Yugoslav Journal of Operations Research
34 (2024), Number 4, 603–626
DOI: https://doi.org/10.2298/YJOR240217014P

A PERTINENT APPROACH: TWO CLASS OF UNCERTAIN PRIORITY QUEUING MODELS UNDER THE STEADY STATE CONDITION

Dragan PAMUCAR
Department of Operations Research and Statistics, Faculty of Organizational Sciences, University of Belgrade, Belgrade, Serbia
dragan.pamucar@fon.bg.ac.rs

Ramya LAKSHMANARAJ
Department of Mathematics, Bharathiar University, Coimbatore, India
ramyalakshmanaraj14@gmail.com

Narayanamoorthy SAMAYAN
Department of Mathematics, Bharathiar University, Coimbatore, India
snmphd@buc.edu.in

Received: February 2024 / Accepted: April 2024

Abstract: The main purpose of this research article is to provide an appropriate interpretation of the priority queuing model. It creates uncertain sequence models with priority discipline, non-priority systems, models with priority systems, and models with non-priority systems. Here, we propose two models. First, we discussed one array with a server and the second, two arrays with a server. These two model customers follow the rule of first come first served in their queue. The presence and service process follows the Poisson process and the high-speed process with ambiguous parameters. Our results are responsible for the best choice of exchange of preferred discipline. Blurred sequence models are more appropriate than uncertain scenes, which are commonly used as part of reality. Numerical results are provided to deal with the current service discipline. Despite the challenge of determining the optimal single-server uncertain queuing model with two class-priority fields, the steady-state performance of the service process is evaluated.

Keywords: Queuing theory, Priority-Discipline, Uncertain parameter, Single server.

MSC: 60K25, 60K30, 68M20, 90B22.

1. INTRODUCTION

Many businesses these days don't deliver things right away. Customers should thus wait to receive the goods they require, as this will result in a queueing-inventory model. Long wait times in line for merchandise can lead to unhappiness and the churn of devoted clients, which can cost businesses a lot of money [1]. For this reason, Abdali et al. [2] studied a priority queueing-inventory approach for inventory management in multi-channel service retailing using machine learning algorithms. Queuing systems square shapes are often used and do not show adequate patterns in many aspects in real life. To provide a completely different quality of service to different types of customers, we continue to manage a queue system with a priority system [3]. It is common to pay attention to this growth. For example, in a telecommunications switching protocol, priority types may appear in the title of the management science suite or in an ATM cell to guarantee a completely different service layer for different customers. Table 1 represent the nomenclature.

Table 1: Nomenclature

FCFS	First Come First Served
FIFO	First In First Out
PD	Poission Distribution
ED	Exponential Distribution
DSW ALGORITHM	Dong Shah Wong Algorithm

Applications for priority management are also common in bandwidth management, manufacturing and traffic management, among other areas [4]. Priority and non-priority items are included in priority management [5]. Considering a queue system with two types of customers, when the first - class customer comes to the server, the server feels that it is serving the second type of customer, puts pressure on customer service and directly gets service [6]. In addition, the same type of customer happiness serves the control of FCFS, this mechanism is designated as a priority priority sequence.

If the customer in the first category sees the server serving the second category customer, then they have to wait for the amount until the customer completes the service and then starts receiving the service, Customer happiness corresponds to the discipline of FCFS same, labeled as a non - priority order for the mechanism [7].

The sequence is usually divided into two groups, one that describes the actual situation of our daily lives with relevant examples based on the presence and service of the customer and server of the other group based on its decision model (design and control model). what the parameters should be in order to highlight the action model. The basic queue system is based on two things, one is the attendance rate and the other is the service rate [8]. This arrival time depends on the length of the queue and the service time depends on the waiting time.

The author conducted several experiments on obscure models [9], [10]. The concept of acquiring some features in consolidated packages with uncertain parameters was introduced. This problem requires the location of new features to be found in a certain compact package, that is, the sum of the weighted stances between new and existing features is reduced [11] and [12]. Note Exploring a multi-server hierarchy by visiting limited or unlimited populations and source populations [13]. And potential delivery after departure. In addition, he recently applied previous results, along with two other scholars, on the issue of machine service and queue decision [14], [15].

The classic method of extended policy and α-cuts is very restrictive as equations often do not require very strong solution or conditions in order to have a solution [16]. Combined distribution of queue length at customer departure age, and the steady state queue-length and stay time distributions [17].

Process for generating performance activity membership functions in complete service ordering systems with obscure attendance and service rate numbers [18]. The level of performance of the system that interferes with a machine with the machine breakdown rate is the obscure number of service rate based on the mathematical programming approach and the expansion policy to build the function performance of the members. The model consists of a single server sequence with two servers. This model is a simple stimulus for performance testing, but it is also imaginatively strange in light of the fact that the utility level sequence has not yet been explored to further understand its disappearance [19]. This functional equation is called a fundamental analysis [20].

The author did so in order of priority [21]. Since this model only depends on service time, the decision maker may not always make the right decision. Time-based high-speed sequence theory was developed in unpredictable models [22], [23]. This article considers the first service model M / G / 6 under the continuous review policy (R, Q) [24]. The proposed model can reduce the expected standby time of the system from 13 hours 30 minutes to 6 hours 2 minutes [25].

Discuss and update a defined parallel multi-server cached sequence system and analyze its two station scenarios [26]. When they studied unique timelines for high and low priority bilateral distribution, they used a cost-effective method to measure system performance. For unpredictable conditions, sequence parameters due to natural disasters are uncertain. In this case the classical sequence was reduced to an obscure expansion theory [27]. Here, the author obtains a single-server uncertain array model with a Bernoulli modified pause using the Z extension policy and the α- cut approach [28].

Researchers such as [29], [30], [31], have described the uncertain sequence model, and we analyze uncertain sequences using [12]. Reasonable sequence models are described perhaps using obscure version theory [32], [33]. This provides a more realistic interpretation of the priority sequence model using uncertain version theory [34]. By prioritizing sequence models by using uncertain synthesis theory, prioritization sequence models are given a more realistic interpretation [35]. A set of nonmetric linear scheme is developed to describe the membership functions of system properties, a family of smooth lines with an unreliable server [36].

One primary goal is to optimize the performance of service systems, such as customer service centers or technical support hotlines, to ensure efficient resource utilization and high customer satisfaction levels. By understanding various factors, such as arrival rates, service times, and priority areas, that impact system performance, the research aims to develop strategies to reduce customer waiting time, reduce queue length, and improve overall service quality. The research goals and motivations behind studying a single-server uncertain queuing model with a two-class priority discipline revolve around improving the efficiency, effectiveness, and resilience of service systems to better meet customer needs and organizational objectives.

This study develops novel mathematical models and analysis techniques to analyze queuing systems with two-class priority categories under uncertain conditions. These models contribute to the theoretical understanding of queuing theory and advance the state-of-the-art in modeling complex service systems. These companies are improving their service operations, providing a better customer experience, and verifying the development of robust and flexible service systems.

2. STEADY STATE CONDITION

The steady state only comes after the steady state. After a sufficient period of time the steady state is usually reached and the level of the system is essentially independent of the initial state (the state distribution remains the same) [37]. Simple queuing system reaches steady state only if, $\rho < 1$ (ρ means Traffic density) [38]. Figure 1 represent the steady state condition.

$$\rho = \frac{\lambda}{\mu} < 1 \tag{1}$$

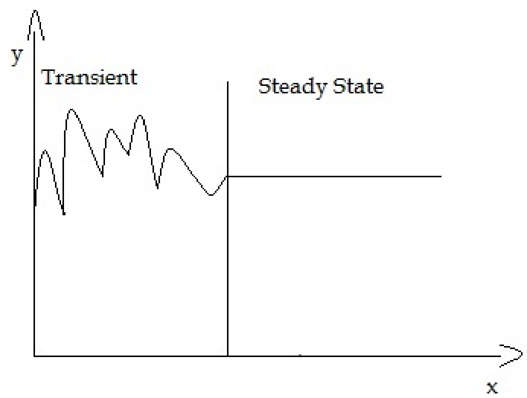

Figure 1: Steady state condition

3. PROPOSED MODELS

3.1. Single Server Queuing System Serving Single Queue Customer - Proposed Model 1

Here, we recommend a single server queue system that caters to individual queue clients. They are assigned when the customer attends. The service rule comes first in all lines and comes first at service sites. Both high-priority and low-priority customers are on the same line.

In queuing, if the highest priority customer interrupts the system, the server will receive the lowest priority customer service. The service will then be suspended and the customer with the highest priority will receive immediate service from the server. After completing high priority customer service, the low priority customer continues to receive service. Figure 2 represent the single server with queue model.

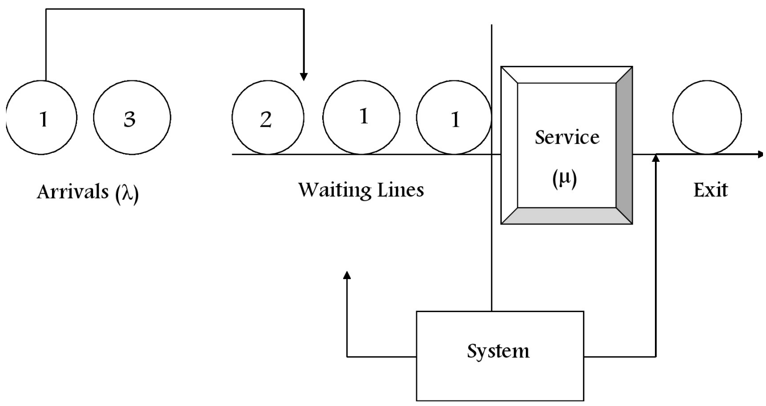

Figure 2: Single Server with single queue model

The arrival process for the customer is state independent, we base our analysis on the following

(i) Arrival for customers are independently, Poisson distribution(PD) process, importation parameters are λ respectively.

(ii) Service time is some exponentially distribution(ED), average service time is $1/\mu$.

(iii) The higher priority customer will interrupt the queue,the lower priority customer service is pause.

3.2. Single Server Queuing System Serving Two Queue Customer - Proposed Model 2

Here, we consider a single server queue system that serves two different clients; Row 1, Row 2. Each row has its own separate set.

One priority is given per line. Suppose each line has a FIFO service rule and the priority mode has already been restarted. (ie) If high priority customers are part of the system during low priority customer service, low priority customer service will be interrupted and restarted when there are no more priority customers on the system.

We will consider both priority and non-priority scenarios, in which case the client server will stop the high priority queue if the client queue becomes a low priority queue, so the priority queue becomes longer than the high priority queue.

In the latter case, the server completes any services started. When completed, one of the customers will be in the longest queue, or if the top two are in the highest priority queue, next.

If the server has completed a service and both lines are uneven, the next line has a long queue of customers to serve, (ie) a different number of customers waiting in the queue. Figure 3 represent the single server with two queue model

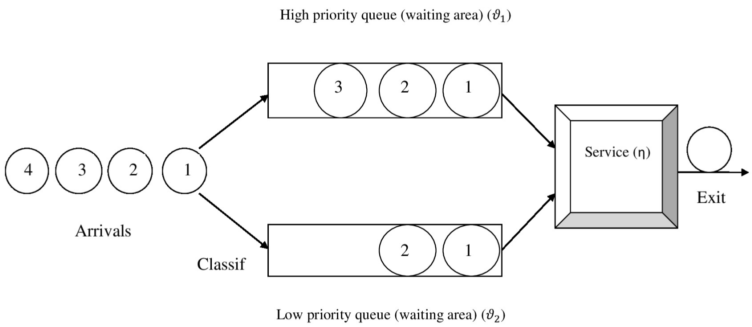

Figure 3: Single Server with two queue model

The arrival process for both classes is state independent, we base our analysis on the following

(i) Arrival for both classes are independently, Poisson distribution(PD) process, importation parameters are ϑ_1, ϑ_2 respectively.

(ii) Service time is some exponentially distribution(ED), average service time is $1/\eta$.

(iii) high priority customer have preemptive priority over low priority customer.

(iv) high priority customer have non-preemptive priority over low priority customer.

4. PRIORITY QUEUING MODEL

The queue priority rule determines which customer is in the next service. The most common rule of thumb is used first. This priority rule selects customers based on long queues. In general, customers who come first consider the first offer to be the best way to prioritize. However, this is not a rule of thumb used alone. That is to say, the best customers come first, the most profitable customers come first, the fastest services first, the biggest service needs first, emergencies first and so on. While each is a priority, it is important to use a priority rule that supports the overall organizational strategy.
(a) For situations where different customers have different preferences. For example, ER activities, VIP customers in nightclubs.
(b) There are two basic principles of priority to consider when assessing a situation with priority classes (where class 1 has a higher priority).
1. Non-Preemptive Priorities A high priority customer should never leave the queue again to leave the service.
2. Preemptive Priorities Low priority customers will be pushed back into the queue to leave space for high priority customers.
The priority-discipline queuing model is presented as follows.:
For Model 1

$$W_i = W_{q,i} + \bar{x}_i \tag{2}$$

$$L_{q,i} = \lambda_i W_{q,i} \tag{3}$$

$$L_i = \lambda_i W_i \tag{4}$$

$$L = \lambda W = \sum_{i=1}^{p} L_i \tag{5}$$

If it is denoted by $\sigma_i = \sum_{i=1}^{P} \rho_j$ with $\sigma_p + 1 = 0$,then W_q, i is:
(1) Preemptive priorities model:

$$W_{q,i} = \frac{\sum_{j=1}^{P} \frac{\lambda_j \bar{x_j}^2}{2}}{(1-\sigma_i)(1-\sigma_i+1)} i = 1, 2, ...P; \tag{6}$$

(2) Nonpreemptive priorities model:

$$W_{q,i} = \frac{\bar{x_i}(1-\sigma_i) + \sum_{j=1}^{P} \frac{\lambda_j \tilde{x_j}^2}{2}}{(1-\sigma_i)(1-\sigma_i+1)} - \tilde{x}_i. \tag{7}$$

For Model 2

$$W_i = W_{q,i} + \bar{x}_i \tag{8}$$

$$L_{q,i} = \vartheta_i W_{q,i} \tag{9}$$

$$L_i = \vartheta_i W_i \tag{10}$$

$$L = \vartheta W = \sum_{i=1}^{p} L_i \tag{11}$$

σ_i denoted as $\sigma_i = \sum_{i=1}^{P} \rho_j$ with $\sigma_p + 1 = 0$,then W_q, i is:
(1) Preemptive priorities model:

$$W_{q,i} = \frac{\sum_{j=1}^{P} \frac{\vartheta_j \tilde{x_j}^2}{2}}{(1-\sigma_i)(1-\sigma_i+1)} i = 1, 2, ...P; \tag{12}$$

(2) Non preemptive priorities model:

$$W_{q,i} = \frac{\bar{x_i}(1-\sigma_i) + \sum_{j=1}^{P} \frac{\vartheta_j \tilde{x_j}^2}{2}}{(1-\sigma_i)(1-\sigma_i+1)} - \tilde{x_i}. \tag{13}$$

5. UNCERTAIN PRIORITY QUEUE DISCIPLINE

Uncertain priority queues are represented by uncertain set theory. This paper constructs a uncertain priority sequence model, in which input sources make the arrival process and service process parameters uncertain. Due to the determination of ambiguity for intermittent valued mapping, the membership functions for yield variables were determined.

The DSW algorithm is one of the approximate methods to use intervals at different α-cut levels to define membership functions. Continuous interval analysis included total α-cut intervals. The DSW algorithm provides highly significant control over the expansion policy, such as a set of uncertain valued variables, uncertain numbers classified on the real line.

This uncertain variable combats the variation in rule by members of the publication process using the teaching of education in the field, and subsequently the functional set of values as changes in functional expression through different events through regular interval analysis methods.

5.1. Interval Analysis Arithmetic

Interval arithmetic involves expressing a real number in two numbers, which refers to numerical intervals. The expression in the form of float arithmetic can be used to denote the lower and upper limits[36].

The lower bounds and upper bounds are A_1 and A_2. These two spatial numbers defined by systematic pairs of real numbers. A_1=[p, q] , $p \leq q$
A_2=[r ,s] , $r \leq s$.

Common arithmetic property code *. * Became [+, -,. ,] Is referred to as. Anr interval is assumed to be the function $A_1 * A_2 = [p, q] * [r, s]$. The size and symptoms of the elements depend on the calculation of the spacing p, q, r, s.
$[p, q]$ + $[r, s] = [p + r, q + s]$
$[p, q]$ - $[r, s] = [p - s, q - r]$

$[p,q]\ .\ [r,s] = [min\,(pr,ps,qr,qs)\,, max\,(pr,ps,qr,qs)]$
$[p,q] \div [r,s] = [p,q]\ .\ \left[\frac{1}{s}\frac{1}{r}\right]$, provided that $0 \notin [r,s]$
$\alpha\,[p,q] = [\alpha p, \alpha q]$ for $\alpha > 0$
$\alpha\,[p,q] = [\alpha q, \alpha p]$ for $\alpha < 0$.
where pr, ps, qr, qs are arithmetic products and quotients.

5.2. Algorithm

One of the approximate ways to use gaps at different alpha-cutting levels in members' activities is to use DSW (Dong, Shaw, Wong) is the process. These are the total α-cut intervals is the standard interval analysis [36].

The DSW algorithm greatly simplifies the ability to handle the principle of describing valuable ambiguous variables in a row. Further defines ambiguous numbers in the actual sequence. As uncertain variables become acceptable in the field, this publication avoids an abnormality in the members' process, which prevents at least the latter's practical expression from being extended to standard methods of interval analysis. This standard interval uses the full α-cut interval in the analysis. Repeat the following steps for different values to complete the α-cut performance of the solution. The procedure of DSW algorithm are given in Figure 4.

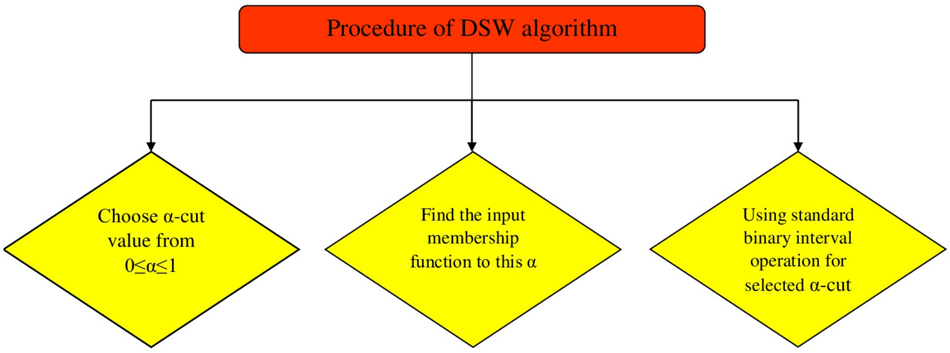

Figure 4: Procedure for DSW Algorithm

6. PROPOSED MODEL 1 - SOLUTION PROCEDURE

The order of intuitionistic uncertain preferences describes the fixed Intuitionistic uncertain principle. Clarity of the following costing process is the decision on the best choice of priority discipline for a queuing system:

$$C = \sum_{i=1}^{P} C_i L_i = \sum_{i=1}^{P} C_i \lambda_i W_i \tag{14}$$

here C_i denotes the unit cost of inactivity, L_i denotes the average length of the system , λ_i denotes the average rate of customer arrivals at the system, W_i denotes the average time in the system and C is the total average cost of system.

Let us consider a queuing model with two unit classes, α_1 is the arrival of one class and α_2 is the arrival of another class (α_2) and (α_2) is represent the membership function). similarly β_1 and β_2 are the arrival of non-membership function. The poison distribution is marked with $\bar{\lambda}$ according to the average arrival rate. $\bar{\mu}$ represents a exponential distribution that follows the service rate.

In general, the priority sequence model consists of three scenarios: (C_a) denotes a no priority discipline , (${C_a}^1$) denotes the preemptive priority discipline, and (${C_a}^2$) denotes the non-preemptive priority discipline. Proximity methods for detail expanded the ambiguity for standard value mapping, which determined membership and non-membership functions for output variables.

The average presence rate of the Poisson process system, which is reported and supplied by the uncertain trapezoidal $\tilde{\lambda}$ number, follows the service rate on the high-speed process system and is known to have $\mu_{\tilde{\mu}}$ uncertain trapezoidal numbers with members, respectively.

$$\mu_{\tilde{\lambda}} = \begin{cases} \frac{\lambda-a}{b-a} & \text{if } a \leq \lambda \leq b; \\ 1 & \text{if } b \leq \lambda \leq c; \\ \frac{d-\lambda}{d-c} & \text{if } c \leq \lambda \leq d; \end{cases} \tag{15}$$

$$\mu_{\tilde{\mu}} = \begin{cases} \frac{\mu-a}{b-a} & \text{if } a \leq \mu \leq b; \\ 1 & \text{if } b \leq \mu \leq c; \\ \frac{d-\mu}{d-c} & \text{if } c \leq \mu \leq d; \end{cases} \tag{16}$$

The non-membership function for $\tilde{\lambda}$ and $\tilde{\mu}$ are denoted by $\nu_{\tilde{\lambda}}$ and $\nu_{\tilde{\mu}}$ respectively.

$$\nu_{\tilde{\lambda}} = \begin{cases} \frac{\lambda-a'}{b'-a'} & \text{if } a' \leq \lambda \leq b'; \\ 1 & \text{if } b' \leq \lambda \leq c'; \\ \frac{d'-\lambda}{d'-c'} & \text{if } c' \leq \lambda \leq d'; \end{cases} \tag{17}$$

$$\nu_{\tilde{\mu}} = \begin{cases} \frac{\mu-a'}{b-a} & \text{if } a' \leq \mu \leq b'; \\ 1 & \text{if } b' \leq \mu \leq c'; \\ \frac{d'-\mu}{d'-c'} & \text{if } c' \leq \mu \leq d'; \end{cases} \tag{18}$$

The uncertain trapezoidal numbers $\tilde{c_A}, \tilde{c_B}$ are established with the possible distribution member function of the unit cost of inactivity per unit in the same class.

$$\mu_{\tilde{c_A}} = \begin{cases} \frac{c_A-a}{b-a} & \text{if } a \leq c_A \leq b; \\ 1 & \text{if } b \leq c_A \leq c; \\ \frac{d-c_A}{d-c_A} & \text{if } c \leq c_A \leq d; \end{cases} \tag{19}$$

$$\mu_{\tilde{c_B}} = \begin{cases} \frac{c_B-a}{b-a} & \text{if } a \leq c_B \leq b; \\ 1 & \text{if } b \leq c_B \leq c; \\ \frac{d-c_B}{d-c_B} & \text{if } c \leq c_B \leq d; \end{cases} \tag{20}$$

And their non-membership functions respectively.

$$\nu_{\tilde{c_A}} = \begin{cases} \frac{c_A - a}{b' - a'} & \text{if } a' \leq c_A \leq b'; \\ 1 & \text{if } b' \leq c_A \leq c'; \\ \frac{d' - c_A}{d' - c'_A} & \text{if } c' \leq c_A \leq d'; \end{cases} \tag{21}$$

$$\nu_{\tilde{c_B}} = \begin{cases} \frac{c_B - a'}{b' - a'} & \text{if } a' \leq c_B \leq b'; \\ 1 & \text{if } b' \leq c_B \leq c'; \\ \frac{d' - c_B}{d' - c_B} & \text{if } c' \leq c_B \leq d'; \end{cases} \tag{22}$$

7. PROPOSED MODEL 2- SOLUTION PROCEDURE

Clarity of the latter costing process is the decision on the preferred priority discipline for a queuing system:

$$C = \sum_{i=1}^{P} C_i L_i = \sum_{i=1}^{P} C_i \vartheta_i W_i \tag{23}$$

When C_i represent the unit cost of inactivity for units in class i, L_i represent the average system length for units in class i, ϑ_i represent the average attendance rate for units in priority classes i, W_i represent the average time in the system for units in the class i and C represent the average total cost of system inactivity.

To set the uncertain queuing model of priority discipline, we need to consider the specific total cost of inactivity for three events: $\tilde{C_l}^1$ denotes the no priority discipline, $\tilde{C_l}^1$ denotes the preemption priority, $\tilde{C_l}^2$ denotes the non preemptive priority discipline. Specified from Proximity methods for expansion extend the ambiguity for standard value mapping set by a member to an output variable.

The average arrival rate of the system follows a Poisson process, which is virtually unknown and gives a trapezoidal uncertain number to an obscure $\tilde{\vartheta}$, It is well known that the service rate in the system follows a exponential process and is provided by the trapezoidal uncertain number $\tilde{\eta}$ with the membership function $\eta_{\tilde{\vartheta}}$, $\eta_{\tilde{\eta}}$ respectively.

$$\eta_{\tilde{\vartheta}} = \begin{cases} \frac{\vartheta - a_1}{b - a_1} & \text{if } a_1 \leq \vartheta \leq b_1; \\ 1 & \text{if } b_1 \leq \vartheta \leq c_1; \\ \frac{d_1 - \vartheta}{d_1 - c_1} & \text{if } c_1 \leq \vartheta \leq d_1; \end{cases} \tag{24}$$

$$\eta_{\tilde{\eta}} = \begin{cases} \frac{\eta - a_1}{b_1 - a_1} & \text{if } a_1 \leq \eta \leq b_1; \\ 1 & \text{if } b_1 \leq \eta \leq c_1; \\ \frac{d_1 - \eta}{d_1 - c_1} & \text{if } c_1 \leq \eta \leq d_1; \end{cases} \tag{25}$$

The possible distribution of the unit cost of inactivity for a unit in the same class as the ambiguous trapezoidal numbers $\tilde{b_A}, \tilde{b_B}$ is established with the member function.

$$\eta_{\tilde{b_A}} = \begin{cases} \frac{b_A - a_1}{b_1 - a_1} & \text{if } a_1 \leq b_A \leq b_1; \\ 1 & \text{if } b_1 \leq b_A \leq c_1; \\ \frac{d_1 - b_A}{d_1 - b_A} & \text{if } c_1 \leq b_A \leq d_1; \end{cases} \tag{26}$$

$$\eta_{\tilde{b_B}} = \begin{cases} \frac{b_B - a_1}{b_1 - a_1} & \text{if } a_1 \leq b_B \leq b_1; \\ 1 & \text{if } b_1 \leq b_B \leq c_1; \\ \frac{d_1 - b_B}{d_1 - b_B} & \text{if } c_1 \leq b_B \leq d_1; \end{cases} \tag{27}$$

8. NUMERICAL EXAMPLE

8.1. An Example of a Single Server Uncertain Queuing Model with Two Class Priority Discipline

Now, we are consider the two unit classes of arrivals.One of the class having 15% of arrivals is denoted by Class A and the second class having 85% of arrivals is denoted by Class B. The average arrival rate of the intuitionistic trapezoidal uncertain numbers are $\tilde{\lambda}$ = [26, 30, 32, 34 ; 27, 30, 32, 35]. Its follows by the poisson distribution.The service rate of the intuitionistic trapezoidal uncertain numbers are $\tilde{\mu}$ = [38, 40, 42, 44 ; 39, 40, 42, 45]. Its follows by the exponential distribution. Two unit classes of cost function are established by an intuitive trapezoidal uncertain number $\tilde{C_A}$ = [15, 17, 19, 20 ; 16, 17, 19, 21] and $\tilde{C_B}$ = [2, 3, 5, 6 ; 1, 3, 5, 7]. The model single server priority queue with intuitionistic uncertain number holds the steady state condition By using (1).

$$\tilde{\rho} = \frac{\tilde{\lambda}}{\tilde{\mu}} < 1, \tilde{\rho} = [\frac{26}{38}, \frac{30}{40}, \frac{32}{42}, \frac{34}{44}] = [0.684, 0.75, 0.761, 0.772] \tag{28}$$

$$\tilde{\rho} = [\frac{27}{39}, \frac{30}{40}, \frac{32}{42}, \frac{35}{45}] = [0.6923, 0.75, 0.761, 0.777], \tag{29}$$

The membership and non-membership function variables are achieve the steady state condition. With three cases of total cost inactivity, we need to compare the priority discipline of the intuitionistic uncertain queuing model.

1. No priority discipline
2. Preemptive priority discipline
3. Non-preemptive priority discipline

The model holds that, So the membership and nonmembership function variables are achives the steady state condition. The membership function of uncertain variable are denoted by $\tilde{C}_a$, $\tilde{C_a}^1$, $\tilde{C_a}^2$. and non-membership function of uncertain variable are denoted by $\tilde{C_a}'$, $\tilde{C_a}'^1$ and $\tilde{C_a}'^2$.This is based on the concept of α-cut

method. Cost functions, calculated from (8) where $\lambda_1 = \alpha_1\lambda$ and $\lambda_2 = \alpha_2\lambda$, are
(a) No priority discipline for membership function, $\tilde{C_a}$

$$\tilde{C_a} = (C_1\lambda_1 + C_2\lambda_2) * \left(\frac{1}{\mu - \lambda}\right) \tag{30}$$

Here we choose the α and β values are 0, 0.5, 1 and the calculation of no priority discipline for membership function:

$$\tilde{C_0} = (C_{1,0}\lambda_{1,0} + C_{2,0}\lambda_{2,0}) * \left(\frac{1}{\mu_0 - \lambda_0}\right) \tag{31}$$

$$\tilde{C_{0.5}} = (C_{1,0.5}\lambda_{1,0.5} + C_{2,0.5}\lambda_{2,0.5}) * \left(\frac{1}{\mu_{0.5} - \lambda_{0.5}}\right) \tag{32}$$

$$\tilde{C_1} = (C_{1,1}\lambda_{1,1} + C_{2,1}\lambda_{2,1}) * \left(\frac{1}{\mu_1 - \lambda_1}\right) \tag{33}$$

(b) Preemptive priority discipline for membership function, $\tilde{C_a}^1$

$$\tilde{C_a}^1 = C_1\lambda_1\left[\frac{\lambda/\mu^2}{(1-\lambda/\mu)(1-\alpha_2\lambda/\mu)} + \frac{1}{\mu}\right] + C_2\lambda_2\left[\frac{\lambda/\mu^2}{1-\alpha_2\lambda/\mu} + \frac{1}{\mu}\right] \tag{34}$$

Calculation of preemptive priority discipline for membership function:

$$\tilde{C_0}^1 = C_{1,0}\alpha_1\lambda_0\left[\frac{\lambda_0/\mu_0^2}{(1-\lambda_0/\mu_0)(1-\alpha_2\lambda_0/\mu_0)} + \frac{1}{\mu_0}\right] + C_{2,0}\alpha_2\lambda_0\left[\frac{\lambda_0/\mu_0^2}{1-\alpha_2\lambda_0/\mu_0} + \frac{1}{\mu_0}\right] \tag{35}$$

$$\tilde{C_{0.5}}^1 = C_{1,0.5}\alpha_1\lambda_{0.5}\left[\frac{\lambda_{0.5}/\mu_{0.5}^2}{(1-\lambda_{0.5}/\mu_{0.5})(1-\alpha_2\lambda_{0.5}/\mu_{0.5})} + \frac{1}{\mu_{0.5}}\right] + C_{2,0.5}\alpha_2\lambda_{0.5}\left[\frac{\lambda_{0.5}/\mu_{0.5}^2}{1-\alpha_2\lambda_{0.5}/\mu_{0.5}} + \frac{1}{\mu_{0.5}}\right] \tag{36}$$

$$\tilde{C_1}^1 = C_{1,1}\alpha_1\lambda_1\left[\frac{\lambda_1/\mu_1^2}{(1-\lambda_1/\mu_1)(1-\alpha_2\lambda_1/\mu_1)} + \frac{1}{\mu_1}\right] + C_{2,1}\alpha_2\lambda_1\left[\frac{\lambda_1/\mu_1^2}{1-\alpha_2\lambda_1/\mu_1} + \frac{1}{\mu_1}\right] \tag{37}$$

(c) Non-preemptive priority discipline for membership function, $\tilde{C_a}^2$

$$\tilde{C_a}^2 = C_1\lambda_1\left[\frac{1/\mu}{(1-\lambda/\mu)(1-\alpha_2\lambda/\mu)}\right] + C_2\lambda_2\left[\frac{1/\mu}{1-\alpha_2\lambda/\mu}\right] \tag{38}$$

Calculation of non-preemptive priority discipline for membership function:

$$\tilde{C_0}^2 = C_{1,0}\alpha_1\lambda_0 \left[\frac{1/\mu_0}{(1-\lambda_0/\mu_0)(1-\alpha_2\lambda_0/\mu_0)}\right] + C_{2,0}\alpha_2\lambda_{2,0}\left[\frac{1/\mu_0}{1-\alpha_2\lambda_0/\mu_0}\right] \quad (39)$$

$$\tilde{C_{0.5}}^2 = C_{1,0.5}\alpha_1\lambda_{0.5}\left[\frac{1/\mu_{0.5}}{(1-\lambda_{0.5}/\mu_{0.5})(1-\alpha_2\lambda_{0.5}/\mu_{0.5})}\right] + C_{2,0.5}\alpha_2\lambda_{2,0.5}\left[\frac{1/\mu_{0.5}}{1-\alpha_2\lambda_{0.5}/\mu_{0.5}}\right] \quad (40)$$

$$\tilde{C_1}^2 = C_{1,1}\alpha_1\lambda_1 \left[\frac{1/\mu_1}{(1-\lambda_1/\mu_1)(1-\alpha_2\lambda_1/\mu_1)}\right] + C_{2,1}\alpha_2\lambda_{2,1}\left[\frac{1/\mu_1}{1-\alpha_2\lambda_1/\mu_1}\right] \quad (41)$$

This same procedure is followed by non-membership function.The functions of uncertain variables $\tilde{C_a}'$,$\tilde{C_a}'^1$,$\tilde{C_a}'^2$ are

(a) No priority discipline for nonmembership function,$\tilde{C_a}'$

$$\tilde{C_a}' = \left(\tilde{C_1'}\tilde{\lambda_1'} + \tilde{C_2'}\tilde{\lambda_2'}\right)\frac{1}{\tilde{\mu'} - \tilde{\lambda'}} \quad (42)$$

(b) Preemptive priority discipline for nonmembership function , $\tilde{C_a}'^1$

$$\tilde{C_a}'^1 = \tilde{C_1'}\tilde{\lambda_1'}\left[\frac{\lambda'/\mu'^2}{(1-\lambda'/\mu')(1-\alpha_2\lambda'/\mu')} + \frac{1}{\mu'}\right] + \tilde{C_2'}\tilde{\lambda_2'}\left[\frac{\lambda'/\mu'^2}{1-\alpha_2\lambda'/\mu'} + \frac{1}{\mu'}\right] \quad (43)$$

(c) Non-preemptive priority discipline for nonmembership function , $\tilde{C_a}'^2$

$$\tilde{C_a}'^2 = \tilde{C_1'}\tilde{\lambda_1'}\left[\frac{1/\mu'}{(1-\lambda'/\mu')(1-\alpha_2\lambda'/\mu')}\right] + \tilde{C_2'}\tilde{\lambda_2'}\left[\frac{1/\mu'}{1-\alpha_2\lambda'/\mu'}\right] \quad (44)$$

Table 1 and Table 2 represent the total costs of inactivity for membership function and non-membership function.

Table 2: The Total Costs of Inactivity for Membership Function

Priority Discipline	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 1$
No Priority Discipline : $\tilde{C_\alpha}$	$[5.705, 68.85]$	$[6.157, 54.27]$	$[6.665, 44.55]$
Preemptive Priority Discipline : $\tilde{C_\alpha^1}$	$[37.410, 136.697]$	$[27.147, 109.493]$	$[25.249, 89.890]$
Non-Preemptive Priority Discipline : $\tilde{C_\alpha^2}$	$[58.556, 178.697]$	$[60.412, 147.862]$	$[62.605, 127.445]$

Membership function of no priority discipline, Membership function of preemptive priority discipline and Membership function of Non-preemptive priority discipline are shown in Figure 5 to Figure 7. Non-membership function of no priority discipline, non-membership function of preemptive priority discipline and non-membership function of Non-preemptive priority discipline are shown in Figure 8 to Figure 10.

Figure 5: Membership Function of No Priority Discipline

Figure 6: Membership Function of Preemptive Priority Discipline

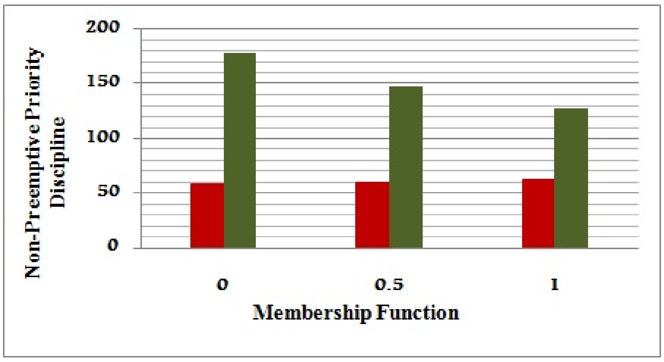

Figure 7: Membership Function of Non-Preemptive Priority Discipline

Table 3: The Total Costs of Inactivity for Non-Membership Function

Priority Discipline	$\beta = 0$	$\beta = 0.5$	$\beta = 1$
No Priority Discipline : $\tilde{C}'_{\beta}$	[4.875, 79.625]	[5.257, 62.79]	[5.6875, 22.666]
Preemptive Priority Discipline : $\tilde{C}'^{1}_{\beta}$	[26.507, 145.789]	[28.479, 116.243]	[30.772, 95.535]
Non-Preemptive Priority Discipline : $\tilde{C}'^{2}_{\beta}$	[59.431, 182.525]	[61.457, 151.879]	[63.466, 130.474]

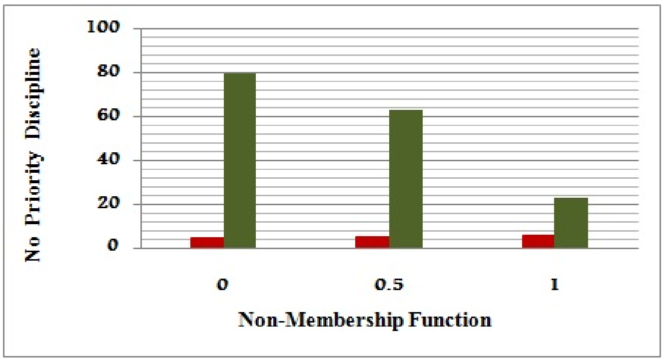

Figure 8: Non-Membership Function of No Priority Discipline

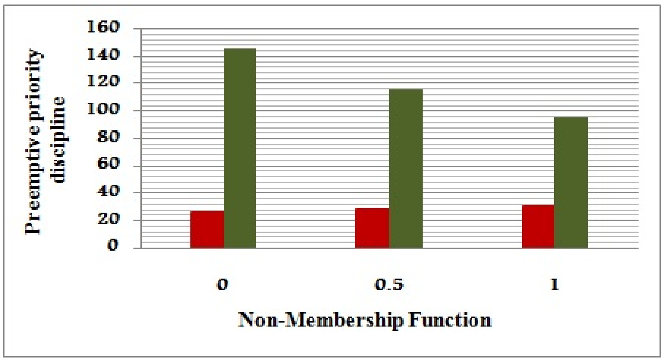

Figure 9: Non-Membership Function of Preemptive Priority Discipline

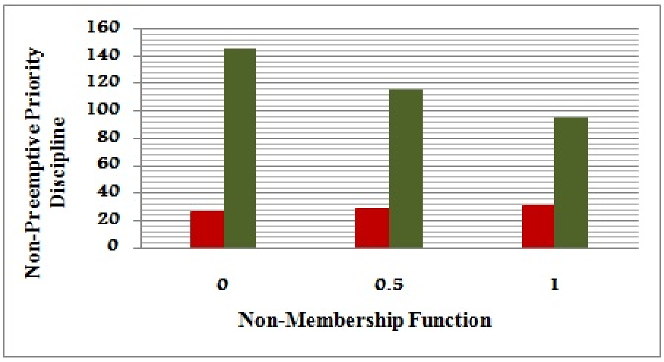

Figure 10: Non-Membership Function of Non-Preemptive Priority Discipline

8.2. An Example of a Two Queue with Single Server Uncertain Queuing Model with Two Class Priority Discipline

Consider a line production of machine receives two types of arrival customers ϑ_1, ϑ_2 and the service time can be represented as a exponential distribution η respectively, noting that all parameters are in a uncertain environment and the management wants to compute the mean of queue length for each class.Trapezoidal uncertain numbers are illustrated for the method as shown in the following subsections. A queuing model that two unit categories arrive at: 15% of arrivals belong to one of the categories its denoted by category A (Class A) and therefore the remaining 85% area unit within the different categories its denoted by category B (Class B). The arrival rates(ϑ) at the system follows a Poisson Distribution (PD) process, is approximately known and is given by the trapezoidal uncertain numbers (TPFN) are $\tilde{\vartheta_1} = [11, 13, 15, 17]$ and $\tilde{\vartheta_2} = [5, 7, 8, 9]$. The service rate($\eta$) from a single server is the same for both unit classes, follows an exponential Distribution(ED) process, is approximately known and is given by trapezoidal uncertain number(TPFN) is $\tilde{\eta}$ = [31, 33, 35, 37]. The possibility distribution of unit cost of inactivity for units in the same class is established by a trapezoidal uncertain number(TPFN), with $\tilde{b_A} = [2,\ 3.5,\ 6,\ 7]$ and $\tilde{b_B} = [1.5,\ 2.5,\ 4,\ 5]$. The model holds that $\tilde{\rho_1} = \frac{\tilde{\vartheta_1}}{\tilde{\eta}} < 1$, $\tilde{\rho_1} = [\frac{11}{31}, \frac{13}{33}, \frac{15}{35}, \frac{17}{37}] = [0.3548,\ 0.3939,\ 0.4285,\ 0.4594]$, and $\tilde{\rho_2} = \frac{\tilde{\vartheta_2}}{\tilde{\eta}} < 1$, $\tilde{\rho_2} = [\frac{5}{31}, \frac{7}{33}, \frac{8}{35}, \frac{9}{37}] = [0.1612,\ 0.2121,\ 0.2285,\ 0.2432]$, So the membership function variables are achieves the steady state condition. To create the priority discipline of the uncertain queuing model, we must compare the average total cost of inactivity for the three cases. The membership function of uncertain variable are denoted by $\tilde{C_l}$, $\tilde{C_l}^1$ and $\tilde{C_l}^2$.This is based on the concept of α-cut method. Cost functions, calculated from (14), $\vartheta = \vartheta_1 + \vartheta_2$. Let $\alpha_1 = 0.15$ and $\alpha_2 = 0.85$. Here $\vartheta_1 = \alpha_1\vartheta$ and $\vartheta_2 = \alpha_2\vartheta$

(a) Average total cost of no priority, $\tilde{C_l}$

$$\tilde{C_l} = \left(\tilde{b_1}\tilde{\vartheta} + \tilde{b_2}\tilde{\vartheta}\right)\tilde{W} \quad with \quad \tilde{W} = \frac{1}{\tilde{\eta} - \tilde{\vartheta}} \tag{45}$$

(b)Average total cost of preemptive priority, $\tilde{C_l}^1$

$$\tilde{C_l}^1 = \tilde{b_1}\tilde{\vartheta}\tilde{W_1} + \tilde{b_2}\tilde{\vartheta}\tilde{W_2} \tag{46}$$

Where, $\tilde{W_1} = \frac{\vartheta/\eta^2}{(1-\vartheta/\eta)(1-\alpha_2\vartheta/\eta)} + \frac{1}{\eta}$, $\quad \tilde{W_2} = \frac{\vartheta/\eta^2}{1-\alpha_2\vartheta/\eta} + \frac{1}{\eta}$

(c) Average total cost of non-preemptive priority, $\tilde{C_l}^2$

$$\tilde{C_l}^2 = \tilde{b_1}\tilde{\vartheta}\tilde{W_1} + \tilde{b_2}\tilde{\vartheta}\tilde{W_2} \tag{47}$$

Where, $\tilde{W_1} = \frac{1/\eta}{(1-\vartheta/\eta)(1-\alpha_2\vartheta/\eta)}$, $\quad \tilde{W_2} = \frac{1/\eta}{1-\alpha_2\vartheta/\eta}$

Comparison of the three total prices shows that the inactivity of priority disciplines reduces the average function of the total cost. Table 4 represent the total costs of inactivity for membership function. Now we can get the performance steps of the uncertain queuing model for each customer class: the average time on the system, $\tilde{W_1}$ and $\tilde{W_2}$ is the average length of units in the system $\tilde{L_1}$ and $\tilde{L_2}$. Table 5 represent the average time of the system and Table 6 represent the average length of the system. The membership functions of the performance measures are obtained applying the Buckley and Qu's method [13] functions of W_1, W_2, L_1 and L_2. During $\vartheta \in \tilde{\vartheta}$ and $\eta \in \tilde{\eta}$ depend on two continuous variables, and they are equivalent to each other (increasing by ϑ and decreasing by η). Membership function of no priority discipline, Membership function of preemptive priority discipline and Membership function of Non-preemptive priority discipline are shown in Figure 11 to Figure 13.

$$W_1 = \frac{1/\eta}{(1 - \vartheta/\eta)(1 - \vartheta_2/\eta)} \tag{48}$$

$$W_2 = \frac{1/\eta}{1 - \vartheta_2/\eta} \tag{49}$$

$$L_1 = \vartheta_1 W_1 \tag{50}$$

$$L_2 = \vartheta_2 W_2 \tag{51}$$

Table 4: The Total Costs of Inactivity for Membership Function

Priority Discipline	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 1$
No Priority Discipline : $\tilde{C_l}$	$[1.19, 9.36]$	$[1.37, 6.921]$	$[1.57, 5.4]$
Preemptive Priority Discipline : $\tilde{C_l^1}$	$[7.40, 43.18]$	$[8.43, 33.01]$	$[9.57, 26.46]$
Non-Preemptive Priority Discipline : $\tilde{C_l^2}$	$[192.67, 544.86]$	$[208.45, 455.51]$	$[224.99, 396.16]$

Figure 11: Membership Function of No Priority Discipline

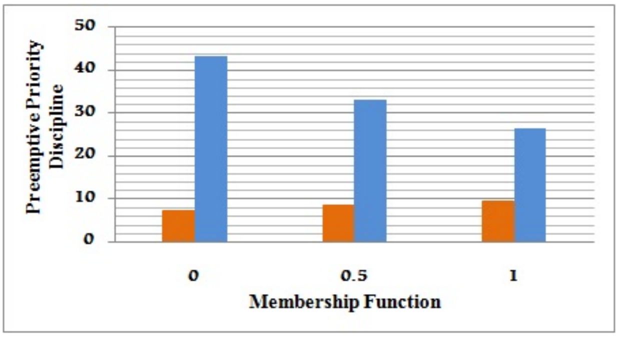

Figure 12: Membership Function of Preemptive Priority Discipline

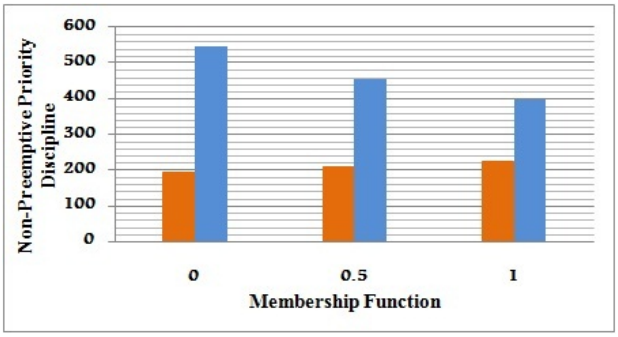

Figure 13: Membership Function of Non-Preemptive Priority Discipline

The average time of the system are shown in Figure 14 and Figure 15. The average length of the system are shown in Figure 16 and Figure 17.

Table 5: Average Time of the System

Average time	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 1$
W_1	$[0.075, 0.695]$	$[0.084, 0.472]$	$[0.096, 0.344]$
W_2	$[0.042, 0.112]$	$[0.045, 0.097]$	$[0.048, 0.086]$

Figure 14: Average time of the system W_1

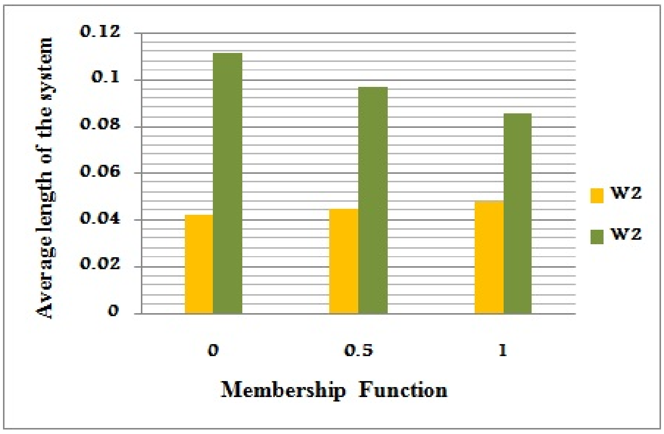

Figure 15: Average time of the system W_2

Table 6: Average Length of the System

Average length	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 1$
L_1	$[0.180, 2.712]$	$[0.215, 1.771]$	$[0.260, 1.239]$
L_2	$[0.579, 2.477]$	$[0.654, 2.069]$	$[0.735, 1.756]$

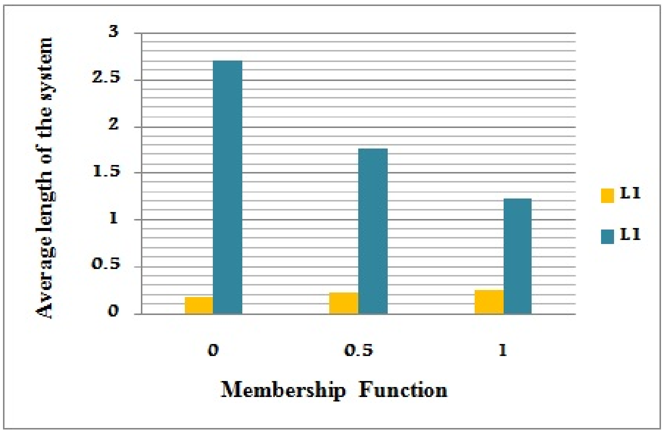

Figure 16: Average length of the system L_1

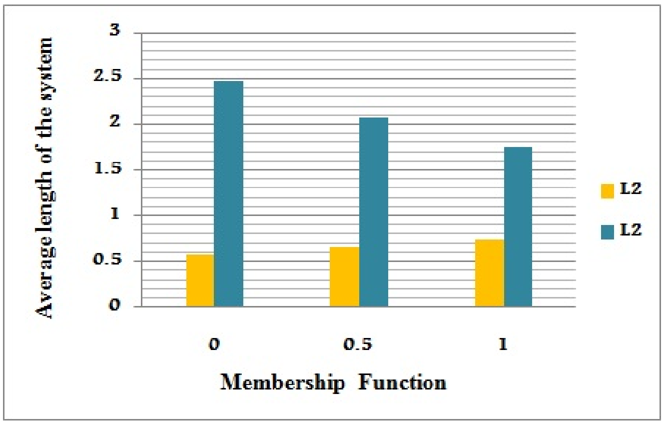

Figure 17: Average length of the system L_2

9. CONCLUSION

In this paper, we have obtained the steady-state solution for the cost of higher and lower priority customers by using an alpha cut method to obtain lower and upper bounds of cost function for No-priority, Preemptive priority and Non-Preemptive priority customer and measures to apply single queue with a single server and two queues with single server uncertain queuing model and function costs are communicated by membership and non-membership functions that completely maintain the uncertainty of the underlying data when a portion of the parameters of the model are uncertain.

The method proposed in this research paper empowers sensible answers to be accomplished for each case, with various levels of probability, running from the most critical to the most idealistic situation. The paper additionally gives more data to help design uncertain priority-discipline queuing systems. This paper just examinations one execution measures, however, the approach we proposed is clearly not restricted to these and can be extended to others. Limitations require more complex queuing models or simulation techniques that can more accurately capture the nuances of real-world service systems. The model considers basic performance metrics such as average queue lengths and waiting times; it may also capture other important performance indicators such as customer satisfaction, service level adherence, or operational costs. In addition, empirical data and sensitivity analyses can help validate and refine queuing models to better reflect actual system behavior. The findings can be incorporated into decision support systems or simulation tools used by service managers to assess different scenarios, forecast future demand, and assess the impact of operational changes on service performance.

Acknowledgement. This research work was supported by Department of Mathematics, Bharathiar University, Coimbatore, India.

Funding. This research received no external funding.

REFERENCES

[1] K. A. Alnowibet, A. F. Alrasheedi, and F. S. Alqahtani, "Queuing models for analyzing the steady-state distribution of stochastic inventory systems with random lead time and impatient customers," *Processes*, vol. 10, no. 4, p. 624, 2022.

[2] N. Abdali, S. Heidari, M. Alipour-Vaezi, F. Jolai, and A. Aghsami, "A priority queueing-inventory approach for inventory management in multi-channel service retailing using machine learning algorithms," *Kybernetes*, 2024.

[3] R. Ramanathan and V. Varadharajan, "Performance approximations of markov modulated poisson process arrival queue with markovian service using matrix geometric approach," *Yugoslav Journal of Operations Research*, 2023.

[4] S. Narayanamoorthy and S. Kalyani, "The intelligence of dual simplex method to solve linear fractional fuzzy transportation problem," *Computational intelligence and neuroscience*, vol. 2015, pp. 1–1, 2015.

[5] M. Yin, M. Yan, Y. Guo, and M. Liu, "Analysis of a pre-emptive two-priority queuing system with impatient customers and heterogeneous servers," *Mathematics*, vol. 11, no. 18, p. 3878, 2023.

[6] R. Yohapriyadharsini and V. Suvitha, "Multi-server markovian heterogeneous arrivals queue with two kinds of working vacations and impatient customers," *Yugoslav Journal of Operations Research*, vol. 33, no. 4, pp. 643–666, 2023.

[7] S. K. Baby and V. Suvitha, "Analysis of single server repairable queueing system with many kinds of service, single working vacation and standby server," *Yugoslav Journal of Operations Research*, no. 00, pp. 29–29, 2023.

[8] S. Narayanamoorthy and P. Anukokila, "Goal programming approach for solving transportation problem with interval cost," *Journal of Intelligent & Fuzzy Systems*, vol. 26, no. 3, pp. 1143–1154, 2014.

[9] S. Barak and M. Fallahnezhad, "Cost analysis of fuzzy queuing systems," 2012.

[10] S. Narayanamoorthy, A. Anuja, V. Murugesan, and D. Kang, "A distinctive analyzation of intuitionistic fuzzy queueing system using erlang service model," in *AIP Conference Proceedings*, vol. 2261, no. 1. AIP Publishing, 2020.

[11] R. E. Bellman and L. A. Zadeh, "Decision-making in a fuzzy environment," *Management science*, vol. 17, no. 4, pp. B–141, 1970.

[12] L. A. Zadeh, "Information and control," *Fuzzy sets*, vol. 8, no. 3, pp. 338–353, 1965.

[13] J. Buckley, "Elementary queueing theory based on possibility theory," *Fuzzy sets and systems*, vol. 37, no. 1, pp. 43–52, 1990.

[14] J. J. Buckley, T. Feuring, and Y. Hayashi, "Fuzzy queueing theory revisited," *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, vol. 9, no. 05, pp. 527–537, 2001.

[15] E. Natarajan, F. Augustin, R. Saraswathy, S. Narayanamoorthy, S. Salahshour, A. Ahmadian, and D. Kang, "A bipolar intuitionistic fuzzy decision-making model for selection of effective diagnosis method of tuberculosis," *Acta Tropica*, vol. 252, p. 107132, 2024.

[16] J. J. Buckley and Y. Qu, "On using alpha-cuts to evaluate fuzzy equations," *Fuzzy sets and Systems*, vol. 38, no. 3, pp. 309–312, 1990.

[17] O. Boxma and D. G. Down, "Dynamic server assignment in a two-queue model," *European Journal of Operational Research*, vol. 103, no. 3, pp. 595–609, 1997.

[18] S.-P. Chen, "Parametric nonlinear programming approach to fuzzy queues with bulk service," *European Journal of Operational Research*, vol. 163, no. 2, pp. 434–444, 2005.

[19] S. Narayanamoorthy and P. Vidhya, "An adoptable technique to solve fuzzy assignment problem int," *Journal of Pure and Applied Mathematics*, vol. 117, no. 14, pp. 199–207, 2017.

[20] J. Cohen, "A two-queue, one-server model with priority for the longer queue," *Queueing systems*, vol. 2, pp. 261–283, 1987.

[21] S. Drekic and D. G. Woolford, "A preemptive priority queue with balking," *European Journal of Operational Research*, vol. 164, no. 2, pp. 387–401, 2005.

[22] R. Groenevelt and E. Altman, "Analysis of alternating-priority queueing models with (cross) correlated switchover times," *Queueing Systems*, vol. 51, pp. 199–247, 2005.

[23] P. G. Harrison and Y. Zhang, "Delay analysis of priority queues with modulated traffic," in *13th IEEE International Symposium on Modeling, Analysis, and Simulation of Computer and Telecommunication Systems*. IEEE, 2005, pp. 280–287.

[24] S. Narayanamoorthy, "Vms and sivakamasundari, k.(2013). fuzzy cetd matrix to estimate the maximum age group victims of pesticide endosulfan problems faced in kerala," *International Journal of Mathematics and Computer*, vol. 3, no. 2, pp. 227–232.

[25] K. Ghafour, R. Ramli, and N. Z. Zaibidi, "Developing a m/g/c-fcfs queueing model with continuous review (r, q) inventory system policy in a cement industry," *Journal of Intelligent & Fuzzy Systems*, vol. 32, no. 6, pp. 4059–4068, 2017.

[26] A. M. Haghighi and D. P. Mishev, "A parallel priority queueing system with finite buffers," *Journal of Parallel and Distributed Computing*, vol. 66, no. 3, pp. 379–392, 2006.

[27] C. Kao, C.-C. Li, and S.-P. Chen, "Parametric programming to the analysis of fuzzy queues," *Fuzzy sets and systems*, vol. 107, no. 1, pp. 93–100, 1999.

[28] R. Kalyanaraman, N. Thillaigovindan, and G. Kannadasan, "A fuzzy bulk queue with modified bernoulli vacation and restricted admissible customers," *Journal of intelligent & Fuzzy Systems*, vol. 24, no. 4, pp. 837–845, 2013.

[29] S.-P. Chen, "Solving fuzzy queueing decision problems via a parametric mixed integer nonlinear programming method," *European Journal of Operational Research*, vol. 177, no. 1, pp. 445–457, 2007.

[30] R.-J. Li and E. Lee, "Analysis of fuzzy queues," *Computers & Mathematics with Applications*, vol. 17, no. 7, pp. 1143–1147, 1989.

[31] H. Prade, "An outline of fuzzy or possibilistic models for queueing systems, fuzzy sets," 1980.

[32] S. Narayanamoorthy, "Application of fuzzy cetd matrix technique to estimate the maximum age group of silk weavers as bonded laborers," *International Journal of Applied Mathematics & Mechanics*, vol. 8, no. 2, pp. 89–98, 2012.

[33] S. Geetha, S. Narayanamoorthy, and D. Kang, "Extended hesitant fuzzy swara techniques to examine the criteria weights and vikor method for ranking alternatives," in *AIP Conference Proceedings*, vol. 2261, no. 1. AIP Publishing, 2020.

[34] M. J. Pardo and D. De La Fuente, "Optimizing a priority-discipline queueing model using fuzzy set theory," *Computers & Mathematics with Applications*, vol. 54, no. 2, pp. 267–281, 2007.

[35] D. Negi and E. Lee, "Analysis and simulation of fuzzy queues," *Fuzzy sets and systems*, vol. 46, no. 3, pp. 321–330, 1992.

[36] S. P. Madheswari and P. Suganthi, "Retrial queue with unreliable server and second optional service under k types of bernoulli vacations," *International Journal of Services and Operations Management*, vol. 40, no. 4, pp. 502–533, 2021.

[37] C. Cadena-Zarate and G. Osma-Pinto, "Study of the variation of operation of a low voltage electric network due to the integration of distributed energy resources—steady state condition," *International Journal of Electrical Power & Energy Systems*, vol. 155, p. 109649, 2024.

[38] B. Lu, Z. Zhang, J. Cai, W. Wang, X. Ju, Y. Xu, X. Lu, H. Tian, L. Shi, and G. Shu, "Integrating engine thermal management into waste heat recovery under steady-state design and dynamic off-design conditions," *Energy*, vol. 272, p. 127145, 2023.