apply NS in actual scientific and engineering contexts, despite the fact that it philosophically generalizes the ideas of FS, IFS, and all existing structures. This idea is crucial in many situations, such as information fusion, which integrates data from several sensors. In recent years, engineering and other industries have mainly used neutrosophic sets to make decisions. A single-valued neutrosophic set (SVNS), which can deal with inaccurate, ambiguous, and incompatible data issues, was proposed by Wang et al. [33]. A SVNS, on the other hand, is an NS that enables us to depict ambiguity, imprecision, incompleteness, and inconsistent behavior in the real world [34]. Making decisions using confusing data and dissimilar metrics would be more acceptable [35, 36, 37, 38]. Contrarily, SVNSs can be used in technical and scientific applications since SVNS theory is effective at modeling ambiguous, imperfect, and inconsistent data [39, 40]. The SVNS can easily capture the ambiguous nature of subjective assessments, making it excellent for gathering vague, ambiguous, and inconsistent data in multi criteria decision-making analysis [41].
Qian et al. (2013) [25] presented a noval notion of HFSs in which IFSs were included as an extended version of HFSs. The HFS satisfies the condition that the total of its MG and NMG of adhesion is less than one. Pythagorean HFS was invented by Khan et al [42]. Batool et al. [43] defined and investigated the usage of a Pythagorean probabilistic hesitant fuzzy set in decision-making algorithms. Ashraf et al. [44, 45], introduced the DM modelling based on sine trigonometric Pythagorean fuzzy aggregate information. Huang et al. [46] proposed the new MULTIMOORA technique in PyFS and discussed its applications in MADM. To solve this, Xu and Zhu [47] proposed the concept of probabilistic hesitant fuzzy sets (PHFSs). The notion of a SV-neutrosophic probabilistic hesitant fuzzy set (SV-NPHFS) is devised to avoid the deficiency in DM difficulties [48].
Kamran, M., Ashraf, S., & Naeem, M. (2023) [48], devolped the Dombi operators for Single-Valued Neutrosophic Probabilistic Hesitant Fuzzy sets. Due to their capacity to represent the compromise between conjunction and disjunction operations, Dombi operators in fuzzy set theory are unique. The Dombi operator is a flexible tool for representing uncertainty and imprecision in contrast to typical fuzzy operators. It does this by introducing a parameter that allows for changing concentration either on the minimum or maximum values. Due to their flexibility, Dombi operators are useful in situations requiring a more customized and flexible approach, including complicated and dynamic system decision-making processes. They are more successful at capturing subtle interactions inside fuzzy sets thanks to their special parameterizations, which makes modeling more precise and customized for a wider range of applications. [49]
Traditional Dombi algorithms in fuzzy sets may face constraints when attempting to address the complex and multifaceted difficulties associated with decision-

making in the complicated areas of economic development, particularly in the rural and urban healthcare sectors, because of the complex interplay of factors like location, infrastructure, and geographical influences. Our book acknowledges the requirement for more resilient tools and suggests new operators that can effectively handle the many uncertainties present in these industries. These novel operators seek to offer a more complete and flexible framework for decision-makers by integrating a wider range of characteristics and improving the depiction of fuzzy interactions. This advancement is essential for managing the complexities of infrastructure planning, policy formulation, and resource allocation, where a nuanced approach is required due to the interconnected nature of several issues. Our manuscript presents novel operators that aim to increase the effectiveness of decision-making processes for economic growth that is sustainable and better healthcare outcomes while recognizing and addressing the complex web of impacts in both rural and urban environments.
In this article, we propose some new AOs like SV-NPHFWA AOs, SV-NPHFWG AOs, SV-NPHFOWA AOs, SV-NPHFOWG, and SV-NPHFHWG AOs because of the following reasons:

1 SV-NPHFSs anticipate more space to decision-makers due to the combined notion of HFS and Probabilistic Fuzzy Sets (PFS).

2 SV-NPHFSs uses all type of hesitation and probability approximation spaces that property lacks in litterateur in SV-NSs environment.

3 SV-NPHFWA AOs, and SV-NPHFWG AOs can incorporate the familiarity degree of experts with evaluated objects for initial assessment and that property lacks in SV-neutrosophic weighted averaging and geometric aggregation operators.

4 This article seeks to cover more advanced and sophisticated data because SV-NPHFWA and SV-NPHFWG operators are simple and cover the decision-making technique.

5 All current drawbacks are limited by the suggested work.

Thus, the contributions of this study are as follows:

1 To begin considering novel AOs like SV-NPHFWA and SV-NPHFWA AOs, as well as their traits like monotonicity, idempotency, and boundedness.

2 We defined all basic algebraic informations of SV-NPHFS.

3 Properties of these aggregation operations have been proposed.

4 To handle increasingly complex data, an algorithm for the MCDM technique has been devised.

5 A medical heathcare unit development algorithm based on SV-NPHFS has also been proposed, and an example is provided to illustrate how well the algorithm works.

The paper is arranged in the prescribed manner. Section 2 contain the definition of FSs, IFSs, HFSs, and SV-NPHFSs, as well as SV-NPHFS aggregation operators. In section 3 is composed the SV-NPHFWA operator, SV-NPHFOWA operator, SV-NPHFHWA operator, SV-NPHFWG operator, SV-NPHFOWG operator, and SV-NPHFHWG operator, as well as their features such as monotonicity, idempotency, and boundedness. In Section 4, we use the SV-NPHF aggregation operators to deal with ambiguity in DM situations using SPHF data. Section 5 demonstrates how to use the well-known MCDM approach. The conclusion and discussion of the work are included in Section 6.

2. PRELIMINARIES

This section contains fundamental definitions and information about Fuzzy set (FS), hesitant FS, intuitionistic FS, Pythagorean FS, and spherical FS.

Definition 1. *[5] Consider the non-empty universal set ρ. A fuzzy set δ in ρ having the form*

$$\delta = \{\langle \Gamma, \mathcal{A}_\delta(\Gamma)\rangle \,|\Gamma \in \rho\}, \tag{1}$$

where $\mathcal{A}_\delta(\ \Gamma) \in [0,1]$ demonstrates the M_G of Γ in δ.

Definition 2. *[6] Consider the non-empty universal set ρ. An intuitionistic FS δ in ρ having the form*

$$\delta = \{\langle \Gamma, \mathcal{A}_\delta(\Gamma), \mathcal{N}_\delta(\Gamma)\rangle \,|\Gamma \in \rho\}, \tag{2}$$

where $\mathcal{A}_\delta : \rho \to [0,1]$ be the M_G and $\mathcal{N}_\delta : \rho \to [0,1]$ be the NM_G with the constraint that $\mathcal{A}_\delta(\ \Gamma) + \mathcal{N}_\delta(\ \Gamma) \leq 1$, for all $\Gamma \in \rho$.

Definition 3. *[33] Consider the non-empty universal set ρ. A Single-Valued Neutrosophic set (SV-NS) δ in ρ having the form*

$$\delta = \{\langle \Gamma, \mathcal{A}_\delta(\Gamma), \mathcal{Z}_\delta(\Gamma), \mathcal{N}_\delta(\Gamma)\rangle \,|\Gamma \in \rho\}, \tag{3}$$

where $\mathcal{A}_\delta : \rho \to [0,1]$ be M_G, $\mathcal{Z}_\delta : \rho \to [0,1]$ be N_G and $\check{N}_\delta : \rho \to [0,1]$ be NM_G with the constraint that $0 \leq \mathcal{A}_\delta(\ \Gamma) + \mathcal{Z}_\delta(\ \Gamma) + \mathcal{N}_\delta(\ \Gamma) \leq 3$, for all $\Gamma \in \rho$.

Definition 4. *[47] Consider the non-empty universal set ρ. A SV-neutrosophic hesitant FS δ in ρ having the form*

$$\delta = \{\langle \Gamma, \mathcal{A}_\delta(\Gamma), \mathcal{Z}_\delta(\Gamma), \mathcal{N}_\delta(\Gamma)\rangle \,|\Gamma \in \rho\}, \tag{4}$$

where

$$\mathcal{A}_\delta(\Gamma) = \{\ell | \ell \in [0,1]\},\ \mathcal{Z}_\delta(\Gamma) = \{\Upsilon | \Upsilon \in [0,1]\} \text{ and } \mathcal{N}_\delta(\Gamma) = \{\check{n} | \check{n} \in [0,1]\},$$

are the three sets of some values in $[0,1]$, represents the M_G, N_G and NM_G with the constraint $0 \leq \ell^+ + \Upsilon^+ + \check{n}^+ \leq 1$, for all $\Gamma \in \rho$, such that

$$\ell^+ = \bigcup_{\ell \in \mathcal{A}_\delta(\Gamma)} \max\{\ell\},\ \Upsilon^+ = \bigcup_{\Upsilon \in \mathcal{Z}_\delta(\Gamma)} \max\{\Upsilon\},\ \text{and } \check{n}^+ = \bigcup_{\check{n} \in \mathcal{N}_\delta(\Gamma)} \max\{\check{n}\}.$$

Definition 5. *[47] For a set ρ. A Probabilistic HFS δ in ρ is described as*

$$\delta = \{\langle \Gamma, \hbar_y(\Gamma)/\alpha_\delta\rangle \,|\Gamma \in \rho\} \tag{5}$$

where $\hbar_y(\,\Gamma)$ is subset of $[0,1]$ and $\hbar_y(\,\Gamma)/\alpha_\delta$ represent the M_G of $\Gamma \in \rho$ in δ. And α_δ represent the possibilities of $\hbar_y(\,\Gamma)$, with constraint that $\sum_\delta \alpha_\delta = 1$.

3. DEVELOPMENT OF SV-NEUTROSOPHIC INFORMATION WITH PROBABILISTIC HESITANT FUZZY SETS

Definition 6. *Consider the non-empty universal set ρ. A SV-NPHFS δ in ρ is described as follows:*

$$\delta = \{\left\langle \Gamma, \mathcal{A}_{\hbar_y}(\Gamma)/\alpha_\delta, \mathcal{Z}_{\hbar_y}(\Gamma)/\beta_\delta, \mathcal{N}_{\hbar_y}(\Gamma)/\gamma_\delta\right\rangle |\Gamma \in \rho\}, \tag{6}$$

for all $\Gamma \in \rho$, $\mathcal{A}_\delta(\,\Gamma), \mathcal{Z}_\delta(\,\Gamma)$ and $\mathcal{N}_\delta(\,\Gamma)$ are sets associated with specific values in $[0,1]$. Where $\mathcal{A}_\delta(\,\Gamma)/\alpha, \mathcal{Z}_\delta(\,\Gamma)/\beta_\delta$ & $\mathcal{N}_\delta(\,\Gamma)/\gamma_\delta$ specifies the possible $\mathrm{M_G}$, $\mathrm{N_G}$ and $\mathrm{NM_G}$ of Γ to the SV-NPHFS δ, individually. $\alpha_\delta, \beta_\delta$ and γ_δ represent the possibilities of the grades. Also, there is $0 \leq \ell_i,\ \Upsilon_i, \check{n}_i \leq 1$ and $0 \leq \alpha_i, \beta_i, \gamma_i \leq 1$ with $\sum_{i=1}^{q} \alpha_i \leq 1, \sum_{i=1}^{q} \beta_i \leq 1,\ \sum_{i=1}^{q} \gamma_i \leq 1$ (q is a positive integer to describe the number of elements contained in SV-NPHFS), where $\ell_i \in \mathcal{A}_{\hbar_y}(\,\Gamma)$, $\Upsilon_i \in \mathcal{Z}_{\hbar_y}(\,\Gamma)$, $\alpha_i \in \alpha_\delta$, $\beta_i \in \beta_\delta, \gamma_i \in \gamma_\delta$. Furthermore, it is required that $\max\left(\mathcal{A}_{\hbar_y}(\Gamma)\right) + \min\left(\mathcal{Z}_{\hbar_y}(\Gamma)\right) + \min\left(\mathcal{N}_{\hbar_y}(\Gamma)\right) \leq 3$ and $\min\left(\mathcal{A}_{\hbar_y}(\Gamma)\right) + \max\left(\mathcal{Z}_{\hbar_y}(\Gamma)\right) + \max\left(\mathcal{N}_{\hbar_y}(\Gamma)\right) \leq 3$.

For simplicity, We shall refer to the SV-NPHF number by the triplet $\left(\mathcal{A}_{\hbar_y}/\alpha_\delta, \mathcal{Z}_{\hbar_y}/\beta_\delta, \mathcal{N}_{\hbar_y}/\gamma_\delta\right)$ .The group of all SV-NPHFs in ρ is represented by $SV - NPHFRS(\rho)$.

Definition 7. *Let $\delta_1 = \left(\mathcal{A}_{\hbar_{\delta_1}}/\alpha_{\delta_1}, \mathcal{Z}_{\delta_1})/\beta_{\delta_1}, \mathcal{N}_{\hbar_{\delta_1}}/\gamma_{\delta_1}\right)$ and $\delta_2 = \left(\mathcal{A}_{\hbar_{\delta_2}}/\alpha_{\delta_2}, \mathcal{Z}_{\delta_2})/\beta_{\delta_2}, \mathcal{N}_{\hbar_{\delta_2}}/\gamma_{\delta_2}\right)$ be SV-NPHFNs. The fundamental operating laws are described as follows:*

$$\delta = \{\left\langle \mathcal{A}_{\hbar_{\delta_1}}/\alpha_{\delta_1}, \mathcal{Z}_{\hbar_{\delta_1}}/\beta_{\delta_1}, \mathcal{N}_{\hbar_{\delta_1}}/\gamma_{\delta_1}\right\rangle |\},$$

$$(1)\ \delta_1 \cup \delta_2 = \left\{\begin{array}{l} \bigcup\limits_{\substack{\ell_1 \in \mathcal{A}_{\hbar_{y_1}}, \alpha_1 \in \alpha_{\delta_1} \\ \ell_2 \in \mathcal{A}_{\hbar_{y_1}}, \alpha_2 \in \alpha_{\delta_2}}} (\max(\ell_1/\alpha_1, \ell_2/\alpha_2)), \bigcup\limits_{\substack{\Upsilon_1 \in \mathcal{Z}_{\hbar_{\delta_1}},\ \beta_1 \in \beta_{\delta_1} \\ \Upsilon_2 \in \mathcal{Z}_{\hbar_{\delta_2}},\ \beta_1 \in \beta_{\delta_2}}} (\min(\Upsilon_1/\beta_1, \Upsilon_2/\beta_2)), \\ \bigcup\limits_{\substack{\check{n}_1 \in \mathcal{Z}_{\hbar_{\delta_1}},\ \gamma_1 \in \gamma_{\delta_1} \\ \check{n}_2 \in \mathcal{Z}_{\hbar_{\delta_2}},\ \gamma_2 \in \gamma_{\delta_2}}} (\min(\check{n}_1/\gamma_1, \check{n}_2/\gamma_2)) \end{array}\right\};$$

$$(2)\ \delta_1 \cap \delta_2 = \left\{\begin{array}{l} \bigcup\limits_{\substack{\ell_1 \in \mathcal{A}_{\hbar_{y_1}}, \alpha_1 \in \alpha_{\delta_1} \\ \ell_2 \in \mathcal{A}_{\hbar_{y_1}}, \alpha_2 \in \alpha_{\delta_2}}} (\min(\ell_1/\alpha_1, \ell_2/\alpha_2)), \bigcup\limits_{\substack{\Upsilon_1 \in \mathcal{Z}_{\hbar_{\delta_1}}, \beta_1 \in \beta_{\delta_1} \\ \Upsilon_2 \in \mathcal{Z}_{\hbar_{\delta_2}},\ \beta_1 \in \beta_{\delta_2}}} (\max(\Upsilon_1/\beta_1, \Upsilon_2/\beta_2)), \\ \bigcup\limits_{\substack{\check{n}_1 \in \mathcal{Z}_{\hbar_{\delta_1}},\ \gamma_1 \in \gamma_{\delta_1} \\ \check{n}_2 \in \mathcal{Z}_{\hbar_{\delta_2}},\ \gamma_2 \in \gamma_{\delta_2}}} (\max(\check{n}_1/\gamma_1, \check{n}_2/\gamma_2)) \end{array}\right\};$$

(3) $\delta_1^c = \left\{\mathcal{N}_{\hbar_{\delta_1}}/\gamma_{\delta_1}, \mathcal{Z}_{\hbar_y}/\beta_\delta, \mathcal{A}_{\hbar_y}/\alpha_\delta\right\}$

Definition 8. *Let* $\delta_1 = \left(\mathcal{A}_{\hbar_{\delta_1}}/\alpha_{\delta_1}, \mathcal{Z}_{\delta_1})/\beta_{\delta_1}, \mathcal{N}_{\hbar_{\delta_1}}/\gamma_{\delta_1}\right)$ *and* $\delta_2 = \left(\mathcal{A}_{\hbar_{\delta_2}}/\alpha_{\delta_2}, \mathcal{Z}_{\delta_2})/\beta_{\delta_2}, \mathcal{N}_{\hbar_{\delta_2}}/\gamma_{\delta_2}\right)$ *be SV-NPHFNs and* $\partial > 0 (\in \mathbb{R})$*, then their operations are presented as:*

$$\textit{(1)}\ \delta_1 \oplus \delta_2 = \left\{ \begin{array}{c} \bigcup\limits_{\substack{\ell_1 \in \mathcal{A}_{\hbar_{\delta_1}}, \ell_2 \in \mathcal{A}_{\hbar_{\delta_2}} \\ \alpha_1 \in \alpha_{\delta_1}, \alpha_2 \in \alpha_{\delta_2}}} \left(\ell_1 + \ell_2 - \ell_1 \ell_2 / \alpha_1 \alpha_2\right), \\ \bigcup\limits_{\substack{\Upsilon_1 \in \mathcal{Z}_{\hbar_{\delta_1}}, \Upsilon_2 \in \mathcal{Z}_{\hbar_{\delta_2}} \\ \beta_1 \in \beta_{\delta_1},\ \beta_2 \in \beta_{\delta_2}}} \left(\Upsilon_1 \Upsilon_2 / \beta_1 \beta_2\right) \\ \bigcup\limits_{\substack{\check{n}_1 \in \mathcal{N}_{\hbar_{\delta_1}}, \check{n}_2 \in \mathcal{N}_{\hbar_{\delta_2}} \\ \gamma_1 \in \gamma_{\delta_1},\ \gamma_2 \in \gamma_{\delta_2}}} \left(\check{n}_1 \check{n}_2 / \gamma_1 \gamma_2\right) \end{array} \right\};$$

$$\textit{(2)}\ \delta_1 \otimes \delta_2 = \left\{ \begin{array}{c} \bigcup\limits_{\substack{\ell_1 \in \mathcal{A}_{\hbar_{\delta_1}}, \ell_2 \in \mathcal{A}_{\hbar_{\delta_2}} \\ \alpha_1 \in \alpha_{\delta_1}, \alpha_2 \in \alpha_{\delta_2}}} \left(\ell_1 \ell_2 / \alpha_1 \alpha_2\right), \\ \bigcup\limits_{\substack{\Upsilon_1 \in \mathcal{Z}_{\hbar_{\delta_1}}, \Upsilon_2 \in \mathcal{Z}_{\hbar_{\delta_2}} \\ \beta_1 \in \beta_{\delta_1}, \beta_2 \in \beta_{\delta_2}}} \left(\Upsilon_1 + \Upsilon_2 - \Upsilon_1 \Upsilon_2 / \beta_1 \beta_2\right) \\ \bigcup\limits_{\substack{\check{n}_1 \in \mathcal{N}_{\hbar_{\delta_1}}, \check{n}_2 \in \mathcal{N}_{\hbar_{\delta_2}} \\ \check{n}_1 \in \gamma_{\delta_1}, \gamma_2 \in \gamma_{\delta_2}}} \left(\check{n}_1 + \check{n}_2 - \check{n}_1 \check{n}_2 / \gamma_1 \gamma_2\right) \end{array} \right\};$$

$$\textit{(3)}\ \partial\ \delta_1 = \left\{ \bigcup_{\ell_1 \in \mathcal{A}_{\hbar_{\delta_1}}, \alpha_1 \in \alpha_{\delta_1}} \left(1 - (1 - \ell_1)^{\partial} / \alpha_1\right), \bigcup_{\Upsilon_1 \in \mathcal{N}_{\hbar_{\delta_1}}, \gamma_1 \in \gamma_{\delta_1}} \left(\Upsilon_1^{\partial} / \gamma_1\right), \bigcup_{\Upsilon_1 \in \mathcal{N}_{\hbar_{\delta_1}}, \gamma_1 \in \gamma_{\delta_1}} \left(\check{n}_1^{\partial} / \gamma_1\right) \right\};$$

$$\textit{(4)}\ \delta_1^{\partial} = \left\{ \bigcup_{\ell_1 \in \mathcal{A}_{\hbar_{\delta_1}}, \alpha_1 \in \alpha_{\delta_1}} \left(\ell_1^{\partial} / \alpha_1\right), \bigcup_{\Upsilon_1 \in \mathcal{Z}_{\beta_1 \in \beta_{\delta_1}}} \left(1 - (1 - \Upsilon_1)^{\partial} / \beta_1\right), \bigcup_{\check{n}_1 \in \mathcal{N}_{\hbar_{\delta_1}}, \gamma_1 \in \gamma_{\delta_1}} \left(1 - (1 - \check{n}_1)^{\partial} / \gamma_1\right) \right\}.$$

Definition 9. *For any SV-NPHFN* $\delta = (\mathcal{A}_{\hbar_\delta}/\alpha_\delta, \mathcal{Z}_\delta/\beta_\delta, \mathcal{N}_{\hbar_\delta}/\gamma_\delta)$ *a score function be defined as*

$$\begin{aligned} s(\delta) \quad = \quad & \left(\frac{1}{l(\mathcal{A}_\delta)} \sum\nolimits_{\ell_i \in \mathcal{A}_{\hbar_{\delta_i}}, \alpha_i \in \alpha_{\delta_i}} (\ell_i \cdot \alpha_i)\right) - \left(\frac{1}{l(\mathcal{Z}_\delta)} \sum\nolimits_{\Upsilon_i \in \mathcal{Z}_{\hbar_{\delta_i}}, \beta_i \in \beta_{\delta_i}} (\Upsilon_i \cdot \beta_i)\right) \quad (7) \\ & \left(\frac{1}{l(\mathcal{N}_\delta)} \sum\nolimits_{\check{n}_i \in \mathcal{N}_{\hbar_{\delta_i}}, \gamma_i \in \gamma_{\delta_i}} (\check{n}_i \cdot \gamma_i)\right) \end{aligned}$$

where $l(\mathcal{A}_\delta)$ *indicates to the number of constituents in* $\mathcal{A}_{\hbar_{\delta_i}}$, $l(\mathcal{Z}_\delta)$ *indicates to the number of constituents in* $\mathcal{Z}_{\hbar_{\delta_i}}$ *and* $l(\mathcal{N}_\delta)$ *indicates to the number of constituents in* $\mathcal{N}_{\hbar_{\delta_i}}$.

Definition 10. *For any SV-NPHFN* $\delta = \left(\mathcal{A}_{\hbar_y}/\alpha_\delta, \mathcal{Z}_{\hbar_y}/\beta_\delta, \mathcal{N}_{\hbar_\delta}/\gamma_\delta\right)$, *an accuracy function is defined as*

$$\begin{aligned} \hbar(\delta) \quad = \quad & \frac{1}{l(\mathcal{A}_\delta)} \sum\nolimits_{\ell_i \in \mathcal{A}_{\hbar_{\delta_i}}, \alpha_i \in \alpha_{\delta_i}} (\ell_i \cdot \alpha_i) + \frac{1}{l(\mathcal{Z}_\delta)} \sum\nolimits_{\Upsilon_i \in \mathcal{Z}_{\hbar_{\delta_i}}, \beta_i \in \beta_{\delta_i}} (\Upsilon_i \cdot \beta_i) \quad (8) \\ & \frac{1}{l(\mathcal{N}_\delta)} \sum\nolimits_{\check{n}_i \in \mathcal{N}_{\hbar_{\delta_i}}, \gamma_i \in \gamma_{\delta_i}} (\check{n}_i \cdot \gamma_i) \end{aligned}$$

where $l(\mathcal{A}_{\delta})$ *indicates to the number of constituents in* $\mathcal{A}_{\hbar_{\delta_i}}$, $l(\mathcal{Z}_{\delta})$ *indicates to the number of constituents in* $\mathcal{Z}_{\hbar_{\delta_i}}$ *and* $l(\mathcal{N}_{\delta})$ *indicates to the number of constituents in* $\mathcal{N}_{\hbar_{\delta_i}}$.

Definition 11. *Let* $\delta_1 = \left(\mathcal{A}_{\hbar_{\delta_1}}/\alpha_{\delta_1}, \mathcal{Z}_{\delta_1})/\beta_{\delta_1}, \mathcal{N}_{\hbar_{\delta_1}}/\gamma_{\delta_1}\right)$ *and* $\delta_2 = \left(\mathcal{A}_{\hbar_{\delta_2}}/\alpha_{\delta_2}, \mathcal{Z}_{\delta_2})/\beta_{\delta_2}, \mathcal{N}_{\hbar_{\delta_2}}/\gamma_{\delta_2}\right)$ *be SV-NPHFNs. Then, using the description above, a comparison of SV-NPHFNs can be stated as*

Definition 12. *(1) If* $s(\delta_1) > s(\delta_2)$, *then* $\delta_1 > \delta_2$.
(2) If $s(\delta_1) = s(\delta_2)$, *and* $\hbar(\delta_1) > \hbar(\delta_2)$ *then* $\delta_1 > \delta_2$

4. SV-NPHF AGGREGATION OPERATORS AND THEIR PROPERTIES

The features of aggregation operators for SV-NPHF Numbers generated from operational rules are discussed in this part.

4.1. SV-NPHF weighted averaging aggregation operators

The current weighted averaging AOs for SV-NPHF data are extended in this part.

Definition 13. *Let* $\delta_\Gamma = \left(\mathcal{A}_{\hbar_{\delta_\Gamma}}/\alpha_{\delta_\Gamma}, \mathcal{Z}_{\delta_\Gamma})/\beta_{\delta_\Gamma}, \mathcal{N}_{\hbar_{\delta_\Gamma}}/\gamma_{\delta_\Gamma}\right)$ $(\Gamma = 1, 2, ..., q)$ *be any group of SV-NPHFNs and* $SV-NP\ HFWA : SV-NP\ HFN^{\Gamma} \longrightarrow SV-NP\ HFN$. *The SV-NPHFWA operator can thus be defined as follows:*

$$SV-NPHFWA(\delta_1, \delta_2, ..., \delta_\Gamma) = \mho_1\delta_1 \oplus \mho_2\delta_2 \oplus ..... \oplus \mho_\Gamma\delta_\Gamma \tag{9}$$

where $\mho = (\mho_1, \mho_2, ..., \mho_\Gamma)^T$ *are the weights of* $\delta_\Gamma \in [0, 1]$ *with* $\sum_{\Gamma=1}^{q} \mho_\Gamma = 1$.

Theorem 14. *Let* $\delta_\Gamma = \left(\mathcal{A}_{\hbar_{\delta_\Gamma}}/\alpha_{\delta_\Gamma}, \mathcal{Z}_{\hbar_{\delta_\Gamma}}/\beta_{\delta_\Gamma}, \mathcal{N}_{\hbar_{\delta_\Gamma}}/\gamma_{\delta_\Gamma}\right)$ $(\Gamma = 1, 2, ..., q)$ *be any group of SV-NPHFNs. The SV-NPHFWA operator can thus be defined as follows, We can accomplish the following goals:*

$$SV-NPHFWA(\delta_1, \delta_2, ..., \delta_q) = \begin{pmatrix} \bigcup_{\ell_\Gamma \in \mathcal{A}_{\delta_\Gamma}, \alpha_{\delta_\Gamma} \in \alpha_{\delta_\Gamma}} 1 - \Pi_{\Gamma=1}^{q}\left(1-(\ell_{\delta_\Gamma})\right)^{\mho_\Gamma} / \Pi_{\Gamma=1}^{q}\alpha_{\delta_\Gamma}, \\ \bigcup_{\Upsilon_{\delta_\Gamma} \in \mathcal{Z}_{\delta_\Gamma}, \beta_{\delta_\Gamma} \in \beta_{\delta_\Gamma}} \Pi_{\Gamma=1}^{q}\left(\Upsilon_{\delta_\Gamma}\right)^{\mho_\Gamma} / \Pi_{\Gamma=1}^{q}\beta_{\delta_\Gamma}, \\ \bigcup_{\check{n}_{\delta_\Gamma} \in \mathcal{N}_{\delta_\Gamma}, \gamma_{\delta_\Gamma} \in \gamma_{\delta_\Gamma}} \Pi_{\Gamma=1}^{q}\left(\check{n}_{\delta_\Gamma}\right)^{\mho_\Gamma} / \Pi_{\Gamma=1}^{q}\gamma_{\delta_\Gamma} \end{pmatrix} \tag{10}$$

where $\mho = (\mho_1, \mho_2, ..., \mho_\Gamma)^T$ *are the weights of* $\delta_\Gamma \in [0, 1]$ *with* $\sum_{\Gamma=1}^{q} \mho_\Gamma = 1$.

Proof. We'll show the theorem by applying mathematical induction to m, and the proof will go like this:

Step-1: When $\Gamma = 2$, we have $\delta_1 = \left(\mathcal{A}_{\hbar_{\delta_1}}/\alpha_{\delta_1}, \mathcal{Z}_{\hbar_{\delta_1}}/\beta_{\delta_1}, \mathcal{N}_{\hbar_{\delta_1}}/\gamma_{\delta_1}\right)$ and $\delta_2 = \left(\mathcal{A}_{\hbar_{\delta_2}}/\alpha_{\delta_2}, \mathcal{Z}_{\hbar_{\delta_2}}/\beta_{\delta_2}, \mathcal{N}_{\hbar_{\delta_2}}/\gamma_{\delta_2}\right)$ As a result of the operation of SV-NPHFEs, we are able to

$$\mho_1\delta_1 = \left\{ \begin{array}{c} \bigcup_{\ell_1 \in \mathcal{A}_{\hbar_{\delta_1}}(l_\delta), \alpha_1 \in \alpha_{\delta_1}} \left(1-(1-\ell_1)^{\mho_1}/\alpha_1\right), \bigcup_{\Upsilon_1 \in \mathcal{Z}_{\hbar_{\delta_1}}(l_\delta), \beta_1 \in \beta_{\delta_1}} \left(\Upsilon_1^{\mho_1}/\beta_1\right), \\ \bigcup_{\check{n}_1 \in \mathcal{N}_{\hbar_{\delta_1}}(l_\delta), \gamma_1 \in \gamma_{\delta_1}} \left(\check{n}_1^{\mho_1}/\gamma_1\right) \end{array} \right\}$$

$$\mho_2\delta_2 = \left\{ \begin{array}{c} \bigcup_{\ell_2 \in \mathcal{A}_{\hbar_{\delta_2}}(l_\delta), \alpha_2 \in \alpha_{\delta_2}} \left(1-(1-\ell_2)^{\mho_2}/\alpha_2\right), \bigcup_{\Upsilon_2 \in \mathcal{Z}_{\hbar_{\delta_2}}(l_\delta), \beta_2 \in \beta_{\delta_2}} \left(\Upsilon_2^{\mho_2}/\beta_2\right), \\ \bigcup_{\check{n}_2 \in \mathcal{N}_{\hbar_{\delta_2}}(l_\delta), \gamma_2 \in \gamma_{\delta_2}} \left(\check{n}_2^{\mho_2}/\gamma_2\right) \end{array} \right\}$$

Then

$$\begin{aligned} SV-NPHFWA(\delta_1,\delta_2) &= \mho_1\delta_1 \oplus \mho_2\delta_2 \\ &= \left\{ \begin{array}{c} \bigcup_{\substack{\ell_1 \in \mathcal{A}_{\hbar_{\delta_1}}(l_\delta), \alpha_1 \in \alpha_{\delta_1} \\ \ell_2 \in \mathcal{A}_{\hbar_{\delta_2}}(l_\delta), \alpha_2 \in \alpha_{\delta_2}}} \left(\begin{array}{c} 1-(1-\ell_1)^{\mho_1}+1-(1-\ell_2)^{\mho_2} \\ -(1-(1-\ell_1)^{\mho_1})(1-(1-\ell_2)^{\mho_2}) \end{array} /\alpha_1\alpha_2 \right), \\ \bigcup_{\substack{\Upsilon_1 \in \mathcal{Z}_{\hbar_{\delta_1}}(l_\delta), \beta_1 \in \beta_{\delta_1} \\ \Upsilon_2 \in \mathcal{Z}_{\hbar_{\delta_2}}(l_\delta), \beta_2 \in \beta_{\delta_2}}} \left(\Upsilon_1^{\mho_1}/\beta_1 \Upsilon_2^{\mho_2}/\beta_2\right), \bigcup_{\substack{\check{n}_1 \in \mathcal{N}_{\hbar_{\delta_1}}(l_\delta), \gamma_1 \in \gamma_{\delta_1} \\ \check{n}_2 \in \mathcal{N}_{\hbar_{\delta_2}}(l_\delta), \gamma_2 \in \gamma_{\delta_2}}} \left(\check{n}_1^{\mho_1}/\gamma_1 \check{n}_2^{\mho_2}/\gamma_2\right) \end{array} \right\} \\ &= \left\{ \begin{array}{c} \bigcup_{\substack{\ell_1 \in \mathcal{A}_{\hbar_{\delta_1}}(l_\delta), \alpha_1 \in \alpha_{\delta_1} \\ \ell_2 \in \mathcal{A}_{\hbar_{\delta_2}}(l_\delta), \alpha_2 \in \alpha_{\delta_2}}} \left(1-(1-\ell_1)^{\mho_1}(1-\ell_2)^{\mho_2}/\alpha_1\alpha_2\right), \\ \bigcup_{\substack{\Upsilon_1 \in \mathcal{Z}_{\hbar_{\delta_1}}(l_\delta), \beta_1 \in \beta_{\delta_1} \\ \Upsilon_2 \in \mathcal{Z}_{\hbar_{\delta_2}}(l_\delta), \beta_2 \in \beta_{\delta_2}}} \left(\Upsilon_1^{\mho_1}\Upsilon_2^{\mho_2}/\beta_1\beta_2\right), \bigcup_{\substack{\check{n}_1 \in \mathcal{N}_{\hbar_{\delta_1}}(l_\delta), \gamma_1 \in \gamma_{\delta_1} \\ \check{n}_2 \in \mathcal{N}_{\hbar_{\delta_2}}(l_\delta), \gamma_2 \in \gamma_{\delta_2}}} \left(\check{n}_1^{\mho_1}\check{n}_2^{\mho_2}/\gamma_1\gamma_2\right) \end{array} \right\} \\ &= \left\{ \begin{array}{c} \bigcup_{\substack{\ell_1 \in \mathcal{A}_{\hbar_{\delta_1}}(l_\delta), \alpha_1 \in \alpha_{\delta_1} \\ \ell_2 \in \mathcal{A}_{\hbar_{\delta_2}}(l_\delta), \alpha_2 \in \alpha_{\delta_2}}} \left(1-\Pi_{\Gamma=1}^2(1-\ell_\Gamma)^{\mho_\Gamma}/\Pi_{\Gamma=1}^2\alpha_\Gamma\right), \\ \bigcup_{\substack{\Upsilon_1 \in \mathcal{Z}_{\hbar_{\delta_1}}(l_\delta), \beta_1 \in \beta_{\delta_1} \\ \Upsilon_2 \in \mathcal{Z}_{\hbar_{\delta_2}}(l_\delta), \beta_2 \in \beta_{\delta_2}}} \left(\Pi_{\Gamma=1}^2\Upsilon_\Gamma^{\mho_\Gamma}/\Pi_{\Gamma=1}^2\beta_\Gamma\right), \\ \bigcup_{\substack{\check{n}_1 \in \mathcal{Z}_{\hbar_{\delta_1}}(l_\delta), \gamma_1 \in \gamma_{\delta_1} \\ \check{n}_2 \in \mathcal{Z}_{\hbar_{\delta_2}}(l_\delta), \gamma_2 \in \gamma_{\delta_2}}} \left(\Pi_{\Gamma=1}^2\check{n}_\Gamma^{\mho_\Gamma}/\Pi_{\Gamma=1}^2\gamma_\Gamma\right) \end{array} \right\} \end{aligned}$$

As a result, the outcome is valid for m=2.

Step-2: Suppose that the outcome is the true for m=n, we obtain

$$SV-NPHFWA(\delta_1,\delta_2,....,\delta_n) = \left(\begin{array}{c} \bigcup_{\ell_\Gamma \in \mathcal{A}_{\delta_\Gamma}, \alpha_{\delta_\Gamma} \in \alpha_{\delta_\Gamma}} 1-\Pi_{\Gamma=1}^q\left(1-(\ell_{\delta_\Gamma})\right)^{\mho_\Gamma}/\Pi_{\Gamma=1}^q\alpha_{\delta_\Gamma}, \\ \bigcup_{\Upsilon_{\delta_\Gamma} \in \mathcal{Z}_{\delta_\Gamma}, \beta_{\delta_\Gamma} \in \beta_{\delta_\Gamma}} \Pi_{\Gamma=1}^q\left(\Upsilon_{\delta_\Gamma}\right)^{\mho_\Gamma}/\Pi_{\Gamma=1}^q\beta_{\delta_\Gamma}, \\ \bigcup_{\check{n}_{\delta_\Gamma} \in \mathcal{N}_{\delta_\Gamma}, \gamma_{\delta_\Gamma} \in \gamma_{\delta_\Gamma}} \Pi_{\Gamma=1}^q\left(\check{n}_{\delta_\Gamma}\right)^{\mho_\Gamma}/\Pi_{\Gamma=1}^q\gamma_{\delta_\Gamma} \end{array} \right)$$

Step-3: When $\Gamma = n+1$, then we have

$$\begin{aligned}
SV-NPHFWA(\delta_1,\delta_2,....,\delta_{n+1}) &= \bigoplus_{\Gamma=1}^{q} \mho_\Gamma \delta_\Gamma \oplus \mho_{q+1}\delta_{q+1} \\
&= \begin{pmatrix} \bigcup_{\ell_\Gamma \in \mathcal{A}_{\delta_\Gamma}, \alpha_{\delta_\Gamma} \in \alpha_{\delta_\Gamma}} 1-\Pi_{\Gamma=1}^{q}\left(1-\ell_{\delta_\Gamma}\right)^{\mho_\Gamma} / \Pi_{\Gamma=1}^{q}\alpha_{\delta_\Gamma}, \\ \bigcup_{\Upsilon_{\delta_\Gamma} \in \mathcal{Z}_{\delta_\Gamma}, \beta_{\delta_\Gamma} \in \beta_{\delta_\Gamma}} \Pi_{\Gamma=1}^{q}\left(\Upsilon_{\delta_\Gamma}\right)^{\mho_\Gamma} / \Pi_{\Gamma=1}^{q}\beta_{\delta_\Gamma}, \\ \bigcup_{\check{n}_{\delta_\Gamma} \in \mathcal{N}_{\delta_\Gamma}, \gamma_{\delta_\Gamma} \in \gamma_{\delta_\Gamma}} \Pi_{\Gamma=1}^{q}\left(\check{n}_{\delta_\Gamma}\right)^{\mho_\Gamma} / \Pi_{\Gamma=1}^{q}\gamma_{\delta_\Gamma} \end{pmatrix} \\
&\quad \oplus \begin{pmatrix} \bigcup_{\ell_{q+1} \in \mathcal{A}_{\delta_{q+1}}, \alpha_{\delta_{q+1}} \in \alpha_{\delta_{q+1}}} 1-\left(1-\ell_{\delta_{q+1}}\right)^{\mho_{q+1}} / \alpha_{\delta_{q+1}}, \\ \bigcup_{\Upsilon_{\delta_{q+1}} \in \mathcal{Z}_{\delta_{q+1}}, \beta_{\delta_{q+1}} \in \beta_{\delta_{q+1}}} \left(\Upsilon_{\delta_{q+1}}\right)^{\mho_\Gamma} / \beta_{\delta_{q+1}}, \\ \bigcup_{\check{n}_{\delta_{q+1}} \in \mathcal{N}_{\delta_{n+1}}, \gamma_{\delta_{n+1}} \in \gamma_{\delta_{n+1}}} \left(\check{n}_{\delta_{n+1}}\right)^{\mho_\Gamma} / \gamma_{\delta_{n+1}} \end{pmatrix} \\
&= \begin{pmatrix} \bigcup_{\ell_\Gamma \in \mathcal{A}_{\delta_\Gamma}, \alpha_{\delta_\Gamma} \in \alpha_{\delta_\Gamma}} 1-\Pi_{\Gamma=1}^{q}\left(1-(\ell_{\delta_\Gamma})\right)^{\mho_\Gamma} / \Pi_{\Gamma=1}^{q}\alpha_{\delta_\Gamma}, \\ \bigcup_{\Upsilon_{\delta_\Gamma} \in \mathcal{Z}_{\delta_\Gamma}, \beta_{\delta_\Gamma} \in \beta_{\delta_\Gamma}} \Pi_{\Gamma=1}^{q}\left(\Upsilon_{\delta_\Gamma}\right)^{\mho_\Gamma} / \Pi_{\Gamma=1}^{q}\beta_{\delta_\Gamma}, \\ \bigcup_{\check{n}_{\delta_\Gamma} \in \mathcal{N}_{\delta_\Gamma}, \gamma_{\delta_\Gamma} \in \gamma_{\delta_\Gamma}} \Pi_{\Gamma=1}^{q}\left(\check{n}_{\delta_\Gamma}\right)^{\mho_\Gamma} / \Pi_{\Gamma=1}^{q}\gamma_{\delta_\Gamma} \end{pmatrix}
\end{aligned}$$

Thus

$$SV-NPHFWA(\delta_1,\delta_2,....,\delta_\Gamma) = \begin{pmatrix} \bigcup_{\ell_\Gamma \in \mathcal{A}_{\delta_\Gamma}, \alpha_{\delta_\Gamma} \in \alpha_{\delta_\Gamma}} 1-\Pi_{\Gamma=1}^{q}\left(1-\ell_{\delta_\Gamma}\right)^{\mho_\Gamma} / \Pi_{\Gamma=1}^{q}\alpha_{\delta_\Gamma}, \\ \bigcup_{\Upsilon_{\delta_\Gamma} \in \mathcal{Z}_{\delta_\Gamma}, \beta_{\delta_\Gamma} \in \beta_{\delta_\Gamma}} \Pi_{\Gamma=1}^{q}\left(\Upsilon_{\delta_\Gamma}\right)^{\mho_\Gamma} / \Pi_{\Gamma=1}^{q}\beta_{\delta_\Gamma}, \\ \bigcup_{\check{n}_{\delta_\Gamma} \in \mathcal{N}_{\delta_\Gamma}, \gamma_{\delta_\Gamma} \in \gamma_{\delta_\Gamma}} \Pi_{\Gamma=1}^{q}\left(\check{n}_{\delta_\Gamma}\right)^{\mho_\Gamma} / \Pi_{\Gamma=1}^{q}\gamma_{\delta_\Gamma} \end{pmatrix}.$$

Proved. □

Clearly, the SV-NPHFWA satisfies certain characteristics.

(1) Idempotency: Let $\delta_\Gamma = \left(\mathcal{A}_{\hbar_{\delta_\Gamma}}/\alpha_{\delta_\Gamma}, \mathcal{Z}_{\hbar_{\delta_\Gamma}}/\beta_{\delta_\Gamma}, \mathcal{N}_{\hbar_{\delta_\Gamma}}/\gamma_{\delta_\Gamma}\right)$ ($\Gamma = 1,2,.....,$ Γ) be any group of SV-NPHFNs. If all $\delta_\Gamma = \left(\mathcal{A}_{\hbar_{\delta_\Gamma}}/\alpha_{\delta_\Gamma}, \mathcal{Z}_{\hbar_{\delta_\Gamma}}/\beta_{\delta_\Gamma}, \mathcal{N}_{\hbar_{\delta_\Gamma}}/\gamma_{\delta_\Gamma}\right)$ ($\Gamma = 1,2,.....,q$) are similar. Then

$$SV-NPHFWA(\delta_1,\delta_2,....,\delta_q) = \delta \tag{11}$$

(2) Boundedness: Let $\delta_\Gamma = \left(\mathcal{A}_{\hbar_{\delta_\Gamma}}/\alpha_{\delta_\Gamma}, \mathcal{Z}_{\hbar_{\delta_\Gamma}}/\beta_{\delta_\Gamma}, \mathcal{N}_{\hbar_{\delta_\Gamma}}/\gamma_{\delta_\Gamma}\right)$ ($\Gamma = 1,2,.....,r$) be any group of SV-NPHFNs. Then

$$\delta^{-} \leq SV-NPHFWA(\delta_1,\delta_2,....,\delta_q) \leq \delta^{+} \tag{12}$$

where

$$\begin{aligned}
\delta^{-} &= (\min \mathcal{A}_{\hbar_{\delta_\Gamma}} / \min \alpha_{\delta_\Gamma}, \max \mathcal{Z}_{\hbar_{\delta_\Gamma}} / \max \beta_{\delta_\Gamma}, \mathcal{N}_{\hbar_{\delta_\Gamma}}/\gamma_{\delta_\Gamma}), \\
\delta^{+} &= (\max \mathcal{A}_{\hbar_{\delta_\Gamma}} / \max \alpha_{\delta_\Gamma}, \min \mathcal{Z}_{\hbar_{\delta_\Gamma}} / \min \beta_{\delta_\Gamma}, \min_{\hbar_{\delta_\Gamma}} \mathcal{N} / \min \gamma_{\delta_\Gamma})
\end{aligned}$$

(3) Monotonicity: Let $\delta_\Gamma = \left(\mathcal{A}_{\hbar_{\delta_\Gamma}}/\alpha_{\delta_\Gamma}, \mathcal{Z}_{\hbar_{\delta_\Gamma}}/\beta_{\delta_\Gamma}, \mathcal{N}_{\hbar_{\delta_\Gamma}}/\gamma_{\delta_\Gamma}\right)$ ($\Gamma = 1, 2, ..., q$) be any group of SV-NPHFNs. If $\delta_\Gamma > \delta_\Gamma^*$, then

$$SV - NPHFWA(\delta_1, \delta_2, ..., \delta_q) \leq SV - NPHFWA(\delta_1^*, \delta_2^*, ..., \delta_q^*) \tag{13}$$

Proof. For any Γ , there are $\mathcal{A}_{\hbar_{\delta_\Gamma}} \leq \mathcal{A}_{\hbar^*_{\delta_\Gamma}}$, $\mathcal{Z}_{\hbar_{\delta_\Gamma}} \geq \mathcal{Z}_{\hbar^*_{\delta_\Gamma}}$ and $\mathcal{N}_{\hbar_{\delta_\Gamma}} \geq \mathcal{N}_{\hbar^*_{\delta_\Gamma}}$. We've used the following terms in the aggregated outcomes:

$$\begin{aligned} 1 - \Pi_{\Gamma=1}^q \left(1 - (\ell_{\delta_\Gamma})\right)^{\mho_\Gamma} &\leq 1 - \Pi_{\Gamma=1}^q \left(1 - \left(\ell_{\delta_\Gamma^*}\right)\right)^{\mho_\Gamma}, \\ \Pi_{\Gamma=1}^q \left(\Upsilon_{\delta_\Gamma}\right)^{\mho_\Gamma} &\geq \Pi_{\Gamma=1}^q \left(\Upsilon_{\delta_\Gamma^*}\right)^{\mho_\Gamma} \text{ and } \Pi_{\Gamma=1}^q \left(\check{n}_{\delta_\Gamma}\right)^{\mho_\Gamma} \geq \Pi_{\Gamma=1}^q \left(\check{n}_{\delta_\Gamma^*}\right)^{\mho_\Gamma} \end{aligned}$$

$$\begin{aligned} 1 - \Pi_{\Gamma=1}^q \left(1 - \ell_{\delta_\Gamma}\right)^{\mho_\Gamma} &\leq 1 - \Pi_{\Gamma=1}^q \left(1 - \ell_{\delta_\Gamma^*}\right)^{\mho_\Gamma}, \\ \Pi_{\Gamma=1}^q \left(\Upsilon_{\delta_\Gamma}\right)^{\mho_\Gamma} &\geq \Pi_{\Gamma=1}^q \left(\Upsilon_{\delta_\Gamma^*}\right)^{\mho_\Gamma} \text{ and } \Pi_{\Gamma=1}^q \left(\check{n}_{\delta_\Gamma}\right)^{\mho_\Gamma} \geq \Pi_{\Gamma=1}^q \left(\check{n}_{\delta_\Gamma^*}\right)^{\mho_\Gamma} \end{aligned}$$

$$\begin{aligned} 1 - \Pi_{\Gamma=1}^q \left(1 - \ell_{\delta_\Gamma}\right)^{\mho_\Gamma} &\leq 1 - \Pi_{\Gamma=1}^q \left(1 - \ell_{\delta_\Gamma^*}\right)^{\mho_\Gamma}, \\ \Pi_{\Gamma=1}^q \left(\Upsilon_{\delta_\Gamma}\right)^{\mho_\Gamma} &\geq \Pi_{\Gamma=1}^q \left(\Upsilon_{\delta_\Gamma^*}\right)^{\mho_\Gamma} \text{ and } \Pi_{\Gamma=1}^q \left(\check{n}_{\delta_\Gamma}\right)^{\mho_\Gamma} \geq \Pi_{\Gamma=1}^q \left(\check{n}_{\delta_\Gamma^*}\right)^{\mho_\Gamma} \end{aligned}$$

$$\begin{aligned} 1 - \Pi_{\Gamma=1}^q \left(1 - \ell_{\delta_\Gamma}\right)^{\mho_\Gamma} / \Pi_{\Gamma=1}^q \alpha_{\delta_\Gamma} &\leq 1 - \Pi_{\Gamma=1}^q \left(1 - \ell_{\delta_\Gamma^*}\right)^{\mho_\Gamma} / \Pi_{\Gamma=1}^q \alpha_{\delta_\Gamma^*}, \\ \Pi_{\Gamma=1}^q \left(\Upsilon_{\delta_\Gamma}\right)^{\mho_\Gamma} / \Pi_{\Gamma=1}^q \beta_{\delta_\Gamma} &\geq \Pi_{\Gamma=1}^q \left(\Upsilon_{\delta_\Gamma^*}\right)^{\mho_\Gamma} / \Pi_{\Gamma=1}^q \beta_{\delta_\Gamma^*} \\ \text{and } \Pi_{\Gamma=1}^q \left(\check{n}_{\delta_\Gamma}\right)^{\mho_\Gamma} / \Pi_{\Gamma=1}^q \gamma_{\delta_\Gamma} &\geq \Pi_{\Gamma=1}^q \left(\check{n}_{\delta_\Gamma^*}\right)^{\mho_\Gamma} / \Pi_{\Gamma=1}^q \gamma_{\delta_\Gamma^*} \end{aligned}$$

Then we have

$$SV - NPHFWA(\delta_1, \delta_2, ..., \delta_q) \leq SV - NPHFWA(\delta_1^*, \delta_2^*, ..., \delta_q^*)$$

with equality iff $\mathcal{A}_{\hbar_{\delta_\Gamma}} = \mathcal{A}_{\hbar^*_{\delta_\Gamma}}$, $\mathcal{Z}_{\hbar_{\delta_\Gamma}} = \mathcal{Z}_{\hbar^*_{\delta_\Gamma}}$ and $\mathcal{N}_{\hbar_{\delta_\Gamma}} = \mathcal{N}_{\hbar^*_{\delta_\Gamma}}$

Proved. □

Definition 15. *Let* $\delta_\Gamma = \left(\mathcal{A}_{\hbar_{\delta_\Gamma}}/\alpha_{\delta_\Gamma}, \mathcal{Z}_{\hbar_{\delta_\Gamma}}/\beta_{\delta_\Gamma}, \mathcal{N}_{\hbar_{\delta_\Gamma}}/\gamma_{\delta_\Gamma}\right)$ ($\Gamma = 1, 2, ..., q$) *be any group of SV-NPHFNs, and* $SV - NP\ HFOWA : SV - NP\ HFN^q \longrightarrow SV - NP\ HFN$. *Then SV-NPHFOWA operator can be described as*

$$SV - NPHFOWA(\delta_1, \delta_2, ..., \delta_q) = \mho_1 \delta_{\ell(1)} \oplus \mho_2 \delta_{\ell(2)} \oplus ..... \oplus \mho_q \delta_{\ell(q)} \tag{14}$$

where $\delta_{\ell(\Gamma)}$ *be the jth largest in them and* $\mho = (\mho_1, \mho_2, ..., \mho_q)^T$ *are the weights of* $\delta_\Gamma \in [0, 1]$ *with* $\sum_{\Gamma=1}^q \mho_\Gamma = 1$.

Theorem 16. *Let* $\delta_\Gamma = \left(\mathcal{A}_{\hbar_{\delta_\Gamma}}/\alpha_{\delta_\Gamma}, \mathcal{Z}_{\hbar_{\delta_\Gamma}}/\beta_{\delta_\Gamma}, \mathcal{N}_{\hbar_{\delta_\Gamma}}/\gamma_{\delta_\Gamma}\right)$ ($\Gamma = 1, 2, ..., q$) *be any group of SV-NPHFNs. Then, applying SV-NPHFOWA to aggregate the results, we may get the following.*

$$SV{-}NPHFOWA(\delta_1, \delta_2, ..., \delta_q) = \begin{pmatrix} \bigcup\limits_{\substack{\ell_{\delta_{\ell(\Gamma)}} \in \mathcal{A}_{\delta_{\ell(\Gamma)}} \\ \alpha_{\delta_{\ell(\Gamma)}} \in \alpha_{\delta_{\ell(\Gamma)}}}} 1 - \Pi_{\Gamma=1}^{q}\left(1 - \ell_{\delta_{\ell(\Gamma)}}\right)^{\mho_\Gamma} / \Pi_{\Gamma=1}^{q}\alpha_{\delta_{\ell(\Gamma)}}, \\ \bigcup\limits_{\substack{\Upsilon_{\delta_{\ell(\Gamma)}} \in \mathcal{Z}_{\delta_{\ell(\Gamma)}} \\ \beta_{\delta_{\ell(\Gamma)}} \in \beta_{\delta_{\ell(\Gamma)}}}} \Pi_{\Gamma=1}^{q}\left(\Upsilon_{\delta_{\ell(\Gamma)}}\right)^{\mho_\Gamma} / \Pi_{\Gamma=1}^{q}\beta_{\delta_{\ell(\Gamma)}}, \\ \bigcup\limits_{\substack{\check{n}_{\delta_{\ell(\Gamma)}} \in \mathcal{N}_{\delta_{\ell(\Gamma)}} \\ \gamma_{\delta_{\ell(\Gamma)}} \in \gamma_{\delta_{\ell(\Gamma)}}}} \Pi_{\Gamma=1}^{q}\left(\check{n}_{\delta_{\ell(\Gamma)}}\right)^{\mho_\Gamma} / \Pi_{\Gamma=1}^{q}\gamma_{\delta_{\ell(\Gamma)}} \end{pmatrix} \quad (15)$$

where $\delta_{\ell(\Gamma)}$ is the jth largest in them and $\mho = (\mho_1, \mho_2, ..., \mho_q)^T$ are the weights of $\delta_\Gamma \in [0, 1]$ with $\sum_{\Gamma=1}^{q} \mho_\Gamma = 1$.

Proof. Similarly as proof of Theorem 14. □

Clearly, the SV-NPHFOWA has some characteristics that it satisfies.

(1) Idempotency: Let $\delta_\Gamma = \left(\mathcal{A}_{\hbar_{\delta_\Gamma}}/\alpha_{\delta_\Gamma}, \mathcal{Z}_{\hbar_{\delta_\Gamma}}/\beta_{\delta_\Gamma}, \mathcal{N}_{\hbar_{\delta_\Gamma}}/\gamma_{\delta_\Gamma}\right)$ ($\Gamma = 1, 2, ..., q$) be any group of SV-NPHFNs. If all $\delta_\Gamma = \left(\mathcal{A}_{\hbar_{\delta_\Gamma}}/\alpha_{\delta_\Gamma}, \mathcal{Z}_{\hbar_{\delta_\Gamma}}/\beta_{\delta_\Gamma}, \mathcal{N}_{\hbar_{\delta_\Gamma}}/\gamma_{\delta_\Gamma}\right)$ ($\Gamma = 1, 2, ..., q$) are identical. Then

$$SV - NPHFOWA(\delta_1, \delta_2, ..., \delta_q) = \delta \quad (16)$$

(2) Boundedness: Let $\delta_\Gamma = \left(\mathcal{A}_{\hbar_{\delta_\Gamma}}/\alpha_{\delta_\Gamma}, \mathcal{Z}_{\hbar_{\delta_\Gamma}}/\beta_{\delta_\Gamma}, \mathcal{N}_{\hbar_{\delta_\Gamma}}/\gamma_{\delta_\Gamma}\right)$ ($\Gamma = 1, 2, ..., q$) be any group of SV-NPHFNs. Then

$$\delta^{-} \leq SV - NPHFOWA(\delta_1, \delta_2, ..., \delta_q) \leq \delta^{+} \quad (17)$$

where

$$\begin{aligned} \delta^{-} &= (\min \mathcal{A}_{\hbar_{\delta_\Gamma}} / \min \alpha_{\delta_\Gamma}, \max \mathcal{Z}_{\hbar_{\delta_\Gamma}} / \max \beta_{\delta_\Gamma}, \max \mathcal{N}_{\hbar_{\delta_\Gamma}} / \max \gamma_{\delta_\Gamma}), \\ \delta^{+} &= (\max \mathcal{A}_{\hbar_{\delta_\Gamma}} / \max \alpha_{\delta_\Gamma}, \min \mathcal{Z}_{\hbar_{\delta_\Gamma}} / \min \beta_{\delta_\Gamma}, \min \mathcal{N}_{\hbar_{\delta_\Gamma}} / \min \gamma_{\delta_\Gamma}) \end{aligned}$$

(3) Monotonicity: Let $\delta_\Gamma = \left(\mathcal{A}_{\hbar_{\delta_\Gamma}}/\alpha_{\delta_\Gamma}, \mathcal{Z}_{\hbar_{\delta_\Gamma}}/\beta_{\delta_\Gamma}, \mathcal{N}_{\hbar_{\delta_\Gamma}}/\gamma_{\delta_\Gamma}\right)$ ($\Gamma = 1, 2, ..., q$) be any group of SV-NPHFNs. If $\delta_\Gamma > \delta_\Gamma^*$, then

$$SV - NPHFOWA(\delta_1, \delta_2, ..., \delta_q) \leq SV - NPHFOWA(\delta_1^*, \delta_2^*, ..., \delta_q^*) \quad (18)$$

Proof. Similarly as proof of Property 4.1. □

Definition 17. *Let* $\delta_\Gamma = \left(\mathcal{A}_{\hbar_{\delta_\Gamma}}/\alpha_{\delta_\Gamma}, \mathcal{Z}_{\hbar_{\delta_\Gamma}}/\beta_{\delta_\Gamma}, \mathcal{N}_{\hbar_{\delta_\Gamma}}/\gamma_{\delta_\Gamma}\right)$ $(\ \Gamma = 1, 2, ..., q)$ *be any group of SV-NPHFNs. Then SV-NPHFHWA operator* $SV - NP\ HF\ \hbar WA : SV - NP\ HFN^q \longrightarrow SV - NP\ HFN$ *with* $\mho = (\mho_1, \mho_2, ..., \mho_q)^T$ *where* $\mho_i \in [0, 1]$, $\sum_{\Gamma=1}^{q} \mho_\Gamma = 1$ *can be described as*

$$SV - NPHF\hbar WA(\delta_1, \delta_2, ..., \delta_q) = \mho_1 \delta_{\ell(1)} \oplus \mho_2 \delta_{\ell(2)} \oplus ..... \oplus \mho_q \delta_{\ell(q)} \tag{19}$$

w here $\mho = (\mho_1, \mho_2, ..., \mho_q)^T$ *are the weights of* $\delta_\Gamma \in [0, 1]$ *with* $\sum_{\Gamma=1}^{r} \mho_\Gamma = 1$ *and* $\delta_{\ell(\Gamma)}$ *is the jth largest of* $\delta_{\ell(\Gamma)} = q\mho_\Gamma\ \delta_\Gamma$ $(\ \Gamma = 1, 2, ..., q)$.

Theorem 18. *Let* $\delta_\Gamma = \left(\mathcal{A}_{\hbar_{\delta_\Gamma}}/\alpha_{\delta_\Gamma}, \mathcal{Z}_{\hbar_{\delta_\Gamma}}/\beta_{\delta_\Gamma}, \mathcal{N}_{\hbar_{\delta_\Gamma}}/\gamma_{\delta_\Gamma}\right)$ $(\ \Gamma = 1, 2, ..., q)$ *be any group of SV-NPHFNs. Then, utilising SV-NPHFHWA, we can get the below aggregation outcome.*

$$SV - NPHF\hbar WA(\delta_1, \delta_2, ..., \delta_q) = \begin{pmatrix} \bigcup\limits_{\substack{\ell_{\delta_{\ell(\Gamma)}} \in \mathcal{A}_{\delta_{\ell(\Gamma)}} \\ \alpha_{\delta_{\ell(\Gamma)}} \in \alpha_{\delta_{\ell(\Gamma)}}}} 1 - \Pi_{\Gamma=1}^{q} \left(1 - \ell_{\delta_{\ell(\Gamma)}}\right)^{\mho_\Gamma} / \Pi_{\Gamma=1}^{q} \alpha_{\delta_{\ell(\Gamma)}}, \\ \bigcup\limits_{\substack{\Upsilon_{\delta_{\ell(\Gamma)}} \in \mathcal{Z}_{\delta_{\ell(\Gamma)}} \\ \beta_{\delta_{\ell(\Gamma)}} \in \beta_{\delta_{\ell(\Gamma)}}}} \Pi_{\Gamma=1}^{q} \left(\Upsilon_{\delta_{\ell(\Gamma)}}\right)^{\mho_\Gamma} / \Pi_{\Gamma=1}^{q} \beta_{\delta_{\ell(\Gamma)}}, \\ \bigcup\limits_{\substack{\check{n}_{\delta_{\ell(\Gamma)}} \in \mathcal{N}_{\delta_{\ell(\Gamma)}} \\ \gamma_{\delta_{\ell(\Gamma)}} \in \beta_{\delta_{\ell(\Gamma)}}}} \Pi_{\Gamma=1}^{q} \left(\check{n}_{\delta_{\ell(\Gamma)}}\right)^{\mho_\Gamma} / \Pi_{\Gamma=1}^{q} \beta_{\delta_{\ell(\Gamma)}} \end{pmatrix} \tag{20}$$

where $\mho = (\mho_1, \mho_2, ..., \mho_q)^T$ *are the weights of* $\delta_\Gamma \in [0, 1]$ *with* $\sum_{\Gamma=1}^{q} \mho_\Gamma = 1$ *and* $\delta_{\ell(\Gamma)}$ *is the largest of* $\delta_{\ell(\Gamma)} = q\mho_\Gamma\ \delta_\Gamma(\ \Gamma = 1, 2, ..., k)$

Proof. Similarly as proof of Theorem 14. □

Clearly, the SV-NPHFHWA satisfies a few characteristics.

(1) Idempotency: Let $\delta_\Gamma = \left(\mathcal{A}_{\hbar_{\delta_\Gamma}}/\alpha_{\delta_\Gamma}, \mathcal{Z}_{\hbar_{\delta_\Gamma}}/\beta_{\delta_\Gamma}, \mathcal{N}_{\hbar_{\delta_\Gamma}}/\gamma_{\delta_\Gamma}\right)$ $(\ \Gamma = 1, 2, ..., q)$ be any group of SV-NPHFNs. If all $\delta_\Gamma = \left(\mathcal{A}_{\hbar_{\delta_\Gamma}}/\alpha_{\delta_\Gamma}, \mathcal{Z}_{\hbar_{\delta_\Gamma}}/\beta_{\delta_\Gamma}, \mathcal{N}_{\hbar_{\delta_\Gamma}}/\gamma_{\delta_\Gamma}\right)$ $(\ \Gamma = 1, 2, ..., q)$ are identical. Then

$$SV - NPHF\hbar WA(\delta_1, \delta_2, ..., \delta_q) = \delta \tag{21}$$

(2) Boundedness: Let $\delta_\Gamma = \left(\mathcal{A}_{\hbar_{\delta_\Gamma}}/\alpha_{\delta_\Gamma}, \mathcal{Z}_{\hbar_{\delta_\Gamma}}/\beta_{\delta_\Gamma}, \mathcal{N}_{\hbar_{\delta_\Gamma}}/\gamma_{\delta_\Gamma}\right)$ $(\ \Gamma = 1, 2, ..., q)$ be any group of SV-NPHFNs. Then

$$\delta^{-} \leq SV - NPHF\hbar WA(\delta_1, \delta_2, ..., \delta_q) \leq \delta^{+} \tag{22}$$

where

$$\begin{aligned} \delta^{-} &= (\min \mathcal{A}_{\hbar_{\delta_\Gamma}} / \min \alpha_{\delta_\Gamma}, \max \mathcal{Z}_{\hbar_{\delta_\Gamma}} / \max \beta_{\delta_\Gamma}, \max \mathcal{N}_{\hbar_{\delta_\Gamma}} / \max \gamma_{\delta_\Gamma}), \\ \delta^{+} &= (\max \mathcal{A}_{\hbar_{\delta_\Gamma}} / \max \alpha_{\delta_\Gamma}, \min \mathcal{Z}_{\hbar_{\delta_\Gamma}} / \min \beta_{\delta_\Gamma}, \min \mathcal{N}_{\hbar_{\delta_\Gamma}} / \min \gamma_{\delta_\Gamma}) \end{aligned}$$

(3) Monotonicity: Let $\delta_\Gamma = \left(\mathcal{A}_{\hbar_{\delta_\Gamma}}/\alpha_{\delta_\Gamma}, \mathcal{Z}_{\hbar_{\delta_\Gamma}}/\beta_{\delta_\Gamma}, \mathcal{N}_{\hbar_{\delta_\Gamma}}/\gamma_{\delta_\Gamma}\right)$ ($\Gamma = 1, 2, ..., q$) be any group of SV-NPHFNs. If $\delta_\Gamma > \delta^*_\Gamma$, then

$$SV - NPHF\hbar WA(\delta_1, \delta_2, ..., \delta_q) \leq SV - NPHF\hbar WA(\delta^*_1, \delta^*_2, ..., \delta^*_q) \quad (23)$$

Proof. Similarly as proof of Property 4.1. □

4.2. SV-NPHF weighted geometric aggregation operators

This section expands the current weighted geometric AOs for SV-Neutrosophic Probabilistic hesitant fuzzy information.

Definition 19. *Let* $\delta_\Gamma = \left(\mathcal{A}_{\hbar_{\delta_\Gamma}}/\alpha_{\delta_\Gamma}, \mathcal{Z}_{\hbar_{\delta_\Gamma}}/\beta_{\delta_\Gamma}, \mathcal{N}_{\hbar_{\delta_\Gamma}}/\gamma_{\delta_\Gamma}\right)$ ($\Gamma = 1, 2, ..., q$) *be any group of SV-NPHFNs, and* $SV-NP\ HFWG : SV-NP\ HFN^q \longrightarrow SV-NP\ HFN$*. Then* $SV - NP\ HFWG$ *operator can be described as*

$$SV - NPHFWG(\delta_1, \delta_2, ..., \delta_q) = \delta_1^{\mho_1} \otimes \delta_2^{\mho_2} \otimes ..... \otimes \delta_q^{\mho_q} \quad (24)$$

and $\mho = (\mho_1, \mho_2, ..., \mho_q)^T$ *are the weights of* $\delta_\Gamma \in [0, 1]$ *with* $\sum_{\Gamma=1}^{q} \mho_\Gamma = 1$.

Theorem 20. *Let* $\delta_\Gamma = \left(\mathcal{A}_{\hbar_{\delta_\Gamma}}/\alpha_{\delta_\Gamma}, \mathcal{Z}_{\hbar_{\delta_\Gamma}}/\beta_{\delta_\Gamma}, \mathcal{N}_{\hbar_{\delta_\Gamma}}/\gamma_{\delta_\Gamma}\right)$ ($\Gamma = 1, 2, ..., q$) *be any group of SV-NPHFNs. Then, using SV-NPHFWG to aggregate the results, we may obtain the following.*

$$SV-NPHFWG(\delta_1, \delta_2, ..., \delta_q) = \begin{pmatrix} \bigcup\limits_{\substack{\ell_{\delta_\Gamma} \in \mathcal{Z}_{\delta_\Gamma} \\ \alpha_{\delta_\Gamma} \in \alpha_{\delta_\Gamma}}} \Pi_{\Gamma=1}^{q} \left(\ell_{\delta_\Gamma}\right)^{\mho_\Gamma} / \Pi_{\Gamma=1}^{q} \alpha_{\delta_\Gamma}, \\ \bigcup\limits_{\substack{\Upsilon_\Gamma \in \mathcal{A}_{\delta_\Gamma} \\ \beta_{\delta_\Gamma} \in \beta_{\delta_\Gamma}}} 1 - \Pi_{\Gamma=1}^{q} \left(1 - \Upsilon_{\delta_\Gamma}\right)^{\mho_\Gamma} / \Pi_{\Gamma=1}^{q} \beta_{\delta_\Gamma}, \\ \bigcup\limits_{\substack{\check{n}_\Gamma \in \mathcal{A}_{\delta_\Gamma} \\ \gamma_{\delta_\Gamma} \in \gamma_{\delta_\Gamma}}} 1 - \Pi_{\Gamma=1}^{q} \left(1 - \check{n}_{\delta_\Gamma}\right)^{\mho_\Gamma} / \Pi_{\Gamma=1}^{q} \gamma_{\delta_\Gamma} \end{pmatrix} \quad (25)$$

where $\mho = (\mho_1, \mho_2, ..., \mho_q)^T$ *are the weights of* $\delta_\Gamma \in [0, 1]$ *with* $\sum_{\Gamma=1}^{q} \mho_\Gamma = 1$.

Proof. Similarly as proof of Theorem 14. □

Clearly, the SV-NPHFWG satisfies certain characteristics.(1) Idempotency: Let $\delta_\Gamma = \left(\mathcal{A}_{\hbar_{\delta_\Gamma}}/\alpha_{\delta_\Gamma}, \mathcal{Z}_{\hbar_{\delta_\Gamma}}/\beta_{\delta_\Gamma}, \mathcal{N}_{\hbar_{\delta_\Gamma}}/\gamma_{\delta_\Gamma}\right)$ ($\Gamma = 1, 2, ..., q$) be any group of SV-NPHFNs. If all $\delta_\Gamma = \left(\mathcal{A}_{\hbar_{\delta_\Gamma}}/\alpha_{\delta_\Gamma}, \mathcal{Z}_{\hbar_{\delta_\Gamma}}/\beta_{\delta_\Gamma}, \mathcal{N}_{\hbar_{\delta_\Gamma}}/\gamma_{\delta_\Gamma}\right)$ ($\Gamma = 1, 2, ..., q$) are identical. Then

$$SV - NPHFWG(\delta_1, \delta_2, ..., \delta_q) = \delta \quad (26)$$

(2) Boundedness: Let $\delta_{\Gamma} = \left(\mathcal{A}_{\hbar_{\delta_{\Gamma}}}/\alpha_{\delta_{\Gamma}}, \mathcal{Z}_{\hbar_{\delta_{\Gamma}}}/\beta_{\delta_{\Gamma}}, \mathcal{N}_{\hbar_{\delta_{\Gamma}}}/\gamma_{\delta_{\Gamma}}\right)$ ($\Gamma = 1, 2, ..., q$) be any group of SV-NPHFNs. Then

$$\delta^{-} \leq SV - NPHFWG(\delta_1, \delta_2, ..., \delta_q) \leq \delta^{+} \tag{27}$$

where

$$\begin{aligned}
\delta^{-} &= (\min \mathcal{A}_{\hbar_{\delta_{\Gamma}}} / \min \alpha_{\delta_{\Gamma}}, \max \mathcal{Z}_{\hbar_{\delta_{\Gamma}}} / \max \beta_{\delta_{\Gamma}}, \max \mathcal{N}_{\hbar_{\delta_{\Gamma}}} / \max \gamma_{\delta_{\Gamma}}), \\
\delta^{+} &= (\max \mathcal{A}_{\hbar_{\delta_{\Gamma}}} / \max \alpha_{\delta_{\Gamma}}, \min \mathcal{Z}_{\hbar_{\delta_{\Gamma}}} / \min \beta_{\delta_{\Gamma}}, \min \mathcal{N}_{\hbar_{\delta_{\Gamma}}} / \min \gamma_{\delta_{\Gamma}})
\end{aligned}$$

(3) Monotonicity: Let $\delta_{\Gamma} = \left(\mathcal{A}_{\hbar_{\delta_{\Gamma}}}/\alpha_{\delta_{\Gamma}}, \mathcal{Z}_{\hbar_{\delta_{\Gamma}}}/\beta_{\delta_{\Gamma}}, \mathcal{N}_{\hbar_{\delta_{\Gamma}}}/\gamma_{\delta_{\Gamma}}\right)$ ($\Gamma = 1, 2, ..., q$) be any group of SV-NPHFNs. If $\delta_{\Gamma} > \delta^{*}_{\Gamma}$, then

$$SV - NPHFWG(\delta_1, \delta_2, ..., \delta_q) \leq SV - NPHFWG(\delta^{*}_1, \delta^{*}_2, ..., \delta^{*}_q) \tag{28}$$

Proof. Similarly as proof of Property 4.1. □

Definition 21. *Let* $\delta_{\Gamma} = \left(\mathcal{A}_{\hbar_{\delta_{\Gamma}}}/\alpha_{\delta_{\Gamma}}, \mathcal{Z}_{\hbar_{\delta_{\Gamma}}}/\beta_{\delta_{\Gamma}}, \mathcal{N}_{\hbar_{\delta_{\Gamma}}}/\gamma_{\delta_{\Gamma}}\right)$ ($\Gamma = 1, 2, ..., q$) *be any group of SPPHFNs, and* $SV-NP\ HFOWG : SV-NP\ HFN^{q} \longrightarrow SV-NP\ HFN$*. Then the SV-NPHFOWG operator is as follows:*

$$SV - NPHFOWG(\delta_1, \delta_2, ..., \delta_q) = \delta^{\mho_1}_{\ell(1)} \otimes \delta^{\mho_2}_{\ell(2)} \otimes ..... \otimes \delta^{\mho_q}_{\ell(q)} \tag{29}$$

where $\delta_{\ell(\Gamma)}$ *be the jth largest in them and* $\mho = (\mho_1, \mho_2, ..., \mho_q)^T$ *are the weights of* $\delta_{\Gamma} \in [0, 1]$ *with* $\sum_{\Gamma=1}^{q} \mho_{\Gamma} = 1$.

Theorem 22. *Let* $\delta_{\Gamma} = \left(\mathcal{A}_{\hbar_{\delta_{\Gamma}}}/\alpha_{\delta_{\Gamma}}, \mathcal{Z}_{\hbar_{\delta_{\Gamma}}}/\beta_{\delta_{\Gamma}}, \mathcal{N}_{\hbar_{\delta_{\Gamma}}}/\gamma_{\delta_{\Gamma}}\right)$ ($\Gamma = 1, 2, ..., q$) *be any group of SV-NPHFNs. Then, using SV-NPHFOWG to aggregate the results, we may get the following.*

$$SV - NPHFOWG(\delta_1, \delta_2, ..., \delta_q) = \begin{pmatrix} \bigcup\limits_{\substack{\ell_{\delta_{\ell(\Gamma)}} \in \mathcal{Z}_{\delta_{\ell(\Gamma)}} \\ \alpha_{\delta_{\ell(\Gamma)}} \in \alpha_{\delta_{\ell(\Gamma)}}}} \Pi_{\Gamma=1}^{q} \left(\ell_{\delta_{\ell(\Gamma)}}\right)^{\mho_{\Gamma}} / \Pi_{\Gamma=1}^{q} \alpha_{\delta_{\ell(\Gamma)}}, \\ \bigcup\limits_{\substack{\Upsilon_{\delta_{\ell(\Gamma)}} \in \mathcal{A}_{\delta_{\ell(\Gamma)}} \\ \beta_{\delta_{\ell(\Gamma)}} \in \beta_{\delta_{\ell(\Gamma)}}}} 1 - \Pi_{\Gamma=1}^{q} \left(1 - \Upsilon_{\delta_{\ell(\Gamma)}}\right)^{\mho_{\Gamma}} / \Pi_{\Gamma=1}^{q} \beta_{\delta_{\ell(\Gamma)}}, \\ \bigcup\limits_{\substack{\check{n}_{\delta_{\ell(\Gamma)}} \in \mathcal{A}_{\delta_{\ell(\Gamma)}} \\ \gamma_{\delta_{\ell(\Gamma)}} \in \gamma_{\delta_{\ell(\Gamma)}}}} 1 - \Pi_{\Gamma=1}^{q} \left(1 - \check{n}_{\delta_{\ell(\Gamma)}}\right)^{\mho_{\Gamma}} / \Pi_{\Gamma=1}^{q} \gamma_{\delta_{\ell(\Gamma)}} \end{pmatrix} \tag{30}$$

where $\delta_{\ell(\Gamma)}$ *is the jth largest in them and* $\mho = (\mho_1, \mho_2, ..., \mho_q)^T$ *are the weights of* $\delta_{\Gamma} \in [0, 1]$ *with* $\sum_{\Gamma=1}^{q} \mho_{\Gamma} = 1$.

Proof. It can be proved in a similar way as 14. □

Clearly, the SV-NPHFOWG satisfies a few characteristics.

(1) Idempotency: Let $\delta_\Gamma = \left(\mathcal{A}_{\hbar_{\delta_\Gamma}}/\alpha_{\delta_\Gamma}, \mathcal{Z}_{\hbar_{\delta_\Gamma}}/\beta_{\delta_\Gamma}, \mathcal{N}_{\hbar_{\delta_\Gamma}}/\gamma_{\delta_\Gamma}\right)$ ($\Gamma = 1, 2, ..., q$) be any group of SV-NPHFNs. If all $\delta_\Gamma = \left(\mathcal{A}_{\hbar_{\delta_\Gamma}}/\alpha_{\delta_\Gamma}, \mathcal{Z}_{\hbar_{\delta_\Gamma}}/\beta_{\delta_\Gamma}, \mathcal{N}_{\hbar_{\delta_\Gamma}}/\gamma_{\delta_\Gamma}\right)$ ($\Gamma = 1, 2, ..., q$) are identical. Then

$$SV - NPHFOWG(\delta_1, \delta_2, ..., \delta_q) = \delta \tag{31}$$

(2) Boundedness: Let $\delta_\Gamma = \left(\mathcal{A}_{\hbar_{\delta_\Gamma}}/\alpha_{\delta_\Gamma}, \mathcal{Z}_{\hbar_{\delta_\Gamma}}/\beta_{\delta_\Gamma}, \mathcal{N}_{\hbar_{\delta_\Gamma}}/\gamma_{\delta_\Gamma}\right)$ ($\Gamma = 1, 2, ..., q$) be any group of SV-NPHFNs. Then

$$\delta^- \leq SV - NPHFOWG(\delta_1, \delta_2, ..., \delta_q) \leq \delta^+ \tag{32}$$

where

$$\begin{aligned} \delta^- &= (\min \mathcal{A}_{\hbar_{\delta_\Gamma}} / \min \alpha_{\delta_\Gamma}, \max \mathcal{Z}_{\hbar_{\delta_\Gamma}} / \max \beta_{\delta_\Gamma}, \max \mathcal{N}_{\hbar_{\delta_\Gamma}} / \max \gamma_{\delta_\Gamma}), \\ \delta^+ &= (\max \mathcal{A}_{\hbar_{\delta_\Gamma}} / \max \alpha_{\delta_\Gamma}, \min \mathcal{Z}_{\hbar_{\delta_\Gamma}} / \min \beta_{\delta_\Gamma}, \min \mathcal{N}_{\hbar_{\delta_\Gamma}} / \min \gamma_{\delta_\Gamma}) \end{aligned}$$

(3) Monotonicity: Let $\delta_\Gamma = \left(\mathcal{A}_{\hbar_{\delta_\Gamma}}/\alpha_{\delta_\Gamma}, \mathcal{Z}_{\hbar_{\delta_\Gamma}}/\beta_{\delta_\Gamma}, \mathcal{N}_{\hbar_{\delta_\Gamma}}/\gamma_{\delta_\Gamma}\right)$ ($\Gamma = 1, 2, ..., q$) be any group of SV-NPHFNs. If $\delta_\Gamma > \delta^*_\Gamma$, then

$$SV - NPHFOWG(\delta_1, \delta_2, ..., \delta_q) \leq SV - NPHFOWG(\delta^*_1, \delta^*_2, ..., \delta^*_q) \tag{33}$$

Proof. It can be proved in a similar way as 4.1. □

Definition 23. *Let* $\delta_\Gamma = \left(\mathcal{A}_{\hbar_{\delta_\Gamma}}/\alpha_{\delta_\Gamma}, \mathcal{Z}_{\hbar_{\delta_\Gamma}}/\beta_{\delta_\Gamma}, \mathcal{N}_{\hbar_{\delta_\Gamma}}/\gamma_{\delta_\Gamma}\right)$ ($\Gamma = 1, 2, ..., q$) *be any group of SV-NPHFNs, and* $SV - NP\ HF\ \hbar WG$ *operator* $SV - NP\ HF\ \hbar WG : SV - NP\ HFN^q \longrightarrow SV - NP\ HFN$ *with* $\mho = (\mho_1, \mho_2, ..., \mho_q)^T$ *where* $\mho_\Gamma \in [0, 1]$, $\sum_{\Gamma=1}^q \mho_\Gamma = 1$ *can be described as*

$$SV - NPHF\hbar WG(\delta_1, \delta_2, ..., \delta_q) = \delta_{\ell(1)}^{\mho_1} \otimes \delta_{\ell(2)}^{\mho_2} \otimes ..... \otimes \delta_{\ell(q)}^{\mho_q} \tag{34}$$

where $\mho = (\mho_1, \mho_2, ..., \mho_q)^T$ *are the weights of* $\delta_\Gamma \in [0, 1]$ *with* $\sum_{\Gamma=1}^q \mho_\Gamma = 1$ *and* $\delta_{\ell(\Gamma)}$ *is the jth largest of* $\delta_{\ell(\Gamma)} = \delta_\Gamma^{q\mho_\Gamma}$ ($\Gamma = 1, 2, ..., q$).

Theorem 24. *Let* $\delta_\Gamma = \left(\mathcal{A}_{\hbar_{\delta_\Gamma}}/\alpha_{\delta_\Gamma}, \mathcal{Z}_{\hbar_{\delta_\Gamma}}/\beta_{\delta_\Gamma}, \mathcal{N}_{\hbar_{\delta_\Gamma}}/\gamma_{\delta_\Gamma}\right)$ ($\Gamma = 1, 2, ..., q$) *be any group of SV-NPHFNs.Then, utilising SV-NPHFHWG, we can get the below aggregation outcome.*

$$SV - NPHF\hbar WG(\delta_1, \delta_2, ..., \delta_q) = \begin{pmatrix} \bigcup\limits_{\substack{\ell_{\delta_{\ell(\Gamma)}} \in \mathcal{Z}_{\delta_{\ell(\Gamma)}} \\ \alpha_{\delta_{\ell(\Gamma)}} \in \alpha_{\delta_{\ell(\Gamma)}}}} \Pi_{\Gamma=1}^q \left(\ell_{\delta_{\ell(\Gamma)}}\right)^{\mho_\Gamma} / \Pi_{\Gamma=1}^q \alpha_{\delta_{\ell(\Gamma)}}, \\ \bigcup\limits_{\substack{\Upsilon_{\delta_{\ell(\Gamma)}} \in \mathcal{A}_{\delta_{\ell(\Gamma)}} \\ \beta_{\delta_{\ell(\Gamma)}} \in \beta_{\delta_{\ell(\Gamma)}}}} 1 - \Pi_{\Gamma=1}^q \left(1 - \Upsilon_{\delta_{\ell(\Gamma)}}\right)^{\mho_\Gamma} / \Pi_{\Gamma=1}^q \beta_{\delta_{\ell(\Gamma)}}, \\ \bigcup\limits_{\substack{\check{n}_{\delta_{\ell(\Gamma)}} \in \mathcal{A}_{\delta_{\ell(\Gamma)}} \\ \gamma_{\delta_{\ell(\Gamma)}} \in \gamma_{\delta_{\ell(\Gamma)}}}} 1 - \Pi_{\Gamma=1}^q \left(1 - \check{n}_{\delta_{\ell(\Gamma)}}\right)^{\mho_\Gamma} / \Pi_{\Gamma=1}^q \gamma_{\delta_{\ell(\Gamma)}} \end{pmatrix} \tag{35}$$

where $\mho = (\mho_1, \mho_2, ..., \mho_q)^T$ *are the weights of* $\delta_\Gamma \in [0,1]$ *with* $\sum_{\Gamma=1}^{q} \mho_\Gamma = 1$*and* $\delta_{\ell(\Gamma)}$ *is the largest of* $\delta_{\ell(\Gamma)} \ = \delta_i^{q\mho_i} (i = 1, 2, ..., q)$

Proof. It can be proved in a similar way as 14. □

Clearly, the SV-NPHFHWG satisfies a few characteristics.

(1) Idempotency: Let $\delta_\Gamma = \left(\mathcal{A}_{\hbar_{\delta_\Gamma}}/\alpha_{\delta_\Gamma}, \mathcal{Z}_{\hbar_{\delta_\Gamma}}/\beta_{\delta_\Gamma}, \mathcal{N}_{\hbar_{\delta_\Gamma}}/\gamma_{\delta_\Gamma}\right)$ ($\Gamma = 1, 2, ..., q$) be any group of SV-NPHFNs. If all $\delta_\Gamma = \left(\mathcal{A}_{\hbar_{\delta_\Gamma}}/\alpha_{\delta_\Gamma}, \mathcal{Z}_{\hbar_{\delta_\Gamma}}/\beta_{\delta_\Gamma}, \mathcal{N}_{\hbar_{\delta_\Gamma}}/\gamma_{\delta_\Gamma}\right)$ ($\Gamma = 1, 2, ..., q$) are identical. Then

$$SV - NPHF\hbar WG(\delta_1, \delta_2, ..., \delta_q) = \delta \tag{36}$$

(2) Boundedness: Let $\delta_\Gamma = \left(\mathcal{A}_{\hbar_{\delta_\Gamma}}/\alpha_{\delta_\Gamma}, \mathcal{Z}_{\hbar_{\delta_\Gamma}}/\beta_{\delta_\Gamma}, \mathcal{N}_{\hbar_{\delta_\Gamma}}/\gamma_{\delta_\Gamma}\right)$ ($\Gamma = 1, 2, ..., q$) be any group of SV-NPHFNs. Then

$$\delta^{-} \leq SV - NPHF\hbar WG(\delta_1, \delta_2, ..., \delta_q) \leq \delta^{+} \tag{37}$$

where

$$\begin{aligned} \delta^{-} &= (\min \mathcal{A}_{\hbar_{\delta_\Gamma}}/\min \alpha_{\delta_\Gamma}, \max \mathcal{Z}_{\hbar_{\delta_\Gamma}}/\max \beta_{\delta_\Gamma}, \max_{\hbar_{\delta_\Gamma}} \mathcal{N}/\max \gamma_{\delta_\Gamma}), \\ \delta^{+} &= (\max \mathcal{A}_{\hbar_{\delta_\Gamma}}/\max \alpha_{\delta_\Gamma}, \min \mathcal{Z}_{\hbar_{\delta_\Gamma}}/\min \beta_{\delta_\Gamma}, \min \mathcal{N}_{\hbar_{\delta_\Gamma}}/\min \gamma_{\delta_\Gamma}) \end{aligned}$$

(3) Monotonicity: Let $\delta_\Gamma = \left(\mathcal{A}_{\hbar_{\delta_\Gamma}}/\alpha_{\delta_\Gamma}, \mathcal{Z}_{\hbar_{\delta_\Gamma}}/\beta_{\delta_\Gamma}, \mathcal{N}_{\hbar_{\delta_\Gamma}}/\gamma_{\delta_\Gamma}\right)$ ($\Gamma = 1, 2, ..., q$) be any group of SV-NPHFNs. If $\delta_\Gamma > \delta_\Gamma^*$, then

$$SV - NPHF\hbar WG(\delta_1, \delta_2, ..., \delta_q) \leq SV - NPHF\hbar WG(\delta_1^*, \delta_2^*, ..., \delta_q^*) \tag{38}$$

Proof. It can be proved in a similar way as 4.1. □

5. DECISION MAKING BASED ON NEUTROSOPHIC PROBABILISTIC HESITANT FUZZY AGGREGATION OPERATORS

In this section, we propose a framework for solving MADM problems under SV-NPHF information. Consider a MADM with a set of m alternatives $\{\Re_1, \Re_2, ..., \Re_\Gamma\}$ and let $\{\alpha_1, \alpha_2, ..., \alpha_q\}$ be a set of attributes with weight vector $\mho = (\mho_1, \mho_2, ..., \mho_q)$ where $\mho_t \in [0,1]$ and $\sum_{t=1}^{q} \mho_t = 1$. To assess the performance of kth alternative $\Re_k$ under the the attribute α_t, let $\{\check{D}_1, \check{D}_2, ..., \check{D}_r\}$ be a set of decision makers and $\Upsilon = (\Upsilon_1, \Upsilon_2, ..., \Upsilon_r)$ be the weighted vector of decision makers with $\Upsilon_s \in [0,1]$ and $\sum_{s=1}^{r} \Upsilon_s = 1$. The SV-NPHF decision matrix can be written as:

$$\begin{array}{c c}
 & \begin{array}{cccc} \alpha_1 & \alpha_2 & \cdots & \alpha_t \end{array} \\
\begin{array}{c} \Re_1 \\ \Re_2 \\ \cdots \\ \Re_k \end{array} &
\left[\begin{array}{cccc}
\left(\begin{array}{c} \mathcal{A}_{\delta_{11}}/\alpha_{\delta_{11}}, \\ \mathcal{Z}_{\delta_{11}}/\beta_{\delta_{11}}, \mathcal{N}_{\delta_{11}}/\gamma_{\delta_{11}} \end{array}\right) & \left(\begin{array}{c} \mathcal{A}_{\delta_{12}}/\alpha_{\delta_{12}}, \\ \mathcal{Z}_{\delta_{12}}/\beta_{\delta_{12}}, \mathcal{N}_{\delta_{12}}/\gamma_{\delta_{12}} \end{array}\right) & \cdots & \left(\begin{array}{c} \mathcal{A}_{\delta t}/\alpha_{\delta_{1t}}, \\ \mathcal{Z}_{\delta_{1t}}/\beta_{\delta_{1t}}, \mathcal{N}_{\delta_{1t}}/\gamma_{\delta_{1t}} \end{array}\right) \\
\left(\begin{array}{c} \mathcal{A}_{\delta_{21}}/\alpha_{\delta_{21}}, \\ \mathcal{Z}_{\delta_{21}}/\beta_{\delta_{21}}, \mathcal{N}_{\delta_{21}}/\gamma_{\delta_{21}} \end{array}\right) & \left(\begin{array}{c} \mathcal{A}_{\delta_{22}}/\alpha_{\delta_{22}}, \\ \mathcal{Z}_{\delta_{22}}/\beta_{\delta_{22}}, \mathcal{N}_{\delta_{22}}/\gamma_{\delta_{22}} \end{array}\right) & \cdots & \left(\begin{array}{c} \mathcal{A}_{\delta_{2t}}/\alpha_{\delta_{2t}}, \\ \mathcal{Z}_{\delta_{2t}}/\beta_{\delta_{2t}}, \mathcal{N}_{\delta_{2t}}/\gamma_{\delta_{2t}} \end{array}\right) \\
\cdots & \cdots & \cdots & \cdots \\
\left(\begin{array}{c} \mathcal{A}_{\delta_{k1}}/\alpha_{\delta_{k1}}, \\ \mathcal{Z}_{\delta_{k1}}/\beta_{\delta_{k1}}, \mathcal{N}_{\delta_{k1}}/\gamma_{\delta_{k1}} \end{array}\right) & \left(\begin{array}{c} \mathcal{A}_{\delta_{k2}}/\alpha_{\delta_{k2}}, \\ \mathcal{Z}_{\delta_{k2}}/\beta_{\delta_{k2}}, \mathcal{N}_{\delta_{k2}}/\gamma_{\delta_{k2}} \end{array}\right) & \cdots & \left(\begin{array}{c} \mathcal{A}_{\delta_{kt}}/\alpha_{\delta_{kt}}, \\ \mathcal{Z}_{\delta t}/\beta_{\delta t}, \mathcal{N}_{\delta t}/\gamma_{\delta t} \end{array}\right)
\end{array}\right]
\end{array}$$

5.1. Algorithm

The following are recommended steps in the created multi-attribute decision making (MADM) problem:

All the below process can be seen in the flow chart in Figure 2.

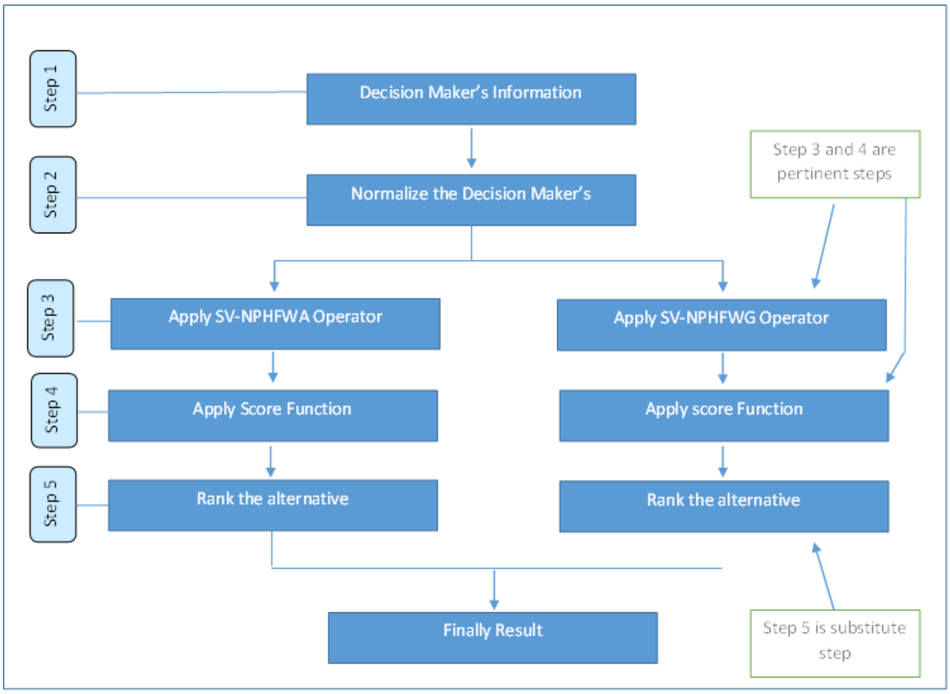

Figure 2: Flow Chart of Decision Maker's Information

Step-1 We create the SV-NPHF decision matrices in this stage

$$\begin{aligned} C &= [\delta_{\Gamma r}]_{k\times t} = [(\mathcal{A}_{\Gamma r}/\alpha_{\Gamma r}, \mathcal{Z}_{\Gamma r}/\beta_{\Gamma r}, \mathcal{N}_{\Gamma r}/\gamma_{\Gamma r})]_{k\times t} \\ & (\Gamma = 1, 2, ..., k; r = 1, 2, ..., t)\,. \end{aligned}$$

Step-2 Normalize the SV-NPHF decision matrix if the attribute contains two types, such as cost and benefit attributes.

$$\check{D}_n = [g_{kt}]_{k\times t}$$

where

$$\begin{aligned} g_{kt} &= \begin{cases} \delta_{kt} & \text{for benefit criteria } \alpha_t, \\ (\delta_{kt})^c & \text{for cost criteria } \alpha_t, \end{cases} \\ k &= 1,2,....,\Gamma; t = 1,2,...,r, \end{aligned}$$

where $(\delta_{kt})^c$ is complement of δ_{kt},that is, $(\delta_{kt})^c = (\mathcal{N}_{\delta_{kt}}, \mathcal{Z}_{\delta_{kt}}, \mathcal{A}_{\delta_{kt}})$. There is no need to normalise the decision matrix if every attribute has the same type.

Step-3 Utilize the structured AOs to obtain the SV-NPHFN $\delta_k(k = 1,2,...., \Gamma)$ for the alternatives $\Re_k$, that is, use the known operators to get the collective overall preference values $\delta_k(k = 1,2,...., \Gamma)$ of the alternative $\Re_k$, where $\mho = (\mho_1, \mho_2, ..., \mho_r)$ is the weighting vector of the attributes.

Step-4 After that, we compute the scores $s(\ \delta_k)$ $(k = 1,2,...., \Gamma)$ and the accuracy degrees $\hbar(\ \delta_k)$ $(k = 1,2,...., \Gamma)$ of all the overall values $\delta_k(k = 1,2,...., \Gamma)$.

Step-5 According to Definition 11, Rank the alternatives $\Re_k(k = 1,2,...., \Gamma)$ and afterward select the best one. weight vector is $\mho = (0.2,\ 0.4,\ 0.3,0.1)^T$.

6. MATHEMATICAL APPLICATION

In this subsection, we show how the suggested neutrosophic probabilistic hesitant fuzzy aggregation information can be used to evaluate the uncertainty to make best decisions for unsure municipality builders using fuzzy mathematical logic.

6.1. Case Study: Urban/Rural healthcare unit site selection

Site selection is one of the first and most important processes that healthcare organisations do when opening a new outpatient service. The size and cost of a parcel of land, as well as its visibility, proximity to other healthcare facilities, and speed of construction, are a few factors that influence where a facility will be built. The success of the completed facility can be affected by site selection, which is a complicated issue that can affect the rest of a project. For health care organisations that have never constructed a new facility before, choosing a location and starting the building might be challenging. Because of this, it is essential to have a complete understanding of all elements before starting a site search. With the correct context and preparation, health care organisations can carry out a site-selection process that works effectively within the project's overall timetable, budget, and vision for success. The first step in developing any new facility is to have a clear vision. Health care organisations must decide how they want the finished facility to function, who it will serve, and how it will fit into the greater community before searching for and analysing potential properties. For instance, a hospital planning to construct an ambulatory surgical centre should consider the location of its current or prospective patient base, the volume of cases it anticipates, the opening date of the facility, and any other ancillary services it may wish to include in the design. These factors will have an impact on everything, from location decisions to facility size and layout. Once its vision has been created, the company may successfully decide the requirements for its facility site and the criteria for its site search. This can be as simple as listing the qualities or characteristics that a site must have or not have in order to be a candidate. For example, if a hospital urgently needs new and expanded medical office space to avoid losing current tenants, finding a shovel-ready site might be the top priority. Organizations can construct their own rubric for quickly evaluating properties by prioritising a list of site-selection criteria. They should be aware, however, that they may not be able to discover a property that meets all of their criteria. In many circumstances, businesses must assess the advantages and disadvantages of imperfect sites and make concessions in order to achieve their goals. That could mean purchasing a larger piece of land than anticipated because it's in a nice location, is zoned properly, and is ready to develop. It might also mean purchasing a piece of real estate whose size and location necessitate deviating from local ordinances. This kind of balancing act occurs frequently during the site-selection process for most projects. Let $\Re = \Re_1; \Re_2; :::; \Re_\Gamma$ be the collection of all obligatory residential progress features. Here we discuss the four alternatives ($\Re_1$ = Pleasant Clinical Environment , $\Re_2$ = Onstage / Offstage Environments, $\Re_3$ = Healthcare Design,and $\Re_4$ = Healthy Occupants), and we want to choose the best one.

1. Pleasant Clinical Environment: Patients and employees both benefit from a well-designed environment. It's simple to focus only on lobbies and waiting areas, but clinical spaces also need to be treated carefully. Imaging suites, procedure rooms where patients are conscious, and blood-draw stations benefit from natural light and uplifting distractions like art, a pleasing material palette, and views. In order to create a tranquil and healing environment, these components are essential.

2. Onstage / Offstage Environments: Disney's onstage/offstage model, where perfect service seems to occur spontaneously, is currently used by a large number of healthcare organisations. Making a circulation and planning diagram that allows for the vertical and horizontal separation of goods and services from patients and their families is equally important when building a new hospital as is separating experience zones from service areas. There are several levels of this isolation, and many different things can affect it. By locating service and patient transport elevators in the patient wing rather than at the ends of the units, the amount of crossover between patients and services can be decreased.

3. Healthcare Design: The arrangement of building approaches, building entrances, and roadways may all be employed as navigational aids in accordance with proper campus planning and architecture. It might be terrifying to try to read signs while driving. In order to reduce travel stress, vehicle entry and approach routes should be designed to be simple and evident. Additionally, the scale, lighting, and material choices made for the main hospital entrance, parking garages, and medical office buildings all help patients and their families get to the front door as fast as feasible. Late-arriving patients and their family are directed to well-lit entrances by vertical circulation towers and large public areas close to the key gates.

4. Healthy Occupants: Healing happens at hospitals, and the building itself ought to contribute to that healing. Using materials that are not on the Red List, providing clean, filtered air, and allowing access to outdoor activities through operable windows or terraces in locations where immune systems are unaffected are all examples of healthy construction practises. Looking beyond patients to a healthy planet, excess heat, rain, and wind should be captured and stored for use. The ideal hospital is a self-contained, net-zero, durable structure since hospitals are mission-critical buildings that must remain open and accessible during calamities like wildfires, tornadoes, and earthquakes.

Many health care organisations choose to hire a development partner with real estate knowledge to swiftly find and rate properties based on an organization's preferences to assist in identifying ideal sites and weighing conflicting variables. The following are the most common criteria: (α_1 =SPEED TO OPENING, α_2 = COST CONTROLS, α_3 =SIZE MATTERS, and α_4 =LOCATION).

1. Speed to Opening: Depending on how quickly the development and opening of its doors might be finished, an organisation may also analyse sites. Perhaps the organisation can't wait for the new or enlarged facilities because it needs to serve a new area right away. The health care organisation or its development partner may restrict the site search to properties that are ready for medical development when time is a top priority. The implication is that development can start without the need for a drawn-out approval process or time-consuming zoning code revisions or variances on properties that are the right size, have the right zoning, and are unrestricted. The organisation should also make sure the property is free of any environmental hazards, like wetlands, or cleanup difficulties, such underground storage tanks on the site of a former gas station. It may take a while to overcome these difficulties before a site is prepared for development.However, some of these challenges might be overcome if the ideal piece of land were available. For instance, to build an outpatient facility in Bluffton, South Carolina, to support the operations of two of the system's hospitals, the Hilton Head (S.C.) Health System partnered with a prominent health care real estate company. Although there were other locations, the preferred one was on the main road of a sizable shopping area, almost equal distances from both hospitals. However, the site was a little bit larger than necessary and was situated in a shopping centre that was anchored by a well-known grocery chain, which maintained significant approval rights over all future uses and centre extension.

However, some of these challenges might be overcome if the ideal piece of land were available. For instance, to build an outpatient facility in Bluffton, South Carolina, to support the operations of two of the system's hospitals, the Hilton Head (S.C.) Health System partnered with a prominent health care real estate company. Although there were other locations, the preferred one was on the main road of a sizable shopping area, almost equal distances from both hospitals. However, the site was a little bit larger than necessary and was situated in a shopping centre that was anchored by a well-known grocery chain, which maintained significant approval rights over all future uses and centre extension. However, because to careful planning and vigilant oversight, the project was completed in just over two years.

2. Cost Controls: The location and size of the property are just two of the many variables that affect property costs. Properties in the most coveted and convenient locations will cost more than those in less convenient locations, assuming all other factors are equal. As a result, in order to achieve its objective, a small-budget organisation might have to make compromises regarding the size or placement of the property. Additionally, the costs of your site may vary slightly depending on whether you own or lease a property. Despite the fact that ground leasing a property might be cost-effective in the short term, owning and maintaining the land is often a better long-term choice for a company. The difference is typically not significant when you take into account the cost of the land and the cost of the long-term ground lease. On the other side, remodelling a historic building might be highly expensive compared to building a new facility from scratch. This is especially true if the project is a smaller retail business, like an urgent care facility or a primary care physician's office, that the organisation wants to locate in a busy area. An organisation can save money by renovating a previous retail location by using the infrastructure that already exists, especially if the floor plan is open.

3. Size Matters: The size of the development site is another important factor. Aiming for a piece of property that can accommodate their present and future square footage needs, parking infrastructure, and other elements without taking up a lot of room is a good idea for healthcare facilities. Unless the organisation has significant expansion plans, there is no justification for excessive spending. If a nonprofit is working with a healthcare real estate development company, the company should be able to determine how much land a facility would need to accommodate the square footage of the structure, parking, drainage, and other issues like possible future growth. This enables a business to narrow down its property search to those that are the right size. In reality, a healthcare business could be driven to a space that is either too big or too little to fully support its original concept. If a site is too tiny but otherwise perfect, it may try to modify its development plan to make the facility fit the parcel. In this case, the organisation might need to think outside the box when it comes to its design and look into requesting a building code exception to allow a building that is taller or has fewer parking spaces than are now permitted by building regulations. Alternately, it could be essential to make certain compromises regarding the practises or programmes that will be housed in the facility. In this case, decide which programme you can live without. It's time to consider other options if your response is "none of them." In order to improve the neighborhood's access to high-quality healthcare, RWJBarnabas Health in West Orange, New Jersey, for instance, planned to construct an ambulatory care facility in Bayonne. The suggested location, which was situated along one of the main streets in downtown Bayonne, had great exposure and was in a place that the city had designated for reconstruction. The structure, which has a standalone emergency room, an imaging centre, and space for future clinical needs, debuted earlier this year.The cost and complexity of the project, as well as the level of governmental engagement and permissions needed, often increase when a medical centre is being developed in an established and highly populated urban area. The project required the demolition of a number of prior mixed-use commercial and residential structures on the Bayonne site. A multistory parking structure was necessary to meet the city's parking needs. To keep costs as low as possible in these conditions, efficiency was key, along with a well-thought-out plan for securing all the necessary permissions at the municipal, regional, and state levels. The ability and willingness of healthcare organisations to pay the increased costs associated with overly big but otherwise adequate properties must be considered. If so, they might desire to keep the excess land for expansion or future growth. If not, they might think about buying the whole parcel and then selling off portions of it.

4. Location: The location of a facility site is one of the most important factors to take into account when setting priorities. Many initiatives are developed as a result of a strategic organization's need to increase its presence and services in a town or neighbourhood where its existing or potential patients reside and work. On the other hand, geographic boundaries are not always the only factor in determining where a property is located. Other specific qualities that a business looks for are typical while selecting a site. Convenience, for instance, is frequently taken into account in healthcare projects. Organizations commonly prioritise patient access, which improves the overall patient experience and raises the likelihood that they will return and refer others. These qualities include being nearby to a freeway interchange, being in or near a busy shopping area, being near a desirable residential neighbourhood, and having plenty and convenient parking. Similar to this, high visibility may be a desirable quality, especially for a company trying to increase the number of patients it serves. The branding of a healthcare organisation may benefit from a building or storefront with visible signage and lots of passing traffic. The challenges of accessibility and visibility have grown in importance for health care organisations during the past decade or so. Health care providers used to be indifferent about convenience because they believed people would be willing to travel to locations where medical services were offered. However, this is no longer the case. Health care organisations are now aware that consumers value convenience. Similar to how they prefer a nearby grocery store to one across town, patients may decide to visit an outpatient location in their neighbourhood rather than travelling downtown to a large hospital. In order to ensure that their facilities are situated along busy routes that are already convenient for their patients, health care organisations are changing their real estate strategy. Organizations may take proximity into account when choosing a site location. In fact, this issue is immediately addressed by two of the most well-known real estate projects in healthcare institutions. One worry for many hospitals is that their outpatient clinics are located too near to their primary hospital. By interacting with their inhabitants, they are essentially "ringing" their region.

Assuming the developers seek to design a decision-making process and undertake a specific development, the management must monitor all index data relating to potential municipality growth and assess the full factor value of hospital development. The assessment results are represented by a neutrosophic hesitant fuzzy detailed assessment matrix, where $\alpha_i (1 \leq i \leq 4)$ represents the various possible outcomes for each element of residential project. And the solution of every element is determined by the linked administrative specialists using a neutrosophic probabilistic hesitant fuzzy linguistic word. Our aim is to determine the municipality development's initial attentiveness rating using a neutrosophic probabilistic hesitant fuzzy value. That is, which of the four factor degrees $\alpha_1; \alpha_2; \alpha_3; \alpha_4$; does the municipality progress relate to. We use the SV-NPHF information measure to determine the likelihood of each fuzzy component happening and the total component value of hospital and commercial developments, and then we assist the relevant hospital planning commission in adopting the appropriate like for to organise and improve the numerous interests of hospital development project.

Step-1 Collection of expert data in the form of neutrosophic probabilistic hesitant fuzzy information is shown in Table-1.

Table-1a: Expert Evaluation Information

	α_1	α_2
$\Re_1$	$\left\{ (0.2/0.4, 0.5/0.3, 0.7/0.3), (0.2/0.8, 0.31/0.1, 0.89/0.1), (0.9/0.4, 0.7/0.3, 0.2/0.3) \right\}$	$\left\{ (0.12/0.4, 0.15/0.6), (0.21/1.0), (0.9/1.0) \right\}$
$\Re_2$	$\left\{ (0.55/0.7, 0.75/0.3), (0.25/0.8, 0.35/0.2), (0.95/1.0) \right\}$	$\left\{ (0.9/1.0), (0.35/0.1, 0.9/0.9), (0.7/0.3, 0.2/0.7) \right\}$
$\Re_3$	$\left\{ (0.2/0.1, 0.15/0.9), (0.28/0.4, 0.39/0.1, 0.85/0.5), (0.19/0.4, 0.72/0.3, 0.25/0.3) \right\}$	$\left\{ (0.35/0.45, 0.95/0.25, 0.75/0.3), (0.82/0.85, 0.38/0.15), (0.75/0.35, 0.82/0.65) \right\}$
$\Re_4$	$\left\{ (0.22/0.25, 0.55/0.35, 0.74/0.4), (0.2/1.0), (0.9/0.65, 0.7/0.35) \right\}$	$\left\{ (0.5/0.35, 0.57/0.65), (0.52/1.0), (0.22/1.0) \right\}$

Table-1b: Expert Evaluation Information

	α_3	α_4
$\Re_1$	$\left\{(0.25/1.0), (0.2/0.45, 0.85/0.55), (0.99/1.0)\right\}$	$\left\{(0.24/1.0), (0.85/1.0), (0.25/1.0)\right\}$
$\Re_2$	$\left\{(0.29/0.01, 0.35/0.99), (0.3/1.0), (0.25/1.0)\right\}$	$\left\{(0.2/0.1, 0.15/0.9), (0.28/1.0), (0.19/0.4, 0.72/0.3, 0.25/0.3)\right\}$
$\Re_3$	$\left\{(0.15/1.0), (0.8/1.0), (0.4/1.0)\right\}$	$\left\{(0.45/1.0), (0.38/1.0), (0.19/1.0)\right\}$
$\Re_4$	$\left\{(0.24/1.0), (0.82/1.0), (0.9/1.0)\right\}$	$\left\{(0.15/1.0), (0.28/1.0), (0.2/0.7, 0.5/0.3)\right\}$

Step-2 Normalised expert information of the considered attributes are presented in Table-2.

Table-2a: Normalised expert Information

	α_1	α_2
$\Re_1$	$\left\{(0.9/0.4, 0.7/0.3, 0.2/0.3), (0.2/0.8, 0.31/0.1, 0.89/0.1), (0.2/0.4, 0.5/0.3, 0.7/0.3)\right\}$	$\left\{(0.9/1.0), (0.21/1.0), (0.12/0.4, 0.15/0.6)\right\}$
$\Re_2$	$\left\{(0.95/1.0), (0.25/0.8, 0.35/0.2), (0.55/0.7, 0.75/0.3)\right\}$	$\left\{(0.7/0.3, 0.2/0.7), (0.35/0.1, 0.9/0.9), (0.9/1.0)\right\}$
$\Re_3$	$\left\{(0.19/0.4, 0.72/0.3, 0.25/0.3), (0.28/0.4, 0.39/0.1, 0.85/0.5), (0.2/0.1, 0.15/0.9)\right\}$	$\left\{(0.75/0.35, 0.82/0.65), (0.82/0.85, 0.38/0.15), (0.35/0.45, 0.95/0.25, 0.75/0.3)\right\}$
$\Re_4$	$\left\{(0.9/0.65, 0.7/0.35), (0.2/1.0), (0.22/0.25, 0.55/0.35, 0.74/0.4)\right\}$	$\left\{(0.22/1.0), (0.52/1.0), (0.5/0.35, 0.57/0.65)\right\}$

Table-2b: Normalised expert Information

	α_3	α_4
$\Re_1$	$\left\{ \begin{array}{c} (0.99/1.0), \\ (0.2/0.45, 0.85/0.55), \\ (0.25/1.0) \end{array} \right\}$	$\left\{ \begin{array}{c} (0.25/1.0), \\ (0.85/1.0), \\ (0.24/1.0) \end{array} \right\}$
$\Re_2$	$\left\{ \begin{array}{c} (0.25/1.0), \\ (0.3/1.0), \\ (0.29/0.01, 0.35/0.99) \end{array} \right\}$	$\left\{ \begin{array}{c} (0.19/0.4, 0.72/0.3, 0.25/0.3), \\ (0.28/1.0), \\ (0.2/0.1, 0.15/0.9) \end{array} \right\}$
$\Re_3$	$\left\{ \begin{array}{c} (0.4/1.0), \\ (0.8/1.0), \\ (0.15/1.0) \end{array} \right\}$	$\left\{ \begin{array}{c} (0.19/1.0), \\ (0.38/1.0), \\ (0.45/1.0) \end{array} \right\}$
$\Re_4$	$\left\{ \begin{array}{c} (0.9/1.0), \\ (0.82/1.0), \\ (0.15/1.0) \end{array} \right\}$	$\left\{ \begin{array}{c} (0.2/0.7, 0.5/0.3), \\ (0.28/1.0), \\ (0.24/1.0) \end{array} \right\}$

Step-3(a) We accomplish the desired preferred values δ_k of the alternative $\Re_k (k = 1, 2, 3, 4)$ by implementing the proposed operators

Case-1: Using $SV - NP\ HFWA$ operator, here $\mho$ is weight vector and, $\mho = 0.2, 0.4, 0.1, 0.3$ the result shown in Table-3.

$$\delta_1 = SV - NPHFWA(\delta_{11}, \delta_{12}, \delta_{13}) = \left(\begin{array}{c} \bigcup_{\ell_\Gamma \in \mathcal{A}_{\delta_\Gamma}, \alpha_{\delta_\Gamma} \in \alpha_{\delta_\Gamma}} 1 - \Pi_{\Gamma=1}^{q} \left(1 - \ell_{\delta_\Gamma}\right)^{\mho_\Gamma} / \Pi_{\Gamma=1}^{q} \alpha_{\delta_\Gamma}, \\ \bigcup_{\Upsilon_{\delta_\Gamma} \in \mathcal{Z}_{\delta_\Gamma}, \beta_{\delta_\Gamma} \in \beta_{\delta_\Gamma}} \Pi_{\Gamma=1}^{q} \left(\Upsilon_{\delta_\Gamma}\right)^{\mho_\Gamma} / \Pi_{\Gamma=1}^{q} \beta_{\delta_\Gamma}, \\ \bigcup_{\check{n}_{\delta_\Gamma} \in \mathcal{N}_{\delta_\Gamma}, \gamma_{\delta_\Gamma} \in \gamma_{\delta_\Gamma}} \Pi_{\Gamma=1}^{q} \left(\check{n}_{\delta_\Gamma}\right)^{\mho_\Gamma} / \Pi_{\Gamma=1}^{q} \gamma_{\delta_\Gamma} \end{array} \right)$$

Table-3: Aggregated Data ($SV - NPHFWA$)

$$\Re_1 = \left\langle \begin{array}{c} \{0.3418, 0.2457, 0.2339\}, \\ \{0.1133, 0.1601, 0.0155, 0.0218, 0.0191, 0.0270\}, \\ \{0.0282, 0.0462, 0.0254, 0.0416, 0.0271, 0.0297\} \end{array} \right\rangle,$$

$$\Re_2 = \left\langle \begin{array}{c} \{0.0829, 0.0697, 0.0628, 0.1517, 0.1400\}, \\ \{0.0241, 0.3166, 0.0064, 0.0847\}, \\ \{0.0003, 0.0027, 0.0328, 0.2704, 0.0001, 0.0012, 0.0149, 0.1233\} \end{array} \right\rangle,$$

$$\Re_3 = \left\langle \begin{array}{c} \{0.0712, 0.1480, 0.0633, 0.1271, 0.0542, 0.1123\}, \\ \{0.1781, 0.0231, 0.0476, 0.0062, 0.2780, 0.0361\}, \\ \{0.0140, 0.1185, 0.0116, 0.1072, \} \end{array} \right\rangle,$$

$$\Re_4 = \left\langle \begin{array}{c} \{0.2619, 0.1231, 0.1155, 0.0568\}, \\ \{0.3734\}, \\ \{0.0264, 0.0517, 0.0444, 0.0869, 0.0539, 0.105400369\} \end{array} \right\rangle.$$

Step-4(a) The score values are calculated as follows in Table 4:

Table-4: Score Values

Operators	$Sc(\Re_1)$	$Sc(\Re_2)$	$Sc(\Re_3)$	$Sc(\Re_4)$
$SV - NP\ HFWA$	0.1813	-0.0598	-0.0616	-0.2955

Step-5(a) Rank the alternatives $\Re_k(k = 1, 2, ..., 4)$ is enclosed

$$s(\Re_1) > s(\Re_2) > s(\Re_3) > s(\Re_4).$$

Therefore

$$\Re_1 > \Re_2 > \Re_3 > \Re_4.$$

The best choice is $\Re_1$.

Data Collection: Through the use of a standardised questionnaire, data were collected. The factors that were examined included location, size metrics, cost restrictions, and speed of opening. The first statistical study used SV-NPHFWA and SV-NPHFWG AOs. This test was designed to statistically disprove the statistical disparities between urban and rural areas.

Step-3(b) We accomplish the desired preferred values δ_k of the alternative $\Re_k(k = 1, 2, 3, 4)$ by implementing the proposed operator.

Case-2: Using $SV - NP\ HFWG$ operator, the result shown in Table-5.

$$SV{-}NPHFWG(\delta_1, \delta_2, ..., \delta_q) = \left(\begin{array}{c} \bigcup\limits_{\substack{\ell_{\delta_\Gamma} \in \mathcal{Z}_{\delta_\Gamma} \\ \alpha_{\delta_\Gamma} \in \alpha_{\delta_\Gamma}}} \Pi_{\Gamma=1}^{q} \left(\ell_{\delta_\Gamma}\right)^{\mho_\Gamma} / \Pi_{\Gamma=1}^{q} \alpha_{\delta_\Gamma}, \\ \bigcup\limits_{\substack{\Upsilon_\Gamma \in \mathcal{A}_{\delta_\Gamma} \\ \beta_{\delta_\Gamma} \in \beta_{\delta_\Gamma}}} 1 - \Pi_{\Gamma=1}^{q} \left(1 - \Upsilon_{\delta_\Gamma}\right)^{\mho_\Gamma} / \Pi_{\Gamma=1}^{q} \beta_{\delta_\Gamma}, \\ \bigcup\limits_{\substack{\check{n}_\Gamma \in \mathcal{A}_{\delta_\Gamma} \\ \gamma_{\delta_\Gamma} \in \gamma_{\delta_\Gamma}}} 1 - \Pi_{\Gamma=1}^{q} \left(1 - \check{n}_{\delta_\Gamma}\right)^{\mho_\Gamma} / \Pi_{\Gamma=1}^{q} \gamma_{\delta_\Gamma} \end{array} \right)$$

Table-5: Aggregative Data $(SV - NPHFWG)$

$$\Re_1 = \left\langle \begin{array}{c} \{0.3418, 0.2457, 0.2339\}, \\ \{0.1133, 0.1601, 0.0155, 0.0218, 0.0191, 0.0270\}, \\ \{0.0282, 0.0462, 0.0254, 0.0416, 0.0271, 0.0297\} \end{array} \right\rangle,$$

$$\Re_2 = \left\langle \begin{array}{c} \{0.0829, 0.0697, 0.0628, 0.1517, 0.1400\}, \\ \{0.0241, 0.3166, 0.0064, 0.0847\}, \\ \{0.0003, 0.0027, 0.0328, 0.2704, 0.0001, 0.0012, 0.0149, 0.1233\} \end{array} \right\rangle,$$

$$\Re_3 = \left\langle \begin{array}{c} \{0.0712, 0.1480, 0.0633, 0.1271, 0.0542, 0.1123\}, \\ \{0.1781, 0.0231, 0.0476, 0.0062, 0.2780, 0.0361\}, \\ \{0.0140, 0.1185, 0.0116, 0.1072, \} \end{array} \right\rangle,$$

$$\Re_4 = \left\langle \begin{array}{c} \{0.2619, 0.1231, 0.1155, 0.0568\}, \\ \{0.3734\}, \\ \{0.0264, 0.0517, 0.0444, 0.0869, 0.0539, 0.105400369\} \end{array} \right\rangle.$$

Step-4(b) The score values are calculated as follows in Table 6:

Table-6: Score Values

Operators	$Sc(\Re_1)$	$Sc(\Re_2)$	$Sc(\Re_3)$	$Sc(\Re_4)$
$SV - NP\ HFWG$	0.0353	-0.1646	-0.1276	-0.5661

Step-5(b) Rank the alternatives $\Re_k (k = 1, 2, ..., 4)$ is enclosed in Table-7:

Table-7: Ranking of the alternatives

Operators	Score
$SV - NP\ HFWG$	$Sc(\Re_1) > Sc(\Re_3) > Sc(\Re_2) > Sc(\Re_4)$

Therefore

$$\Re_1 > \Re_3 > \Re_2 > \Re_4.$$

The best choice is $\Re_1$.

Table-8: Ranking of Obtained Operators

Operators	Score	Best Alternative
$SV - NPHFWA$	$Sc(\Re_1) > Sc(\Re_2) > Sc(\Re_3) > Sc(\Re_4)$	$\Re_1$
$SV - NPHFWG$	$Sc(\Re_1) > Sc(\Re_3) > Sc(\Re_2) > Sc(\Re_4)$	$\Re_1$

Therefore the best choice is $\Re_1$ as shown in comparison Table 8.

6.2. Comparison Analysis

In order to clarify the validity of the suggested operators, a comparative analysis is very helpful as it offers a methodical framework for comparing their performance to current methodologies. This allows for a more in-depth analysis of the advantages and disadvantages of the proposed operators in managing the complex uncertainties that are present in decision-making processes, especially in the intricate areas of economic development in the rural and urban medical sectors. Here, we will compare our work with SV-neutrosophic (SV-N) weighted averaging (SV-NWA) AO [50], the SV-N weighted geometric (SV-NWG) AO [50], Interval-Valued neutrosophic weighted averaging (IV-NWA) AO [51], Interval-Valued neutrosophic weighted geometric (IV-NWG) AO [51], SV-N Dombi weighted geometric (SV-NDWG) AO [52], the SV-NHF weighted averaging (SV-NHWA) AO, and the SV-NH the SV-NHF weighted averaging (SV-NHWA) AO [53], the SV-NHF weighted geometric (SV-NHWG) AO [53] solving only lower approximations, the SVN probabalistic hesitant fuzzy rough (SV-NPHFR)weighted averaging (SV-NPHFRWA) and weighted geometric (SV-NPHFRWG) [54], SV-NPHF Dombi weighted arithmetic average (SV-NPHFDWAA) operator and SV-NPHF Dombi weighted arithmetic geometric (SV-NPHFDWAG) [48] and SVN Hesitant Fuzzy Rough Weighted Averaging (SV-NHFRWA), and hyper weighted averaging (SV-NHFRHWA) [55]. The overall analysis of the comparative study is given in Table

9. From the analysis, we can observe that

Table-8: Comparison Analysis

Operators	Ranking	Best Alternative
$SV-NWA$	Impossible to access	No result
$SV-NDWG$	Impossible to access	No result
$SV-NHWA$	Impossible to access	No result
$SV-NHWG$	Impossible to access	No result
$IV-NWA$	Impossible to access	No result
$IV-NWG$	Impossible to access	No result
$SV-NPHFRWA$	$Sc(\Re_1) > Sc(\Re_3) > Sc(\Re_2) > Sc(\Re_4)$	$\Re_1$
$SV-NPHFRWG$	$Sc(\Re_2) > Sc(\Re_1) > Sc(\Re_3) > Sc(\Re_4)$	$\Re_2$
$SV-NPHFDWAA$	$Sc(\Re_1) > Sc(\Re_2) > Sc(\Re_3) > Sc(\Re_4)$	$\Re_1$
$SV-NPHFDWAG$	$Sc(\Re_1) > Sc(\Re_3) > Sc(\Re_2) > Sc(\Re_4)$	$\Re_1$
$SV-NHFRWA$	$Sc(\Re_1) > Sc(\Re_2) > Sc(\Re_4) > Sc(\Re_3)$	$\Re_1$
$SV-NHFRHWA$	$Sc(\Re_1) > Sc(\Re_2) > Sc(\Re_3) > Sc(\Re_4)$	$\Re_1$
$SV-NPHFWA$	$Sc(\Re_1) > Sc(\Re_2) > Sc(\Re_3) > Sc(\Re_4)$	$\Re_1$
$SV-NPHFWG$	$Sc(\Re_1) > Sc(\Re_3) > Sc(\Re_2) > Sc(\Re_4)$	$\Re_1$

1 The SV-NWG , SV-NWA, SV-NHWA, SV-NHWG, IV-NWA, and IV-NWG operators can only deal with neutrosophic data, and these notions lack the extra characteristic of handling probabilistic information. While our initiated work has the advantages of using hesitant probabilistic information in their stricture.

2 Additionally, we can show that prior work, specifically the SV-NPHF framework, might give decision-makers additional room. While ideas currently held cannot. As a result, our initiated work is more broad.

3 After being considered, we ended up with the same top choice, $\Re_1$ things we suggested for our numerical case analysis. In order to achieve this, we first divided the MADM challenge into three more manageable sub-problems, such as $\{\Re_1, \Re_3, \Re_4\}$ and $\{\Re_1, \Re_3, \Re_2\}$. Now that the larger challenges have been reduced into smaller ones, we apply our recommended decision-making methodology to them and arrive at the ranking of options shown below: $\Re_1 > \Re_3 > \Re_2$, $\Re_3 > \Re_2 > \Re_4$ and $\Re_1 > \Re_3 > \Re_4$ respectively. We find that $\Re_1 > \Re_3 > \Re_2 > \Re_4$ is the same as the outcomes of applying the conventional decision-making process when giving a thorough ranking.

6.3. Limitations of the proposed work

1 The computational complexity of the integrated decision-making framework is a drawback because it may impede real-time decision-making in dynamic decision-making scenarios when single-valued neutrosophic probabilistic hesitant fuzzy sets and unconventional aggregation operators are incorporated.

2 The single-valued neutrosophic probabilistic hesitant fuzzy approach's accuracy may be undermined by uncertainties and information gaps, which could impact the framework's reliance on complete and accurate data and, in circumstances where data availability is limited, the decision outcomes' dependability.

3 It may be necessary to give considerable thought to and validate the framework's performance in a variety of real-world applications because its efficacy may depend on the selection of creative aggregation operators and the extent to which their performance can be generalized across distinct decision scenarios.

4 In circumstances where there are swift changes in technology or healthcare paradigms, the framework might not be as flexible as it should be, and it might find it difficult to take into account new factors and changing standards that are necessary for sound decision-making in the dynamic field of hospital development.

5 The framework's intricacy and need on specialized knowledge for execution could potentially impede its extensive integration, especially in settings with limited resources. This would restrict its applicability and accessibility in contexts where ease of use and simplicity are crucial.

7. CONCLUSION

The SV-NPHFS was created for circumstances where every value has a range of possible values that are determined by MD, indeterminacy, and non-MD. It is a powerful mix of an SV-NS and Probabilistic HFS (PHFS). This paper proposes an SV-NPHFEWA operator, SV-NPHFEWG operator, SV-NPHFEOWG operator, and SV-NPHFEOWA operator. A novel MADM strategy based on the SV-NPHFEWA and SV-NPHFEWG operators was also suggested. Below is further information regarding the benefits of these strategies.

1 It's crucial to start by pointing out that both the SV-NPHFWG and SV-NPHFWA operations possess a number of crucial traits. Idempotency, commutativity, boundedness, and monotonicity are a few of them; others include them but are not restricted to them. All of these have a big impact on how they behave and are used.

2 The ability of the proposed AOs to be modified further demonstrates their adaptability, as demonstrated by the conversion of the SV-NPHFWA and SV-NPHFWG operators into the already existing AOs created for SV-NPHFSs. This presentation highlights how adaptable these operators are and how they may be used in a variety of situations.

3 Thirdly, compared to previous methodologies used for Multiple Attribute Decision Making (MADM) situations within the SV-NPHF framework, the results produced by the SV-NPHFWA and SV-NPHFWG operators offer an elevated level of accuracy and dependability. This comparison demonstrates their efficiency and implies that they are suitable for use in realistic situations.

4 The methods for MADM in this study are capable of recognizing a higher level of interconnectivity between characteristics and options. When compared to current approaches, their increased utility and precision are a result of this improved discernment. The complex interdependencies between qualities in real-world circumstances are frequently not sufficiently taken into consideration by these current approaches. The results demonstrate that not only do the MADM approaches proposed in this study excel in this regard, but they also have the ability to reveal even more complex correlations between features.

5 The aggregation operators presented in this paper are also successfully used in real-world contexts to analyze symmetry analysis while choosing features for the development of urban and rural hospitals. The relevance and usefulness of the suggested operators in addressing decision-making difficulties connected to building projects in various situations is demonstrated by this actual application.

6 Further studies on modified human uniformity advancement consensus problems, acceptance development including decision-making problems with uncooperative actions, and decision-making problems involving two-sided associated with multi-granular and unfinished criteria weight data might all benefit from the indicated aggregation operators. It should be noted that the proportions of involvement, abstention, and non-membership have little influence on the assessment of the limits imposed by the proposed aggregation operators. A novel combination of prioritised interactions aggregation operators is being created alongside to the planned aggregation operators. This new structure is intended to significantly enhance the effectiveness and impact of aggregation approaches.

7 In future study, we will investigate the mathematical foundations of SV-NPHFSs for Einstein operations using cutting-edge decision-making approaches such as TOPSIS, VIKOR, TODAM, GRA, and EDAS. We'll also discuss how these strategies are applied in a variety of fields, such as computer science, robotics, horticulture, artificial intelligence, social science, finance, and management of human resources.

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