Yugoslav Journal of Operations Research
28 (2018), Number 2, 237–248
DOI: https://doi.org/10.2298/YJOR170212005B

AN APPROACH TO SOLVE MULTI-OBJECTIVE LINEAR PRODUCTION PLANNING GAMES WITH FUZZY PARAMETERS

Hamid BIGDELI
Institute for the Study of War, Army Command and Staff University, Tehran, I.R. Iran
hamidbigdeli92@gmail.com

Hassan HASSANPOUR
Department of Mathematics, University of Birjand, Birjand, I.R. Iran
hhassanpour@birjand.ac.ir

Received: February 2017 / Accepted: February 2018

Abstract: In this research, n-person cooperative games, arising from multi-objective linear production planning problem with fuzzy parameters, are considered. It is assumed that the fuzzy parameters are fuzzy numbers. The fuzzy multi-objective game problem is transformed to a single-objective game problem by group AHP method. The obtained problem is converted to a problem with interval parameters by considering the nearest interval approximation of the fuzzy numbers. Then, optimistic and pessimistic core concepts are introduced. The payoff vectors of the players are obtained by the duality theorem of linear programming. Finally, validity and applicability of the method are illustrated by a practical example.

Keywords: Cooperative Game, Production Planning, Optimistic and Pessimistic Core, Nearest Interval Approximation.

MSC: 90B85, 90C26.

1. INTRODUCTION

Game theory is a formal way to analyze interaction among a group of rational decision makers who behave strategically. Games are broadly classified into two major categories: cooperative and non-cooperative games. In cooperative games,

coalitions are organized by group agreement among some or all of the players and many coalitions are possible in the n-person case. Any player participating in a coalition must accept completely the decisions of the coalition, in other words, a coalition behaves like an individual decision maker [26].
In the field of fuzzy single-objective cooperative games, considerable studies have been made (for example see [2, 3, 11, 12]).
Sakawa and Nishizaki [26] extended the least core and the nucleolus in n-person cooperative games with fuzzy coalitions. Nishizaki and Sakawa [26] constructed cooperative games with fuzzy coalition values arising from the linear programming problems with fuzzy parameters, and they investigated the core and a solution concept based on the fuzzy goals for the coalition's payoffs. The multi-objective games were investigated by Bergstresser and Yu [6]. They mainly considered the core, defined by the domination structures and referred to a couple of solution concepts that yield a unique solution such as the nucleolus in n-person cooperative games.
Sakawa and Nishizaki [26] considered the nucleolus in n-person cooperative games with multiple scenarios. Multi-objective n-person cooperative games are defined by the set of players and the sets in multi-dimensional payoff space corresponding to the objective space.
Several articles have been devoted to the study of multi-objective cooperative games (e.g. see [6, 17, 19, 20, 23, 24, 34, 35, 36]). Owen [28] considered linear production planning problems in which multiple decision makers pool resources to make several products. Sakawa and Nishizaki [26] considered multi-objective linear production planning problem in which multiple decision makers pool resources to make several products. The authors ([7, 8, 9, 10]) considered some of fuzzy multiobjective games in other researchs. We consider the n-person cooperative games arising from the multi-objective linear production planning problem with fuzzy parameters. The remainder of the paper is organized as follows. In section 2, some preliminaries and necessary definitions about fuzzy sets and n-person cooperative games are presented. In section 3, an n-person cooperative game arising from the multi-objective linear production planning problem with fuzzy parameters is introduced. Then a method is proposed to find optimistic and pessimistic core. In section 4, validity and applicability of the method is illustrated by a practical example. Finally, conclusion is made in section 5.

2. PRELIMINARIES

2.1. Fuzzy sets

In this subsection, we review some definitions and preliminaries of fuzzy sets according to [29]. Let X denote a universal set. A fuzzy subset $\tilde{a}$ of X is defined by its membership function $\mu_{\tilde{a}} : X \to [0,1]$, which assigns to each element $x \in X$ a real number $\mu_{\tilde{a}}(x)$ in the interval $[0,1]$; $\mu_{\tilde{a}}(x)$ is the grade of membership of x in the set $\tilde{a}$. The support of $\tilde{a}$, denoted by $supp(\tilde{a})$, is the set of points $x \in X$ at which $\mu_{\tilde{a}}(x)$ is positive; $\tilde{a}$ is said to be normal if there is $x \in X$ such that $\mu_{\tilde{a}}(x) = 1$. The α-cut of the fuzzy set $\tilde{a}$, denoted by $\tilde{a}_\alpha$, is a crisp set defined by $\tilde{a}_\alpha = \{x|\mu_{\tilde{a}}(x) \geq \alpha\}$ for each $\alpha \in (0,1]$, and $\tilde{a}_0 = closure\{x|\mu_{\tilde{a}}(x) > 0\}$; $\tilde{a}$ is said to be a convex fuzzy set if its α-cuts are convex. A fuzzy number is a convex normalized fuzzy set of the real line $\mathbb{R}$ whose membership function is piecewise continuous. From the definition of a fuzzy number $\tilde{a}$, it is significant to note that each α-cut $\tilde{a}_\alpha$ of a fuzzy number $\tilde{a}$ is a closed interval $\left[a_\alpha^L, a_\alpha^R\right]$. A triangular fuzzy number $\tilde{a} = \left(a^l, a^m, a^r\right)$ is a special fuzzy number, whose membership function is given by

$$\mu_{\tilde{a}}(x) = \begin{cases} \frac{x-a^l}{a^m-a^l} & a^l \leq x \leq a^m \\ \frac{a^r-x}{a^r-a^m} & a^m \leq x \leq a^r \\ 0 & o.w. \end{cases}$$

where a^m is the mean of $\tilde{a}$, and a^l and a^r are the left and right end-points of $supp(\tilde{a})$, respectively. In order to simply represent and handle fuzzy numbers, a natural need is to replace fuzzy numbers with some simpler approximations.

Proposition 1. *[16] Suppose $\tilde{a}$ is a fuzzy number with α-cut $\left[a_\alpha^L, a_\alpha^R\right]$. The nearest interval approximation of $\tilde{a}$ is* $\left[\int_0^1 a_\alpha^L d\alpha, \int_0^1 a_\alpha^R d\alpha\right]$.

2.2. Preliminaries of cooperative games

In this subsection, we review some definitions and preliminaries of cooperative games.

A cooperative game (transferable utility game) is a pair (N, v), where $N = \{1, \ldots, n\}$ is a finite set of players and v is a real-valued function defined on the power set of N, i.e., $v: 2^N \to \mathbb{R}$ satisfying $v(\phi) = 0$. Each subset S of N is called a coalition and the value $v(S)$ is called the worth of S. We denote by G^N the set of all games on N.

Definition 2. *[26] A game $v \in G^N$ is said to be superadditive if*

$$v\left(S \cup T\right) \geq v(S) + v(T) \quad \forall S, T \subseteq N : S \cap T = \phi$$

In the cooperative game theory, the most important topic is to find an appropriate rule for allocating the worth of the grand coalition among the players. Such a rule is usually called a solution of the cooperative game. The allocated profit vector is denoted by $x = (x_1, \ldots, x_n)$, where x_i is the profit of the i-th player. It is quite natural that this vector satisfies the following efficiency condition

$$\sum_{i \in N} x_i = v(N).$$

Two approaches have been taken in developing solutions of transferable utility games. One of them is based on the objections of the coalitions, and the other is based on the contributions of the players. A typical example of the former is the core defined below, while that of the latter is the Shapley value [1]. A payoff vector $x = (x_1, \ldots, x_n)$ which satisfies both efficiency and individual rationality, defined by

$$x_i \geq v\left(i\right) \;\; for\, all\, i \in N,$$
$$\sum_{i \in N} x_i = v\left(N\right),$$

is called an imputation, and the set of all imputations of the game $v \in G^N$ is denoted by $I(v)$, i.e.,

$$I\left(v\right) = \left\{ x \in \mathbb{R}^n | \sum_{i \in N} x_i = v\left(N\right), x_i \geq v\left(i\right) \;\; \forall i \in N \right\}.$$

Definition 3. *[26] The core of the game $v \in G^N$, denoted by $C(v)$, is defined as*

$$C\left(v\right) = \left\{ x \in \mathbb{R}^n | \sum_{i \in N} x_i = v\left(N\right), \sum_{i \in S} x_i \geq v\left(S\right) \quad \forall S \subseteq N \right\}.$$

It is clear that the core is a convex polyhedron, since it is represented by a linear equation and $2^n - 2$ linear inequalities. In general, unfortunately, the core of a game may be empty. The existence of the core is characterized by the concept of the balanced collections.

Definition 4. *[26] Let $B = \{S_1, \ldots, S_m\}$ be a collection of nonempty subsets of N. B is called a balanced collection if there exists a vector of positive numbers, the balancing vector $y = (y_1, \ldots, y_m)$, such that*

$$\sum_{\substack{S \in B \\ i \in S}} y_S = 1, \quad \forall i \in N$$

Theorem 5. *[26] A game (N, v) is said to be balanced if, for every balanced collection $B = \{S_1, \ldots, S_m\}$ with any balancing vector $y = (y_1, \ldots, y_m)$,*

$$\sum_{j=1}^{m} y_j v\left(S_j\right) \leq v\left(N\right).$$

Theorem 6. *[26] For a game (N, v), a necessary and sufficient condition that $C(v) \neq \varnothing$ is that the game is balanced.*

3. MULTI-OBJECTIVE LINEAR PRODUCTION PLANNING PROBLEM WITH FUZZY PARAMETERS

Owen [28] considered linear production planning problems in which multiple decision makers pool resources to make several products. The objective function of linear production planning problem was represented as the total revenue from selling certain kinds of products, and the problem was formulated as the linear programming problem in which, the revenue is maximized subject to resource constraints.

In this section, for an n-person cooperative game arising from the multi-objective linear production planning problem with fuzzy parameters, we introduce the concepts of the core solutions. We study multi-objective linear production planning problems with fuzzy parameters which reflect the players ambiguous or fuzzy understanding of the nature of the parameters in the problem formulation process. We assume that the parameters of the objective functions and constraints of the problem are fuzzy numbers. A multi-objective linear production planning game with fuzzy parameters is described as follows.

Let $N = \{1, \ldots, n\}$ be the set of all players. Each player is in possession of a resource vector $\tilde{b}^i = \left(\tilde{b}^i_1, \ldots, \tilde{b}^i_m\right),\ i = 1, \ldots, n,$ and p kinds of products are made by cooperation of the players. Let S be a coalition. This coalition will have a total of

$$\tilde{b}_r(S) = \sum_{i \in S} \tilde{b}^i_r$$

units of the r-th resource. A unite of the j-th product, $j = 1, \ldots, p$, requires $\tilde{a}_{rj}$ units of the r-th resource $r = 1, \ldots, m$ and a unit of the j-th product for the k-th objective can be sold at a price $\tilde{c}_{kj}$. We assume that the parameters $\tilde{b}^i_r, \tilde{a}_{rj}, \tilde{c}_{kj}$ are fuzzy numbers. Using all of their resources, the members of S can produce any vector $(u_1, u_2, \ldots, u_p)$ of products which satisfies

$$\begin{array}{ll}
\tilde{a}_{11}u_1 + \ldots + \tilde{a}_{1p}u_p \tilde{\leq} \tilde{b}_1(S) & \\
\vdots & \\
\tilde{a}_{m1}u_1 + \ldots + \tilde{a}_{mp}u_p \tilde{\leq} \tilde{b}_m(S) & \\
u_j \geq 0 & j = 1, \ldots, p,
\end{array}$$

where the symbol "$\tilde{\leq}$" denotes a relaxed or fuzzy version of the ordinary inequality "$\leq$". If they wish to maximize their revenues, then they will look for u to maximize the following l objectives.

$$\begin{aligned}
\tilde{v}^1(S) &= \tilde{c}_{11}u_1 + \ldots + \tilde{c}_{1p}u_p, \\
&\vdots \\
\tilde{v}^l(S) &= \tilde{c}_{l1}u_1 + \ldots + \tilde{c}_{lp}u_p.
\end{aligned}$$

Thus for a coalition S, the multi-objective linear production planning problem with fuzzy parameters can be expressed as

$$\begin{array}{lll}
maximize\ \tilde{c}_{11}u_1 + \ldots + \tilde{c}_{1p}u_p & & \\
\vdots & & \\
maximize\ \tilde{c}_{l1}u_1 + \ldots + \tilde{c}_{lp}u_p & & \\
s.t. & \tilde{a}_{11}u_1 + \ldots + \tilde{a}_{1p}u_p \tilde{\leq} \tilde{b}_1(S) & \\
& \vdots & \\
& \tilde{a}_{m1}u_1 + \ldots + \tilde{a}_{mp}u_p \tilde{\leq} \tilde{b}_m(S) & \\
& u_j \geq 0 \qquad j = 1, \ldots, p. &
\end{array} \tag{1}$$

The decision makers will be asked to participate in a group decision making to prioritize the objectives. This can be done by "Expert Choice" Software that uses a group AHP methodology [18] for prioritization purposes. Assume that w_k, $k = 1, \ldots, l$ indicate the relative importance of the k-th objective function. Therefore, the problem (1), rewritten, is as follows.

$$\begin{array}{ll} maximize\ w_1\ \tilde{v}^1(S) + \ldots + w_l \tilde{v}^l(S) & \\ s.t. \qquad \tilde{a}_{11}u_1 + \ldots + \tilde{a}_{1p}u_p \tilde{\leq} \tilde{b}_1(S) & \\ \qquad\qquad \vdots & \qquad (2) \\ \qquad\qquad \tilde{a}_{m1}u_1 + \ldots + \tilde{a}_{mp}u_p \tilde{\leq} \tilde{b}_m(S) & \\ \qquad\qquad u_j \geq 0 \qquad j = 1, \ldots, p. & \end{array}$$

By solving the problem (2) (in the following a solution approach is proposed), we obtain the optimal solution $\left(u_1^*, \ldots, u_p^*\right)$ and the optimal value of the objective as $\tilde{V}_w(S)$ for the coalition S. Thus the fuzzy cooperative game $(N, \tilde{V}_w)$ with the fuzzy coalition values arises from the multi-objective linear production planning problem with fuzzy parameters. We will refer to this game as a fuzzy weighted linear production planning game.

For solving the problem (2), we first use the nearest interval approximation of the fuzzy numbers. Thus the problem (2) is transformed to the following weighted linear programming problem with interval parameters.

$$\begin{array}{ll} maximize\ w_1\ (c_{11} + \ldots + c_{l1})\, u_1 + \ldots + w_l\, (c_{1p} + \ldots + c_{lp})\, u_p & \\ s.t. \qquad a_{11}u_1 + \ldots + a_{1p}u_p \leq b_1(S) & \\ \qquad\qquad \vdots & \qquad (3) \\ \qquad\qquad a_{m1}u_1 + \ldots + a_{mp}u_p \leq b_m(S) & \\ \qquad\qquad u_j \geq 0 \qquad j = 1, \ldots, p, & \end{array}$$

where the parameters of the problem are the intervals. To solve the problem (3), we consider two point of views.

1) Optimistic case

In this case, the players consider the most values of sold prices of products in market over the largest feasible region of the problem. Therefore the players must solve the following linear programming problem. We call it optimistic linear programming problem.

$$\begin{array}{ll} maximize\ w_1\ \left(c_{11}^R + \ldots + c_{l1}^R\right) u_1 + \ldots + w_l \left(c_{1p}^R + \ldots + c_{lp}^R\right) u_p & \\ s.t. \qquad a_{11}^L u_1 + \ldots + a_{1p}^L u_p \leq b_1^R(S) & \\ \qquad\qquad \vdots & \qquad (4) \\ \qquad\qquad a_{m1}^L u_1 + \ldots + a_{mp}^L u_p \leq b_m^R(S) & \\ \qquad\qquad u_j \geq 0 \qquad j = 1, \ldots, p. & \end{array}$$

By solving the problem (4), we obtain the most possible values of benefits of the objectives for the coalition S.

2) Pessimistic case
In this case, the players consider the least values of sold prices of products in market over the smallest feasible region of the problem. Therefore the players must solve the following linear programming problem. We call it pessimistic linear programming problem.

$$\begin{array}{lll} \textit{maximize}\ w_1 & \left(c_{11}^L + \ldots + c_{l1}^L\right) u_1 + \ldots + w_l \left(c_{1p}^L + \ldots + c_{lp}^L\right) u_p & \\ s.t. & a_{11}^R u_1 + \ldots + a_{1p}^R u_p \leq b_1^L (S) & \\ & \vdots & \\ & a_{m1}^R u_1 + \ldots + a_{mp}^R u_p \leq b_m^L (S) & \\ & u_j \geq 0 \qquad\qquad j = 1, \ldots, p & \end{array} \tag{5}$$

By solving the problem (5), we obtain the least possible values of benefits of the objectives for the coalition.
Thus, the fuzzy cooperative game $(N, \tilde{V}_w)$ is transformed to the two cooperative games $\left(N, V_w^L\right)$ and $\left(N, V_w^R\right)$. The cooperative game $\left(N, V_w^R\right)$ arises from the optimistic linear programming problem (4), and the cooperative game $\left(N, V_w^L\right)$ arises from the pessimistic linear programming problem (5). Therefore, we define the optimistic and pessimistic cores of problem (1) as follows.

Definition 7. *The optimistic and pessimistic cores of the problem (1) are the obtained cores of solving the two cooperative games $\left(N, V_w^R\right)$ and $\left(N, V_w^L\right)$, respectively.*

Owen [28] showed that linear production planning problems have nonempty cores. According to the fact that the cooperative games $\left(N, V_w^R\right)$ and $\left(N, V_w^L\right)$ are crisp, the following theorem is established. The proof of the following theorem is similar to the presented proof by Owen [28] with some little changes.

Theorem 8. *The weighted linear production planning games $\left(N, V_w^L\right)$ and $\left(N, V_w^R\right)$ have nonempty cores.*

Proof. We present the proof for the game $\left(N, V_w^R\right)$. Let $u^R(S) = \left(u_1^R(S), \ldots, u_p^R(S)\right)$ be an optimal solution to the problem (4). Then

$$V_w^R(S) = w_1 \left(c_{11}^R + \ldots + c_{l1}^R\right) u_1^R(S) + \ldots + w_1 \left(c_{1p}^R + \ldots + c_{lp}^R\right) u_p^R(S).$$

We have

$$\sum_{S \in B} y_S V_W^R(S) = \sum_{S \in B} \left(y_S \sum_{j=1}^{p} \sum_{k=1}^{l} w_j c_{kj}^R u_j^R(S) \right)$$
$$= \sum_{j=1}^{p} \sum_{k=1}^{l} w_j c_{kj}^R \left(\sum_{S \in B} y_S u_j^R(S)\right) = \sum_{j=1}^{p} \sum_{k=1}^{l} w_j c_{kj}^R \hat{u}_j^R$$

where $\hat{u}_j^R = \sum\limits_{S \in B} y_S u_j^R(S)$. Now, we have

$$\begin{aligned}
\sum_{j=1}^{p} a_{rj}^L \hat{u}_j^R &= \sum_{j=1}^{p} a_{rj}^L \sum_{S \in B} y_S u_j^R(S) \\
&= \sum_{S \in B} y_S \sum_{j=1}^{p} a_{rj}^L u_j^R(S) \\
&\leq \sum_{S \in B} y_S b_r^R(S) = \sum_{S \in B} \sum_{i \in S} y_S b_r^{iR} = \sum_{i \in N} \Big(\sum_{\substack{S \in B \\ i \in S}} y_S \Big) b_r^{iR} = b_r^R(N)
\end{aligned}$$

Since $u_j^R(S) \geq 0$, $j = 1, \ldots, p$ and $y_S > 0$, $\forall S \in B$, we have $\hat{u}_j^R \geq 0$. Thus the vector $\hat{u}_j^R$ satisfies the constraints (4) for $S = N$, and we must have

$$V_w^R(N) \geq w_1 \left(c_{11}^R + \ldots + c_{l1}^R \right) \hat{u}_1^R + \ldots + w_1 \left(c_{1p}^R + \ldots + c_{lp}^R \right) \hat{u}_p^R$$

so V_w^R is a balanced game. Therefore the core of the game $\left(N, V_w^R \right)$ is nonempty. □

To find payoff vectors in the optimistic and pessimistic core, we utilize the duality theory for the linear programming problems (4) and (5). The duals of the problems (4) and (5) are given by the problems (6) and (7), respectively.

$$\begin{array}{ll}
\textit{minimize} & b_1^R(S)\, y_1 + \ldots + b_m^R(S)\, y_m \\
\textit{s.t.} & a_{11}^L y_1 + \ldots + a_{m1}^L y_m \geq w_1 \left(c_{11}^R + \ldots + c_{l1}^R \right) \\
& \vdots \\
& a_{1p}^L y_1 + \ldots + a_{mp}^L y_m \geq w_l \left(c_{1p}^R + \ldots + c_{lp}^R \right) \\
& y_r \geq 0 \qquad r = 1, \ldots, m,
\end{array} \tag{6}$$

$$\begin{array}{ll}
\textit{minimize} & b_1^L(S)\, y_1 + \ldots + b_m^L(S)\, y_m \\
\textit{s.t.} & a_{11}^R y_1 + \ldots + a_{m1}^R y_m \geq w_1 \left(c_{11}^L + \ldots + c_{l1}^L \right) \\
& \vdots \\
& a_{1p}^R y_1 + \ldots + a_{mp}^R y_m \geq w_l \left(c_{1p}^L + \ldots + c_{lp}^L \right) \\
& y_r \geq 0, \qquad r = 1, \ldots, m.
\end{array} \tag{7}$$

We indicate the optimal solutions of the problem (6) and (7) by $\left(y_1^R, \ldots, y_m^R \right)$ and $\left(y_1^L, \ldots, y_m^L \right)$.
As we know, dual variables are marginal worth of the resources. Thus the worst and the best possible values of the payoff for i-th player are obtained as follows.

$$x_i^L = \sum_{r=1}^{m} y_r^L b_r^{iL} \tag{8}$$

$$x_i^R = \sum_{r=1}^{m} y_r^R b_r^{iR} \tag{9}$$

Now, we show that the obtained x_i^L and x_i^R by (8) and (9) are in the core. Assume $\left(y_1^R, \ldots, y_m^R\right)$ be an optimal solution of the problem (6) for $S = N$. Then

$$V_w^R(N) = b_1^R(N)\, y_1^R + \ldots + b_m^R(N)\, y_m^R. \tag{10}$$

Also, we have

$$V_w^R(S) \leq b_1^R(S)\, y_1^R + \ldots + b_m^R(S)\, y_m^R \tag{11}$$

Since $V_w^R(S)$ is the minimum over all feasible vectors y. Consider $x^R = \left(x_1^R, \ldots, x_n^R\right)$, where

$$x_i^R = \sum_{r=1}^{m} y_r^R b_r^{iR} \quad i = 1, \ldots, n.$$

For any S, we have

$$\sum_{i \in S} x_i^R = \sum_{i \in S} \sum_{r=1}^{m} b_r^{iR} y_r^R = \sum_{r=1}^{m} \sum_{i \in S} b_r^{iR} y_r^R = b_1^R(S)\, y_1^R + \ldots + b_m^R(S)\, y_m^R$$

so

$$\sum_{i \in N} x_i^R = V_w^R(S).$$

From (11), it follows that

$$\sum_{i \in S} x_i^R \geq V_w^R(S).$$

Therefore, x^R is an imputation in the core. Similarly $x^L = \left(x_1^L, \ldots, x_n^L\right)$ is an imputation in the core.

4. NUMERICAL EXAMPLE

Consider a linear production planning problem in which five players make three kinds of products P_1, P_2, and P_3. Each of the players initially possesses a certain number of resources R_1, R_2 and R_3. We formulate a multi-objective linear programming problem with three objective functions. The initial resources b^i, $i = 1, 2, 3$ of the players and the resources of the grand coalition are shown in Table 1. The coefficients of the objective functions and technological coefficients are represented by the following matrices.

$$\tilde{C} = \begin{bmatrix} (1.5, 2.5, 3) & (4.75, 5, 5.5) & (3.5, 4, 4.75) \\ (2, 3, 4.5) & (2, 3, 4.5) & (4.5, 5, 5.25) \\ (0.5, 1, 1.5) & (4.25, 5, 5.75) & (-.25, 0, .25) \end{bmatrix},$$

$$\tilde{A} = \begin{bmatrix} (1.5, 2, 3) & (8, 9, 10) & (2, 3.5, 3.75) \\ (5.75, 6, 6.75) & (3.5, 4, 4.75) & (8.5, 9, 9.75) \\ (7, 8, 8.75) & (8, 9, 10.5) & (6.75, 7, 8) \end{bmatrix}.$$

Resources	R_1	R_2	R_3
PlayerI	(118,119,120)	(56,57,58)	(109,110,110.5)
PlayerII	(104.5,105,105.5)	(74.5,75,75.5)	(94.5,95,95.75)
PlayerIII	(85.75,86,86.5)	(79.75,80,80.75)	(139,140,141)
PlayerIV	(69.5,70,70.5)	(87.5,88,88.25)	(104.5,105,106)
PlayerV	(49,50,51)	(109.25,110,110.5)	(119,120,120.75)
Grand coalition	(426.75,430,433.5)	(407,410,413)	(566,570,574)

Table 1: Initial resources

To solve this problem, we first use the nearest interval approximation of fuzzy numbers. The problems (6) and (7) are as follows (assume that the decision makers have agreed to $w_1 = w_2 = w_3 = \frac{1}{3}$).

$$\begin{array}{ll} minimize & 431.75y_1 + 411.5y_2 + 572y_3 \\ s.t. & 1.75y_1 + 5.875y_2 + 7.5y_3 \geq 2.54 \\ & 8.5y_1 + 3.75y_2 + 8.5y_3 \geq 4.79 \\ & 2.75y_1 + 8.75y_2 + 6.875y_3 \geq 3.17 \\ & y_1, y_2, y_3 \geq 0, \end{array} \tag{12}$$

and

$$\begin{array}{ll} minimize & 428.375y_1 + 408.5y_2 + 568y_3 \\ s.t. & 2.5y_1 + 6.375y_2 + 8.375y_3 \geq 1.75 \\ & 9.5y_1 + 4.375y_2 + 9.75y_3 \geq 4.79 \\ & 3.625y_1 + 9.625y_2 + 7.5y_3 \geq 2.83 \\ & y_1, y_2, y_3 \geq 0, \end{array} \tag{13}$$

The optimal solutions of the problems (12) and (13) are respectively

$$\begin{array}{l} \left(y_1^L, y_2^L, y_3^L\right) = (0.367, 0.156, 0)\,, \\ \left(y_1^R, y_2^R, y_3^R\right) = (0.354, 0.135, 0.150)\,. \end{array}$$

To solve the above problems Lingo software is used. Then, according to (8) and (9), the payoffs vectors for players I,II,III,IV,V are obtained as

$$\begin{array}{l} \left(x_1^L, x_1^R\right) = (52.304, 66.603) \\ \left(x_2^L, x_2^R\right) = (50.104, 61.724) \\ \left(x_3^L, x_3^R\right) = (43.977, 62.4581) \\ \left(x_4^L, x_4^R\right) = (39.443, 52.607) \\ \left(x_5^L, x_5^R\right) = (35.268, 50.817) \end{array}$$

where x_i^L, x_i^R are the worst and best values for player i.

5. CONCLUSION

The problem of cooperative game arising from the multi-objective production planning problem with fuzzy parameters has not been considered in previous re-

searches, based on the best knowledge of the authors. We converted the multi-objective game to the single-objective game by using group AHP method. Then the obtained weighted single-objective games were transformed to the two, optimistic and pessimistic, games arising from optimistic and pessimistic linear programming problems. The payoffs vectors of the players were obtained using the duality theorem of linear programming. In comparison with the existing approach (for single-objective problem), the proposed method has the following advantages:
(1) The nearest interval approximation is introduced to replace a fuzzy number, which captures essential features of original fuzzy quantities to a great extent.
(2) The proposed approach requires fewer intermediate models and less computational effort.
Finally, validity and applicability of the method are illustrated by a practical example.

REFERENCES

[1] Ansari, Q. H., Yao, J.-, *Recent Developments in Vector Optimization*, Springer, New York, 2012.

[2] Aubin, J.-P., *Mathematical Methods of Game and Economic Theory*, North Holland, 1979.

[3] Aubin, J.-P., "Cooperative fuzzy game: the static and dynamic points of view", *TIMS/Studies in the Management Science*, 20 (1984) 407-428.

[4] Aumann, RJ., Peleg, B., "Von Neumann-Morgenstern solutions to cooperative games without side payments", *Bulletin of the American Mathematical Society*, 66 (3) (1960) 173-179.

[5] Bellman, RE., Zadeh, L.A., "Decision making in a fuzzy environment", *Management Science*, 17 (1970) 141-164.

[6] Bergstresser, K., Yu, P.L., "Domination structure and multicriteria problems in n-person games", *Theory and Decision*, 8 (1977) 5-48.

[7] Bigdeli, H., Hassanpour, H., "A satisfactory strategy of multiobjective two person matrix games with fuzzy payoffs", *Iranian Journal of Fuzzy Systems*, 13 (2016) 17-33.

[8] Bigdeli, H., Hassanpour, H., "Modeling and solving of multi-objective security game using multiobjective bilevel programming and its application in security of Metro system", *Journal of Electronical & Cyber Defence*, (In Persian) (2017) 31-38.

[9] Bigdeli, H., Hassanpour, H., Tayyebi, J., "Constrained bimatrix games with fuzzy goals and its application in nuclear negotiations", *Iranian Journal of Numerical Analysis and Optimization*, Accepted paper.

[10] Bigdeli, H., Hassanpour, H., Tayyebi, J., " The optimistic and pessimistic solutions of single and multiobjective matrix games with fuzzy payoffs and analysis of some of military problems, *Defence Sci. & Tech*, (In Persian), 2 (2016) 133-145.

[11] Butnariu, D., "Fuzzy games; a description of the concept", *Fuzzy Sets and Systems*, 1 (3) (1978) 181-192.

[12] Butnariu, D., "Stability and Shapley value for an n-persons fuzzy game", *Fuzzy Sets and Systems*, 4 (1) (1980) 63-72.

[13] Butnariu, D., "Values and cores of fuzzy games with infinitely many players", *International Journal of Game Theory*, 16 (1) (1987) 43-68.

[14] Charens, A., Granat, D., "Coalitional and chance-constrained solution to n person games I: The satisficing probabilistic nucleolus", *SIAM Journal on Applied Mathematics*, 31 (1976) 358-367.

[15] Charens, A., Granat, D., "Coalitional and chance-constrained solution to nperson games II: Two stage solutions", *Operations Research*, 27 (1977) 1013- 1019.

[16] Grzegorewski, P., "Nearest interval approximation of a fuzzy number", *Fuzzy Sets and Systems*, 130 (3) (2002) 321-330.

[17] Hinojosa, M. A., Mrmol, A. M., Monroy, L., Fernndez, F. R., "A multi-objective approach to fuzzy linear production games", *International Journal of Information Technology & Decision Making*, 12 (5) (2013) 927-943.

[18] Hwang, C.-L., Lin, M.-J., *Group Decision Making under Multiple Criteria: Methods and Applications*, Lecture Notes in Economics and Mathematical Systems, Springer-Verlag, Berlin, 1987.

[19] Jornsten, K., Lind, M., Tind, J., "Stable payment schemes of TU-games with multiple criteria", *Optimization*, 40 (1) (1997) 57-78.

[20] Lind, M., "Cooperative game theory and multiple criteria decision making", Ph.D. Thesis, Department of Operations Research, University of Aarhus, 1996.

[21] Littelchild, S.C., Vaidya, K.G., "Propensity to disrupt and the disruption nucleolus of a characteristic function game", *International Journal of Game Theory*, 5 (2) (1976) 151-161.

[22] Maschler, M., Peleg, B., Shapley, I.S., "Geometric properties of the kernel, nucleolus and related solution concepts", *Mathematics of Operations Research*, 4 (4) (1979) 303-338.

[23] Meng, F. Y., Zhang, Q, "The Shapley function for fuzzy cooperative games with multilinear extension form", *Applied Mathematics Letters*, 23 (5) (2010) 644-650.

[24] Michener, H.A., Yuen, K., Sakuari, M. M., "On the comparative accuracy of lexicographical solutions in cooperative games", *International Journal of Game Theory*, 10 (2) (1981) 75-89.

[25] Mishra, S., "Cooperative Fuzzy Game Theoretic Approach to Multi Objective Linear Fractional Programming Problems", *American International Journal of Research in Science, Technology, Engineering & Mathematics*, 14 (2) (2016) 98-107.

[26] Nishizaki, I., Sakawa, M., *Fuzzy and multiobjective games for conflict resolution*, Springer-Verlag Berlin Heidelberg, 2001.

[27] Nishizaki, I., Sakawa, M., "Fuzzy cooperative games arising from linear production programming problems with fuzzy parameters", *Fuzzy Sets and Systems*, 114 (1) (2000) 11-21.

[28] Owen, G., "On the core of linear production games", *Mathematical Programming*, 9 (1) (1975) 358-370.

[29] Sakawa, M., *Fuzzy sets and Interactive multiobjective optimization*, New York and London, 1993.

[30] Schmeidler, D., "The nucleolus of a characteristic function", *SIAM Journal on Applied Mathematics*, 17 (6) (1969) 1163-1170.

[31] Seo, F., Nishizaki, I., "The nucleolus of a characteristic function", *SIAM Journal on Applied Mathematics*, 17 (1969) 1163-1170.

[32] Seo, F., Nishizaki, I., "Conflict resolution with robustness in international negotiations: a game theoretic approach", *Group Decision and Negotiation Journal*, 3 (1) (1994) 47-68.

[33] Spinetto, R., "The geometry of solution concepts for n-person cooperative games", *Management Science*, 20 (9) (1974) 1292-1299.

[34] Tanino, T., Muranaka, Y., Tanaka, M., "On multiple criteria characteristic mapping games", *Proceedings of MCDM* , (1992) 63-72.

[35] Tsurumi, M., Tanino, T., Inuiguchi, M., "A Shapley function on a class of cooperative fuzzy games", *European Journal of Operational Research*, 129 (3) (2001) 596-618.

[36] Yu, X., Zhang, Q., "An extension of cooperative fuzzy games", *Fuzzy Sets and Systems*, 161 (11) (2010) 1614-1634.