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DATA NORMALIZATION IN RAWEC METHOD: LIMITATIONS AND REMEDIES

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Abstract: Multi-Criteria Decision Making (*MCDM*) methods play a significant role in evaluating and comparing options based on various criteria, helping optimize decisions in complex situations. They enhance transparency and fairness in the decision-making process while minimizing risks and optimizing decision performance. Ranking of Alternatives with Weights of Criterion (*RAWEC*) is a *MCDM* method recently proposed in early 2024. Its advantage lies in the minimal steps required for implementation. Simultaneously employing two data normalization approaches is also a distinctive feature of *RAWEC* compared to most other *MCDM* methods. However, if there is at least one zero element in the decision matrix, the normalization method using available data in *RAWEC* cannot be utilized. This study aims to address these limitations. Investigating the suitability of data normalization methods when combined with the *RAWEC* method has been conducted in various scenarios. The results of this study have addressed the limitations of the *RAWEC* method. Specifically, an alternative normalization method has been identified as suitable to replace the normalization method using available data in *RAWEC* when there is at least one zero element in the decision matrix.

Keywords: MCDM, RAWEC, data normalization.

1. INTRODUCTION

MCDM is a topic garnering increasing attention from researchers as it finds applications in various fields such as construction [1, 2], sports [3], education [4], petroleum [5], mechanical engineering [6-8], financial management [9-14], and more. There is no specific figure on the number of existing *MCDM* methods, but it is known that there are over 200 methods proposed [15]. No study claims any *MCDM* method to be superior to others. Instead, newer methods aim to either expand the application scope of previous methods or create methods that are more convenient to apply. Some studies aimed at the first purpose involve combining fuzzy theory with original *MCDM* methods, such as fuzzy *MARCOS* [16], fuzzy *COCOSO* [17], fuzzy *EDAS* [18], etc., or combining design of experiments (*DOE*) methods with *MCDM* methods for quickly ranking alternatives when the number of alternatives changes [19, 20], or extending data normalization methods for original *MCDM* methods [21-24], etc.

RAWEC is considered the youngest method among the existing methods. This method was discovered in February 2024 [25]. Compared to other *MCDM* methods, *RAWEC* is relatively simple, with only four simple steps. *RAWEC* also has a distinguishing feature from other methods in that it utilizes dual normalization techniques. In the first normalization, all criteria are transformed into "the larger, the better" form (*B*-form criteria). In the second normalization, all criteria are transformed into "the smaller, the better" form (*C*-form criteria). The utilization of dual normalization technique to leverage the strengths of different methods, enhancing the accuracy and reliability of decisions, while also yielding the best possible decision by evaluating and comparing options more carefully and comprehensively [26, 27]. However, as it only emerged in February 2024, no studies have been published yet on the application of the *RAWEC* method. Furthermore, the issue of utilizing the available data normalization method in this approach has not been extensively discussed. Specifically, the available normalization method in *RAWEC* cannot be applied if there exists at least one zero value in the decision-making matrix. Further clarification on this normalization issue will be provided in the subsequent section of this paper. Leveraging the benefits of the *RAWEC* method, such as its simplicity with just four steps and the utilization of dual normalization technique, as well as overcoming the limitations when unable to utilize the available data normalization method, serves as the primary motivation behind this research. To address this limitation of the *RAWEC* method, a survey of the suitability of alternative data normalization methods when combined with *RAWEC* will be conducted. This involves firstly identifying data normalization methods that can be applied when there is at least one zero value in the matrix. Secondly, investigating the suitability of these data normalization methods when combined with the *RAWEC* method. This study has identified an alternative data normalization method to the one available in *RAWEC*. When combined with *RAWEC*, this new method consistently ensures the accuracy of decisions. Particularly noteworthy is that the new normalization method can be applied in situations where the available data normalization method in *RAWEC* cannot be utilized. In section 2 of this article, there is a summary of commonly used data normalization methods and a summary of the steps involved in applying the *RAWEC* method. This section also briefly analyzes the characteristics of each data normalization method and clarifies the cases where the data normalization method available in the *RAWEC* method cannot be used. In section 3, investigating the suitability of combining data normalization methods with the *RAWEC* method will be conducted in various cases. Finally, there is the conclusion of the study.

2. METHODOLOGY AND METHODS

In *MCDM* actions, data normalization is crucial to ensure fairness and objectivity. This helps standardize data from different sources, facilitating comparison and analysis. Consequently, decisions will be based on more consistent and reliable data, aiding more effective management in strategizing and directing organizational efforts. The normalization of data significantly influences the ranking of alternatives. Each *MCD*M method may utilize different data normalization techniques, but not all are suitable. Most studies confirm that the ranking of alternatives depends on the data normalization method used [28-32]. Thus, assessing the suitability of data normalization methods when combined with each specific *MCDM* method is essential [21-24]. This assessment also expands the scope of application of *MCDM* methods in cases where the default data normalization method cannot be used. Many *MCDM* methods include at least one data normalization method by default, while some integrate more than one. For instance, the *RAWEC* method being examined in this study incorporates two data normalization methods [25], *COCOSO* integrates two [26], and *MACONT* integrates three [27]. Integrating multiple data normalization methods in an *MCDM* method reflects the full range of initial information, reduces the deviation of evaluation values, and enhances the reliability of the final decision results. This approach also aids in accurately determining the distance from alternatives to the ideal solution [26, 27].

Twelve data normalization methods commonly used in *MCDM* include linear normalization (*N1*), Weitendorf normalization (*N2*), vector normalization (*N3*), Z-score normalization (*N4*), enhanced accuracy normalization (*N5*), sum linear normalization (*N6*), logarithmic normalization (*N7*), max linear normalization (*N8*), min linear normalization (*N9*), Jüttler-Körth normalization (*N10*), Peldschus normalization (*N11*), and stop normalization (*N12*) [33]. Assuming the decision matrix has *m* rows and *n* columns, where *m* is the number of alternatives to be ranked, and *n* is the number of criteria for the alternatives. The value of criterion *j* for alternative *i* is denoted as x_{ij}, with $j = 1\text{-}n$, $i = 1\text{-}m$. The symbols $max(x_{ij})$, $min(x_{ij})$, and μ_j correspondingly represent the maximum, minimum, and mean values of criterion *j* across all alternatives. The normalization formula for beneficial criteria (*B*-form criteria) and non-beneficial criteria (*C*-form criteria) for each method is summarized in Table 1.

Some basic characteristics of each normalization method are summarized as follows. Methods *N1, N8, N9, N11*, and *N12* are easy to implement but cannot be applied when there is at least one xij value equal to zero, known as an outlier. All four methods *N2, N3, N4*, and *N5* can be applied to handle outlier values. Method *N3* maintains the direction of the data but may lose information about the magnitude of the data. *N4* is effective when the data is normally distributed, but it shows limitations with non-normally distributed data. *N5* improves data quality but involves complex computational processes. The use of *N*6 preserves the original data sum but is susceptible to outliers. *N7* is suitable for data with large variations but cannot be applied when there is at least one x_{ij} value ≤ 0 [34-37].

Table 1: Some data normalization methods [33]

Normalization method	$if\ j \in B$	$if\ j \in C$
N1	$\frac{x_{ij}}{max(x_{ij})}$	$\frac{min(x_{ij})}{x_{ij}}$
N2	$\frac{x_{ij} - min(x_{ij})}{max(x_{ij}) - min(x_{ij})}$	$\frac{max(x_{ij}) - x_{ij}}{max(x_{ij}) - min(x_{ij})}$
N3	$\frac{x_{ij}}{\sqrt{\sum_{i=1}^{m}(x_{ij})^2}}$	$1 - \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m}(x_{ij})^2}}$
N4	$\frac{x_{ij} - \frac{\sum_{i=1}^{m} x_{ij}}{m}}{\sqrt{\frac{\sum_{i=1}^{m}(x_{ij} - \mu_j)^2}{m}}}$	$-\frac{x_{ij} - \frac{\sum_{i=1}^{m} x_{ij}}{m}}{\sqrt{\frac{\sum_{i=1}^{m}(x_{ij} - \mu_j)^2}{m}}}$
N5	$1 - \frac{max(x_{ij}) - x_{ij}}{\sum_{i=1}^{m}(max(x_{ij}) - x_{ij})}$	$1 - \frac{x_{ij} - min(x_{ij})}{\sum_{i=1}^{m}\left(x_{ij} - min(x_{ij})\right)}$
N6	$\frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}}$	$\frac{1/x_{ij}}{\sum_{i=1}^{m} 1/x_{ij}}$
N7	$\frac{lnx_{ij}}{ln(\prod_{i=1}^{m} x_{ij})}$	$1 - \frac{lnx_{ij}}{ln(\prod_{i=1}^{m} x_{ij})}$
N8	$\frac{x_{ij}}{max(x_{ij})}$	$1 - \frac{x_{ij}}{max(x_{ij})}$
N9	$1 - \frac{min(x_{ij})}{x_{ij}}$	$\frac{min(x_{ij})}{x_{ij}}$
N10	$1 - \left\lvert\frac{max(x_{ij}) - x_{ij}}{max(x_{ij})}\right\rvert$	$1 - \left\lvert\frac{min(x_{ij}) - x_{ij}}{max(x_{ij})}\right\rvert$
N11	$\left(\frac{x_{ij}}{max(x_{ij})}\right)^2$	$\left(\frac{x_{ij}}{max(x_{ij})}\right)^3$
N12	$\frac{100x_{ij}}{max(x_{ij})}$	$\frac{100min(x_{ij})}{x_{ij}}$

In Table 1, *N1* represents the normalization method available within the *RAWEC* method. Also in this table, it is noted that only four methods, *N2*, *N3*, *N4*, and *N5*, can be used when there is at least one component equal to 0 in the decision matrix. When normalized using methods *N1*, *N2*, *N3*, and *N5*, the normalized values will fall within the range from -1 to 1. This is also a common characteristic of these four methods. Normalized values using method *N4* may fall within the range of -1 to 1 or may not fall within this range (less than -1 or greater than 1).

The *RAWEC* method employs four simple steps to rank alternatives [25].

Step 1: Establish the decision matrix as in equation (1).

$$X = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \cdots & \cdots & \ddots & \cdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix} \quad (1)$$

Step 2: Perform dual normalization according to equations (2) and (3). This is the *N1* normalization method.

$$n_{ij} = \frac{x_{ij}}{max(x_{ij})}, \text{ and } n'_{ij} = \frac{\min(x_{ij})}{x_{ij}}, \text{ if j } \in \text{B} \quad (2)$$

$$n_{ij} = \frac{\min(x_{ij})}{x_{ij}}, \text{ and } n'_{ij} = \frac{x_{ij}}{max(x_{ij})}, \text{ if j } \in \text{C} \quad (3)$$

Step 3: Let w_j be the weight of criterion *j*, the deviation (v_{ij}) from the weight of criteria is calculated according to equations (4) and (5).

$$v_{ij} = \sum_{j=1}^{n} w_j \cdot \left(1 - n_{ij}\right) \quad (4)$$

$$v'_{ij} = \sum_{j=1}^{n} w_j \cdot \left(1 - n'_{ij}\right) \quad (5)$$

Step 4: Calculate the scores (*Qi*) of alternatives according to equation (6). The alternative with the highest score is ranked 1, and so forth.

$$Q_i = \frac{v'_{ij} - v_{ij}}{v'_{ij} + v_{ij}} \quad (6)$$

Observing equations (2) and (3), it is noted that if there exists at least one value of x_{ij} equal to 0, then neither of these formulas can be used. This is the limitation of *RAWEC* that this study aims to overcome.

3. CASE STUDIES

To overcome the limitation of the *RAWEC* method as mentioned above, this section will investigate the suitability of data normalization methods when combined with the *RAWEC* method. Therefore, besides *N1*, which is the normalization method available in *RAWEC*, four other methods, including *N2*, *N3*, *N4*, and *N5*, will also be explored. Thus, step 2 of the *RAWEC* method will be executed in five scenarios as follows:

Scenario 1: Use *N1* to normalize the data according to equations (2) and (3).

Scenario 2: Use *N2* to normalize the data according to equations (7) and (8).

$$n_{ij} = \frac{x_{ij} - min(x_{ij})}{max(x_{ij}) - min(x_{ij})}, \text{ and } n'_{ij} = \frac{max(x_{ij}) - x_{ij}}{max(x_{ij}) - min(x_{ij})}, \text{ if j } \in \text{B} \quad (7)$$

$$n_{ij} = \frac{max(x_{ij}) - x_{ij}}{max(x_{ij}) - min(x_{ij})}, \text{ and } n'_{ij} = \frac{x_{ij} - min(x_{ij})}{max(x_{ij}) - min(x_{ij})}, \text{ if j } \in \text{C} \quad (8)$$

Scenario 3: Use *N3* to normalize the data according to equations (9) and (10).

$$n_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} (x_{ij})^2}}, \text{ and } n'_{ij} = 1 - \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} (x_{ij})^2}}, \text{ if } j \in B \quad (9)$$

$$n_{ij} = 1 - \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} (x_{ij})^2}}, \text{ and } n'_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} (x_{ij})^2}}, \text{ if } j \in C \quad (10)$$

Scenario 4: Use *N4* to normalize the data according to equations (11) and (12), where μ_j is the mean value of criterion *j*.

$$n_{ij} = \frac{x_{ij}-\mu_j}{\sqrt{\frac{\sum_{i=1}^{m}\left(x_{ij}-\mu_j\right)^2}{m}}}, \text{ and } n'_{ij} = -\frac{x_{ij}-\mu_j}{\sqrt{\frac{\sum_{i=1}^{m}\left(x_{ij}-\mu_j\right)^2}{m}}}, \text{ if } j \in B \quad (11)$$

$$n_{ij} = -\frac{x_{ij}-\mu_j}{\sqrt{\frac{\sum_{i=1}^{m}\left(x_{ij}-\mu_j\right)^2}{m}}}, \text{ and } n'_{ij} = \frac{x_{ij}-\mu_j}{\sqrt{\frac{\sum_{i=1}^{m}\left(x_{ij}-\mu_j\right)^2}{m}}}, \text{ if } j \in C \quad (12)$$

Scenario 5: Use *N5* to normalize the data according to equations (13) and (14).

$$n_{ij} = 1 - \frac{max(x_{ij})-x_{ij}}{\sum_{i=1}^{m}\left(max(x_{ij})-x_{ij}\right)}, \text{ and } n'_{ij} = 1 - \frac{x_{ij}-min(x_{ij})}{\sum_{i=1}^{m}\left(x_{ij}-\min(x_{ij})\right)}, \text{ if j} \in \text{B} \quad (13)$$

$$n_{ij} = 1 - \frac{x_{ij}-min(x_{ij})}{\sum_{i=1}^{m}\left(x_{ij}-\min(x_{ij})\right)}, \text{ and } n'_{ij} = 1 - \frac{max(x_{ij})-x_{ij}}{\sum_{i=1}^{m}\left(max(x_{ij})-x_{ij}\right)}, \text{ if j} \in \text{C} \quad (14)$$

These five scenarios will be applied in various cases. Differences in cases relate to the application fields, the number of *B* form criteria and *C* form criteria in each case, and in the last case, a problem arises when the *N1* normalization method cannot be used.

CASE 1: Ranking alternatives when all criteria are *C* form

In this scenario, data on seventeen different drilling methods were utilized [8, 38]. All six criteria describing the alternatives are in *C* form, including machining time (s), height of the chip layer on the drill entry surface (mm), height of the chip layer on the drill exit surface (mm), thickness of the chip layer on the drill entry surface (mm), thickness of the chip layer on the drill exit surface (mm), and surface roughness (µm). These six criteria are denoted as *C1*, *C2*, *C3*, *C4*, *C5*, and *C6*, respectively. The values of the criteria for each alternative and the weights of each criterion have been compiled in Table 2.

Table 2: Metal drilling alternatives [8, 38].

No.	C1	C2	C3	C4	C5	C6
A1	14.03	0.051	0.058	0.105	0.21	0.479
A2	7.59	0.053	0.058	0.155	0.245	1.211
A3	7.34	0.035	0.06	0.165	0.215	0.916
A4	4.06	0.033	0.075	0.18	0.215	0.535
A5	5.4	0.048	0.078	0.25	0.195	0.601
A6	5.5	0.05	0.084	0.185	0.185	0.703
A7	2.81	0.033	0.058	0.185	0.185	0.466
A8	2.62	0.028	0.048	0.2	0.19	0.577
A9	2.88	0.028	0.05	0.18	0.15	0.417
A10	2.75	0.043	0.051	0.23	0.195	0.675
A11	2.84	0.043	0.055	0.165	0.205	0.418
A12	1.59	0.028	0.074	0.145	0.17	0.601
A13	1.88	0.038	0.064	0.185	0.175	0.563
A14	3.44	0.049	0.066	0.19	0.185	0.391
A15	2.04	0.023	0.059	0.16	0.18	0.493
A16	2.1	0.043	0.05	0.235	0.185	0.675
A17	1.25	0.04	0.049	0.44	0.19	0.65
Weight	0.3	0.1	0.1	0.1	0.1	0.3

The task of *MCDM* is to rank the seventeen alternatives in Table 2 to find the top-ranked alternative. The top-ranked alternative is the one with the smallest values for all six criteria. This task has been previously performed using *TOPSIS*, *COPRAS* [38], and *FUCA* methods [8].

In Figure 1 represents the ranking results of the seventeen alternatives using the *RAWEC* method with five different scenarios (corresponding to five normalization methods *N1*, *N2*, *N3*, *N4*, and *N5*) and other *MCDM* methods.

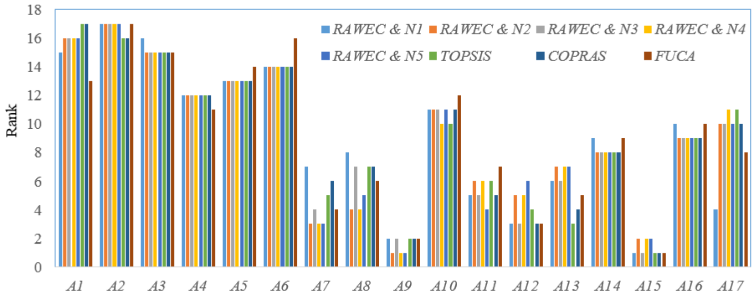

Figure 1: Ranking of alternatives in case 1

Methods used in previous studies including *TOPSIS*, *COPRAS*, and *FUCA* all indicate *A15* as the best alternative. Combining *N1* and *N3* with *RAWEC* also identifies *A15* as the best alternative. This suggests that the effectiveness in achieving the task of finding the best alternative in this scenario is equivalent when using the combination of *RAWEC & N1*, *RAWEC & N3*, and when using *TOPSIS*, *COPRAS*, and *FUCA* methods. Conversely, other combinations such as *RAWEC & N2*, *RAWEC & N4*, and *RAWEC & N5* do not yield desired results. In this case, a preliminary conclusion can be drawn that *N1* and *N3* are suitable for combining with *RAWEC*. It is worth noting that *N1* is the method available within *RAWEC*.

CASE 2: Ranking alternatives when all criteria are *B* form

Ranking of seven personnel in a garment company was conducted in this scenario. All five criteria describing each individual are in *B* form. These five criteria include work experience, language proficiency, problem-solving skills, communication skills, and teamwork abilities. These criteria are denoted as *C1*, *C2*, *C3*, *C4*, and *C5*, respectively. The values of the criteria for each individual and the weights of the criteria have been compiled in Table 3 [22, 39].

Table 3: Personnel data [22, 39].

No.	C1	C2	C3	C4	C5
A1	2	110	3	2	3
A2	5	100	5	3	3
A3	3	90	4	5	2
A4	10	80	3	4	4
A5	4	85	2	4	5
A6	8	80	3	4	4
A7	5	95	2	4	3
Weight	0.257	0.129	0.214	0.196	0.204

The task of *MCDM* is to find an individual with the highest values in all five criteria. In published studies, this task has also been performed using the *CODAS* and *PSI* methods [39], and the *CRADIS* method [22].

In Figure 2, the ranking results of personnel have been synthesized using the combination of data normalization methods with the *RAWEC* method, as well as with other *MCDM* methods.

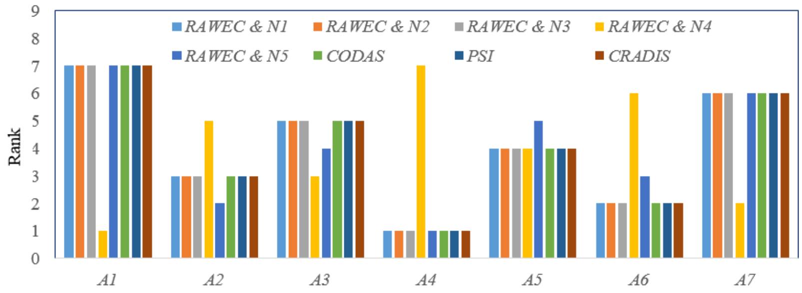

Figure 2: Ranking of alternatives in case 2

The results indicate that when using combinations of *RAWEC & N1*, *RAWEC & N2*, *RAWEC & N3*, *RAWEC & N5*, along with the *CODAS*, *PSI*, *CRADIS* methods, all identify *A4* as the best personnel. Conversely, using the combination of *RAWEC & N4* does not yield the desired results. Therefore, it can be concluded that in this case, *N1*, *N2*, *N3*, and *N5* are suitable for combining with *RAWEC*.

CASE 3: Ranking of alternatives when the number of *C* form criteria is greater than the number of *B* form criteria

In this scenario, the ranking of ten locations for establishing agricultural produce distribution centers is conducted. Out of the ten criteria for evaluating the alternatives, seven criteria are in *C* form and three criteria are in *B* form.

Table 4: Data on locations for establishing agricultural produce distribution centers [25].

No.	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
	B	B	C	C	C	C	C	C	B	C
A1	5.8	35.9	2.6	10	3.2	75	35	8	6.6	3.4
A2	6.6	36	2.6	25	3	70	25	5	6.4	4.2
A3	3.6	20.8	4.8	5	4.8	70	35	10	3.4	4.6
A4	5.4	37	4.6	6	4	90	35	10	3.2	4.2
A5	5.8	105	4.4	6	3.6	70	40	15	5.8	3.8
A6	4.6	88	4.2	6	3.8	70	30	15	4.4	4.6
A7	7.2	170	2.6	15	2.6	80	40	5	7	4.2
A8	7.4	21	1.6	120	2.4	80	20	3	7.4	4.8
A9	6	36	3.6	6	3.4	70	40	15	6	4.4
A10	5	44	4.6	6	4	70	25	11	3.4	4.2

These ten criteria include Feasibility of multimodal transit, Scale and adaptability, Vicinity to clientele and suppliers, Land expenditures, Transportation overheads, Adjacency to air hub, Proximity to freeway, Accessibility to rail lines, Developed infrastructure facilities, and Ecological ramifications. The values of the criteria, their forms (*B* form or *C* form), and their weights have been compiled in Table 4 [25].

Please note that the data in Table 4 is cited from reference [25], where the ranking of locations for establishing agricultural produce distribution centers was conducted using the combination of *RAWEC & N*1 and some *MCDM* methods such as *MABAC*, *TOPSIS*, *WASPAS*. Figure 3 illustrates the ranking results of the alternatives using different methods.

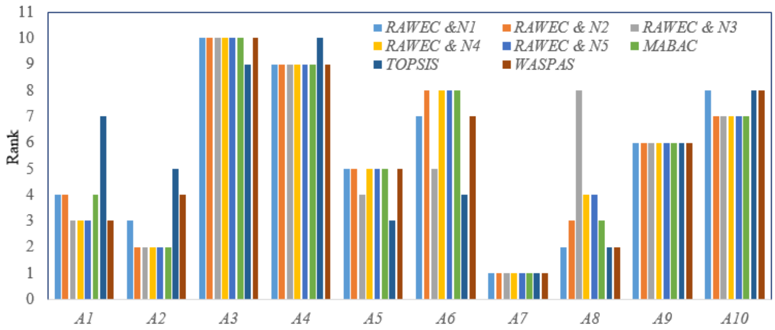

Figure 3: Ranking of alternatives in case 3

All five combinations *RAWEC & N1*, *RAWEC & N2*, *RAWEC & N3*, *RAWEC & N4*, and *RAWEC & N5*, when used, indicate that *A7* is the best location for distributing agricultural produce. This result is also consistent with the findings when using the *MABAC*, *TOPSIS*, and *WASPAS* methods [25]. Therefore, in this scenario, all five normalization methods *N1*, *N2*, *N3*, *N4*, and *N5* are evaluated as suitable for combining with *RAWEC*.

CASE 4: Ranking of alternatives when the number of *B* form criteria is greater than the number of *C* form criteria

Table 5: Specifications of ten types of electric bicycles [40].

No	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
	C	B	C	B	B	B	B	B	B	B
	Vietnam dong	km	hour	km/h	kg	kg	mm	mm	mm	mm
A1	7500000	45	7	35	56	160	750	1593	635	1015
A2	7900000	45	7	35	50	150	750	1640	640	1200
A3	9900000	50	7	35	50	180	750	1640	640	1200
A4	9900000	50	7	35	46	180	750	1640	640	1200
A5	11500000	50	7	35	52	180	750	1640	640	1200
A6	13990000	45	7	30	45	75	550	1550	650	1040
A7	13990000	45	8	30	45	75	600	1530	750	1000
Weight	1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10

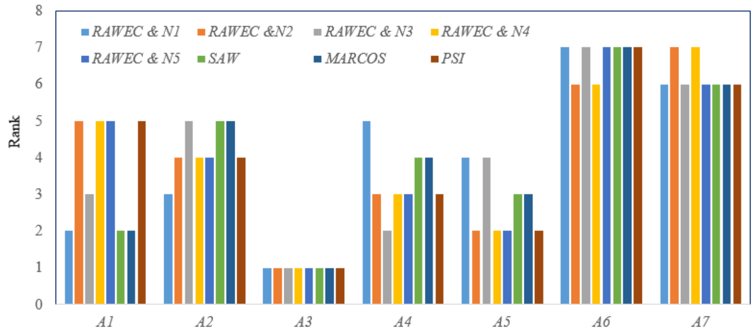

Figure 4: Ranking of alternatives in case 4

In this scenario, the ranking of alternatives is conducted with the subject being seven types of electric bicycles. Ten criteria for each alternative include Price, Range per charge, Charging time, Maximum speed, Weight of bicycle, Payload capacity, Saddle height, Overall length of bicycle, Overall width of bicycle, Overall height of bicycle. Among these ten criteria, eight are in *B* form, and two are in *C* form. The units, values, and forms of each criterion have been compiled in Table 5.

In Figure 4, the ranking chart of seven electric bicycles using five combinations *RAWEC & N1*, *RAWEC & N2*, *RAWEC & N3*, *RAWEC & N4*, *RAWEC & N5*, and using the *SAW*, *MARCOS*, and *PSI* methods is presented [40]. *A3* has been identified as the best electric bicycle when ranked by all different methods. This indicates that all five normalization methods *N1*, *N2*, *N3*, *N4*, and *N5* are considered suitable for combining with *RAWEC*.

After conducting the four scenarios above, the compatibility of combining data normalization methods with *RAWEC* is summarized in Table 6.

Table 6: Compatibility of combining data normalization methods with RAWEC

Case	RAWEC & N1	RAWEC & N2	RAWEC & N3	RAWEC & N4	RAWEC & N5
Case 1	yes	No	yes	No	No
Case 2	yes	yes	yes	No	yes
Case 3	yes	yes	yes	yes	yes
Case 4	yes	yes	yes	yes	yes

One observation is that in all four cases conducted, only *N1* and *N3* are consistently evaluated as suitable for combining with *RAWE*C. *N3*, which is the vector normalization type, can be applied to normalize the values of criteria in all cases (refer to equations (9) and (10)). The combination of *RAWEC & N3* has been confirmed to be accurate in ranking alternatives, indicating that using *N3* as a replacement for *N1* when at least one value x_{ij} equals zero is a valuable discovery. This discovery could mitigate the limitations of the *RAWEC* method. To verify this result, another example will be conducted immediately.

CASE 5: Ranking alternatives when the normalization method *N1* cannot be used.

A numerical example is generated as shown in Table 7. Four alternatives to be ranked are *A1*, *A2*, *A3*, and *A4*. Four criteria considered are *C1*, *C2*, *C3*, and *C4*. The type and value of each criterion in each alternative have been synthesized in Table 7.

In this case, a scenario was deliberately created where the *N1* method could not be used, namely by assigning the value of criterion *C1* at *A1*, criterion *C2* at *A4*, and criterion *C3* at *A4* to zero. Here's the proof that method *N1* cannot be applied in this case. Applying formulas (2) and (3) to calculate the normalized values n_{ij} and n'_{ij} using method *N1*, the data is illustrated in Table 8.

Table 7: Example of Case 5

No.	C1	C2	C3	C4
	C	C	B	B
A1	0	2	5	4
A2	3	2	6	4
A3	3	2.5	6	3
A4	4.5	0	0	3

Table 8: Values of n_{ij} and n'_{ij} calculated using method N1 for case 5

No.	n_{ij}				n'_{ij}			
	C1	C2	C3	C4	C1	C2	C3	C4
A1	0/0	0/2	5/6	4/4	0/4.5	2/2.5	0/5	3/4
A2	0/3	0/2	6/6	4/4	3/4.5	2/2.5	0/6	3/4
A3	0/3	0/2.5	6/6	3/4	3/4.5	2.5/2.5	0/6	3/3
A4	0/4.5	0/0	0/6	3/4	4.5/4.5	0/2.5	0/0	3/3

In Table 8, it is evident that the quantities n_{11}, n_{42}, and n'_{43} are meaningless. This clearly demonstrates that applying method *N1* cannot normalize the data in this case. The use of the *RAWEC* method can only be done when combined with the *N3* method. The combination of *RAWEC & N3* is employed in this scenario with five different scenarios. Generating these different scenarios is achieved by changing the weights of the criteria as shown in Table 9.

Table 9: Weights of criteria in different scenarios

Scenario	C1	C2	C3	C4
S1	1/4	1/4	1/4	1/4
S2	0.5	0.5/3	0.5/3	0.5/3
S3	0.5/3	0.5	0.5/3	0.5/3
S4	0.5/3	0.5/3	0.5	0.5/3
S5	0.5/3	0.5/3	0.5/3	0.5

In each scenario, the *ROV*, *FUCA*, *TOPSIS*, *MOORA*, and *PIV* methods have also been used to rank the alternatives. The *ROV, TOPSIS*, *MOORA*, and *PIV* methods were chosen because the normalization methods they provide can be used when x_{ij} equals zero. The *FUCA* method was also employed because its application does not require data normalization. The Spearman rank correlation coefficient has also been utilized to compare the alternatives in each scenario. This coefficient is calculated according to equation (15) [41, 42].

$$S = 1\frac{6\sum_{i=1}^{m} D_i^2}{2(m^2-1)} \quad (15)$$

In equation (15), D_i represents the difference in ranking of alternative *i* when ranked by different methods, where *m* is the number of alternatives to be ranked. The rankings of the alternatives by different methods as well as the values of the Spearman coefficient in each scenario are listed below.

In scenario 1, the rankings of the alternatives are completely identical when ranked using all five methods, which include the combination of *RAWEC & N1*, *ROV*, *FUCA*, *MOORA*, and *PIV* (Table 10). The correlation coefficient *S* always equals 1 when comparing any two methods with each other. Therefore, in this scenario, it demonstrates that *N3* is entirely suitable to be combined with *RAWEC*.

Table 10: Ranking of alternatives for scenario S1

No.	RAWEC & N3	ROV	FUCA	TOPSIS	MOORA	PIV
A1	1	1	1	1	1	1
A2	2	2	2	2	2	2
A3	3	3	3	3	3	3
A4	4	4	4	4	4	4

Table 11: Ranking of alternatives for scenario S2

No.	RAWEC & N3	ROV	FUCA	TOPSIS	MOORA	PIV
A1	1	1	1	1	1	1
A2	2	2	2	2	2	2
A3	3	3	3	3	3	3
A4	4	4	4	4	4	4

All achievements in scenario 2 are identical to scenario 1. The correlation coefficient *S* remains equal to 1 when comparing any two methods with each other. Therefore, in this scenario as well, it indicates that *N3* is entirely suitable for combination with *RAWEC*.

Table 12: Ranking of alternatives for scenario S3

No.	RAWEC & N3	ROV	FUCA	TOPSIS	MOORA	PIV
A1	2	1	2	2	2	2
A2	3	3	1	3	3	3
A3	4	4	4	4	4	4
A4	1	2	3	1	1	1

Table 13: Spearman coefficient for scenario S3

Method	RAWEC & N3	ROV	FUCA	TOPSIS	MOORA	PIV
RAWEC & N3	1	0.8	0.2	1	1	1
ROV		1	0.4	0.8	0.8	0.8
FUCA			1	0.2	0.2	0.2
TOPSIS				1	1	1
MOORA					1	1
PIV						1

In scenario 3, the rankings of the alternatives when ranked by the combination of *RAWEC & N3* are identical to those when using three methods: *TOPSIS*, *MOORA*, and *PIV*. Additionally, the Spearman correlation coefficient between *RAWEC & N3* and *ROV* is 0.8, indicating that the rankings of the alternatives when ranked by the combination of *RAWEC & N3* change very little compared to when ranked by the *ROV* method. The significant difference in the rankings of the alternatives when ranked by the *FUCA* method compared to other methods can be attributed to the fact that the *FUCA* method does not perform data normalization [43, 44]. All of these observations lead us to conclude that *N3* is suitable for combining with *RAWEC*.

Table 14: Ranking of alternatives for scenario S4

No.	RAWEC & N3	ROV	FUCA	TOPSIS	MOORA	PIV
A1	1	1	2	1	1	1
A2	2	2	1	2	2	2
A3	3	3	3	3	3	3
A4	4	4	4	4	4	4

Table 15: Spearman coefficient for scenario S4

Method	RAWEC & N3	ROV	FUCA	TOPSIS	MOORA	PIV
RAWEC & N3	1	1	0.8	1	1	1
ROV		1	0.8	1	1	1
FUCA			1	0.8	0.8	0.8
TOPSIS				1	1	1
MOORA					1	1
PIV						1

In scenario 4, the rankings of the alternatives when ranked by the combination of *RAWEC & N3* are identical to those when using the *ROV*, *TOPSIS*, *MOORA*, and *PIV* methods. The Spearman correlation coefficient between *RAWEC & N3* and *FUCA* is also 0.8. Therefore, *N3* is also concluded to be suitable for combining with *RAWEC* in this scenario.

Table 16: Ranking of alternatives for scenario S5

No.	RAWEC & N3	ROV	FUCA	TOPSIS	MOORA	PIV
A1	1	1	1	1	1	1
A2	2	2	1	2	2	2
A3	3	3	3	3	3	3
A4	4	4	4	4	4	4

Table 17: Spearman coefficient for scenario S5

Method	RAWEC & N3	ROV	FUCA	TOPSIS	MOORA	PIV
RAWEC & N3	1	1	0.9	1	1	1
ROV		1	0.9	1	1	1
FUCA			1	0.9	0.9	0.9
TOPSIS				1	1	1
MOORA					1	1
PIV						1

Once again, the rankings of the alternatives when ranked by the combination of *RAWEC & N3* are identical to those when using the *ROV*, *TOPSIS*, *MOORA*, and *PIV* methods. The Spearman correlation coefficient between *RAWEC & N3* and *FUCA* is also very high, at 0.9. Therefore, *N3* is also concluded to be suitable for combining with *RAWEC* in this scenario.

Comparing the ranking results of the alternatives when using the combination of *RAWEC & N3* with other *MCDM* methods in the five scenarios above allows us to come to a definite conclusion that *N3* is entirely suitable for combining with *RAWEC*. This discovery is highly significant in expanding the scope of application of the *RAWEC* method. When there are no elements equal to zero in the decision matrix, data normalization can use either *N1* or *N3*. Conversely, if there is at least one element in the decision matrix equal to zero, using *N1* becomes infeasible. In this case, users can resort to *N3* as a replacement while still being assured of the accuracy of the ranking of alternatives.

CONCLUSION

If there is at least one zero element in the decision matrix, the normalization method *N1* available in the *RAWEC* method cannot be used. The use of data normalization method *N3* in combination with the *RAWEC* method opens up a more flexible and reliable approach for situations where the decision matrix contains zero elements. In cases where the matrix has no zero elements, users can confidently use either *N1* or *N3* for data normalization. However, in the presence of zero elements, applying *N1* becomes unfeasible. In such instances, users can employ *N3* as a substitute while ensuring the accuracy of the results. This research has contributed to expanding the scope of application of the *RAWEC* method.

In this study, the situation where elements in the decision matrix are fuzzy sets has not been considered. In the context where elements in the decision matrix are fuzzy sets, whether method *N3* is still suitable for combination with the *RAWEC* method remains a question to be answered. This can be addressed by developing a *fuzzy-RAWEC* method in future research.

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