Yugoslav Journal of Operations Research
xx (20xx), Number xx, xxx–xxx
DOI: https://doi.org/10.2298/YJOR240316025S

A NOVEL APPROACH FOR SOLVING THE TRIANGULAR FUZZY NEUTROSOPHIC ASSIGNMENT PROBLEM

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Received: March 2024 / Accepted: June 2024

Abstract: The assignment problem (AP) is a fundamental challenge in linear programming and operations research focused on optimizing assignments to minimize costs or maximize profits. This study extends the traditional AP to address uncertainties and ambiguities through the neutrosophic assignment problem (NAP). Further advancing this concept, the triangular fuzzy neutrosophic assignment problem (TFNAP) is introduced, utilizing triangular fuzzy neutrosophic numbers (TFNNs) to represent fuzziness and neutrosophy, thus offering a more comprehensive depiction of uncertainty and indeterminacy. The primary aim of this research is to identify the optimal assignment that either minimizes cost or maximizes profit within the TFNAP framework, a task known for its computational complexity. The TFNAP is represented through a triangular fuzzy neutrosophic assignment matrix (TFNAM), which employs TFNNs as its core elements. This study develops and refines algorithms tailored to address these complexities, ensuring originality through a stepwise procedure that simplifies computations in a neutrosophic manner. The methodology includes resolving the problem using a neutrosophic approach and incorporating a score function to convert triangular fuzzy neutrosophic values to their equivalent crisp numbers for comparative purposes. The proposed method's effectiveness is validated through its application to real-world problems, with the results compared against previously established solutions. The findings demonstrate that the TFNAP framework provides more accurate and insightful outcomes in dealing with uncertainties compared to traditional methods. This study introduces significant innovations in handling ambiguity and indeterminacy in assignment problems, offering a robust tool for optimization in complex, uncertain environments. Conclusively, the developed approach not only enhances the understanding of neutrosophic assignments but also presents a practical solution for real-world applications.

Keywords: Neutrosophic number, triangular fuzzy neutrosophic number, triangular fuzzy Neutrosophic Assignment problem, score function, range, optimal solution.

MSC: 03E72, 90B99.

1. INTRODUCTION

In the course of our daily lives, we frequently encounter a range of vague, confusing, and inadequate situations. As a result, Zadeh [1] proposed the idea of fuzzy sets (FS) in 1965 as an extension of classical sets that permits partial membership, i.e., assigns a membership grade for each element. The fuzzy set theory has had remarkable success in a variety of domains because of its capacity to handle inconsistency. In 1983, Atanassov [2] developed the concept of intuitionistic fuzzy sets(IFS) as an enhancement of fuzzy sets. Due to certain constraints, these sets include both the membership grade and the non-membership grade of each element. The membership grades for each element's truth (T), indeterminacy (I), and falsity (F) are included in neutrosophic sets (NS), an extension of intuitionistic fuzzy sets. In 1995, Smarandache [3] first put forward the concept.

Numerous neutrosophic theories and applications have been established as a consequence of the introduction of neutrosophic numbers (NNs), including those in the areas of mathematics, decision-making, artificial intelligence, and linguistics. It offers a more adaptable and thorough framework to deal with ambiguous and imprecise data, making it useful in a variety of academic fields and real-world problem-solving. Fuzzy sets, neutrosophic sets, and triangular fuzzy numbers are all combined in the idea of triangular fuzzy neutrosophic numbers (TFNNs), bringing about a more comprehensive and customizable framework for managing uncertainty and ambiguity in modeling and decision-making. It is more expressive and flexible to real-world issues owing to its hybrid character. It is an effective tool for handling intricate and unclear data. De and Nandi [4] developed the exact defuzzification method under polynomial approximation of various fuzzy sets. A promising approach for decision modeling with single-valued neutrosophic probabilistic hesitant fuzzy Dombi operators was provided by Kamran et al [5]. Biswas [6] aggregated the triangular fuzzy neutrosophic set information and applied it to MADM. An extended MABAC method for multiple-criteria group decision making problems based on triangular fuzzy neutrosophic numbers was introduced by Irvanizam [7]. Das and Edalatpanah [8] proposed a new ranking function of triangular neutrosophic number and applied it in integer programming. Geetha and Narayanamoorthy [9] solved the MCDM problems by employing superiority and inferiority ranking method with hesitant Pythagorean fuzzy sets. An integrating model with DEMATEL and VIKOR methods was proposed by Narayanamoorthy et al [10] to identify and evaluate the criteria and alternatives in the selection of renewable energy resources. A score function was devised by Chakraborty et al [11] and was applied to defuzzify triangular fuzzy neutrosophic numbers to tackle neutrosophic assignment challenges as well as a few other issues.

Research on the transportation problem and travelling salesman problem has focused on developing efficient algorithms and methodologies to solve large-scale instances and adapt to real-world complexities. The Dhouib-matrix-TP1 heuristic was first put forward by Dhouib [12] to address the trapezoidal fuzzy transportation difficulties. Sikkannanl and Shanmugavel [13] answered the fuzzy transportation problems using ECCT and standard deviation. Narayanamoorthy and Kalyani [14] proposed a new method for obtaining the initial basic feasible solution of a fuzzy transportation problem. Pratihar [15] investigated the transportation issue in a neutrosophic setting. Using the Ones Assignment Method, Subasri and Selvakumari [16] answered the Neutrosophic Travelling Salesman Problem in triangular fuzzy numbers. Employing the Dhouib-Matrix-TSP1 Heuristic, Dhouib [17] optimized the travelling salesman problem for single-valued triangular neutrosophic numbers. The Neutrosophic Travelling Salesman Problem built on trapezoidal fuzzy numbers was solved by Subasri and Selvakumari [18] utilizing the Branch and Bound technique. Dhouib [19] fixed the travelling salesman problem in triangular neutrosophic numbers.

The American mathematician Harold W. Kuhn originally stated and addressed the assignment issue in 1955. Determining the most effective and economical approach to match each task with a resource is the ultimate goal of the assignment problem, which aims to minimize or maximize the overall cost or benefit. The assignment issue was a ground-breaking research project by Kuhn that later became a central theme in the field of operations research and optimization. Due to its extensive applicability and significance in several real-world settings, including workforce scheduling, transportation optimization, project planning, and more, the assignment issue has drawn a lot of attention. In addition to the multiple locations already indicated, the AP can also be resolved using the neutrosophic numbers. Due in large part to Kuhn's outstanding work, several techniques and approaches have been developed to effectively tackle the assignment problem. As a result of the triangular fuzzy neutrosophic numbers' ability to represent a wide range of uncertainty (mentioned above), it may be utilized in a variety of activities in a multitude of fields, including decision-making, artificial intelligence, and expert systems. This sparks curiosity and encourages exploration and experimentation with the triangular fuzzy neutrosophic assignment issue. Afroz and Hossen [20] adopted the Divide Column and Subtract One Assignment Method for solving the assignment problem. Mass - A New Ones Assignment Method was implemented by Esakkiammal and Murugesan [21] for finding the optimal solution of assignment problems. Kar and Shaw [22] constructed a new approach to obtain the optimal solution of triangular fuzzy assignment problem using Hungarian method. The fully fuzzy assignment problem was resolved by Muruganandam and Hema [23] utilizing Branch and Bound Technique. Dhouib [24] unravelled the intuitionistic triangular fuzzy assignment problem by the novel heuristic Dhouib-Matrix-AP1. Lone et al [25] handled the intuitionistic fuzzy assignment problem and applied it to agriculture. The intuitionistic fuzzy assignment problem was fixed by a method involving Branch and Bound method by Srinivas and Ganesan [26]. Bera and Mahapatra [27] explored the neutrosophic assignment problem by offering its solution methodology. Prabha and Vimala [28] evaluated the neutrosophic assignment problem through the BnB algorithm.

The applications of TFNNs are indeed diverse, reflecting their versatility and effectiveness in complex decision-making and analysis tasks. By integrating TFNNs into different domains, organizations and researchers can achieve more accurate, reliable, and informed results. The neutrosophic MAGDM was dealt by Mallick et al [29] based on critic-EDAS strategy using geometric aggregation operator. Abdel-Basset et al [30] designed a novel group decision-making model based on triangular neutrosophic numbers. The triangular single-valued neutrosophic data was analyzed and applied by Yang [31] to hospital performance measurement. Hamza et al [32] studied cryptography interms of triangular neutrosophic numbers with real life applications. Xie [33] developed the Modified GRA methodology for MADM under triangular fuzzy neutrosophic sets and applied it to blended teaching effect evaluation of college English courses.

Here comes a small discussion about the limitations along with the gaps of the existing algorithms related to this study, which has lead to the need for using the proposed algorithm for solving the triangular fuzzy neutrosophic assignment problem (TFNAP): The current literature on the assignment problem (AP) under fuzzy and neutrosophic environments have several limitations that create gaps in their applicability and effectiveness and needs more comprehensive exploration, particularly when dealing with the complexities of triangular fuzzy neutrosophic assignment problems (TFNAP) (which requires utilizing the advanced representation capabilities of triangular fuzzy neutrosophic numbers (TFNNs)) as given below:

Limitations and Gaps in the Existing Research:

- Handling Combined Uncertainty and Indeterminacy.
- Scalability and Efficiency.
- Adaptability to Real-World Variability.
- Accuracy and Optimality.
- Integration of Triangular Fuzzy Neutrosophic Numbers.
- Comparative Analysis and Validation.
- Application to Diverse Domains.

The above limitations and gaps highlight the need for developing a more robust and tailored algorithm. Hence, this research aims to address the aforementioned gap by investigating the assignment problem within a neutrosophic framework and leveraging the distinctive features of TFNNs to develop a more efficient and precise solution methodology.

Need for the Proposed Algorithm/Objectives: To address these limitations and gaps, the proposed algorithm for solving the triangular fuzzy neutrosophic assignment problem (TFNAP) is designed with the following objectives:

- **Comprehensive Modeling of TFNNs:** To offer unique features of TFNNs for assignment problems, effectively managing combined uncertainty and indeterminacy for accurate real-world representation.
- **Improved Efficiency:** The algorithm aims to enhance scalability and efficiency for large-scale applications through optimized computational processes and advanced techniques.
- **Develop and Propose TFNAP Methodology:** To formulate various kinds of TFNAPs and present a step-by-step algorithm to solve these problems using TFNNs.
- **Compare and Validate:** To formulate the TFNAP as a matrix called the TFNAM, apply the method for the optimal solution, and compare results to showcase efficacy and improvements.

Contribution: This research paper contributes to the field in a few significant ways:

- **Novel Formulation:** The TFNAP is formulated as a matrix with TFNNs, providing a more comprehensive representation of uncertainty and indeterminacy in assignment problems.
- **Proposed Algorithm:** An algorithm is developed to solve TFNAPs, employing a score function to compare TFNNs and derive the optimal solution efficiently.
- **Practical Application:** The considered TFNAM is solved using the suggested method, and the results are compared to earlier findings, demonstrating the practical advantages and improvements over traditional approaches.
- **Enhanced Decision-Making:** By applying the proposed methodology, this research will enhance decision-making processes (like the illustration solved in this paper, which itself is a decision-making problem) in various domains, including logistics, resource allocation, and project management, where uncertainty and indeterminacy are prevalent.

By addressing these critical areas, the proposed algorithm aims to fill the gaps in the existing literature and overcome the limitations of current approaches, achieves its objectives of developing and validating a more robust and effective approach and solution for triangular fuzzy neutrosophic assignment problems, and makes substantial contributions to the field of optimization under uncertainty. The paper is structured as follows: Section 1 comprises the abstract and introduction. In Section 2's Preliminaries section, we provide a few basic definitions. In Section 3, we discuss neutrosophic numbers (NNs), triangular fuzzy neutrosophic numbers, and both their properties and operations. The triangular fuzzy neutrosophic assignment issue is addressed in Section 4, along with its traits, kinds, and solutions, as well as the necessary score function. In Section 5, a technique is offered for tackling the triangular fuzzy neutrosophic assignment problem. This procedure requires defuzzifying the neutrosophic data and employing the recommended algorithm step-by-step to arrive at the optimal solution. In Section 6, the proposed approach (PA) for solving the triangular fuzzy neutrosophic assignment issue is demonstrated. This article is concluded in Section 7, which highlights the key observations and outcomes.

2. PRELIMINARIES

Fuzzy set: Let X be a non-empty set. A fuzzy set H in X is characterized by its membership function $\mu_H : X \longrightarrow [0,1]$ and $\mu_H(x)$ is interpreted as the degree of membership of element x in fuzzy set H, for each $x \in X$, given by,

$$H = \{(x, \mu_H(x)) : x \in X\}$$

Fuzzy number: The fuzzy set H defined on the set of real numbers is said to be a fuzzy number(FN) if H and its membership function $\mu_H(X)$ has the following properties:

1. H is normal and convex
2. H is bounded
3. $\mu_H(X)$ is piece - wise continuous

Neutrosophic set: Let X be a non empty set. A Neutrosophic set $H \in X$ is of the form $H = \{(x, T_H(x), I_H(x), F_H(x)) : x \in X\}$, where the functions $T_H, I_H, F_H : X \longrightarrow ^{-}]0,1[^{+}$ define respectively the degree of truth membership, the degree of indeterminacy and the degree of falsity membership for every element $x \in X$ to the set H, which is a subset of X.

$$^{-}0 \leq T_H(x) + I_H(x) + F_H(x) \leq 3^{+}.$$

3. NEUTROSOPHIC NUMBERS AND ITS PROPERTIES

3.1. Neutrosophic numbers

A neutrosophic set H defined on the universal set of real numbers R is called a neutrosophic number, if it has the following properties:

1. H is normal if there exists $x_0 \in R$, such that $T_H(x_0) = 1$, $I_H(x_0) = F_H(x_0) = 0$.
2. H is a convex set for the truth function $T_H(x)$, i.e., $T_H(\mu x_1 + (1-\mu)x_2) \geq min(T_H(x_1), T_H(x_2))$, $\forall x_1, x_2 \in R$ and $\mu \in [0,1]$.

3. H is a concave set for the indeterministic function and false function $I_H(x)$ and $F_H(x)$, i.e., $I_H(\mu x_1 + (1-\mu)x_2) \geq max(I_H(x_1), I_H(x_2))$, $\forall x_1, x_2 \in R$ and $\mu \in [0,1]$ and $F_H(\mu x_1 + (1-\mu)x_2) \geq max(F_H(x_1), F_H(x_2))$, $\forall x_1, x_2 \in R$ and $\mu \in [0,1]$.

Remark 1. *If H is a neutrosophic set in a non empty set X, then for convenience, we denote a neutrosophic number by, $H = (T_H(x), I_H(x), F_H(x))$.*

3.1.1. Properties of Neutrosophic numbers

Let $G, H \in X$. Then their operations are defined as,

1. $(T_G(x), I_G(x), F_G(x)) + (T_H(x), I_H(x), F_H(x)) = (T_G(x)+T_H(x)-T_G(x)T_H(x), I_G(x)I_H(x), F_G(x)F_H(x))$
2. $(T_G(x), I_G(x), F_G(x)).(T_H(x), I_H(x), F_H(x)) = (T_G(x)T_H(x), I_G(x)+I_H(x)-I_G(x)I_H(x), F_G(x) + F_H(x) - F_G(x)F_H(x))$
3. $k(T_G(x), I_G(x), F_G(x)) = (1-(1-T_G(x))k, I_G(x)k, F_G(x)k), (k \in R)$
4. $(T_G(x), I_G(x), F_G(x))k = (T_G(x)k, 1-(1-I_G(x))k, 1-(1-F_G(x))k), (k \in R)$

3.2. Triangular fuzzy neutrosophic number

Let X be the universal set and let the set of all triangular fuzzy numbers on $[0,1]$ be denoted by $F[0,1]$. A triangular fuzzy neutrosophic set, H in X is written as,

$$H = \{x : (T_H(x), I_H(x), F_H(x)), x \in X\},$$

where, $T_H(x), I_H(x), F_H(x) : X \longrightarrow F[0,1]$. The triangular fuzzy numbers $T_H(x) = (T_H^1(x), T_H^2(x), T_H^3(x))$, $I_H(x) = (I_H^1(x), I_H^2(x), I_H^3(x))$ and $F_H(x) = (F_H^1(x), F_H^2(x), F_H^3(x))$ denote respectively the truth-membership, indeterminacy-membership and falsity-membership of x in H and for every $x \in X$, $0 \leq T_H^3(x) + I_H^3(x) + F_H^3(x) \leq 3$. For convenience, we indicate the triangular fuzzy neutrosophic number H as, $H = ((h_1, h_2, h_3), (h_4, h_5, h_6), (h_7, h_8, h_9))$, where, $(T_H^1(x), T_H^2(x), T_H^3(x)) = (h_1, h_2, h_3)$, $(I_H^1(x), I_H^2(x), I_H^3(x)) = (h_4, h_5, h_6)$, $(F_H^1(x)$ and $F_H^2(x), F_H^3(x)) = (h_7, h_8, h_9)$.

3.2.1. Operations on Triangular fuzzy neutrosophic numbers

Let $G = ((g_1, g_2, g_3), (g_4, g_5, g_6), (g_7, g_8, g_9))$ and $H = ((h_1, h_2, h_3), (h_4, h_5, h_6), (h_7, h_8, h_9))$ be two triangular fuzzy neutrosophic numbers in the set of real numbers and $\lambda > 0$. Then, the operations involving them are listed as follows:

1. $G + H = ((min(g_1 + h_1, U_l), min(g_2 + h_2, U_l), min(g_3 + h_3, U_l)), (min(g_4 + h_4, U_l), min(g_5 + h_5, U_l), min(g_6 + h_6, U_l)), (min(g_7 + h_7, U_l), min(g_8 + h_8, U_l), min(g_9 + h_9, U_l)))$.
2. $G - H = ((max(g_1 - h_3, L_l), max(g_2 - h_2, L_l), max(g_3 - h_1, L_l)), (max(g_4 - h_6, L_l), max(g_5 - h_5, L_l), max(g_6 - h_4, L_l)), (max(g_7 - h_9, L_l), max(g_8 - h_8, L_l), max(g_9 - h_7, L_l)))$.
3. $-G = ((-g_3, -g_2, -g_1), (-g_6, -g_5, -g_4), (-g_9, -g_8, -g_7))$.
4. $\lambda G = ((\lambda g_1, \lambda g_2, \lambda g_3), (\lambda g_4, \lambda g_5, \lambda g_6), (\lambda g_7, \lambda g_8, \lambda g_9))$,

where, L_l and U_l denote the lower limit and the upper limit of the considered triangular fuzzy neutrosophic number.

4. TRIANGULAR FUZZY NEUTROSOPHIC ASSIGNMENT PROBLEM (TFNAP)-ATTRIBUTES, KINDS AND SOLUTIONS

4.1. Attributes of the TFNAP

4.1.1. Description of TFNAP

The assignment issue is a particular instance of the linear programming problem, and its primary objective is to assign m resources (often workers/people) to n tasks (typically jobs) in such a way as to minimize the total assignment cost or maximize the total assignment profit. In other words, "Given m persons, n jobs, and the effectiveness of each person for each job, the problem is to assign every resource to one and only one job in such a way that the measure of effectiveness is optimized (maximized or minimized)". If there are as many people as there are jobs, the situation is/becomes fixed. The assignment problem is referred to as a neutrosophic assignment problem if it is examined in a neutrosophic setting. Additionally, the problem can more exactly be called as a triangular fuzzy neutrosophic assignment problem if the components of the specified neutrosophic assignment problem are triangular fuzzy neutrosophic numbers.

4.1.2. A few basic notations

1. m - number of persons/workers $(i = 1, 2, ...m)$.
2. n - number of jobs $(j = 1, 2, ...n)$.
3. c_{ij}^N - neutrosophic unit cost of assigning worker i to job j.
4. x_{ij} - worker i assigned to job j (1 if assigned, 0 otherwise).

4.1.3. Mathematical formulation of TFNAP

The Triangular Fuzzy Neutrosophic Assignment Problem (TFNAP) is a type of assignment problem that deals with assigning tasks to agents under conditions of uncertainty, imprecision, and indeterminacy. It is a complex assignment problem that utilizes the concepts of fuzzy logic, neutrosophic sets and triangular fuzzy neutrosophic numbers, extending the traditional assignment problem to better model real-world scenarios in cost evaluations where data may be vague or incomplete. This formulation aids in making more robust and flexible decisions by accounting for the truth, indeterminacy, and falsity in cost evaluations. **Parameters -** $c_{ij}^N = (c_{ij}^{(T)^N}, c_{ij}^{(I)^N}, c_{ij}^{(F)^N})$: Triangular fuzzy neutrosophic cost of assigning task j to agent i.

Decision Variables - x_{ij}: Binary variable indicating whether task j is assigned to agent i (1 if assigned, 0 otherwise).

Objective Function: The objective is to minimize the overall fuzzy neutrosophic cost of assignments, ensuring each task is assigned exactly once and each agent handles at most one task, which can be expressed as:

$$\text{Minimize} \quad Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^N x_{ij}$$

This involves minimizing the aggregate cost considering the truth, indeterminacy, and falsity components. **Constraints:**
1. Assignment Constraints: Ensure each task is assigned to exactly one agent.

$$\sum_{i=1}^{m} x_{ij} = 1, \quad \forall j = 1, 2, \ldots, n$$

2. Capacity Constraints: Ensure each agent is assigned to at most one task.

$$\sum_{j=1}^{n} x_{ij} \leq 1, \quad \forall i = 1, 2, \ldots, m$$

3. Binary Constraints: Ensure x_{ij} is binary.

$$x_{ij} \in \{0, 1\}, \quad \forall i = 1, 2, \ldots, m, \forall j = 1, 2, \ldots, n$$

Hence, the TFNAP can be formulated mathematically as follows:

$$MinZ = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^{N} \; x_{ij},$$

subject to,

$$\sum_{i=1}^{n} x_{ij} = 1, \quad \forall j = 1, 2, ...n$$

$$\sum_{j=1}^{m} x_{ij} \leq 1, \quad \forall i = 1, 2, ...m$$

where, $x_{ij} \in \{0, 1\}, \quad \forall i = 1, 2, \ldots, n, \quad \forall j = 1, 2, \ldots, m$ and all the components c_{ij}^{N} are triangular fuzzy neutrosophic numbers. TFNAP models the assignment of tasks to agents while dealing with uncertainties in the cost estimates through the use of TFNNs, that seeks to find the optimal assignment that minimizes the total neutrosophic cost, respecting the constraints. This approach is beneficial in scenarios where costs are not precisely known. This problem can be solved using advanced optimization techniques that handle fuzzy and neutrosophic data, providing a robust framework for decision-making under uncertainty.

4.1.4. The Structure of the Problem

Assume there are n jobs that need to be done and there are m people available to complete them. Assume that each individual is capable of performing each task, albeit to various degrees of efficiency. The neutrosophic cost of placing the i^{th} employee in the j^{th} position is represented by c_{ij}^{N}. Let x_{ij} represent the worker i who is working on job j. Identifying a work assignment (which task should be given to which individual on a one-on-one basis) is the challenge in order to keep the cost of carrying out all tasks to a minimum. The triangular fuzzy neutrosophic assignment issue may be expressed as follows in the form of a matrix, which is termed the triangular fuzzy neutrosophic assignment matrix:

$$\mathbf{A} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ : \\ \mathrm{i} \\ : \\ \mathrm{m} \end{array} \begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & .. & \mathrm{j} & .. & \mathrm{n} \end{array} \\ \begin{pmatrix} c_{11}^N & c_{12}^N & c_{13}^N & .. & c_{1j}^N & .. & c_{1n}^N \\ c_{21}^N & c_{22}^N & c_{23}^N & .. & c_{2j}^N & .. & c_{2n}^N \\ c_{31}^N & c_{32}^N & c_{33}^N & .. & c_{3j}^N & .. & c_{3n}^N \\ : & : & : & .. & : & .. & : \\ c_{i1}^N & c_{i2}^N & c_{i3}^N & .. & c_{ij}^N & .. & c_{in}^N \\ : & : & : & .. & : & .. & : \\ c_{m1}^N & c_{m2}^N & c_{m3}^N & .. & c_{mj}^N & .. & c_{mn}^N \end{pmatrix} \end{array}$$

4.2. Kinds of triangular fuzzy neutrosophic assignment problem

Depending on the number of columns and rows that are present in the triangular fuzzy neutrosophic assignment matrix, situations can be generally categorized as balanced or unbalanced.

1. **Balanced triangular fuzzy neutrosophic assignment problem:** A balanced triangular fuzzy neutrosophic assignment problem is what occurs when the number of rows (workers) equals the number of columns (jobs).

2. **Unbalanced triangular fuzzy neutrosophic assignment problem:** The term "unbalanced triangular fuzzy neutrosophic assignment problem" describes a situation in which the number of employees (rows) and the number of jobs (columns) are not equal.

    - When there exist more rows than columns, a dummy column with zero cost components is included to the triangular fuzzy neutrosophic assignment matrix to bring the number of rows and columns to an even number.
    - On the other hand, if the number of columns exceeds the number of rows, a dummy row with zero cost components is included to the triangular fuzzy neutrosophic assignment matrix to make it equal to the number of columns.

Remark 2. *It is necessary and sufficient for a TFNAP to contain the identical number of rows and columns, or to be a balanced TFNAP. The TFNAM comprises a dummy row or a dummy column, depending on the necessity, to balance an imbalanced problem. Following this, the problem can be resolved in a manner similar to the balanced problem.*

4.3. Solutions of the TFNAP

4.3.1. Different types of solutions of the TFNAP

The possible forms of solutions for a specific triangular fuzzy neutrosophic assignment issue include the following:

Triangular Fuzzy Neutrosophic Feasible Solution: A "triangular fuzzy neutrosophic feasible solution" in TFNAP ensures that each job is assigned to a valid resource and each resource to a valid task, meeting all constraints and requirements.

Triangular Fuzzy Neutrosophic Basic Feasible Solution: A "triangular fuzzy neutrosophic basic feasible solution" in TFNAP initially assigns tasks to resources, ensuring all jobs are assigned, all resources utilized without overload, and each task assigned exactly one resource. It can be classified as degenerate or non-degenerate based on resource-task assignments and zero entries in the TFNAM.

- **Triangular Fuzzy Neutrosophic Degenerate Basic Feasible Solution:** Triangular fuzzy neutrosophic degenerate basic feasible solutions arise when a resource is assigned multiple tasks, violating the one-task-per-resource requirement. Strategies are employed to resolve degeneracy and ensure convergence to an ideal solution.
- **Triangular Fuzzy Neutrosophic Non-Degenerate Basic Feasible Solution:** Triangular fuzzy neutrosophic non-degenerate basic feasible solutions adhere closely to the requirement of assigning one resource to each job and one task to each resource. They offer a balanced and direct assignment of tasks without resource duplication or underutilization, making them preferred choices.

Triangular Fuzzy Neutrosophic Optimal Solution: The triangular fuzzy neutrosophic optimal solution of a TFNAP is the assignment of tasks to resources that minimizes total cost or maximizes total benefit, depending on the objective. It's the best assignment, achieved through specialized algorithms like the Hungarian algorithm (which is listed in the next sub-section).

4.3.2. Some common methods for solving a triangular fuzzy neutrosophic assignment problem

Numerous methods have been devised to address the assignment problem, each with distinct advantages tailored to various types of assignment scenarios. These methods are effective for both small-scale, straightforward tasks and large-scale, intricate problems. The selection of an appropriate method hinges on the specific characteristics and demands of the given problem. Here are some commonly used methods:

- Hungarian method [19]
- One's assignment method [18]
- Divide column and subtract method [17]
- Mass - A New One's assignment method [18] and
- Branch and bound technique [20].

Remark 3. *The Hungarian method [19] is versatile and adaptable, allowing for extensions to handle variations such as fuzzy or neutrosophic data, which enhances its applicability across diverse fields and complex problem settings. This fact makes it a widely favored approach to solve various problems involving assignments, neutrosophic assignments, and triangular fuzzy neutrosophic assignments.*

4.3.3. Score function

The score function employed here to transform the neutrosophic data of the triangular fuzzy neutrosophic assignment issue into crisp data (the same as that in [11]), is as follows:

$$S(H) = \frac{1}{12}[(h_1 + 2h_2 + h_3) + (h_4 + 2h_5 + h_6) + (h_7 + 2h_8 + h_9)] \quad (1)$$

5. AN APPROACH FOR RESOLVING THE TRIANGULAR FUZZY NEUTROSOPHIC ASSIGNMENT PROBLEM (TFNAP)

5.1. Defuzzification of the neutrosophic data

Each entry (a triangular fuzzy neutrosophic number) of the given triangular fuzzy neutrosophic assignment problem, which is expressed in the form of a triangular fuzzy neutrosophic assignment matrix, is defuzzified using the above score function (1) as the first step in solving the problem, as given below:

$$S(H) = \tfrac{1}{12}[(h_1 + 2h_2 + h_3) + (h_4 + 2h_5 + h_6) + (h_7 + 2h_8 + h_9)],$$

hence being converted into their respective crisp numbers.

5.2. The proposed technique for tackling the triangular fuzzy neutrosophic assignment problem (TFNAP)

The phases of the suggested approach to solving the triangular fuzzy neutrosophic assignment issue are as follows:

Step 1 : From the provided triangular fuzzy neutrosophic assignment matrix (TFNAM), the first step is to verify if the considered triangular fuzzy neutrosophic assignment problem (TFNAP) is balanced (i.e., to check whether the number of rows(m) is equal to the number of columns (n)).

- If yes, go to step 2.
- Orelse, if it is unbalanced, make it a balanced one, by either including a dummy row or a dummy column, appropriate to the situation's requirement and then go to step 2.

Step 2 : Now, after making the problem balanced., i.e., after making the overall number of rows (m) meet the overall number of columns (n), the next step is to include a row underneath, called the Range Demand Column (RDC) and a column at the right, called the Range Supply Row (RSR), in the triangular fuzzy neutrosophic assignment matrix.

Step 3 : Now, after introducing the above mentioned row (RDC) and column (RSR), find out the corresponding values for the newly added row and column, by calculating the range value, for every row and column, utilizing the formula, Range = Highest value - Least value (Here, since all the elements are in the form of triangular fuzzy neutrosophic numbers, we have to first convert all the neutrosophic data into their respective crisp data using the above score function (1), because, only then, we would be able to compare them and find out the maximum and minimum values among the found out crisp data, take their corresponding neutrosophic data and perform their required operations (as provided in section 3) to calculate the range).

Step 4 : On having obtained all the values of the RSR and the RDC, now, spot and select the highest value among all the found out new entries of the RSR and the RDC (Here, again since all the elements are in the form of triangular fuzzy neutrosophic numbers, we have to first convert all the neutrosophic data into their respective crisp data using the above score function (1), because, only then, we would be able to compare them and find out the maximum value among all the found out crisp data and select the corresponding neutrosophic data as the highest value).

- If this selected value is in the RSR, then, choose the least element c_{ij}^N of its corresponding row (which is again found out after defuzzifying the neutrosophic data of that considered row using the above score function (1) and comparing their respective crisp values) and make an assignment to that chosen element by boxing it.
- Else, if this highest value lies in the RDC, then, choose the least element c_{ij}^N of its corresponding column (which is again found out after defuzzifying the neutrosophic data of that considered column using the above score function (1) and comparing their respective crisp values) and make an assignment to that chosen element by boxing it.

Step 5 : The next task after spotting the minimal element either through the RSR or the RDC and making an assignment to it is to delete both the corresponding row and column in which it is present.

Step 6 : Now, with the resulting matrix, re - perform steps 2 - 5, until all columns disappear. Then, verify if the overall number of assignments precisely equals the total number of rows (m) and columns (n) and in addition to it, if every row and column has exactly one assignment, which guarantees the prevalence of a triangular fuzzy neutrosophic non-degenerate basic feasible solution to the given triangular fuzzy neutrosophic assignment problem.

- If yes, move forward to step 7.
- Orelse, make it non - degenerate, by making the overall number of assignments equal to the total number of rows and columns and also by making every row and column have exactly one assignment and then proceed to step 7.

Step 7 : Now, compute the triangular fuzzy neutrosophic assignment schedule (TFNAS) as well as the total minimal triangular fuzzy neutrosophic assignment cost (TMTFNAC), for the given triangular fuzzy neutrosophic assignment problem, from the considered triangular fuzzy neutrosophic assignment matrix, which will themselves serve as the triangular fuzzy neutrosophic optimal solution (TFNOS) and the triangular fuzzy neutrosophic optimal assignment cost (TFNOAC) respectively.
Step 8 : Finally, obtain the respective crisp assignment schedule (CAS) and the total minimal crisp assignment cost (TMCAC)(which is calculated by defuzzifying the already obtained total minimal triangular fuzzy neutrosophic assignment cost using the score function (1)), for the given triangular fuzzy neutrosophic assignment problem, which will themselves serve as the crisp optimal solution (COS) and the crisp optimal assignment cost (COAC) respectively.

6. ILLUSTRATIONS FOR THE SUGGESTED APPROACH

6.1. Illustration 1

Let us consider a problem of assigning three trucks (T_1, T_2 and T_3) to three destinations (D_1, D_2 and D_3) as in [11] under a neutrosophic environment as follows:

$$\mathbf{R} = \begin{array}{c} \\ T_1 \\ T_2 \\ T_3 \end{array} \begin{array}{c} \begin{array}{ccc} D_1 & D_2 & D_3 \end{array} \\ \begin{pmatrix} c_{11}^N & c_{12}^N & c_{13}^N \\ c_{21}^N & c_{22}^N & c_{23}^N \\ c_{31}^N & c_{32}^N & c_{33}^N \end{pmatrix} \end{array}$$

Furthermore, since the components involved here are in the form of triangular fuzzy neutrosophic numbers (TFNNs), it can be explicitly expressed through the following triangular fuzzy neutrosophic assignment matrix (TFNAM), taken from [11]:

$$\mathbf{R} = \begin{array}{c|ccc} & D_1 & D_2 & D_3 \\ \hline T_1 & \begin{array}{c}(1,4,7),\\(1,3,5),\\(3.5,6,7.5)\end{array} & \begin{array}{c}(0.5,2.5,4.5),\\(1,2,3),\\(1.5,3.5,5.5)\end{array} & \begin{array}{c}(1,3,5),\\(0.5,1.5,3.5),\\(2,4,6)\end{array} \\ T_2 & \begin{array}{c}(1,2,3),\\(0.5,1.5,2.5),\\(1.5,2.5,3.5)\end{array} & \begin{array}{c}(1,1.5,4),\\(0.5,1,2.5),\\(1.25,3,4.25)\end{array} & \begin{array}{c}(1.5,2.5,3.5),\\(1,1.5,3),\\(2,3,4)\end{array} \\ T_3 & \begin{array}{c}(2,4,6),\\(1.5,2.5,4.5),\\(3,5,7)\end{array} & \begin{array}{c}(1,5,8),\\(1.5,4.5,7.5),\\(4,6.5,9)\end{array} & \begin{array}{c}(1,5,8),\\(1.5,4.5,7.5),\\(4,6.5,9)\end{array} \end{array}$$

The assignment costs which include travel expenses are listed here in rupees. How should the trucks be routed such that the overall cost of the assignment is reduced?
Step 1 : From the above considered TFNAM, the first step is to verify if it is balanced. Here, we see that, the number of rows and columns are equal, since it is a 3×3 square matrix. Hence, the considered TFNAP is balanced. Thus, we can move forward to step 2.
Step 2 : Now, after knowing that the problem is balanced, the next step is to include a row underneath, called the Range Demand Column (RDC) and a column at the right, called the Range Supply Row (RSR), in the TFNAM as shown below:

$$\begin{array}{c|ccc|c} & D_1 & D_2 & D_3 & RSR \\ \hline T_1 & (1,4,7), & (0.5,2.5,4.5), & (1,3,5), & \\ & (1,3,5), & (1,2,3), & (0.5,1.5,3.5), & \\ & (3.5,6,7.5) & (1.5,3.5,5.5) & (2,4,6) & \\ T_2 & (1,2,3), & (1,1.5,4), & (1.5,2.5,3.5), & \\ & (0.5,1.5,2.5), & (0.5,1,2.5), & (1,1.5,3), & \\ & (1.5,2.5,3.5) & (1.25,3,4.25) & (2,3,4) & \\ T_3 & (2,4,6), & (1,5,8), & (1,5,8), & \\ & (1.5,2.5,4.5), & (1.5,4.5,7.5), & (1.5,4.5,7.5), & \\ & (3,5,7) & (4,6.5,9) & (4,6.5,9) & \\ \hline RDC & & & & \end{array}$$

Step 3 : Now, after having introduced the above mentioned row (RDC) and column (RSR), the next task is to find out the corresponding values for the newly added row and column, by calculating the range value, for every row and column, utilizing the formula, Range = Highest value - Least value.
Hence, now since all the elements are in the form of TFNNs, for calculating the range of the first row, the first task is to defuzzify the neutrosophic data (c_{11}^N, c_{12}^N and c_{13}^N) of that row into their respective crisp values (c_{11}, c_{12} and c_{13}) using the above score function (1) for comparing them and find their maximum and minimum values. After performing the above task, they are found to be c_{11} and c_{12} respectively. Now, take the corresponding original neutrosophic data of the found out maximum and minimum values (c_{11}^N and c_{12}^N) and thereby, calculate the range of that row by performing the required operations (as given in section 3) as follows:

$R(T_1)^N = ((1,4,7),(1,3,5),(3.5,6,7.5)) - ((0.5,2.5,4.5),(1,2,3),(1.5,3.5,5.5))$
$= ((max(1-4.5,0), max(4-2.5,0), max(7-0.5,0)), (max(1-3,0), max(3-2,0), max(5-1,0)),$
$(max(3.5-5.5,0), max(6-3.5,0), max(7.5-1.5,0))) = ((max(-3.5,0), max(1.5,0), max(6.5,0)),$
$(max(-2,0), max(1,0), max(4,0)), (max(-2,0), max(2.5,0), max(6,0))) = ((0,1.5,6.5),(0,1,4),$
$(0,2.5,6))$

Similarly, calculate the range for the other rows and columns of the RSR and RDC respectively in the same way as mentioned above, after which the respective required neutrosophic range values ($R(T_2)^N$, $R(T_3)^N$, $R(D_1)^N$, $R(D_2)^N$ and $R(D_3)^N$) for all the rows and columns are obtained as shown below:

$$\begin{array}{c|ccc|c} & D_1 & D_2 & D_3 & RSR \\ \hline T_1 & (1,4,7), & (0.5,2.5,4.5), & (1,3,5), & R(T_1)^N \\ & (1,3,5), & (1,2,3), & (0.5,1.5,3.5), & \\ & (3.5,6,7.5) & (1.5,3.5,5.5) & (2,4,6) & \\ T_2 & (1,2,3), & (1,1.5,4), & (1.5,2.5,3.5), & R(T_2)^N \\ & (0.5,1.5,2.5), & (0.5,1,2.5), & (1,1.5,3), & \\ & (1.5,2.5,3.5) & (1.25,3,4.25) & (2,3,4) & \\ T_3 & (2,4,6), & (1,5,8), & (1,5,8), & R(T_3)^N \\ & (1.5,2.5,4.5), & (1.5,4.5,7.5), & (1.5,4.5,7.5), & \\ & (3,5,7) & (4,6.5,9) & (4,6.5,9) & \\ \hline RDC & R(D_1)^N & R(D_2)^N & R(D_3)^N & \end{array}$$

where, the neutrosophic values of both RSR and RDC are as follows:
$R(T_1)^N = ((0,1.5,6.5),(0,1,4),(0,2.5,6))$; $R(T_2)^N = ((0,1,2.5),(0,0.5,2.5),(0,0,2.75))$
$R(T_3)^N = ((0,1,6),(0,2,6),(0,1.5,6))$; $R(D_1)^N = ((0,2,6),(0,1.5,4.5),(0,3.5,6))$
$R(D_2)^N = ((0,3.5,7),(0,3.5,7),(0,3.5,7.75))$; $R(D_3)^N = ((0,2.5,6.5),(0,1.5,5.5),(0,4,9))$

Step 4 : On having obtained all the values of the RSR and the RDC, our next work is to spot and select the highest value among all the found out new entries of the RSR and the RDC.
Hence, now again since all the elements of the RSR and the RDC are in the form of TFNNs, the first task is to defuzzify all the neutrosophic data ($R(T_1)^N$, $R(T_2)^N$, $R(T_3)^N$, $R(D_1)^N$, $R(D_2)^N$ and $R(D_3)^N$) into their respective crisp values ($R(T_1)$, $R(T_2)$, $R(T_3)$, $R(D_1)$, $R(D_2)$ and $R(D_3)$) using the above score function (1) for comparing them and find the maximum value among them. After performing the above task, it is found to be $R(D_2)$. Now, take the corresponding original neutrosophic data of the found out maximum value, which is $R(D_2)^N$ and thereby, select it as the highest value among all the neutrosophic values of the RSR and RDC as shown in the following matrix:

$$
\begin{array}{c} \\ T_1 \\ \\ \\ T_2 \\ \\ \\ T_3 \\ \\ \\ RDC \end{array}
\begin{array}{c}
\begin{array}{ccc} D_1 & D_2 & D_3 \end{array} \\
\left(\begin{array}{ccc}
(1,4,7), & (0.5,2.5,4.5), & (1,3,5), \\
(1,3,5), & (1,2,3), & (0.5,1.5,3.5), \\
(3.5,6,7.5) & (1.5,3.5,5.5) & (2,4,6) \\
(1,2,3), & (1,1.5,4), & (1.5,2.5,3.5), \\
(0.5,1.5,2.5), & (0.5,1,2.5), & (1,1.5,3), \\
(1.5,2.5,3.5) & (1.25,3,4.25) & (2,3,4) \\
(2,4,6), & (1,5,8), & (1,5,8), \\
(1.5,2.5,4.5), & (1.5,4.5,7.5), & (1.5,4.5,7.5), \\
(3,5,7) & (4,6.5,9) & (4,6.5,9)
\end{array}\right) \\
\begin{array}{ccc} R(D_1)^N & \boxed{R(D_2)^N} & R(D_3)^N \end{array}
\end{array}
\begin{array}{c} RSR \\ R(T_1)^N \\ \\ \\ R(T_2)^N \\ \\ \\ R(T_3)^N \\ \\ \\ \\ \end{array}
$$

Now, since this highest value ($R(D_2)^N$) lies in the RDC, we have to now choose the least element c_{i2}^N of its corresponding column (i.e., column 2) (which is again found out after defuzzifying the neutrosophic data of column 2 (c_{12}^N, c_{22}^N and c_{32}^N) using the above score function (1) and comparing their respective crisp values (c_{12}, c_{22} and c_{32})). The required least value is found to be c_{22} and hence, we select its corresponding neutrosophic value, which is c_{22}^N and box it as an act of making an assignment to it as shown below:

$$
\begin{array}{c} \\ T_1 \\ \\ \\ T_2 \\ \\ \\ T_3 \\ \\ \\ RDC \end{array}
\begin{array}{c}
\begin{array}{ccc} D_1 & D_2 & D_3 \end{array} \\
\left(\begin{array}{ccc}
(1,4,7), & (0.5,2.5,4.5), & (1,3,5), \\
(1,3,5), & (1,2,3), & (0.5,1.5,3.5), \\
(3.5,6,7.5) & (1.5,3.5,5.5) & (2,4,6) \\
(1,2,3), & \boxed{(1,1.5,4),} & (1.5,2.5,3.5), \\
(0.5,1.5,2.5), & \boxed{(0.5,1,2.5),} & (1,1.5,3), \\
(1.5,2.5,3.5) & \boxed{(1.25,3,4.25)} & (2,3,4) \\
(2,4,6), & (1,5,8), & (1,5,8), \\
(1.5,2.5,4.5), & (1.5,4.5,7.5), & (1.5,4.5,7.5), \\
(3,5,7) & (4,6.5,9) & (4,6.5,9)
\end{array}\right) \\
\begin{array}{ccc} R(D_1)^N & R(D_2)^N & R(D_3)^N \end{array}
\end{array}
\begin{array}{c} RSR \\ R(T_1)^N \\ \\ \\ R(T_2)^N \\ \\ \\ R(T_3)^N \\ \\ \\ \\ \end{array}
$$

Step 5 : The next task after spotting the minimal element either through the RSR or the RDC and making an assignment to it is to delete both the corresponding row and column in which it is present. Here, since the chosen minimal element is c_{22}^N, we thereby delete both the row 2 and column 2, after which the resulting matrix looks like the one shown below:

$$\begin{array}{c} \\ T_1 \\ \\ \\ T_3 \\ \\ \\ \end{array} \begin{pmatrix} D_1 & D_3 \\ (1,4,7), & (1,3,5), \\ (1,3,5), & (0.5,1.5,3.5), \\ (3.5,6,7.5) & (2,4,6) \\ (2,4,6), & (1,5,8), \\ (1.5,2.5,4.5), & (1.5,4.5,7.5), \\ (3,5,7) & (4,6.5,9) \end{pmatrix}$$

Step 6 : Now, with the resulting matrix, re - perform steps 2 - 5, until all columns disappear. Hence, here since not all columns are discarded, from the above matrix, we again have to start the same procedure from step 2 and carry on till step 5. Thus, we first include a row underneath (RDC) and a column at the right (RSR), in the triangular fuzzy neutrosophic assignment matrix with their respective required values (which are calculated in the same way as mentioned previously in step - 3, by defuzzifying the neutrosophic data into their respective crisp values using the above score function (1), finding the maximum and minimum values by comparing their corresponding crisp values, taking their respective neutrosophic values and then, computing their corresponding range values) as provided below:

$$\begin{array}{cccc} & D_1 & D_3 & RSR \\ T_1 & (1,4,7), & (1,3,5), & R(T_1)^N \\ & (1,3,5), & (0.5,1.5,3.5), & \\ & (3.5,6,7.5) & (2,4,6) & \\ T_3 & (2,4,6), & (1,5,8), & R(T_3)^N \\ & (1.5,2.5,4.5), & (1.5,4.5,7.5), & \\ & (3,5,7) & (4,6.5,9) & \\ RDC & R(D_1)^N & R(D_3)^N & \end{array}$$

where, the values of both the RSR and RDC are as follows:

$R(T_1)^N = ((0,1,6),(0,1.5,4.5),(0,2,5.5))$; $R(T_3)^N = ((0,1,6),(0,0.5,5),(0,2,6))$

$R(D_1)^N = ((0,0,5),(0,0.5,3.5),(0,1,4.5))$; $R(D_3)^N = ((0,2,7),(0,1.5,6),(0,3,7))$

Our next work is to spot and select the highest value among all the found out new entries of the RSR and the RDC, which is done by first defuzzifying the considered neutrosophic data ($R(T_1)^N$, $R(T_3)^N$, $R(D_1)^N$ and $R(D_3)^N$) into their respective crisp values ($R(T_1)$, $R(T_3)$, $R(D_1)$ and $R(D_3)$) using the above score function (1), in order to be able to compare them and then find the maximum value among them. After performing the above task, it is found to be $R(D_3)$. Now, take the corresponding original neutrosophic data of the found out maximum value, which is $R(D_3)^N$ and thereby, select it as the highest value among all the neutrosophic values of the RSR and RDC as shown in the following matrix:

$$\begin{array}{cccc} & D_1 & D_3 & RSR \\ T_1 & (1,4,7), & (1,3,5), & R(T_1)^N \\ & (1,3,5), & (0.5,1.5,3.5), & \\ & (3.5,6,7.5) & (2,4,6) & \\ T_3 & (2,4,6), & (1,5,8), & R(T_3)^N \\ & (1.5,2.5,4.5), & (1.5,4.5,7.5), & \\ & (3,5,7) & (4,6.5,9) & \\ RDC & R(D_1)^N & \boxed{R(D_3)^N} & \end{array}$$

Now, since this highest value ($R(D_3)^N$) lies in the RDC, we have to now choose the least element c_{i3}^N of its corresponding column (i.e., column 3) (which is again found out after defuzzifying the neutrosophic data of column $3(c_{13}^N$ and $c_{33}^N)$ using the above score function (1) and comparing their respective crisp values (c_{13} and c_{33})). The required least value is found to be c_{13} and hence, we select its corresponding neutrosophic value, which is c_{13}^N and box it as an act of making an assignment to it as shown below:

$$\begin{array}{cc} & \begin{array}{ccc} D_1 & D_3 & RSR \end{array} \\ \begin{array}{c} T_1 \\ \\ \\ T_3 \\ \\ \\ RDC \end{array} & \left(\begin{array}{cc} (1,4,7), & \boxed{(1,3,5),} \\ (1,3,5), & \boxed{(0.5,1.5,3.5),} \\ (3.5,6,7.5) & \boxed{(2,4,6)} \\ (2,4,6), & (1,5,8), \\ (1.5,2.5,4.5), & (1.5,4.5,7.5), \\ (3,5,7) & (4,6.5,9) \end{array} \right) \begin{array}{c} R(T_1)^N \\ \\ \\ R(T_3)^N \\ \\ \\ \end{array} \\ & \begin{array}{cc} R(D_1)^N & R(D_3)^N \end{array} \end{array}$$

The next task after spotting the minimal element either through the RSR or the RDC and making an assignment to it is to delete both the corresponding row and column in which it is present. Here, since the chosen minimal element is c_{13}^N, we thereby delete both the row 1 and column 3, after which the resulting matrix looks like the one shown below:

$$\begin{array}{cc} & D_1 \\ T_3 & \left(\begin{array}{c} (2,4,6), \\ (1.5,2.5,4.5), \\ (3,5,7) \end{array} \right) \end{array}$$

Now, since all columns are discarded (since there is only one element left out, which is c_{31}^N), we can stop repeating steps 2 - 5, select it and box it as an act of making an assignment to it in the resulting TFNAM as shown below:

$$\begin{array}{cc} & D_1 \\ T_3 & \left(\begin{array}{c} \boxed{(2,4,6),} \\ \boxed{(1.5,2.5,4.5),} \\ \boxed{(3,5,7)} \end{array} \right) \end{array}$$

Thus, the final resulting TFNAM with all assignments is as follows:

$$\mathbf{R} = \begin{array}{c} \\ T_1 \\ T_2 \\ T_3 \end{array} \begin{array}{c} \begin{array}{ccc} D_1 & D_2 & D_3 \end{array} \\ \left(\begin{array}{ccc} c_{11}^N & c_{12}^N & \boxed{c_{13}^N} \\ c_{21}^N & \boxed{c_{22}^N} & c_{23}^N \\ \boxed{c_{31}^N} & c_{32}^N & c_{33}^N \end{array} \right) \end{array}$$

Further, the final resulting TFNAM displaying explicitly all its elements along with the required assignments is as shown below:

$$
\mathbf{R} = \begin{array}{c} \\ T_1 \\ \\ \\ T_2 \\ \\ \\ T_3 \\ \\ \\ \end{array}
\begin{array}{ccc} D_1 & D_2 & D_3 \\ \end{array}
\left(\begin{array}{ccc}
(1,4,7), & (0.5,2.5,4.5), & \boxed{(1,3,5),} \\
(1,3,5), & (1,2,3), & \boxed{(0.5,1.5,3.5),} \\
(3.5,6,7.5) & (1.5,3.5,5.5) & \boxed{(2,4,6)} \\
(1,2,3), & \boxed{(1,1.5,4),} & (1.5,2.5,3.5), \\
(0.5,1.5,2.5), & \boxed{(0.5,1,2.5),} & (1,1.5,3), \\
(1.5,2.5,3.5) & \boxed{(1.25,3,4.25)} & (2,3,4) \\
\boxed{(2,4,6),} & (1,5,8), & (1,5,8), \\
\boxed{(1.5,2.5,4.5),} & (1.5,4.5,7.5), & (1.5,4.5,7.5), \\
\boxed{(3,5,7)} & (4,6.5,9) & (4,6.5,9)
\end{array}\right)
$$

Our next work is to verify if the overall number of assignments precisely equals the total number of rows (m) and columns (n) and in addition to it, if every row and column has exactly one assignment. Here, we have made a total of 3 assignments which equals the total number of rows and columns and also, every row and column has exactly one assignment as seen above, which guarantees the prevalence of a triangular fuzzy neutrosophic non-degenerate basic feasible solution to the considered TFNAP. Hence, we can move forward to step 7.

Step 7 : Now, we have to compute the TFNAS and the TMTFNAC, for the given TFNAP, from the considered TFNAM, which will themselves serve as the TFNOS and the TFNOAC respectively. The TFNAS (optimal solution) is, $T_1 \longrightarrow D_3$, $T_2 \longrightarrow D_2$ and $T_3 \longrightarrow D_1$ and the TMTFNAC (optimal) $= c_{13} + c_{22} + c_{31} = ((1,3,5),(0.5,1.5,3.5),(2,4,6)) + ((1,1.5,4),(0.5,1,2.5),(1.25,3,4.25)) + ((2,4,6),(1.5,2.5,4.5),(3,5,7)) = ((4,7,7),(2.5,5,7),(6,7,7))$

Step 8 : Finally, obtain the respective CAS and the TMCAC (which is calculated by defuzzifying the already obtained TMTFNAC using the score function (1)), for the given TFNAP, which will themselves serve as the COS and the COAC respectively. Hence, by doing so, we obtain the CAS (optimal solution) as, $T_1 \longrightarrow D_3$, $T_2 \longrightarrow D_2$ and $T_3 \longrightarrow D_1$ and the TMCAS (optimal) $= \frac{1}{12}[(4+2(7)+7)+(2.5+2(5)+7)+(6+2(7)+7)] = \frac{1}{12}[(4+14+7)+(2.5+10+7)+(6+14+7)] = \frac{1}{12}[25+19.5+27] = \frac{71.5}{12} =$ Rs.5.9583

6.2. Illustration 2

Consider a triangular fuzzy neutrosophic assignment problem (TFNAP), as in [28], where a farmer plans to plant four different crops in each of four equal-sized paddocks. The crops are S_1, S_2, S_3, and S_4, and the paddocks are G_1, G_2, S_3, and G_4, respectively. Distinct crops have distinct nutritional needs, and the soil fertility in the paddocks varies. As a result, the price of the fertilizers that must be employed varies depending on the crop that is cultivated in each field. Let the cost matrix be $[c_{ij}^N]$ with triangular fuzzy neutrosophic numbers (TFNNs) as its components. The farmer's goal is to determine the appropriate distribution of paddocks for crops in order to reduce the cost of fertilizer overall. The following matrix describes the issue:

$$\mathbf{K} = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \\ S_4 \end{array} \begin{array}{c} \begin{array}{cccc} G_1 & G_2 & G_3 & G_4 \end{array} \\ \begin{pmatrix} c_{11}^N & c_{12}^N & c_{13}^N & c_{14}^N \\ c_{21}^N & c_{22}^N & c_{23}^N & c_{24}^N \\ c_{31}^N & c_{32}^N & c_{33}^N & c_{34}^N \\ c_{41}^N & c_{42}^N & c_{43}^N & c_{44}^N \end{pmatrix} \end{array}$$

Furthermore, since the components involved in the above matrix are in the form of TFNNs, it can be explicitly expressed through the following triangular fuzzy neutrosophic assignment matrix (TFNAM), taken from [28]:

$$\mathbf{K} = \begin{array}{c|cccc} & G_1 & G_2 & G_3 & G_4 \\ \hline S_1 & (3,4,5), & (8,12,16), & (20,22,24), & (34,38,40), \\ & (2,3,5), & (4,6,8), & (7,9,11), & (10,12,14), \\ & (1,2,3) & (6,7,8) & (9,11,13) & (12,14,16) \\ S_2 & (8,12,16), & (2,3,5), & (23,26,28), & (27,30,32), \\ & (4,6,8), & (1,2,3), & (10,11,12), & (10,11,12), \\ & (6,7,8) & (2,3,4) & (11,12,13) & (11,12,13) \\ S_3 & (13,15,17), & (34,38,40), & (2,3,5), & (3,4,5), \\ & (6,7,8), & (10,12,14), & (1,2,3), & (2,3,5), \\ & (3,5,7) & (12,14,16) & (2,3,4) & (1,2,3) \\ S_4 & (27,30,32), & (8,12,16), & (19,22,24), & (19,22,24), \\ & (10,11,12), & (4,6,8), & (10,12,14), & (10,12,14), \\ & (11,12,13) & (6,7,8) & (8,10,12) & (8,10,12) \end{array}$$

Step 1 : It is clear that the above considered TFNAP is balanced since it is a 4×4 square matrix. Thus, we can move forward to step 2.

Step 2 : Now, the Range Demand Column (RDC) and the Range Supply Row (RSR) are included in the TFNAM as shown below:

$$\begin{array}{c|cccc|c} & G_1 & G_2 & G_3 & G_4 & RSR \\ \hline S_1 & (3,4,5), & (8,12,16), & (20,22,24), & (34,38,40), & \\ & (2,3,5), & (4,6,8), & (7,9,11), & (10,12,14), & \\ & (1,2,3) & (6,7,8) & (9,11,13) & (12,14,16) & \\ S_2 & (8,12,16), & (2,3,5), & (23,26,28), & (27,30,32), & \\ & (4,6,8), & (1,2,3), & (10,11,12), & (10,11,12), & \\ & (6,7,8) & (2,3,4) & (11,12,13) & (11,12,13) & \\ S_3 & (13,15,17), & (34,38,40), & (2,3,5), & (3,4,5), & \\ & (6,7,8), & (10,12,14), & (1,2,3), & (2,3,5), & \\ & (3,5,7) & (12,14,16) & (2,3,4) & (1,2,3) & \\ S_4 & (27,30,32), & (8,12,16), & (19,22,24), & (19,22,24), & \\ & (10,11,12), & (4,6,8), & (10,12,14), & (10,12,14), & \\ & (11,12,13) & (6,7,8) & (8,10,12) & (8,10,12) & \\ \hline RDC & & & & & \end{array}$$

Step 3 : Now, the range values are calculated for every row and column as shown below:

$$\begin{array}{c|cccc|c} & G_1 & G_2 & G_3 & G_4 & RSR \\ \hline S_1 & (3,4,5), & (8,12,16), & (20,22,24), & (34,38,40), & R(S_1)^N \\ & (2,3,5), & (4,6,8), & (7,9,11), & (10,12,14), & \\ & (1,2,3) & (6,7,8) & (9,11,13) & (12,14,16) & \\ S_2 & (8,12,16), & (2,3,5), & (23,26,28), & (27,30,32), & R(S_2)^N \\ & (4,6,8), & (1,2,3), & (10,11,12), & (10,11,12), & \\ & (6,7,8) & (2,3,4) & (11,12,13) & (11,12,13) & \\ S_3 & (13,15,17), & (34,38,40), & (2,3,5), & (3,4,5), & R(S_3)^N \\ & (6,7,8), & (10,12,14), & (1,2,3), & (2,3,5), & \\ & (3,5,7) & (12,14,16) & (2,3,4) & (1,2,3) & \\ S_4 & (27,30,32), & (8,12,16), & (19,22,24), & (19,22,24), & R(S_4)^N \\ & (10,11,12), & (4,6,8), & (10,12,14), & (10,12,14), & \\ & (11,12,13) & (6,7,8) & (8,10,12) & (8,10,12) & \\ \hline RDC & R(G_1)^N & R(G_2)^N & R(G_3)^N & R(G_4)^N & \end{array}$$

where, the neutrosophic values of both RSR and RDC are as follows:
$R(S_2)^N = ((22,27,30),(7,9,11),(7,9,11))$; $R(S_3)^N = ((29,35,38),(7,10,13),(8,11,14))$
$R(S_4)^N = ((11,18,24),(2,5,8),(3,5,7))$; $R(G_1)^N = ((22,26,29),(5,8,10),(8,10,12))$
$R(G_2)^N = ((29,35,38),(7,10,13),(8,11,14))$; $R(G_3)^N = ((18,23,26),(7,9,11),(7,9,11))$
$R(G_4)^N = ((29,34,37),(5,9,12),(9,12,15))$

Step 4 : Next, the highest value among all the found out new entries of the RSR and the RDC ($R(S_3)^N$) is spotted and selected as shown in the following matrix:

	G_1	G_2	G_3	G_4	RSR
S_1	(3,4,5), (2,3,5), (1,2,3)	(8,12,16), (4,6,8), (6,7,8)	(20,22,24), (7,9,11), (9,11,13)	(34,38,40), (10,12,14), (12,14,16)	$R(S_1)^N$
S_2	(8,12,16), (4,6,8), (6,7,8)	(2,3,5), (1,2,3), (2,3,4)	(23,26,28), (10,11,12), (11,12,13)	(27,30,32), (10,11,12), (11,12,13)	$R(S_2)^N$
S_3	(13,15,17), (6,7,8), (3,5,7)	(34,38,40), (10,12,14), (12,14,16)	(2,3,5), (1,2,3), (2,3,4)	(3,4,5), (2,3,5), (1,2,3)	$\boxed{R(S_3)^N}$
S_4	(27,30,32), (10,11,12), (11,12,13)	(8,12,16), (4,6,8), (6,7,8)	(19,22,24), (10,12,14), (8,10,12)	(19,22,24), (10,12,14), (8,10,12)	$R(S_4)^N$
RDC	$R(G_1)^N$	$R(G_2)^N$	$R(G_3)^N$	$R(G_4)^N$	

Now, since this highest value ($R(S_3)^N$) lies in the RSR, the required least element (c_{33}^N) of its corresponding row is chosen and boxed as an act of making an assignment to it as shown below:

	G_1	G_2	G_3	G_4	RSR
S_1	(3,4,5), (2,3,5), (1,2,3)	(8,12,16), (4,6,8), (6,7,8)	(20,22,24), (7,9,11), (9,11,13)	(34,38,40), (10,12,14), (12,14,16)	$R(S_1)^N$
S_2	(8,12,16), (4,6,8), (6,7,8)	(2,3,5), (1,2,3), (2,3,4)	(23,26,28), (10,11,12), (11,12,13)	(27,30,32), (10,11,12), (11,12,13)	$R(S_2)^N$
S_3	(13,15,17), (6,7,8), (3,5,7)	(34,38,40), (10,12,14), (12,14,16)	$\boxed{(2,3,5),}$ $\boxed{(1,2,3),}$ $\boxed{(2,3,4)}$	(3,4,5), (2,3,5), (1,2,3)	$R(S_3)^N$
S_4	(27,30,32), (10,11,12), (11,12,13)	(8,12,16), (4,6,8), (6,7,8)	(19,22,24), (10,12,14), (8,10,12)	(19,22,24), (10,12,14), (8,10,12)	$R(S_4)^N$
RDC	$R(G_1)^N$	$R(G_2)^N$	$R(G_3)^N$	$R(G_4)^N$	

Step 5 : As a next task, both the corresponding row and column in which the chosen minimal element (c_{33}^N) is present are deleted, after which the resulting matrix looks like the one shown below:

	G_1	G_2	G_4
S_1	(3,4,5), (2,3,5), (1,2,3)	(8,12,16), (4,6,8), (6,7,8)	(34,38,40), (10,12,14), (12,14,16)
S_2	(8,12,16), (4,6,8), (6,7,8)	(2,3,5), (1,2,3), (2,3,4)	(27,30,32), (10,11,12), (11,12,13)
S_4	(27,30,32), (10,11,12), (11,12,13)	(8,12,16), (4,6,8), (6,7,8)	(19,22,24), (10,12,14), (8,10,12)

Step 6 : Now, with the resulting matrix, since not all columns are discarded, steps 2-5 are re-performed, until all columns disappear. Hence, after repeating the same above procedure again for a few number of times until all columns are discarded, the final resulting TFNAM with all the assignments is obtained as follows:

$$\mathbf{K} = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \\ S_4 \end{array} \begin{array}{c} \begin{array}{cccc} G_1 & G_2 & G_3 & G_4 \end{array} \\ \begin{pmatrix} \boxed{c_{11}^N} & c_{12}^N & c_{13}^N & c_{14}^N \\ c_{21}^N & \boxed{c_{22}^N} & c_{23}^N & c_{24}^N \\ c_{31}^N & c_{32}^N & \boxed{c_{33}^N} & c_{34}^N \\ c_{41}^N & c_{42}^N & c_{43}^N & \boxed{c_{44}^N} \end{pmatrix} \end{array}$$

Further, the final resulting TFNAM displaying explicitly all its elements along with the required assignments is as shown below:

$$\mathbf{K} = \begin{array}{c|cccc} & G_1 & G_2 & G_3 & G_4 \\ \hline S_1 & \boxed{(3,4,5),} & (8,12,16), & (20,22,24), & (34,38,40), \\ & \boxed{(2,3,5),} & (4,6,8), & (7,9,11), & (10,12,14), \\ & \boxed{(1,2,3)} & (6,7,8) & (9,11,13) & (12,14,16) \\ S_2 & (8,12,16), & \boxed{(2,3,5),} & (23,26,28), & (27,30,32), \\ & (4,6,8), & \boxed{(1,2,3),} & (10,11,12), & (10,11,12), \\ & (6,7,8) & \boxed{(2,3,4)} & (11,12,13) & (11,12,13) \\ S_3 & (13,15,17), & (34,38,40), & \boxed{(2,3,5),} & (3,4,5), \\ & (6,7,8), & (10,12,14), & \boxed{(1,2,3),} & (2,3,5), \\ & (3,5,7) & (12,14,16) & \boxed{(2,3,4)} & (1,2,3) \\ S_4 & (27,30,32), & (8,12,16), & (19,22,24), & \boxed{(19,22,24),} \\ & (10,11,12), & (4,6,8), & (10,12,14), & \boxed{(10,12,14),} \\ & (11,12,13) & (6,7,8) & (8,10,12) & \boxed{(8,10,12)} \end{array}$$

Our next work is to verify if the overall number of assignments precisely equals the total number of rows (m) and columns (n) and in addition to it, if every row and column has exactly one assignment. Here, we have made a total of 4 assignments which equals the total number of rows and columns and also, every row and column has exactly one assignment as seen above, which guarantees the prevalence of a triangular fuzzy neutrosophic non-degenerate basic feasible solution to the considered TFNAP. Hence, we can move forward to step 7.

Step 7 : Now, we have to compute the TFNAS and the TMTFNAC, for the given TFNAP, from the considered TFNAM, which will themselves serve as the TFNOS and the TFNOAC respectively . The TFNAS (optimal solution) is, $S_1 \longrightarrow G_1$, $S_2 \longrightarrow G_2$, $S_3 \longrightarrow G_3$ and $S_4 \longrightarrow G_4$ and the TMTFNAC (optimal) $=$ $c_{11} + c_{22} + c_{33} + c_{44} = ((3,4,5),(2,3,5),(1,2,3)) + ((2,3,5),(1,2,3),(2,3,4)) + ((2,3,5),(1,2,3),(2,3,4)) + ((19,22,24),(10,12,14),(8,10,12)) = ((11,18,20),(3,9,10),(3,6,7))$

Step 8 : Finally, obtain the respective CAS and the TMCAC (which is calculated by defuzzifying the already obtained total minimal triangular fuzzy neutrosophic assignment cost using the score function (1)), for the given TFNAP, which will themselves serve as the COS and the COAC respectively. Hence, by doing so, we obtain the CAS (optimal solution) as, $S_1 \longrightarrow G_1$, $S_2 \longrightarrow G_2$, $S_3 \longrightarrow G_3$ and $S_4 \longrightarrow G_4$ and the TMCAC (optimal) $= \frac{1}{12}[(11+2(18)+20)+(3+2(9)+10)+(3+2(6)+7)]$ $= \frac{1}{12}[(11+36+20)+(3+18+10)+(3+12+7)] = \frac{1}{12}[67+31+22] = \frac{120}{12} =$ Rs.10

Remark 4. *It has been observed that the TMCAC for illustrations 1 and 2 obtained here using the proposed method (PM) differ from those of the corresponding illustrations acquired using existing methods (EMs) in [11] and [28] respectively. The following section discusses further insights and justifications on the TFNAP that was taken into account in this study.*

7. RESULTS AND CONCLUSION

7.1. Results

The following Tables 1, 2 and Figures 1-5 provide the solutions of the TFNAP obtained using the proposed approach and a comparison of the solutions of the proposed approach here to solve the TFNAP, with a few other existing methods as in [11] and [28] respectively. Some of the significant results are shown in these tables and figures.

Table 1: Solutions of the TFNAP obtained using the proposed approach

Solutions	**Illustration 1**	**Illustration 2**
TFNAS/CAS	$T_1 \longrightarrow D_3$, $T_2 \longrightarrow D_2$ and $T_3 \longrightarrow D_1$	$S_1 \longrightarrow G_1$, $S_2 \longrightarrow G_2$, $S_3 \longrightarrow G_3$ and $S_4 \longrightarrow G_4$
TMTFNAC	$((4,7,7),(2.5,5,7),(6,7,7))$	$((11,18,20),(3,9,10),(3,6,7))$
TMCAC	Rs.5.9583	Rs.10

Table 2: Comparison of the solutions of the TFNAP obtained using the proposed approach, with a few other existing methods

Solutions	**Illustration 1**	**Illustration 2**
CAS of EMs	$T_1 \longrightarrow D_1$, $T_2 \longrightarrow D_3$ and $T_3 \longrightarrow D_2$ [11]	$S_1 \longrightarrow G_4$, $S_2 \longrightarrow G_3$, $S_3 \longrightarrow G_2$ and $S_4 \longrightarrow G_1$ [28]
CAS of PM	$T_1 \longrightarrow D_3$, $T_2 \longrightarrow D_2$ and $T_3 \longrightarrow D_1$	$S_1 \longrightarrow G_1$, $S_2 \longrightarrow G_2$, $S_3 \longrightarrow G_3$ and $S_4 \longrightarrow G_4$
TMCAC of EMs	Rs.8.55 [11]	11.16 [28]
TMCAC of PM	Rs.5.9583	Rs.10

- The Table 1 showcases the triangular fuzzy neutrosophic/crisp assignment schedules (TFNAS/CAS), TMTFNAC and the TMCAC of both the above considered two illustrations.
- The Table 2 compares the CAS and TMCAC calculated using the PM with those of the two illustrations, expressed as real-world problems taken into consideration above, using a few other EMs in [11] and [28].

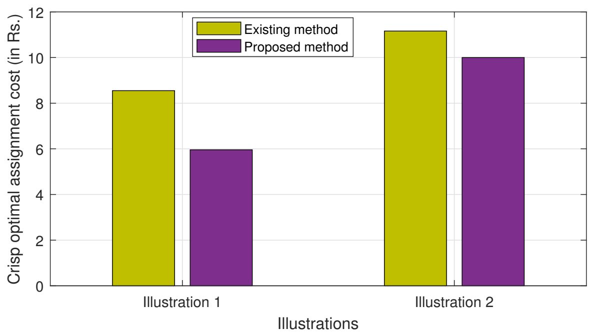

Figure 1: An overview of the solutions (COAC) of the TFNAP using the proposed approach and a few other existing methods

- Thus, the above comparison ensures that for the given TFNAPs, the CAS and the TMCAC found by applying the PM itself serve as the COS and the COAC, respectively.

- The same above conclusions can be drawn from the Figure 1 which provides an overview of the solutions (COACs) to the two TFNAPs that were previously taken into consideration and solved using the two previously mentioned EMs with that of the same two problems using the PM. It does so by demonstrating a sizable amount of variation in the values of the solutions, through the usage of the PM.

- Figure 1 also shows that the solutions to the aforementioned two TFNAPs obtained using the PM (the violet bar) are better (in terms of TMCAC/COAC) than those obtained using the other EMs (the green bar) as in [11] and [28].

- Comparison of Proposed Method (Employing Range) Vs Hungarian Method in [11]: Knowing that the Hungarian method (used in [11]), which is frequently used to test the optimality of the given AP, provides better solutions than the other methods (guarantees the optimal solution), we can infer from the Figure 1 that the PM also seems to fulfil the same purpose (thereby providing the best possible solution (or) making the total crisp assignment costs as minimal as possible) by giving lower values for the TMCAC of the two TFNAPs taken into consideration in this article, when compared to the Hungarian method thus making it as the COS along with the COAC. Unlike the Hungarian Method, which operates on precise crisp data, the proposed method effectively deals with the uncertainty and imprecision in triangular fuzzy neutrosophic environments. By capturing the variability and range of possible costs, the proposed method allows for a more nuanced optimization, potentially reducing the total minimal crisp optimal assignment costs when compared to the crisp optimal assignment obtained using the Hungarian Method. By employing the range to address the dispersion in triangular

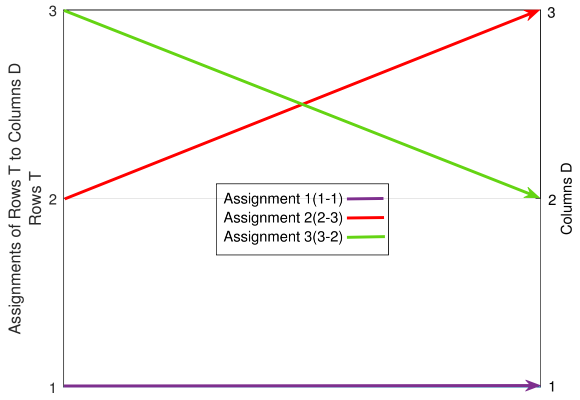

Figure 2: A glimpse of the CAS of illustration 1 using the existing method as in [11]

fuzzy neutrosophic environments, the proposed method can identify a more nuanced optimal assignment schedule. This approach considers the variability and potential extreme values, leading to a reduction in the total minimal crisp optimal assignment costs of the corresponding final total minimal triangular fuzzy neutrosophic optimal assignment costs. As a result, the proposed method offers a best crisp optimal assignment schedule that reflects a more accurate and resilient solution compared to the crisp optimal assignment schedule derived from the Hungarian Method employed in [11]. In summary, while the Hungarian Method excels in providing optimal solutions for crisp and well-defined assignment problems, the proposed method employing the range stands out in handling uncertainty and variability in triangular fuzzy neutrosophic assignment problems.

- The two Figures 2 and 3 present a glimpse of the CASs of illustration 1 using an EM ($T_1 \longrightarrow D_1$, $T_2 \longrightarrow D_3$ and $T_3 \longrightarrow D_2$) as in [11] and the PM ($T_1 \longrightarrow D_3$, $T_2 \longrightarrow D_2$ and $T_3 \longrightarrow D_1$) respectively, where the violet arrow line represents the assignment 1, the red arrow line represents the assignment 2 and the green arrow line represents the assignment 3 of the TFNAP taken into consideration.

- The CASs of the TFNAP under consideration differ when the two aforementioned graphs are compared. Since we have already established that the PM gives us the COS along with the COAC, the Figure 3 can be viewed as the COS of Illustration 1 using the PM.

- The two Figures 4 and 5 show a glimpse of the CASs of illustration 2 using an EM ($S_1 \longrightarrow G_4$, $S_2 \longrightarrow G_3$, $S_3 \longrightarrow G_2$ and $S_4 \longrightarrow G_1$) as in [28] and the PM ($S_1 \longrightarrow G_1$, $S_2 \longrightarrow G_2$, $S_3 \longrightarrow G_3$ and $S_4 \longrightarrow G_4$) respectively,

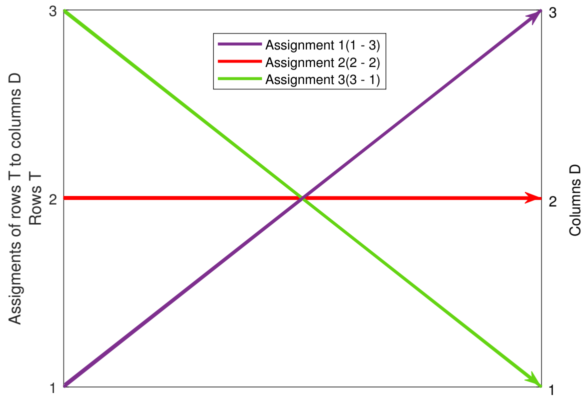

Figure 3: A glimpse of the CAS (COS) of illustration 1 using the proposed approach

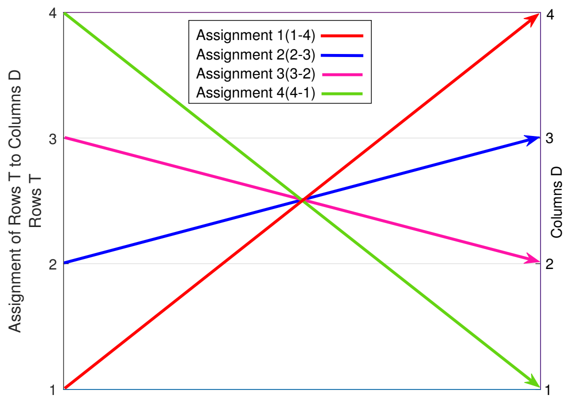

Figure 4: A glimpse of the CAS of illustration 2 using the existing method as in [28]

where the red arrow line represents the assignment 1, the blue arrow line represents the assignment 2, the pink arrow line represents the assignment 3 and the green arrow line represents the assignment 4 of the TFNAP under consideration.

- The CASs of the TFNAP under consideration differ when the two aforementioned graphs are compared. Since we have already established that the PM gives us the COS along with the COAC, the Figure 5 can be viewed as the COS of Illustration 2 using the PM.

Advantages of using the proposed methodology: The following given

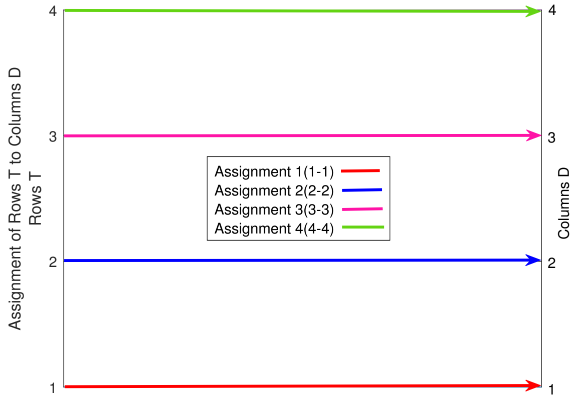

Figure 5: A glimpse of the CAS (COS) of illustration 2 using the proposed approach

are certain distinct advantages and significance that the proposed methodology offers:

Significance of employing Range in the proposed method for solving the TFNAP: The range is a measure of dispersion that represents the difference between the maximum and minimum values in a dataset. It provides an indication of the spread or variability within the data. The following are some of the advantages of including the range concepts in the suggested methodology of this research study:

- **Simplicity:** Range is a simple and easy-to-understand measure of dispersion, which makes it a practical choice for initial analysis in complex problems like TFNAP.
- **Applicability to Various Data Sets:** Can be used with any type of data, whether it's a simple numerical dataset or complex fuzzy and neutrosophic data.
- **Quick Insight:** It provides a quick overview of the spread of the data, helping to identify the extent of variability in the assignment costs or times.
- **Identifying Extremes:** Range highlights the difference between the maximum and minimum values, helping to identify outliers and extreme values that might influence the assignment decision.
- **Decision Making and Determining the Spread:** Range helps in understanding the variability and consistency of the assignment costs, which is crucial for optimizing the assignment schedule and making informed decisions in assignment problems.

- **Resource Allocation:** By assessing the range of costs, it is possible to allocate resources more effectively and anticipate potential variations in costs.
- **Optimization:** In optimization problems, knowing the range can help in setting realistic bounds and constraints, leading to more efficient and feasible solutions.
- **Simplified Comparison:** Facilitates the comparison between different assignment scenarios by providing a straightforward measure of dispersion.

Benefits of Including Range Demand Column (RDC) and Range Supply Row (RSR):

- **Enhanced Data Analysis:** Incorporating RDC and RSR into the methodology allows for a more detailed analysis of the variability in demand and supply, respectively and provides a richer context for decision-making.
- **Improved Clarity and Insight:** RDC and RSR add additional layers of clarity, helping to understand how demand and supply ranges impact overall costs and assignments, leading to a more accurate TFNAS/TFNOS.
- **Comprehensive Representation:** By including RDC and RSR, the proposed methodology can better represent the triangular fuzzy neutrosophic conditions, ensuring that all aspects of both fuzzy and crisp costs are considered, providing a holistic view of the assignment problem and its solutions.

Novelty of the proposed approach and a comparison with the existing methods: The proposed methodology of incorporating range measures like RDC and RSR into TFNAP solving enhances its simplicity and practicality in managing complex uncertainty and hence is novel. Unlike traditional methods relying on intricate statistical measures such as standard deviation and various means, this approach offers deeper insights into cost variability in both fuzzy and crisp forms, streamlining computation and interpretation. In contrast, range offers a simple, easily interpretable measure that effectively captures data spread. It quickly reveals variability in assignment costs or benefits and identifies outliers. Seamlessly integrating with existing models, this method is flexible and robust for managing complexities in fuzzy and neutrosophic environments. It stands out for its simplicity, sensitivity to extreme values, and ease of integration. This approach aids in obtaining accurate and realistic fuzzy neutrosophic assignment schedules and costs, addressing inherent uncertainties. It offers a more intuitive, less computationally demanding compared to existing methods, ensuring practical applicability while maintaining analytical depth. The range provides a baseline for comparing different scenarios and highlighting assignment process uncertainty. This, in turn, supports informed, robust decision-making, ensuring strategies are resilient to variability and capable of addressing best and worst-case outcomes. Incorporating the range into the TFNAP enhances clarity, flexibility, and robustness by quickly and effectively assessing data variability, crucial for managing TFNAP complexity and uncertainty.

Limitations of the proposed method:

- Limited Representation of Uncertainty: Range alone might not fully capture the uncertainty inherent in TFNNs, potentially leading to oversimplified models.
- Loss of Precision: Relying solely on range might result in a loss of precision, as it provides only a crude approximation of the variability within TFNNs, neglecting the values in between the extreme bounds.
- Interpretability Challenges: The interpretation of results based solely on ranges can be challenging, making it difficult for decision-makers to understand and act upon the solutions.
- Handling Indeterminacy: Range might not effectively handle the indeterminacy aspect of TFNNs, potentially overlooking important nuances in decision-making.
- Scalability Issues: As the problem size increases, managing ranges for numerous TFNNs can become computationally expensive and resource-intensive.
- Risk of Conservative Estimates: Ranges tend to be conservative estimates, potentially leading to overly cautious decision-making and suboptimal solutions.
- Complexity in Implementation: Developing algorithms and software tools to handle range-based dispersion measures for TFNNs can be complex and require specialized expertise.

These limitations underscore the need for careful consideration and possibly the integration of other measures of dispersion or uncertainty to effectively address the TFNAP.

Relevance of Triangular Fuzzy Neutrosophic Assignment Problem and the Proposed Method in Real-World Scenarios:

In real-world scenarios, decision-makers often face situations where data is imprecise, uncertain, or incomplete. Traditional crisp models fall short in such contexts because they cannot capture the inherent vagueness and ambiguity. The TFNAP addresses this by incorporating three aspects: fuzziness, indeterminacy, and membership, which reflect the uncertainty and incomplete information more accurately.

Key Real-World Applications:

- **Supply Chain Management:** Assigning suppliers to orders where delivery times and costs are not precisely known.
- **Project Management:** Allocating resources to tasks when task durations and costs are uncertain.

- **Healthcare Management:** Scheduling doctors or allocating medical resources under uncertain patient needs and resource availability.
- **Human Resources:** Assigning employees to projects based on skills and availability, both of which may be uncertain.

Insights and Implications for Future Research:

- **Explore Dispersion Measures:** Compare effectiveness of other measures like variance, IQR, median and mode.
- **Hybrid Models:** Combine range with other techniques for robust solutions.
- **Algorithmic Enhancements:** Develop efficient algorithms for large-scale problems.
- **Practical Implementations:** Conduct case studies in various industries.
- **Technology Integration:** Incorporate AI and machine learning for enhanced decision-making.
- **Impact Analysis:** Validate practical benefits in cost reduction and robustness.

The TFNAP and the proposed range-based method address real-world uncertainties effectively. Offering enhanced decision-making, quick insights, and adaptability, this method holds significant advantages over traditional approaches. Future research can expand on this to develop comprehensive solutions for complex environments.

8. CONCLUSION

The TFNAP significantly extends the classical AP by incorporating uncertainty and ambiguity through TFNS, enabling decision-makers to handle complex scenarios more effectively. It represents uncertain data realistically and considers multiple perspectives to enhance understanding and problem-solving. Researchers have developed new algorithms for triangular fuzzy neutrosophic data, exploring optimization, heuristics, and metaheuristics for efficient solutions and better computational efficiency. This research article explores the attributes, kinds, solutions of TFNAP and introduces a suggested technique for solving TFNAP, comparing it with a few methods in existence as in [11] and [28] through real-life examples to demonstrate its efficiency and significance. The proposed method consistently reveals its effectiveness, offering reduced assignment costs, optimal solutions, and computational simplicity in commparison with that of the existing methods in [11] and [28] without compromising quality. It effectively balances accuracy and computational efficiency, making it suitable for time-sensitive and resource optimization scenarios in domains like supply chain management and logistics. Despite its advantages, ongoing research aims to refine algorithms and address scalability challenges. Overall, TFNAP significantly contributes to real-world problem-solving and advances decision theory and optimization techniques. Future work will explore applying this suggested technique to multi-objective neutrosophic assignment problems, enhancing the findings and benefiting the triangular fuzzy neutrosophic research environment.

Acknowledgements. The authors would like to express their gratitude to the editors and the anonymous referees for their thoughtful feedback and suggestions that helped enhance the paper's presentation and content.

Funding. This research received no external funding.

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