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A MULTI-ATTRIBUTE DECISION-MAKING CONTEXT FOR HOSPITAL SITE EVALUATION USING DISTANCE MEASURES UNDER COMPLEX PICTURE FUZZY SOFT SET ENVIRONMENT

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Abstract: In scenarios containing complex or high-dimensional data, in which conventional distance measures might not be able to accurately portray the subtleties of information interactions, a complex picture fuzzy soft distance measure is crucial. The potential of complex picture fuzzy soft distance measures to manage intricacy, imperfection, and uncertainty across a range of domains is what makes them significant. This article proposes a distance formula based on complex picture fuzzy soft sets (CPiFSs) along with related results in its first phase. Following that, the second phase presents the η-equalities for the proposed distances of CPiFSs. Establishing a hospital is complicated, especially when it comes to choosing the ideal site because it affects mobility and the efficacy of community service. Even though there are several algorithms in the literature those deals with these location-related decision-making challenges, every algorithm has intrinsic flaws that affect how decisions are made. To overcome these challenges, an algorithm based on CPiFS distance measures is presented in the third phase for the evaluation of hospital sites. Finally, the paper concludes by emphasizing the significance

of this research and its potential applications in various contexts.

Keywords: Decision-making, optimization, fuzzy soft set, complex fuzzy soft set, picture fuzzy soft set, complex picture fuzzy soft set.

MSC: 03B52,90B50,03E72.

1. INTRODUCTION

Classical set theory was the foundational framework for understanding collections of elements without considering uncertainty or vagueness. It did not involve the notion of membership functions. Zedah [1] introduced fuzzy set theory, which extended classical set theory. In this novel framework, each element is associated with a membership function that allows for degrees of belongingness. Building upon Zadeh's work, Atanassov [2] introduced the concept of intuitionistic fuzzy sets, enriching the field by incorporating both membership and non-membership functions for elements. Taking the theory further, Cuong et al. [3] contributed by introducing picture fuzzy sets, a more generalized version of both fuzzy and intuitionistic fuzzy sets. In picture fuzzy sets, elements are characterized by three distinct functions: degree of membership, degree of non-membership, and degree of neutrality. The sum of all these functions lies within the closed unit interval. Molodtsov [4] introduced the new concept of soft set theory, that aimed of the addressing uncertainties in parametric form. The theory of soft set is a generalization of fuzzy set theory, providing a versatile framework for handling imprecise information. The fundamental properties, operations and relations have been discussed by several researchers but the significant contributions have been reported by the authors [5, 6, 7, 8, 9, 10]. By working on the Molodtsov's work, Çağman, et al. [11] introduced fuzzy soft set theory, a significant extension that finds practical utility in decision-making problems. This theory blends the characteristics of fuzzy sets and soft sets, offering a more comprehensive approach to uncertainty management. Further advancements in this field led to the development of the intuitionistic fuzzy soft set theory by Xu et al. [12], which extends the principles of intuitionistic fuzzy sets into the soft set framework, enriching the toolbox for handling imprecise information. Cuong et al. [13] contributed by introducing the theory of picture fuzzy soft sets, expanding upon the principles of both picture fuzzy sets and soft sets. This novel concept provides a broader and more expressive representation for handling uncertainty in various applications. These advancements in set theories have significantly broadened our capacity to handle imprecise and uncertain information, offering valuable tools for decision-making and problem-solving across diverse domains. The idea of picture fuzzy soft sets have been employed in several domains of study, however, the contributions made by Jayaraman et al. [14], Khan et al. [15], Lu et al. [16], Khan et al. [17], and Rehman & Mahmood [18] in different decision making situations using generalized picture fuzzy soft sets. Verma & Rohtagi [19] formulated similarity measures based on picture fuzzy sets and then applied them to pattern recognition and medical diagnosis.

Ramot et al. [20] gave the theory of complex fuzzy set in which each element has a complex valued membership function instead of real valued function. Alkouri and Salleh [21] is extended the concept of intuitionistic fuzzy set to the complex intuitionistic fuzzy set by setting the all functions in complex plane. After that they presented an example based on the distance measure of complex intuitionistic fuzzy set. The theory of complex fuzzy set and picture fuzzy set is used by Akram, et al. [22] to develop the theory of complex picture fuzzy set. The idea of complex fuzzy soft Sets was formally introduced by Thirunavukarasu in [23] which is the combination of fuzzy sets and soft sets to address the shortcomings of existing models. The goal was to create a unified framework that could accommodate uncertain and complex data in a more comprehensive manner. The complex intuitionistic fuzzy soft sets were introduced by Kumar & Bajaj [24], as an advanced extension of intuitionistic fuzzy sets and soft sets, enriched by the inclusion of complex numbers. Khan et al. [25] formulated distance measures based complex picture fuzzy sets and then applied the idea to medical diagnosis, and pattern recognition. This framework aims to handle uncertainty, vagueness, and complex relationships in a more comprehensive manner.
After careful analysis of the above reviewed literature, it can concluded that these are inadequate for the following challenges:

1. Information or a few portions can occasionally be duplicated in the form of raw data, necessitating a specific configuration. Thus far, the most suitable context in the literature for dealing with two-dimensional issues is the complex setting, which offers phase and amplitude terms.
2. Decision-makers must have access to a dynamic context in which they may readily offer knowledgeable judgments based on suitable parameters, exhibiting objectivity or neutrality. This necessitates a specific setting that provides a neutral, dependent membership grade. Up till now, the idea of a picture fuzzy soft context has been considered in this context.

In recent research, scholars have addressed distance measurement problems pertinent to practical applications. The literature contains numerous algorithms designed to handle such issues, aiding decision-making in real-life scenarios. However, each algorithm exhibits certain limitations and drawbacks in decision-making processes. To address these challenges, an algorithm based on CPiFSs has been proposed. CPiFSs build upon the complex picture fuzzy set framework, enabling decision-makers to consider individual attributes, qualities, and nuances of elements when making choices. By integrating CPiFSs into the realm of soft sets, this algorithm provides decision-makers with a versatile tool to enhance the precision and informativeness of their decisions. This approach helps bridge the gap in decision-making processes between macro- and micro-level considerations. Asghar et al. [26] introduced the CPiFSs as an extension of picture fuzzy sets and soft sets.

1.1. Research Motivation

A comprehensive structure to support decision-making that is capable of handling the multifaceted nature of contemporary issues and producing superior decision integrity, as well as outcomes,is provided by CPiFSs-based MADM context. Some motivational cum advantageous aspects are mentioned below:

1. Comparing CPiFSs to straightforward models enables intricate and accurate representation. To produce superior decision results, they cancapture intricate interactions, connections, and interconnections between selection parameters and options. The CPiFSs have the potential to produce better decisions by taking into account a greater variety of variables and intricate interactions. They help decision-makers make better decisions by allowing them to examine the issue from several angles.
2. MADM models that are based on CPiFSs offer more versatility in managing various kinds of information as well as assessment situations. They are appropriate for sophisticated practical decision-making scenarios because they can handle different kinds of uncertainty, inaccuracies, and ambivalence.
3. MADM models that are based on CPiFSs take into account not only the characteristics of options but also the repercussions and interconnections, enabling a thorough examination of decision challenges. Using a comprehensive strategy, managers can better comprehend the broader context of their decisions and the possible outcomes of their choices. To help decision-makers explore many situations, perform sensitivity studies, and find the best answers, they can be used as sophisticated decision support tools. They enable administrators in challenging and unpredictable situations to make more calculated and wise choices.
4. The CPiFSs-based context gives decision makers a three-dimensional membership grading system with truth, falsehood, and neutrality gradings, allowing them to express their thoughts objectively.

Moreover, CPiFSs are utilized to handle hospital site selection challenges because theyregulate the uncertainty and unpredictability associated with such decision problems. Many factors need to be carefully considered when determining the ideal location for a hospital, including population density, accessibility, demand for healthcare, financial situation, and transit accessibility. Hospitals are necessary buildings. Certain traits are inherently ambiguous and imprecise, making it occasionally challenging for traditional methods to accurately record them. Decision-makers may express and analyze complex interactions among these characteristics while taking into account both qualitative and quantitative data, thanks to the robust framework provided by CPiFSs context. By employing this technique, stakeholders can make better-informed decisions by selecting hospital locations that best meet the diverse needs of the community while optimizing resource utilization and accessibility to healthcare services.

1.2. Salient Contributions

The significant contributions of the study are:

1. The study tackles the shortcomings of traditional distance measures in situations requiring complicated or high-dimensional data by introducing a novel distance measure based on CPiFSs.

2. It provides a distance formula using CPiFSs and describes related results, providing an empirical breakthrough in managing complexity, andinadequacy, including uncertainties in several domains.
3. To improve the theoretical comprehension and practicality of CPiFS-based distance measures, this research presents η-equalities for the suggested distances of CPiFSs.
4. To fix the intrinsic flaws in currently available algorithms, the study examines the challenges of hospital site selection and suggests a method based on CPiFS distance measures. This work offers an opportunity to improve decision-making in hospital-buildingendeavors efficiently.

2. PRELIMINARIES

Definition 1. *[1] A fuzzy set $\mathcal{F}$ defined as $\mathcal{F} = \{(\hat{u}, A_{\mathcal{F}}(\hat{u}))|\hat{u} \in \hat{\mathcal{H}}\}$ such that $A_{\mathcal{F}} : \hat{\mathcal{H}} \to \mathbb{I}$ where $A_{\mathcal{F}}(\hat{u})$ denotes the belonging value of $\hat{u}$ in $\mathcal{F}$.*

Definition 2. *[4] For a set of attributes $\check{K}$, the soft set over the universe $\check{T}$ is defined as pair $(\hat{P}, \check{K})$, where $\hat{P} : \check{K} \to \mathbb{P}(\check{T})$ and $\mathbb{P}(\check{T})$ is power set of $\check{T}$.*

Definition 3. *[11] Let $\check{T}$ is the universal set, $\check{K}$ be the set of parameters and $p(\check{T})$ is the collection of all fuzzy subsets of $\check{T}$. The pair (F, A) is called a fuzzy soft set over the set $\check{T}$, where $A \subset \check{K}$ and F is a set valued mapping given by $F : A \to P(\check{T})$.*

Definition 4. *[26] For the universal set $\check{T}$ and the subset $\check{Q}$ of set of attributes $\check{K}$, the pair $(\mathcal{P}_c, \check{Q})$ is known as the CPiFSs over $\check{T}$, in which $\mathcal{P}_c : \check{Q} \to CPF(\check{T})$ is defined by*

$$\mathcal{P}_c(\check{q}) = \left\{ \begin{array}{l} \left(\frac{\left(\mathcal{X_P}^+(\check{t}), \mathcal{Y_P}^-(\check{t}), \mathcal{Z_P}(\check{t})\right)}{\check{t}} \right) : \\ \mathcal{X_P}^+(\check{t}) = \mathfrak{f}^+(\check{t})e^{\mathfrak{j}^+(\check{t})}, \mathcal{Y_P}^-(\check{t}) = \mathfrak{f}^-(\check{t})e^{\mathfrak{j}^-(\check{t})}, \mathcal{Z_P}(\check{t}) = \mathfrak{f}(\check{t})e^{\mathfrak{j}(\check{t})}, \\ 0 \le \mathfrak{f}^+(\check{t}) + \mathfrak{f}^-(\check{t}) + \mathfrak{f}(\check{t}) \le 1;\ 0 \le \mathfrak{j}^+(\check{t}) + \mathfrak{j}^-(\check{t}) + \mathfrak{j}(\check{t}) \le 2\pi \end{array} \right\}$$

where $\check{q} \in \check{Q}$, $\check{t} \in \check{T}$, $CPF(\check{T})$ represents the set of all complex picture fuzzy subsets over $\check{T}$, the components $\mathcal{X_P}^+(\check{t})$, $\mathcal{Y_P}^-(\check{t})$ and $\mathcal{Z_P}(\check{t}))$ represents the membership, non-membership and neutral grades respectively. The collection of all CPiFSs over $\check{T}$ is denoted by $CP_i(\check{T})$.

3. DISTANCE MEASURE ON CPiFSs

This section introduces the definition of the distance measure of CPiFSs along with related results that contribute to its practical applications.

Definition 5. *Consider $CP_i(\check{T})$ be the collection of all CPiFSs over $\check{T}$, then distance function for CPiFSs is defined as $d : CP_i(\check{T}) \times CP_i(\check{T}) \to [0, 1]$ such that for the sets $\mathcal{P}_c^1, \mathcal{P}_c^2, \mathcal{P}_c^3 \in CP_i(\check{T})$ the following axioms hold:*

(i) $d(\mathcal{P}_c^1, \mathcal{P}_c^2) \geq 0$,

(ii) $d(\mathcal{P}_c^1, \mathcal{P}_c^2) = 0 \iff \mathcal{P}_c^1 = \mathcal{P}_c^2$,

(iii) $d(\mathcal{P}_c^1, \mathcal{P}_c^2) = d(\mathcal{P}_c^2, \mathcal{P}_c^1)$,

(iv) $d(\mathcal{P}_c^1, \mathcal{P}_c^2) \leq d(\mathcal{P}_c^1, \mathcal{P}_c^3) + d(\mathcal{P}_c^3, \mathcal{P}_c^1)$.

Theorem 6. *Say that $CP_i(\check{T})$ be the collection of all CPiFSs over $\check{T}$ and define a function $d : CP_i(\check{T}) \times CP_i(\check{T}) \to [0,1]$ by,*

$$\begin{aligned} d(\mathcal{P}_c^1, \mathcal{P}_c^2) &= max.\Big(sup.\big(|\mathfrak{f}_\kappa^{1+}(\check{t}_\tau) - \mathfrak{f}_\kappa^{2+}(\check{t}_\tau)|, \frac{1}{2\pi}|\mathfrak{j}_\kappa^{1+}(\check{t}_\tau) - \mathfrak{j}_\kappa^{2+}(\check{t}_\tau)|\big), \\ & sup.\big(|\mathfrak{f}_\kappa^{1-}(\check{t}_\tau) - \mathfrak{f}_\kappa^{2-}(\check{t}_\tau)|, \frac{1}{2\pi}|\mathfrak{j}_\kappa^{1-}(\check{t}_\tau) - \mathfrak{j}_\kappa^{2-}(\check{t}_\tau)|\big), sup.\big(|\mathfrak{f}_\kappa^{1}(\check{t}_\tau) - \mathfrak{f}_\kappa^{2}(\check{t}_\tau)|, \\ & \frac{1}{2\pi}|\mathfrak{j}_\kappa^{1}(\check{t}_\tau) - \mathfrak{j}_\kappa^{2}(\check{t}_\tau)|\big)\Big) \end{aligned}$$

then d is the distance function of CPiFSs, where $\kappa = 1,2,3\cdots,m$ is the number of elements in the universe and $\tau = 1,2,3,\cdots,n$ is the number of elements in the set of attributes.

Proof. 1. Since the absolute value of any number is non-negative, the maximum and supremum of non-negative numbers is also a non-negative, therefore, $d(\mathcal{P}_c^1, \mathcal{P}_c^2) \geq 0$.

2.
$$\begin{aligned} & d(\mathcal{P}_c^1, \mathcal{P}_c^2) = 0 \\ \iff & \max.\Big(\sup.\big(|\mathfrak{f}_\kappa^{1+}(\check{t}_\tau) - \mathfrak{f}_\kappa^{2+}(\check{t}_\tau)|, \frac{1}{2\pi}|\mathfrak{j}_\kappa^{1+}(\check{t}_\tau) - \mathfrak{j}_\kappa^{2+}(\check{t}_\tau)|\big), \\ & \sup.\big(|\mathfrak{f}_\kappa^{1-}(\check{t}_\tau) - \mathfrak{f}_\kappa^{2-}(\check{t}_\tau)|, \frac{1}{2\pi}|\mathfrak{j}_\kappa^{1-}(\check{t}_\tau) - \mathfrak{j}_\kappa^{2-}(\check{t}_\tau)|\big), \sup.\big(|\mathfrak{f}_\kappa^{1}(\check{t}_\tau) - \mathfrak{f}_\kappa^{2}(\check{t}_\tau)|, \\ & \frac{1}{2\pi}|\mathfrak{j}_\kappa^{1}(\check{t}_\tau) - \mathfrak{j}_\kappa^{2}(\check{t}_\tau)|\big)\Big) = 0 \\ \iff & \sup.\big(|\mathfrak{f}_\kappa^{1+}(\check{t}_\tau) - \mathfrak{f}_\kappa^{2+}(\check{t}_\tau)|, \frac{1}{2\pi}|\mathfrak{j}_\kappa^{1+}(\check{t}_\tau) - \mathfrak{j}_\kappa^{2+}(\check{t}_\tau)|\big) = 0, \\ & \sup.\big(|\mathfrak{f}_\kappa^{1-}(\check{t}_\tau) - \mathfrak{f}_\kappa^{2-}(\check{t}_\tau)|, \frac{1}{2\pi}|\mathfrak{j}_\kappa^{1-}(\check{t}_\tau) - \mathfrak{j}_\kappa^{2-}(\check{t}_\tau)|\big) = 0,\ \sup.\big(|\mathfrak{f}_\kappa^{1}(\check{t}_\tau) - \mathfrak{f}_\kappa^{2}(\check{t}_\tau)|, \\ & \frac{1}{2\pi}|\mathfrak{j}_\kappa^{1}(\check{t}_\tau) - \mathfrak{j}_\kappa^{2}(\check{t}_\tau)|\big) = 0 \\ \iff & \mathcal{X}^{1\,+}_{\mathcal{P}\,\kappa\eta} = \mathcal{X}^{2\,+}_{\mathcal{P}\,\kappa\eta} \text{ and } \mathcal{Y}^{1\,+}_{\mathcal{P}\,\kappa\eta} = \mathcal{Y}^{2\,+}_{\mathcal{P}\,\kappa\eta} \text{ and } \mathcal{Z}^{1\,+}_{\mathcal{P}\,\kappa\eta} = \mathcal{Z}^{2\,+}_{\mathcal{P}\,\kappa\eta}. \end{aligned}$$

It follows that $d(\mathcal{P}_c^1, \mathcal{P}_c^2) = 0 \iff \mathcal{P}_c^1 = \mathcal{P}_c^2$.

3. By using the property $|f - g| = |g - f|$ for all functions f, g, it can easily be proved that $d(\mathcal{P}_c^1, \mathcal{P}_c^2) = d(\mathcal{P}_c^2, \mathcal{P}_c^1)$.

4. By using the triangular inequality $|f+g| \leq |f+h|+|h+g|$ for all functions f, g, h, it can easily be proved that $d(\mathcal{P}_c^1, \mathcal{P}_c^2) \leq d(\mathcal{P}_c^1, \mathcal{P}_c^3) + d(\mathcal{P}_c^3, \mathcal{P}_c^1)$. Since d satisfy the all the axioms of definition 5, therefore, it is distance function of CPiFSs.
□

Example 7. *Consider $\hat{\mathcal{T}} = \{\check{t}_1, \check{t}_2, \check{t}_3\}$ be the universal set and $\check{K} = \{\check{q}_1, \check{q}_2, \check{q}_3, \check{q}_4\}$ be the set of attributes. Then the elements of CPiFSs are given as:*

$$\aleph(\check{q}_1) = \left\{ \begin{array}{l} \left(\check{t}_1, \left(0.13e^{(2\pi)(0.20)}, 0.21e^{(2\pi)(0.02)}, 0.16e^{(2\pi)(0.3)}\right)\right), \\ \left(\check{t}_2, \left(0.11e^{(2\pi)(0.31)}, 0.41e^{(2\pi)(0.16)}, 0.19e^{(2\pi)(0.23)}\right)\right), \\ \left(\check{t}_3, \left(0.04e^{(2\pi)(0.21)}, 0.31e^{(2\pi)(0.21)}, 0.21e^{(2\pi)(0.19)}\right)\right) \end{array} \right\}$$

$$\aleph(\check{q}_2) = \left\{ \begin{array}{l} \left(\check{t}_1, \left(0.3e^{2\pi(0.13)}, 0.19e^{2\pi(0.3)}, 0.21e^{2\pi(0.2)}\right)\right), \\ \left(\check{t}_2, \left(0.23e^{2\pi(0.3)}, 0.14e^{2\pi(0.19)}, 0.23e^{2\pi(0.31)}\right)\right), \\ \left(\check{t}_3, \left(0.3e^{2\pi(0.18)}, 0.17e^{2\pi(0.2)}, 0.09e^{2\pi(0.14)}\right)\right) \end{array} \right\}$$

$$\aleph(\check{q}_3) = \left\{ \begin{array}{l} \left(\check{t}_1, \left(0.17e^{2\pi(0.13)}, 0.23e^{2\pi(0.16)}, 0.16e^{2\pi(0.22)}\right)\right), \\ \left(\check{t}_2, \left(0.09e^{2\pi(0.06)}, 0.4e^{2\pi(0.21)}, 0.3e^{2\pi(0.23)}\right)\right), \\ \left(\check{t}_3, 0.21e^{2\pi(0.32)}, 0.21e^{2\pi(0.14)}, 0.22e^{2\pi(0.24)}\right) \end{array} \right\}$$

So the $\mathcal{P}_c^1 \in CP_i(\check{T})$ in matrix form is defined as:

$$\mathcal{P}_c^1 = \begin{pmatrix} \hat{\mathcal{G}} & \check{t}_1 & \check{t}_2 & \check{t}_3 \\ \check{q}_1 & \aleph(\check{q}_1) & \aleph(\check{q}_1) & \aleph(\check{q}_1) \\ \check{q}_2 & \aleph(\check{q}_2) & \aleph(\check{q}_2) & \aleph(\check{q}_2) \\ \check{q}_3 & \aleph(\check{q}_3) & \aleph(\check{q}_3) & \aleph(\check{q}_3) \end{pmatrix}$$

Then the CPiFSs distances are given as:

$$d(\aleph(\check{q}_1), \aleph(\check{q}_2)) = 0.214, \; d(\aleph(\check{q}_1), \aleph(\check{q}_3)) = 0.134, \; d(\aleph(\check{q}_2), \aleph(\check{q}_3)) = 0.311$$

Theorem 8. *The CPiFSs distance satisfy the following.*
(i). $d\left(\mathcal{P}_c^1, (\mathcal{P}_c^2)^{'}\right) = d\left((\mathcal{P}_c^1)^{'}, \mathcal{P}_c^2\right)$
(ii). $d\left((\mathcal{P}_c^1))^{'}, (\mathcal{P}_c^2)^{'}\right) = d\left(\mathcal{P}_c^1, \mathcal{P}_c^2\right)$
(iii).$d\left(\mathcal{P}_c^1, \mathcal{P}_c^2\right) = d\left(\mathcal{P}_c^1 \cap \mathcal{P}_c^2, \mathcal{P}_c^1 \cup \mathcal{P}_c^2\right)$
(iv). $d\left(\mathcal{P}_c^1, \mathcal{P}_c^1 \cap \mathcal{P}_c^2\right) = d\left(\mathcal{P}_c^2, \mathcal{P}_c^1 \cup \mathcal{P}_c^2\right)$
Where $(\mathcal{P}_c^1)^{'}$ represents the complement of CPiFSs $\mathcal{P}_c^1$.

Proof. (i). By theorem 6 the CPiFSs distance between $\mathcal{P}_c^1$ and $\left(\mathcal{P}_c^2\right)'$ is given as:

$$\begin{aligned}
d(\mathcal{P}_c^1, (\mathcal{P}_c^2)') &= \max.\Big(\sup.\big(|\mathfrak{f}_\kappa^{1+}(\check{t}_\tau) - \mathfrak{f}_\kappa^{2-}(\check{t}_\tau)|, \frac{1}{2\pi}|\mathfrak{j}_\kappa^{1+}(\check{t}_\tau) - \mathfrak{j}_\kappa^{2-}(\check{t}_\tau)|\big), \\
&\quad \sup.\big(|\mathfrak{f}_\kappa^{1-}(\check{t}_\tau) - \mathfrak{f}_\kappa^{2+}(\check{t}_\tau)|, \frac{1}{2\pi}|\mathfrak{j}_\kappa^{1-}(\check{t}_\tau) - \mathfrak{j}_\kappa^{2+}(\check{t}_\tau)|\big), \\
&\quad \sup.\big(|\mathfrak{f}_\kappa^{1}(\check{t}_\tau) - \big(1 - \mathfrak{f}_\kappa^{2}(\check{t}_\tau)\big)|, \frac{1}{2\pi}|\mathfrak{j}_\kappa^{1}(\check{t}_\tau) - \big(1 - \mathfrak{j}_\kappa^{2}(\check{t}_\tau)\big)|\big)\Big) \\
&= \max.\Big(\sup.\big(|\mathfrak{f}_\kappa^{1-}(\check{t}_\tau) - \mathfrak{f}_\kappa^{2+}(\check{t}_\tau)|, \frac{1}{2\pi}|\mathfrak{j}_\kappa^{1-}(\check{t}_\tau) - \mathfrak{j}_\kappa^{2+}(\check{t}_\tau)|\big), \\
&\quad \sup.\big(|\mathfrak{f}_\kappa^{1+}(\check{t}_\tau) - \mathfrak{f}_\kappa^{2-}(\check{t}_\tau)|, \frac{1}{2\pi}|\mathfrak{j}_\kappa^{1+}(\check{t}_\tau) - \mathfrak{j}_\kappa^{2-}(\check{t}_\tau)|\big), \\
&\quad \sup.\big(|\big(1 - \mathfrak{f}_\kappa^{1}(\check{t}_\tau)\big) - \mathfrak{f}_\kappa^{2}(\check{t}_\tau)|, \frac{1}{2\pi}|\big(1 - \mathfrak{j}_\kappa^{1}(\check{t}_\tau)\big) - \mathfrak{j}_\kappa^{2}(\check{t}_\tau)\big)|\big)\Big) \\
&= d\big(\big(\mathcal{P}_c^1\big)', \mathcal{P}_c^2\big)
\end{aligned}$$

(ii). By theorem 6 the CPiFSs distance between $\left(\mathcal{P}_c^1\right)'$ and $\left(\mathcal{P}_c^2\right)'$ is given as:

$$\begin{aligned}
&\mathrm{d}\big((\mathrm{P}_c^1)', (\mathcal{P}_c^2)'\big) = \max.\Big(\sup.\big(|\mathfrak{f}_\kappa^{1-}(\check{t}_\tau) - \mathfrak{f}_\kappa^{2-}(\check{t}_\tau)|, \tfrac{1}{2\pi}|\mathfrak{j}_\kappa^{1-}(\check{t}_\tau) - \mathfrak{j}_\kappa^{2-}(\check{t}_\tau)|\big), \\
&\quad \sup.\big(\text{—}\mathrm{f}_\kappa^{1+}(\check{t}_\tau) - \mathfrak{f}_\kappa^{2+}(\check{t}_\tau)|, \tfrac{1}{2\pi}|\mathfrak{j}_\kappa^{1+}(\check{t}_\tau) - \mathfrak{j}_\kappa^{2+}(\check{t}_\tau)|\big), \\
&\quad \sup.\big(\text{—}\big(1\text{-}\mathrm{f}_\kappa^{1}(\check{t}_\tau)\big) - \big(1 - \mathfrak{f}_\kappa^{2}(\check{t}_\tau)\big)|, \tfrac{1}{2\pi}|\big(1 - \mathfrak{j}_\kappa^{1}(\check{t}_\tau)\big) - \big(1 - \mathfrak{j}_\kappa^{2}(\check{t}_\tau)\big)|\big)\Big) \\
&\quad = \max.\Big(\sup.\big(\text{—}\mathrm{f}_\kappa^{1+}(\check{t}_\tau) - \mathfrak{f}_\kappa^{2+}(\check{t}_\tau)|, \tfrac{1}{2\pi}|\mathfrak{j}_\kappa^{1+}(\check{t}_\tau) - \mathfrak{j}_\kappa^{2+}(\check{t}_\tau)|\big), \\
&\quad \sup.\big(\text{—}\mathrm{f}_\kappa^{1-}(\check{t}_\tau) - \mathfrak{f}_\kappa^{2-}(\check{t}_\tau)|, \tfrac{1}{2\pi}|\mathfrak{j}_\kappa^{1-}(\check{t}_\tau) - \mathfrak{j}_\kappa^{2-}(\check{t}_\tau)|\big), \\
&\quad \sup.\big(\text{—}\mathrm{f}_\kappa^{1}(\check{t}_\tau) - \mathfrak{f}_\kappa^{2}(\check{t}_\tau)|, \tfrac{1}{2\pi}|\mathfrak{j}_\kappa^{1}(\check{t}_\tau) - \mathfrak{j}_\kappa^{2}(\check{t}_\tau)|\big)\Big) \\
&\quad = \mathrm{d}\big(\mathrm{P}_c^1, \mathcal{P}_c^2\big)
\end{aligned}$$

(iii). By theorem 6 the CPiFSs distance between $\mathcal{P}_c^1$ and $\mathcal{P}_c^2$ is given as:

$$\begin{aligned}
d\big(\mathcal{P}_c^1,\mathcal{P}_c^2\big) &= \max.\Big\{\sup.\big(|\mathfrak{f}_\kappa^{1+}(\check{t}_\tau)-\mathfrak{f}_\kappa^{2+}(\check{t}_\tau)|,\frac{1}{2\pi}|\mathfrak{j}_\kappa^{1+}(\check{t}_\tau)-\mathfrak{j}_\kappa^{2+}(\check{t}_\tau)|\big),\\
&\sup.\big(|\mathfrak{f}_\kappa^{1-}(\check{t}_\tau)-\mathfrak{f}_\kappa^{2-}(\check{t}_\tau)|,\frac{1}{2\pi}|\mathfrak{j}_\kappa^{1-}(\check{t}_\tau)-\mathfrak{j}_\kappa^{2-}(\check{t}_\tau)|\big)\\
&,\sup.\big(|\mathfrak{f}_\kappa^{1}(\check{t}_\tau)-\mathfrak{f}_\kappa^{2}(\check{t}_\tau)|,\frac{1}{2\pi}|\mathfrak{j}_\kappa^{1}(\check{t}_\tau)-\mathfrak{j}_\kappa^{2}(\check{t}_\tau)|\big)\Big\}\\
&= \max.\Big\{\sup.\Big(|\min.\big(\mathfrak{f}_\kappa^{1+}(\check{t}_\tau),\mathfrak{f}_\kappa^{2+}(\check{t}_\tau)\big)-\max.\big(\mathfrak{f}_\kappa^{1+}(\check{t}_\tau),\mathfrak{f}_\kappa^{2+}(\check{t}_\tau)\big)|,\\
&\frac{1}{2\pi}|\min.\big(\mathfrak{j}_\kappa^{1+}(\check{t}_\tau),\mathfrak{j}_\kappa^{2+}(\check{t}_\tau)\big)-\max.\big(\mathfrak{j}_\kappa^{1+}(\check{t}_\tau),\mathfrak{j}_\kappa^{2+}(\check{t}_\tau)\big)|\Big),\\
&\sup.\Big(|\min.\big(\mathfrak{f}_\kappa^{1-}(\check{t}_\tau),\mathfrak{f}_\kappa^{2-}(\check{t}_\tau)\big)-\max.\big(\mathfrak{f}_\kappa^{1-}(\check{t}_\tau),\mathfrak{f}_\kappa^{2-}(\check{t}_\tau)\big)|,\\
&\frac{1}{2\pi}|\min.\big(\mathfrak{j}_\kappa^{1-}(\check{t}_\tau),\mathfrak{j}_\kappa^{2-}(\check{t}_\tau)\big)-\max.\big(\mathfrak{j}_\kappa^{1-}(\check{t}_\tau),\mathfrak{j}_\kappa^{2-}(\check{t}_\tau)\big)|\Big),\\
&\sup.\Big(|\min.\big(\mathfrak{f}_\kappa^{1}(\check{t}_\tau),\mathfrak{f}_\kappa^{2}(\check{t}_\tau)\big)-\max.\big(\mathfrak{f}_\kappa^{1}(\check{t}_\tau),\mathfrak{f}_\kappa^{2}(\check{t}_\tau)\big)|,\\
&\frac{1}{2\pi}|\min.\big(\mathfrak{j}_\kappa^{1}(\check{t}_\tau),\mathfrak{j}_\kappa^{2}(\check{t}_\tau)\big)-\max.\big(\mathfrak{j}_\kappa^{1}(\check{t}_\tau),\mathfrak{j}_\kappa^{2}(\check{t}_\tau)\big)|\Big)\Big\},\\
&= d\big(\mathcal{P}_c^1\cap\mathcal{P}_c^2,\mathcal{P}_c^1\cup\mathcal{P}_c^2\big)
\end{aligned}$$

(iv).By using same procedure, $d\big(\mathcal{P}_c^1,\mathcal{P}_c^1\cap\mathcal{P}_c^2\big)=d\big(\mathcal{P}_c^2,\mathcal{P}_c^1\cup\mathcal{P}_c^2\big)$ can be proved. □

Theorem 9. *For the CPiFSs distance function d, the following hold:*
(i). $d\big(\mathcal{P}_c^1\cup\mathcal{P}_c^2,\mathcal{P}_c^3\big)+d\big(\mathcal{P}_c^1\cap\mathcal{P}_c^2,\mathcal{P}_c^3\big)=d\big(\mathcal{P}_c^1,\mathcal{P}_c^3\big)+d\big(\mathcal{P}_c^2,\mathcal{P}_c^3\big)$
(ii). $d\big(\mathcal{P}_c^1\cup\mathcal{P}_c^2,\mathcal{P}_c^1\big)+d\big(\mathcal{P}_c^1\cap\mathcal{P}_c^2,\mathcal{P}_c^1\big)=d\big(\mathcal{P}_c^1,\mathcal{P}_c^2\big)$
(iii).(ii). $d\big(\mathcal{P}_c^1\cup\mathcal{P}_c^2,\mathcal{P}_c^2\big)+d\big(\mathcal{P}_c^1\cap\mathcal{P}_c^2,\mathcal{P}_c^2\big)=d\big(\mathcal{P}_c^1,\mathcal{P}_c^2\big)$

Proof. It can easily be proved as similar to theorem 8. □

4. η-EQUALITIES OF DISTANCE MEASURES OF CPiFSs

Definition 10. *The two distances* $d_1\big(\mathcal{P}_c^1,\mathcal{P}_c^2\big),d_2\big(\mathcal{P}_c^3,\mathcal{P}_c^4\big)$ *of CPiFSs are known as* η*-equal if and only if* $\max\{d_1\big(\mathcal{P}_c^1,\mathcal{P}_c^2\big),d_2\big(\mathcal{P}_c^3,\mathcal{P}_c^4\big)\}\leq 1-\eta$*, where* $\eta\in[0,1]$*. This* η*-equal distances are represented by notation* $d_1=(\eta)d_2$ *or* $d_2=(\eta)d_1$*.*

Definition 11. *The complement of the CPiFSs distance* $d\big(\mathcal{P}_c^1,\mathcal{P}_c^2\big)$ *is denoted by* $d'\big(\mathcal{P}_c^1,\mathcal{P}_c^2\big)$ *and is defined as:*

$$d'\big(\mathcal{P}_c^1,\mathcal{P}_c^2\big)=1-d\big(\mathcal{P}_c^1,\mathcal{P}_c^2\big)$$

Theorem 12. *For $\eta_1, \eta_2, \eta_3 \in [0,1]$ then $\eta_1 * \eta_2 = \max\{0, \eta_1 + \eta_2 - 1\}$ satisfy the following: (i).* $0 * \eta_1 = 0$
(ii). $1 * \eta_1 = \eta_1$
(iii). $0 \le \eta_1 * \eta_2 \le 1$
(iv). if $\eta_1 \le \eta_2$ then $\eta_1 * \eta_3 \le \eta_1 * \eta_3$.
(v). $(\eta_1 * \eta_2) * \eta_3 = \eta_1 * (\eta_2 * \eta_3)$

Proof. Each property can easily be proved by using $\eta_1 * \eta_2 = \max\{0, \eta_1 + \eta_2 - 1\}$. □

Theorem 13. *If $d_1 = (\eta_1)d_2$ and $d_2 = (\eta_2)d_3$, then $d_1 = (\eta)d_3$, where $\eta = \eta_1 * \eta_2$.*

Proof. Since $d_1 = (\eta_1)d_2$ and $d_2 = (\eta_2)d_3$, therefore, by theorem 10 it follows that, $\max\{d_1(\mathcal{P}_c^1, \mathcal{P}_c^2), d_2(\mathcal{P}_c^3, \mathcal{P}_c^4)\} \le 1-\eta_1$ and $\max\{d_2(\mathcal{P}_c^3, \mathcal{P}_c^4), d_4(\mathcal{P}_c^5, \mathcal{P}_c^6)\} \le 1-\eta_2$. Since distance cannot be negative and $1 - \eta_1 \ge 0, 1 - \eta_2 \ge 0$, therefore,

$$\max\{d_1(\mathcal{P}_c^1, \mathcal{P}_c^2), d_2(\mathcal{P}_c^3, \mathcal{P}_c^4)\} + \max\{d_2(\mathcal{P}_c^3, \mathcal{P}_c^4), d_4(\mathcal{P}_c^5, \mathcal{P}_c^6)\} \le (1-\eta_1) + (1-\eta_2)$$

Without loss of generality,

$$\begin{aligned} \max\{d_1(\mathcal{P}_c^1, \mathcal{P}_c^2), d_3(\mathcal{P}_c^5, \mathcal{P}_c^6)\} \quad &\le \quad \max\{d_1(\mathcal{P}_c^1, \mathcal{P}_c^2), d_2(\mathcal{P}_c^3, \mathcal{P}_c^4)\} + \\ &\quad\ \max\{d_2(\mathcal{P}_c^3, \mathcal{P}_c^4), d_4(\mathcal{P}_c^5, \mathcal{P}_c^6)\} \le 1 - (\eta_1 + \eta_2 - 1) \end{aligned}$$

Say $\eta_1 + \eta_2 - 1 = \eta$,

$$\max\{d_1(\mathcal{P}_c^1, \mathcal{P}_c^2), d_3(\mathcal{P}_c^5, \mathcal{P}_c^6)\} \le 1 - \eta$$

It follows that $d_1 = (\eta)d_3$. □

5. APPLICATION RELATED TO DISTANCE MEASURES OF THE CPiFSs

5.1. Problem based on the selection of a place for the construction of a hospital

Ensuring the provision of basic needs and facilities is an essential responsibility of any government, as it significantly contributes to the overall well-being of its citizens. In the contemporary context, the establishment of quality healthcare facilities, particularly hospitals, stands out as a crucial requirement for a better life. Constructing a hospital is a complex task, and one of the primary challenges lies in selecting an optimal location for it. The location of a hospital plays a pivotal role in determining its accessibility and effectiveness in serving the community. While the construction phase may pose certain challenges, the careful consideration of the hospital's location involves a multi-attribute decision-making process. Several factors must be taken into account to ensure that the chosen location aligns with the diverse needs of the population it aims to serve. In summary, the quest for an ideal location for hospital space involves thoughtful consideration of distances to crucial amenities, and the proposed decision-making algorithm offers a systematic means to navigate the complexities involved in selecting the perfect hospital's location, ensuring it aligns with various individual preferences and requirements.

5.2. Step-wise procedure based on distance measure of CPiFSs

To address the complexities involved in selecting an ideal location based on various factors, a multi-attribute decision-making algorithm provides a systematic approach. In every society or city, there exists an ideal place that is easy to approach for the majority. The algorithmic approach, based on the Complex Picture Fuzzy Soft Sets (CPiFSs) distance measure, offers a methodical way to navigate these complexities.
The algorithm involves the following steps:

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1. Construct the CPiFSs for the ideal location by utilizing relevant information pertaining to those ideal locations.

2. Construct the CPiFSs for the available locations by utilizing the information associated with each of those locations.

3. Determine the distance of each available location from the ideal location using the CPiFSs distance measure formula.

4. Select the location whose distance from the ideal location is minimum, indicating its closer alignment with the desired attributes.

By following these steps, the algorithm helps individuals systematically evaluate and compare different locations based on a variety of factors, facilitating the selection of the most suitable place for a hospital. The utilization of the CPiFSs distance measure enhances the precision and objectivity of the decision-making process, providing a valuable tool for an optimal space.

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The mentioned algorithm in pictorial form is shown in Figure 1.

5.3. Case Study: MADM-based hospital site selection

The selection of where to build a hospital is a multifaceted, unpredictable decision that requires negotiating a complicated web of variables. It is necessary to carefully balance several factors, including infrastructure, accessibility, healthcare needs locally, and demographics. The decision-making process is further complicated by uncertainties resulting from projected population increase, modifications to healthcare regulations, and advancements in medical technologies. Robust analytical methods that can account for these uncertainties are necessary for effective site selection to guarantee the best possible outcomes when it comes to community healthcare services.
The growing requirement for more hospitals is caused by several interrelated factors that shape the needs of modern healthcare. In developing countries, like Pakistan, the current healthcare infrastructure is under more stress due to population growth, especially in urban areas, which has resulted in crowded facilities

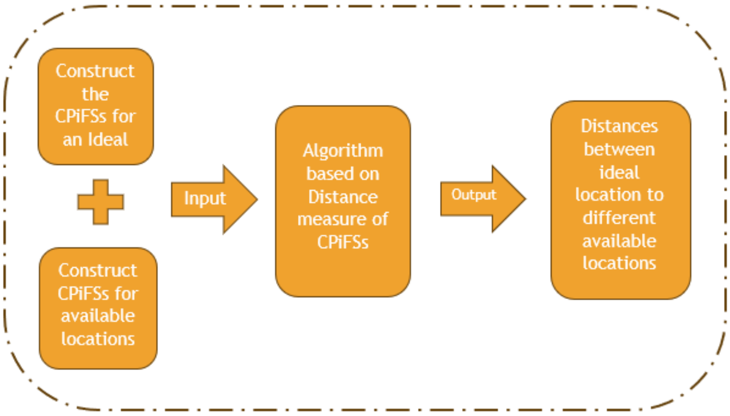

Figure 1: Algorithm based on CPiFSs

and longer wait times for patients. Furthermore, longer life expectancies brought about by improvements in medical technology and therapies have increased the number of older people in need of complicated healthcare. Additionally, changes in the patterns of disease, such as the increase in pandemics and chronic illnesses, emphasize the need for greater healthcare services to address changing public health issues. The need for more hospitals grows stronger as societies and healthcare expectations change and more healthcare facilities are needed to provide everyone with timely, accessible, and high-quality care.
There are some backward cities in the South Punjab region of the province of Punjab, Pakistan, where there is a dire need for hospitals to provide healthcare services. The Government of Punjab Health Department intends to construct a children's hospital in a particular city, "Rajanpur." After the survey made by experts, three potential sites, $\check{q}_1$, $\check{q}_2$, and $\check{q}_3$ for the construction of a new hospital, were shortlisted. To select the unique site, the decision makers decided on some parameters like Future Growth and Expansion ($\check{t}_1$), Safety and Security ($\check{t}_2$), Environmental Considerations ($\check{t}_3$) and Demographics and Population Health ($\check{t}_4$). To represent these decision-making factors, we define the set $\check{T} = \{\check{t}_1, \check{t}_2, \check{t}_3, \check{t}_4\}$, encompassing the four main factors. The set of potential locations for the new hospital is denoted as $\check{Q} = \{\check{q}_1, \check{q}_2, \check{q}_3\}$.

1. Since an ideal location $\check{q}$ is considered with the maximum positive membership, minimum negative membership and accordingly neutral membership degree. So the the CPiFSs element for the ideal location $\check{q}$ by utilizing rele-

vant information pertaining to those ideal locations is given as:

$$\aleph(\check{q}) = \left\{ \begin{array}{l} \left(\check{t}_1, \left(0.94e^{(2\pi)(0.45)}, 0.04e^{(2\pi)(0.32)}, 0.01e^{(2\pi)(0.12)}\right)\right), \\ \left(\check{t}_2, \left(0.91e^{(2\pi)(0.51)}, 0.06e^{(2\pi)(0.29)}, 0.02e^{(2\pi)(0.19)}\right)\right), \\ \left(\check{t}_3, \left(0.93e^{(2\pi)(0.49)}, 0.03e^{(2\pi)(0.36)}, 0.03e^{(2\pi)(0.16)}\right)\right), \\ \left(\check{t}_4, \left(0.90e^{(2\pi)(0.53)}, 0.05e^{(2\pi)(0.28)}, 0.02e^{(2\pi)(0.18)}\right)\right) \end{array} \right\}.$$

2. The CPiFSs elements for the available locations $\check{q}_1, \check{q}_2, \check{q}_3$ by utilizing the information associated with each of those locations are given as:

$$\aleph(\check{q}_1) = \left\{ \begin{array}{l} \left(\check{t}_1, \left(0.53e^{(2\pi)(0.32)}, 0.14e^{(2\pi)(0.17)}, 0.10e^{(2\pi)(0.31)}\right)\right), \\ \left(\check{t}_2, \left(0.41e^{(2\pi)(0.34)}, 0.12e^{(2\pi)(0.15)}, 0.23e^{(2\pi)(0.10)}\right)\right), \\ \left(\check{t}_3, \left(0.34e^{(2\pi)(0.46)}, 0.11e^{(2\pi)(0.21)}, 0.19e^{(2\pi)(0.19)}\right)\right), \\ \left(\check{t}_4, \left(0.47e^{(2\pi)(0.16)}, 0.35e^{(3\pi)(0.13)}, 0.09e^{(2\pi)(0.13)}\right)\right) \end{array} \right\},$$

$$\aleph(\check{q}_2) = \left\{ \begin{array}{l} \left(\check{t}_1, \left(0.49e^{(2\pi)(0.22)}, 0.09e^{(2\pi)(0.19)}, 0.08e^{(2\pi)(0.21)}\right)\right), \\ \left(\check{t}_2, \left(0.43e^{(2\pi)(0.31)}, 0.15e^{(2\pi)(0.21)}, 0.19e^{(2\pi)(0.23)}\right)\right), \\ \left(\check{t}_3, \left(0.41e^{(2\pi)(0.56)}, 0.20e^{(2\pi)(0.16)}, 0.11e^{(2\pi)(0.16)}\right)\right), \\ \left(\check{t}_4, \left(0.59e^{(2\pi)(0.16)}, 0.32e^{(3\pi)(0.13)}, 0.17e^{(2\pi)(0.30)}\right)\right) \end{array} \right\},$$

$$\aleph(\check{q}_3) = \left\{ \begin{array}{l} \left(\check{t}_1, \left(0.51e^{(2\pi)(0.45)}, 0.11e^{(2\pi)(0.20)}, 0.21e^{(2\pi)(0.16)}\right)\right), \\ \left(\check{t}_2, \left(0.61e^{(2\pi)(0.14)}, 0.19e^{(2\pi)(0.16)}, 0.21e^{(2\pi)(0.23)}\right)\right), \\ \left(\check{t}_3, \left(0.39e^{(2\pi)(0.26)}, 0.21e^{(2\pi)(0.14)}, 0.09e^{(2\pi)(0.24)}\right)\right), \\ \left(\check{t}_4, \left(0.69e^{(2\pi)(0.16)}, 0.21e^{(3\pi)(0.13)}, 0.04e^{(2\pi)(0.24)}\right)\right) \end{array} \right\}.$$

3. The distance from each available location $\check{q}_1, \check{q}_2, \check{q}_3$ to the ideal location $\check{q}$ using the CPiFSs distance measure formula given in 6, is given as:

$$d\big(\aleph(\check{q}_1), \aleph(\check{q})\big) = 1.3110, \quad d\big(\aleph(\check{q}_1), \aleph(\check{q})\big) = 1.5325, \quad d\big(\aleph(\check{q}_3), \aleph(\check{q})\big) = 1.3241.$$

4. Since the distance of location $\check{q}_1$ is the smallest as compared to the other distances. Therefore, the most appropriate location for the construction

of an idea hospital is $\check{q}_1$. A graphical representation of above mentioned example is shown in Figure 2.

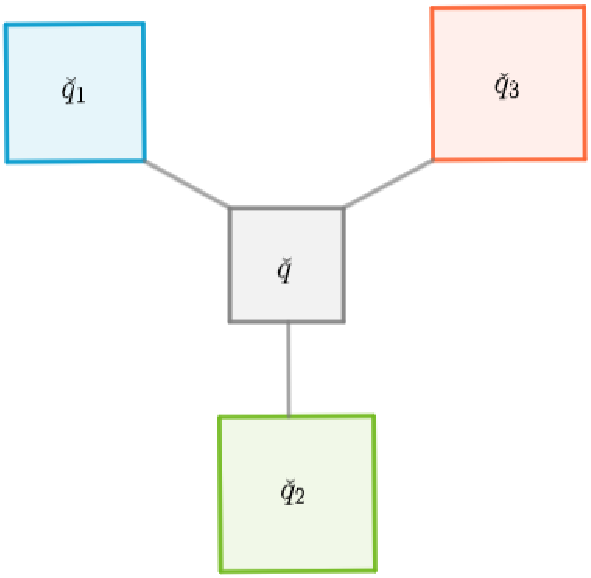

Figure 2: Graphical representation of Example 1

5.4. Comparison and Discussion

A significant divergence from conventional distance measures like Euclidean, normalized Euclidean, Hamming, and normalized Hamming distances is provided by the CPiFSs distance measures. To compare and validate our proposed algorithm, which is based on the distance function defined in Theorem 6, we proceed with the same algorithm by using the extended distance functions, that is, Hamming distance, Euclidean distance, Normalized Hamming distance, and Normalized Euclidean distance, as specified in the work of Kumar et al. [24]. Subsequently, the obtained results and their ranking in a tabulated format are shown in Table 1 and Table 2 for thorough analysis. As compared to such distance measures, the CPiFSs distance measures are excellent at handling complexity, imperfection, and uncertainty in a variety of domains. As such, they are useful instruments for examining intricate data structures and assisting with well-informed decision-making.

Table 1: Comparison and validation of results

Distances	$d_1 = d\big(\aleph(\check{q}_1), \aleph(\check{q})\big)$	$d_2 = d\big(\aleph(\check{q}_2), \aleph(\check{q})\big)$	$d_3 = d\big(\aleph(\check{q}_3), \aleph(\check{q})\big)$
CPiFSs distance	1.3110	1.5325	1.3141
Hamming distance	1.3108	1.5298	1.3145
Normalized Hamming distance	1.3185	1.5423	1.3201
Euclidean distance	1.3121	1.5328	1.3133
Normalized Euclidean distance	1.3201	1.5366	1.3176

Table 2: Ranking

Distances	Ranking
CPiFSs distance	$d_2 > d_3 > d_1$
Hamming distance	$d_2 > d_3 > d_1$
Normalized Hamming distance	$d_2 > d_3 > d_1$
Euclidean distance	$d_2 > d_3 > d_1$
Normalized Euclidean distance	$d_2 > d_1 > d_3$

As can be seen from Tables 1 and 2, the CPiFSs distance measures formulation is a strong measure that shows findings that are equivalent to those of known metrics like Euclidean distance, normalized Euclidean distance, Hamming distance, and normalized Hamming distance. The CPiFSs distance is valid for capturing dissimilarity in complex fuzzy soft sets, as evidenced by the consistency seen across different distance metrics. This thorough comparison sheds light on the effectiveness and dependability of the CPiFSs distance and validates its value as a useful metric for determining dissimilarity across intricate fuzzy soft sets.

5.5. Beneficial features of the research

Compared to conventional fuzzy sets or soft sets, the CPiFSs and their distance measures have severalbenefits, especially when processing complicated and multidimensional information. Some of them are provided below:

1. Enhanced Representation: By combining complex numbers with fuzzy and soft-set theories, the CPiFSs offer a more thorough depiction of ambiguity, imperfection, and complexity in data. A more sophisticated comprehension of actual occurrences is made possible by this improved depiction.
2. Ability to Capture Subtleties: When dealing with complicated or high-dimensional data, the CPiFSs are capable of detecting subtle linkages and interconnections that standard measurements could miss. Developing sound decisions in situations with complex information dynamics requires this capacity.

3. Robustness to Data Complexity: Because of their adaptable structure, which takes into account different levels of membership and uncertainty, they are resistant to the complications present in real-world data, such as uncertainty, and vagueness.
4. Versatility Across Domains: The CPiFSs have proven to be useful in many different fields, such as machine learning, image processing, pattern identification, and decision-making. Their adaptability renders them invaluable instruments for tackling intricate issues across several domains.
5. Theoretical Advancements: By establishing new avenues for study and advancements in adjacent domains, the development of CPiFSs distance measures advances theoretical frameworks for managing complicated data structures.

6. CONCLUSION

Uncertain and ambiguous information is frequently used in site selection procedures. The CPiFSs can deal with this kind of fuzziness and ambiguity, which makes them appropriate in scenarios where accurate data may be hard to come by or unavailable. The suggested algorithm, which considers the distance measure of CPiFSs, offers decision-makers a flexible method by letting them consider a variety of factors and characteristics. This adaptability is crucial when choosing a hospital location because there are many variables to take into account, such as community needs, transportation, and infrastructure. Decision-makers can use this improved decision assistance to help them select a location that complies with the objectives and specifications of the healthcare system. Over time, changes in healthcare policies, technological improvements, and population growth can bring about changes in hospital facilities. Because of their flexibility, CPiFSs enable decision-makers to revise and alter the site selection standards as circumstances change. Additionally, this method can be used in a variety of real-world scenarios where judgments are made based on the separations between distinct entities. It is not just restricted to the selection of hospital locations. The algorithm's adaptability makes it possible to apply it in a variety of scenarios involving decision-making outside of the particular setting of hospital planning. Think of situations where the location is important, such as choosing a site for a new school, business, factory, or other establishment. In these situations, decision-makers must assess several issues, including environmental concerns, transportation infrastructure, and proximity to potential users. Decision-makers may incorporate complex characteristics and traits associated with these entities into their decisions with greater precision and knowledge thanks to the framework that the CPiFS-based algorithm offers. Furthermore, the algorithm's adaptability makes it useful in a variety of domains outside of actual geographic location determinations. It could be applied, for example, to supply chain management to maximize supplier selection based on costs, distances, and other pertinent variables. When making decisions about conservation efforts in environmental research, the algorithm could help by taking into account prospective threats, biodiversity hotspots, and the distances between protected regions.

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