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ECONOMICAL HEURISTICS FOR FULLY INTERVAL INTEGER MULTI-OBJECTIVE FUZZY AND NON-FUZZY TRANSPORTATION PROBLEMS

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Abstract: The single-objective fuzzy or non-fuzzy transportation problem (TP) is not capable of dealing with real-life decision-making problems due to our current competitive market state. In this article, we investigate a fully integer interval multi-objective transportation problem (FIIMOTP) and a fully fuzzy integer multi-objective transportation problem (FFIMOTP). We also provide two solution approaches for solving the FIIMOTP and FFIMOTP. Numerical examples are provided to validate these two approaches. Our results show that the proposed algorithm hugely outperforms the best solution approaches.

Keywords: Fuzzy integer multi-objective transportation problem, solution approach, integer interval multi-objective transportation problem, decision making.

MSC: 90B06, 90C29, 03B52.

1. INTRODUCTION

TP is one of the well-known problems in the field of mathematical programming. A transportation problem is one that arises when goods must be transported from one source to another. This concept was first introduced by Hitchcock [8] and then by Koopman [11]. Juman and Hoque [29] also explored this single objective TP. However, because of its unique mathematical structure, the transportation simplex approach is frequently used to solve the TP problem. These transportation problems have a single objective in common. All real-life TPs cannot be made with a single objective TP. However, under actual circumstances, every organization seeks to accomplish multiple objectives while organizing the movement of goods.

Several real-world problems can be solved using operational research approaches, but they struggle to deal with imprecise data. However, by using fuzzy linear programming, many researchers have been successful in obtaining imprecise and fuzzy linear programming problems (LPP). Bellman and Zadeh [4] were the first to put forth the idea of fuzzy decision-making. Fuzzy optimization techniques have been used for LPPs with multiple objectives by Zimmermann [28].

In this research, we focus on the FIIMOTP solution method, where the decision-maker expresses the source and destination parameters' interval values as well as the value coefficients of the objective functions. First, we calculate it using a method that Akilbasha et al. [2] adapted. The problem is then solved using the suggested technique. We also provide a fuzzy method for solving FFIMOTP.

2. LITERATURE REVIEW

In this section, we will discuss some pertinent research on problems that deal with inexact coefficients in TPs.

Many scholars have suggested fuzzy and interval programming approaches for solving TP, including Chanas et al. [5], Ishibuchi [10], Moore [13], Oliveira et al. [14], and Tong [25]. A novel method for resolving a fuzzy TP was put out by Ebrahimnejad [7]. In order to determine the value interval of the cost parameters of the interval TP, Panda and Das [15] created a two-vehicle cost variable interval transportation model. To unravel interval TPs, Sengupta and Pal [23] suggested a replacement fuzzy-oriented solution approach. Hosseinzade and Hassanpour [9] derived Karush Kuhn Tucker's conditions for an objective function problem with interval values and used them to solve the problem.

For integer TPs, Pandian and Natarajan [16,17,18,19] found a compromise solution, offered a substitute method for finding an optimal solution, and presented two new algorithms, a fuzzy zero-point algorithm and a level-bound algorithm for finding fuzzy TPs and fully fuzzy interval integer TPs, respectively. A method for supporting weighted sum and resolving multi-objective stochastic TPs was created by Roy and Mahapatra [22]. A MOMILP was presented by Singh and Goh [24] and incorporates a number of conflicting objectives.

By using a mid-width method, Akilbasha et al. [1,2] have introduced a novel exact method for solving fully interval integer TPs and fully fuzzy integer TPs. By using an interactive method, Vincent Yu et al. [26] were able to arrive at a compromise solution for the multi-objective interval TP. Annie Christi and Kalpana [3] were able to unravel multi-objective fuzzy TPs with non-linear membership functions using compromise solutions. Fuzzy programming was developed by Rani [21] and Dalman [6] to solve multi-objective TP and multi-objective solid TP, respectively. Fuzzy programming has been suggested as a strategy to solve multi-objective TP by Yeola and Jahav [27]. Multi-objective interval TP was offered as a compromise by Patel and Dhodiya [20].

In addition, the research demonstrates many optimization techniques using sensitivity analysis, which greatly enhances the accuracy of the computational findings. It has been suggested by Ali et al. [30] that a method be used for multiobjective optimization, in particular, nonlinear supply chains under uncertainty, to attain the lowest transportation cost while regulating all other degradation variables. Also, many researchers including Pratihar et al. [33], Veeramani et al. [34], Kumar et al. [35], and Akram et al. [36] studied various multiobjective TPs under uncertainty. More MATLAB code has been executed for nonlinear and stochastic situations using various validation procedures. Das and Lee [31] offer research utilizing the stochastic technique with uncertainties for a multiobjective allocation issue involving a transport with a Weibull distribution. Palanivel and Das [32] proposed a nonlinear programming problem for optimizing an objective function with multiple constraints. Wang [37] has invented a smart parking system to avoid traffic congestion through a smart transportation approach. Edalatpanah [38] studied the multidimensional solution approach to solve fuzzy linear programming problems. Khalifa et al. [39] have proposed the min-max goal programming approach for solving piecewise quadratic fuzzy multi-objective de novo programming problems. Many researchers including Alburaikan et al. [44], Sheikhi and Ebadi [45], El-Wahed Khalifa and Ali Yousif [48], and Akram et al. [50] have proposed various methods to solve different types of linear programming problems. Kané et al. [42,43] and Akram et al. [46,49] have developed different methods to solve various transportation problems. Ghasemi et al. [40] and Komijan [47] have studied multi-objective and multi-level vehicle routing problems in different environments. Abdou El-Morsy [41] proposed a zero-base budgeting method for the selection and management of budgets.

Here, we present a technique for solving FIIMOTP and FFIMOTP. The benefits of the suggested approaches, the conclusion, and novel methods for solving FIIMOTP and FFIMOTP are described in the following sections of the study, together with numerical examples used to verify the methods.

3. A MATHEMATICAL MODEL AND A DEVELOPED SOLUTION METHODOLOGY WITH EXAMPLES

Fully interval integer multi-objective transportation problems:

$$Minimize\ Z^m = [z_1{}^m, z_2{}^m] = \sum_{i=1}^{M} \sum_{j=1}^{N} [c_{ij}{}^m, d_{ij}{}^m] \otimes [x_{ij}, y_{ij}]$$

Subject to

$$\sum_{j=i}^{N} [x_{ij}, y_{ij}] = [a_i{}^m, p_i{}^m] \qquad i = 1,2,3,\dots,M\ \ and\ \ m = 1,2,3,\dots,L \qquad (1)$$

$$\sum_{i=1}^{M} [x_{ij}, y_{ij}] = [b_j{}^m, q_j{}^m] \qquad j = 1,2,3,\dots,N\ \ and\ \ m = 1,2,3,\dots,L \qquad (2)$$

$$x_{ij} \geq 0, \qquad y_{ij} \geq 0, \qquad i = 1,2,3,\dots,M;\ m = 1,2,3,\dots,L\ \ and\ \ j = 1,2,3,\dots,N\ and\ are\ integer \qquad (3)$$

where $c_{ij}{}^m$ and $d_{ij}{}^m$ are positive real numbers $\forall$ i, j and *m*. $a_i{}^m, p_i{}^m, b_j{}^m, and\ q_j{}^m$ are positive real numbers $\forall$ i, j and *m*.

3.1. A new approach for solving a fully interval integer multi-objective transportation problem (FIIMOTP)

An algorithm to solve a fully interval integer multi-objective TP is presented in this section.

New Approach: *Heuristic for solving a fully interval integer multi-objective TP*

Step 1: First, consider one objective at a time and ignore the other objectives. Then solve this fully interval integer single-objective TP using Akilbasha et al. [2]'s approach. For L different objective functions, repeat this for L times. Let $\{[{x_1}^*, {y_1}^*], [{x_2}^*, {y_2}^*], \dots, [{x_L}^*, {y_L}^*]\}$ be the respective optimal solutions of the L objective functions.

Step 2: Build a pay-off matrix of order L⊗L by computing all the objective functions of all the optimal solutions obtained in Step 1:

$$\begin{bmatrix} Z^1 = [Z_1, Z_2]_{({x_1}^*, {y_1}^*)} & Z^2 = [Z_1, Z_2]_{({x_1}^*, {y_1}^*)} & \cdots & Z^L = [Z_1, Z_2]_{({x_1}^*, {y_1}^*)} \\ Z^1 = [Z_1, Z_2]_{({x_2}^*, {y_2}^*)} & Z^2 = [Z_1, Z_2]_{({x_2}^*, {y_2}^*)} & \cdots & Z^L = [Z_1, Z_2]_{({x_2}^*, {y_2}^*)} \\ Z^1 = [Z_1, Z_2]_{({x_3}^*, {y_3}^*)} & Z^2 = [Z_1, Z_2]_{({x_3}^*, {y_3}^*)} & \cdots & Z^L = [Z_1, Z_2]_{({x_3}^*, {y_3}^*)} \\ \vdots & \vdots & \vdots & \vdots \\ Z^1 = [Z_1, Z_2]_{({x_L}^*, {y_L}^*)} & Z^2 = [Z_1, Z_2]_{({x_L}^*, {y_L}^*)} & \cdots & Z^L = [Z_1, Z_2]_{({x_L}^*, {y_L}^*)} \end{bmatrix}$$

Step 3:

1. From the payoff matrix provided in Step 2, create the following payoff matrix by taking into account all upper limit values for all intervals:

$$Z_U = \begin{bmatrix} Z_U^1 = [Z_2]_{({y_1}^*)} & Z_U^2 = [Z_2]_{({y_1}^*)} \cdots & Z_U^m = [Z_2]_{({y_1}^*)} \cdots & Z_U^L = [Z_2]_{({y_1}^*)} \\ Z_U^1 = [Z_2]_{({y_2}^*)} & Z_U^2 = [Z_2]_{({y_2}^*)} \cdots & Z_U^m = [Z_2]_{({y_2}^*)} \cdots & Z_U^L = [Z_2]_{({y_2}^*)} \\ Z_U^1 = [Z_2]_{({y_3}^*)} & Z_U^2 = [Z_2]_{({y_3}^*)} \cdots & Z_U^m = [Z_2]_{({y_3}^*)} \cdots & Z_U^L = [Z_2]_{({y_3}^*)} \\ \vdots & \vdots \quad \vdots & \vdots \quad \vdots & \vdots \\ Z_U^1 = [Z_2]_{({y_L}^*)} & Z_U^2 = [Z_2]_{({y_L}^*)} \cdots & Z_U^m = [Z_2]_{({y_L}^*)} \cdots & Z_U^L = [Z_2]_{({y_L}^*)} \end{bmatrix}$$

In the m^{th} column of the payoff matrix Z_u mentioned above, find the L_m (Lower bound) and U_m (Upper bound) values that correspond to the m^{th} objective function, $Z_U^m(y)$. Formulate the membership function as follows using Zimmermann's [28] method for each objective function $Z_U^m(y_m), m = 1,2,3, \dots, L$:

$$\mu(Z_U^m(y_m)) = \begin{cases} 0, & if\ Z_U^m(y_m) \geq U_m \\ \frac{U_m - Z_U^m(y_m)}{U_m - L_m}, & if\ L_m < Z_U^m(y_m) < U_m, \quad m = 1,2,3, \dots, L \\ 1, & if\ Z_U^m(y_m) \leq L_m \end{cases} \quad (4)$$

2. From the payoff matrix provided in Step 2, create the following payoff matrix by taking into account the lower limit values of all intervals:

$$Z_L = \begin{bmatrix} Z_L^1 = [Z_1]_{(x_1^*)} & Z_L^2 = [Z_1]_{(x_1^*)} & \cdots & Z_L^m = [Z_1]_{(x_1^*)} \cdots & Z_L^L = [Z_1]_{(x_1^*)} \\ Z_L^1 = [Z_1]_{(x_2^*)} & Z_L^2 = [Z_1]_{(x_2^*)} & \cdots & Z_L^m = [Z_1]_{(x_2^*)} \cdots & Z_L^L = [Z_1]_{(x_2^*)} \\ Z_L^1 = [Z_1]_{(x_3^*)} & Z_L^2 = [Z_1]_{(x_3^*)} & \cdots & Z_L^m = [Z_1]_{(x_3^*)} \cdots & Z_L^L = [Z_1]_{(x_3^*)} \\ \vdots & \vdots & \vdots & \vdots \quad \vdots & \vdots \\ Z_L^1 = [Z_1]_{(x_L^*)} & Z_L^2 = [Z_1]_{(x_L^*)} & \cdots & Z_L^m = [Z_1]_{(x_L^*)} \cdots & Z_L^L = [Z_1]_{(x_L^*)} \end{bmatrix}$$

Obtain l_m (Lower bound) and u_m (Upper bound) corresponding to the m^{th} objective function, $Z_L^m(x)$ in the m^{th} column of the above payoff matrix Z_L. Then by using Zimmermann's [28] approach for each objective function $Z_L^m(x_m)$, $m = 1,2,3, \dots, L$, formulate the membership function as given below:

$$\mu\left(Z_L^m(x_m)\right) = \begin{cases} 0, & if\ Z_L^m(x_m) \geq u_m \\ \frac{u_m - Z_L^m(x_m)}{u_m - l}, & if\ L_m < Z_L^m(x_m) < u_m,\ m = 1,2,3, \dots, L \\ 1, & if\ Z_L^m(x_m) \leq l_m \end{cases} \tag{5}$$

Step 4: Formulate the corresponding integer LPPs as follows by adding the auxiliary variable (see Maity and Roy [12]):

1. In the m^{th} column of the payoff matrix Z_U mentioned above, for the m^{th} objective function, $Z_U^m(y)$, $m = 1,2,3, \dots, L$

$Maximum\ \lambda$

Subject to

$$\lambda \leq \mu\left(Z_U^m(y_m)\right), \mathrm{m} = 1,2,3, \dots, \mathrm{L}$$

$$\sum_{j=1}^{N} y_{ij} = p_i{}^m \quad i = 1,2,3, \dots, M \quad and \ \ m = 1,2,3, \dots, L \tag{6}$$

$$\sum_{i=1}^{M} y_{ij} = q_j{}^m \quad j = 1,2,3, \dots, N \quad and \ \ m = 1,2,3, \dots, L \tag{7}$$

$$y_{ij} \geq 0, \quad i = 1,2,3, \dots, M\ and\ j = 1,2,3, \dots, N\ and\ are\ integers \tag{8}$$

Let [y*] be its associated solutions and let λ^U be the optimal upper bound cost.

2. In the m^{th} column of the payoff matrix Z_L mentioned above, for the m^{th} objective function, $Z_L^m(x)$, $m = 1,2,3, \dots, L$

$Maximum\ \lambda$

Subject to

$$\lambda \leq \mu(Z_L^m(x_m)), \qquad \mathrm{m} = 1,2,3, \dots, \mathrm{L}$$

$$\sum_{j=1}^{N} x_{ij} = a_i{}^m \quad i = 1,2,3, \dots, M \ \ and \ \ m = 1,2,3, \dots, L \tag{9}$$

$$\sum_{i=1}^{M} x_{ij} = b_j^{\,m} \qquad j = 1,2,3,\dots,N \;\; and \;\; m = 1,2,3,\dots,L \tag{10}$$

$$x_{ij} \geq 0, \;\; i = 1,2,3,\dots,M \; and \; j = 1,2,3,\dots,N \; and \; are \; integers \tag{11}$$

Let [x*] be the solutions that are associated with it, and let λ^L be the optimal Lower bound cost.

Here, $\mu(Z_L^m(x_m))$ *and* $\mu(Z_U^m(y_m))$ are the membership functions of the mth objective function for $m = 1,2,3,\dots,L$ as stated in Step 3.

Step 5: Determine the ultimate interval compromise solution to the specified FIIMOTP by solving both models obtained in step 4.

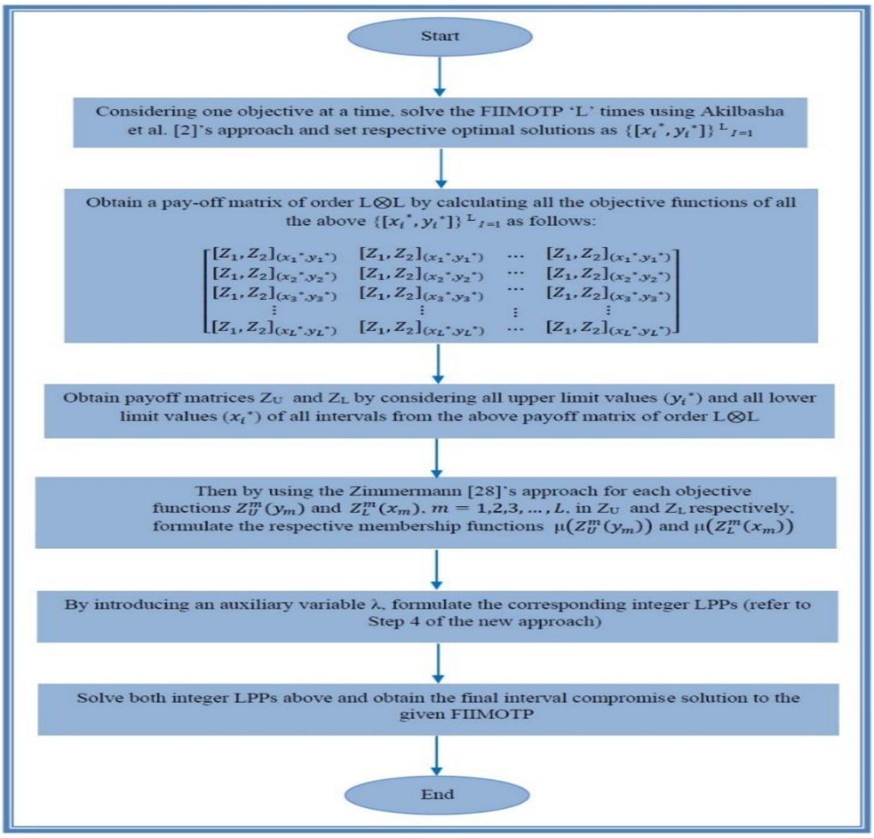

Figure 1: Flowchart representation of our approach 1 for solving FIIMOTP

3.2. Demonstration via a numerical example

Utilizing some data that was provided at random, the aforementioned solution strategy is illustrated, and a numerical example is provided in Vincent Yu et al. [26]. Data for this benchmark instance along with its certain randomly generated data are given below:

A drug company produces a variety of medicines in its three factories A, B and C. The product will be sent to four destinations D1, D2, D3 and D4 from the three factories. Determine a shipping plan for the company from three factories to four destinations such that the total shipping cost and time should be minimal using the following numerical data obtained from the company:

The minimum supply from A, B and C are 15,000, 18,000 and 10,000 respectively and the maximum supply from A, B and C are 17,000, 20,000 and 11,000 respectively. The minimum demand for D1, D2, D3 and D4 are 8,000, 10,000, 11,000 and 14,000 respectively and the maximum demand for D1, D2, D3 and D4 are 9,000, 11,000, 12,000 and 16,000 respectively.

The unit shipping cost and time range from each supply point to each demand point are given below:

	D1	D2	D3	D4
	[2,4] [4,6]	[1,2] [5,8]	[2,3] [7,9]	[2,3] [8,12]
	[1,3] [6,8]	[2,4] [7,13]	[2,5] [10,14]	[2,6] [15,19]
	[1,2] [8,12]	[2,3] [10,14]	[2,4] [16,20]	[1,7] [8,10]

In each cell, the 1st interval is the shipping cost per unit on the corresponding route and the 2nd interval is the time of shipping per unit on that route.

The interval compromise solution to this problem, FIIMOTP is [68000, 581000]. A detailed computation of this problem is given in Appendix A.

3.4. Fully fuzzy integer multi-objective transportation problems (FFIMOTP)

Consider the following FFIMOTP:

$$Minimize\ \tilde{Z}^m = \sum_{i=1}^{M}\sum_{j=1}^{N} \tilde{c}_{ij}^m\, \tilde{x}_{ij}$$

Subject to

$$\sum_{j=1}^{N} \tilde{x}_{ij} = \tilde{a}_j^m \qquad i = 1,2,\dots,M \ \ and\ \ m = 1,2,\dots,L \tag{12}$$

$$\sum_{i=1}^{M} \tilde{x}_{ij} = \tilde{b}_j^m \qquad j = 1,2,\dots,N \ \ and\ m = 1,2,\dots,L \tag{13}$$

$$\tilde{x}_{ij} \geq \tilde{0} \quad i = 1,2,\dots,M \ \ and \ \ j = 1,2,\dots,N\ and\ are\ integers. \tag{14}$$

Where the decision variables $\tilde{x}_{ij}$, $\forall\ i,\ j$ and m are triangular fuzzy numbers and parameters $\tilde{c}_{ij}^m$, $\tilde{a}_j^m$ and $\tilde{b}_j^m$ are positive triangular fuzzy numbers $\forall\ i,\ j$ and m.

A triangular fuzzy number (p,q,r) can be represented as an interval number as follows:

$$relation : (p,q,r) = [p + (q-p)\alpha, r - (r-q)\alpha]; \quad 0 \leq \alpha \leq 1 \tag{15}$$

3.5. A fuzzy approach for solving the FFIMOTP

This section presents a fuzzy approach to solving the FFIMOTP

New fuzzy Approach: *Heuristic for solving an FFIMOTP*

Step A: Transform the given FFIMOTP into a FIIMOTP using the above relation.

Step B: Applying the approach we suggested in section B, find the optimal interval solution to this FIIMOTP.

Step C: Apply the aforementioned relation to the given FFIMOTP to find the optimal fuzzy solution.

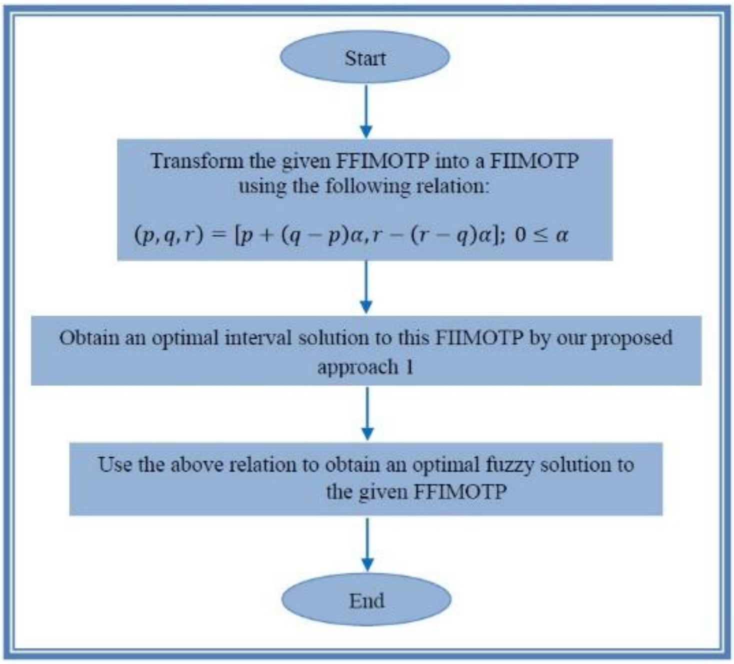

Figure 2: Flowchart representation of our approach 2 for solving FFIMOTP

3.6. Demonstration via a numerical example

The above-mentioned fuzzy approach is illustrated by using some randomly contributed data and the numerical example is given in Akilbasha et al. [2]. Data for this numerical example problem is provided in Appendix B. Besides, a detailed computation of this problem is given in Appendix B.

The literature review indicates that no research has been done on the problems with the FIIMOTP and FFIMOTP. The decision-makers can choose a suitable strategy for shipping the goods in accordance with their financial condition.

4. A COMPARATIVE ASSESSMENT

To show the potential significance of our approach, the new approach of this paper is compared with Akilbasha et al. [2] approach. Both approaches are used to solve two single–objective fuzzy and non-fuzzy instances and two multi-objective fuzzy and non-fuzzy instances and the obtained results are provided in Table 18 of Appendix C. Table 18 clearly shows that the proposed methods of this paper can be used to solve both the single and multi-objective whereas the existing method can solve only single-objective ones. Thus, our method scores over the existing one.

5. MERITS AND LIMITATIONS OF THE PROPOSED METHODS

- Finding the best shipping and transportation costs for the provided FIIMOTP and FFIMOTP is the major goal of this study.
- Due to our current competitive market state, the single-objective fuzzy or non-fuzzy TP is insufficient to handle real-life decision-making problems. So, we look into FIIMOTP and FFIMOTP.
- The current method does not rely on decisions variable upper bounded conditions because it is based on two independent TPs that are derived from the provided FIIMOTP.
- The suggested approach has the potential to be a useful tool and will provide decision-makers with the optimum transportation plan when they are tackling various logistic issues with interval or fuzzy parameters as a multi-objective TP.
- It will assist future researchers in extending this problem to other problems like trapezoidal fuzzy sets, rough sets, and so on with parameters.
- The inherent limitations or constraints of this approach are doubtful for unbalanced cases. So, it is an open problem to extend this work for unbalanced cases as well.

6. CONCLUSION

In this study, FIIMOTP and FFIMOTP are explored, and two techniques for their solution are offered. These approaches are supported by two numerical examples. Finally, it is claimed that our novel method outperformed the current method. In addition, the interval parameter is considered to be the quantities of supply and demand. In reality, though, variations in demands and supplies could adhere to a specific pattern. Thus, assuming a suitable trend in supply and demand distributions, future research could be conducted to enhance this fully integer interval multi-objective fuzzy and non-fuzzy TP. We will fully commit to the current course of future study.

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APPENDICES

Appendix A

Appendix A gives the mathematical formulation for this problem.

Notation

Index set

i Index for factory, $\forall\ i = 1, 2, \ldots, m$

j Index for DC, $\forall\ j = 1, 2, \ldots, n$

Decision variable

x_{ij} Quantity transported from factory i to DC j

Parameter

PC_{ij} Production cost per unit delivered from factory i to DC j

TC_{ij} Transportation cost/per unit transported from factory i to DC j

DT_{ij} Delivery time from factory i to DC j

S_i Total supply available for each factory i

D_j Total demand for each DC j

B_i Budget allocated to each factory i

Objective functions

$$\min Z^1\ (total\ transportation\ cost) = \sum_{i=1}^{m}\sum_{j=1}^{n}(PC_{ij} + TC_{ij})x_{ij} \qquad (i)$$

$$\min Z^2\ (total\ delivery\ time) = \sum_{i=1}^{m}\sum_{j=1}^{n} DT_{ij}\, x_{ij} \qquad (ii)$$

Constraints

$$\sum_{j=1}^{n} x_{ij} \le S_i \quad \forall i \qquad (iii)$$

$$\sum_{i=1}^{m} x_{ij} = D_j \quad \forall j \qquad (iv)$$

$$\sum_{j=1}^{n}(PC_{ij} + TC_{ij})x_{ij} \le B_i \quad \forall i \qquad (v)$$

$$x_{ij} \ge 0 \quad \forall i, j \qquad (vi)$$

The objective functions in Formulae (i)-(ii) are to minimize the TP cost and delivery time. Constraint (iii) ensures that the maximum available supply is no more than all factories' combined capacity. Constraint (iv) guarantees that the available quantities of transported products from each factory to DC can meet total demand. Constraint (v) makes sure that the Production–TP costs of all factories do not exceed the total budget. Constraint (vi) ensures that all decision variables are non-negative.

Table 1: Transportation information of the drug company

	D1	D2	D3	D4	Supply
A	[2,4] [4,6]	[1,2] [5,8]	[2,3] [7,9]	[2,3] [8,12]	[15,000,17,000]
B	[1,3] [6,8]	[2,4] [7,13]	[2,5] [10,14]	[2,6] [15,19]	[18,000,20,000]
C	[1,2] [8,12]	[2,3] [10,14]	[2,4] [16,20]	[1,7] [8,10]	[10,000,11,000]
Demand	[8000,9000]	[10000,11000]	[11000,12000]	[14000,16000]	

Table 2: Information about the unit transportation cost (Z^1)

	D1	D2	D3	D4	Supply
A	[2,4]	[1,2]	[2,3]	[2,3]	[15,000,17,000]
B	[1,3]	[2,4]	[2,5]	[2,6]	[18,000,20,000]
C	[1,2]	[2,3]	[2,4]	[1,7]	[10,000,11,000]
Demand	[8000,9000]	[10000,11000]	[11000,12000]	[14000,16000]	

Table 3: Information on the delivery time (Z^2)

	D1	D2	D3	D4	Supply
A	[4,6]	[5,8]	[7,9]	[8,12]	[15,000,17,000]
B	[6,8]	[7,13]	[10,14]	[15,19]	[18,000,20,000]
C	[8,12]	[10,14]	[16,20]	[8,10]	[10,000,11,000]
Demand	[8000,9000]	[10000,11000]	[11000,12000]	[14000,16000]	

Now, by Step 1, consider one objective at a time and ignore the remaining objectives and the mid-value TP (M) of the problem is given below:

Table 4: The mid-value TP of Z^1

	D1	D2	D3	D4	Supply
A	3	1.5	2.5	2.5	16000
B	2	3	3.5	4	18000
C	1.5	2.5	3	4	10500
Demand	8500	10500	11500	15000	

The optimal solution to the problem (M) constraints are ${m_{13}}^0 = 1000$, ${m_{14}}^0 = 15000$, ${m_{21}}^0 = 8500, {m_{23}}^0 = 10500$, ${m_{32}}^0 = 10500$.

Table 5: The mid-value TP of Z^2

	D1	D2	D3	D4	Supply
A	5	6.5	8	10	16000
B	7	10	12	17	18000
C	10	22	18	9	10500
Demand	8500	10500	11500	15000	

The optimal solution to the problem (M) constraints are ${m_{12}}^0 = 0$, ${m_{13}}^0 = 11500$, ${m_{14}}^0 = 4500$, ${m_{21}}^0 = 8500, {m_{22}}^0 = 10500$, ${m_{24}}^0 = 0$ & ${m_{34}}^0 = 10500$.

Now, by Step 1, consider one objective at a time and ignore the remaining objectives and the half-width TP (W) of the problem is given below:

Table 6: The half-width TP of Z^1

	D1	D2	D3	D4	Supply
A	1	0.5	0.5	0.5	1000
B	1	1	1.5	2	1000
C	0.5	0.5	1	3	500
Demand	500	500	500	1000	

The optimal solution to the problem (W) constraints are ${w_{13}}^0 = 0$, ${w_{14}}^0 = 1000$, ${w_{21}}^0 = 500, {w_{23}}^0 = 500$ & ${w_{32}}^0 = 500$.

Table 7: The half-width TP of Z^2

	D1	D2	D3	D4	Supply
A	1	1.5	1	2	1000
B	1	3	2	2	2000
C	2	2	2	1	500
Demand	500	500	500	1000	

The optimal solutions to the problem (W) constraints are ${w_{12}}^0 = 500$, ${w_{13}}^0 = 500$, ${w_{14}}^0 = 0$, ${w_{21}}^0 = 500, {w_{22}}^0 = 0$, ${w_{24}}^0 = 500$ & ${w_{34}}^0 = 500$.

Now, using Step 1, an optimal solution to the transportation unit cost is $[{x_{13}}^0, {y_{13}}^0] = [1000, 1000]$, $[{x_{14}}^0, {y_{14}}^0]$ = [14000, 16000], $[{x_{21}}^0, {y_{21}}^0]$ = [8000, 9000], $[{x_{23}}^0, {y_{23}}^0]$ = [10000, 11000] &$[{x_{32}}^0, {y_{32}}^0]$ = [10000, 11000].

Now, using Step 1, an optimal solution to the delivery time is $[{x_{12}}^0, {y_{12}}^0]$ = [-500,500],$[{x_{13}}^0, {y_{13}}^0] = [11000, 12000]$, $[{x_{14}}^0, {y_{14}}^0]$ = [4500, 4500], $[{x_{21}}^0, {y_{21}}^0]$ = [8000, 9000], $[{x_{22}}^0, {y_{22}}^0]$ = [10500, 10500], $[{x_{24}}^0, {y_{24}}^0]$ = [-500,500], $[{x_{34}}^0, {y_{34}}^0]$ = [10000, 11000].

Now, by Step 2, the payoff matrix is given below:

Table 8: Payoff matrix of a given problem

		Objectives	
		Z^1	Z^2
Solutions	x_1^*, y_1^*	[78000,166000]	[367000,581000]
	x_2^*, y_2^*	[68500,199500]	[304500,494000]

Now, by Step 3, consider all upper limit values of all intervals from the above payoff matrix

$$Z_U = \begin{bmatrix} 166000 & 581000 \\ 199500 & 494000 \end{bmatrix}$$

By using this upper limit payoff matrix, we get

$$\mu(Z_U^1(y_1)) = \begin{cases} 0, & if\ Z_U^1(y_1) \geq U_1 \\ \dfrac{199500 - Z_U^1(y_1)}{199500 - 166000}, & if\ L_1 < Z_U^1(y_1) < U_1 \\ 1, & if\ Z_U^1(y_1) \leq L_1 \end{cases}$$

$$\mu(Z_U^2(y_2)) = \begin{cases} 0, & if\ Z_U^2(y_2) \geq U_2 \\ \dfrac{581000 - Z_U^1(y_1)}{581000 - 494000}, & if\ L_2 < Z_U^2(y_2) < U_2 \\ 1, & if\ Z_U^2(y_2) \leq L_2 \end{cases}$$

Now, by Step 3, consider all lower limit values of all intervals from the above payoff matrix

$$Z_L = \begin{bmatrix} 78000 & 367000 \\ 68500 & 304500 \end{bmatrix}$$

By using this lower limit payoff matrix, we get

$$\mu(Z_L^1(x_1)) = \begin{cases} 0, & if\ Z_L^1(x_1) \geq u_1 \\ \dfrac{78000 - Z_L^1(x_1)}{78000 - 68500}, & if\ L_1 < Z_L^1(x_1) < u_1 \\ 1, & if\ Z_L^1(x_1) \leq 1 \end{cases}$$

$$\mu(Z_L^2(x_2)) = \begin{cases} 0, & if\ Z_L^2(x_2) \geq u_2 \\ \dfrac{367000 - Z_L^2(x_2)}{367000 - 304500}, & if\ L_2 < Z_L^2(x_2) < u_2 \\ 1, & if\ Z_L^2(x_2) \leq 1 \end{cases}$$

Now, by Step 4, the corresponding integer LPPs are as follows for upper limit values.

$Maximum\ \lambda$

Subject to:

$$\begin{aligned}
&87000\lambda \leq 581000 - (4y_{11} + 2y_{12} + 3y_{13} + 3y_{14} + 3y_{21} + 4y_{22} + 5y_{23} + 6y_{24} \\
&\qquad + 2y_{31} + 3y_{32} + 4y_{33} + 7y_{34}) \\
&(4y_{11} + 2y_{12} + 3y_{13} + 3y_{14} + 3y_{21} + 4y_{22} + 5y_{23} + 6y_{24} + 2y_{31} + 3y_{32} + 4y_{33} \\
&\qquad + 7y_{34}) \leq 581000 \\
&33500\lambda \leq 199500 - (6y_{11} + 8y_{12} + 9y_{13} + 12y_{14} + 8y_{21} + 13y_{22} + 14y_{23} + 19y_{24} \\
&\qquad + 12y_{31} + 14y_{32} + 20y_{33} + 10y_{34}) \\
&6y_{11} + 8y_{12} + 9y_{13} + 12y_{14} + 8y_{21} + 13y_{22} + 14y_{23} + 19y_{24} + 12y_{31} + 14y_{32} \\
&\qquad + 20y_{33} + 10y_{34} \leq 80350 \\
&y_{11} + y_{12} + y_{13} + y_{14} = 17000 \\
&y_{21} + y_{22} + y_{23} + y_{24} = 18000 \\
&y_{31} + y_{32} + y_{33} + y_{34} = 11000 \\
&y_{11} + y_{21} + y_{31} = 9000 \\
&y_{12} + y_{22} + y_{32} = 11000 \\
&y_{13} + y_{23} + y_{33} = 12000 \\
&y_{14} + y_{24} + y_{34} = 16000 \\
&y_{11} + y_{12} + y_{13} + y_{14} = 17000 \\
&y_{21} + y_{22} + y_{23} + y_{24} = 18000 \\
&y_{31} + y_{32} + y_{33} + y_{34} = 11000 \\
&y_{11} + y_{21} + y_{31} = 9000 \\
&y_{12} + y_{22} + y_{32} = 11000 \\
&y_{13} + y_{23} + y_{33} = 12000
\end{aligned}$$

$y_{14} + y_{24} + y_{34} = 16000$
$y_{ij} \geq 0 \quad i = 1,2,3 \ \& \ j = 1,2,3,4$
$0 < \lambda < 1.$

The above model is a LPP. We get the following compromise optimal solution by using lingo 18.0 software:

$[y^*]$ = [0,0,1000,16000,9000,0,11000,0,0,11000,0,0]

The minimum value of the objective functions of the upper bound (λ^U) is 581000.

Now, by Step 4, the corresponding integer LPPs are as follows for lower limit values.

$Maximum\ \lambda$

Subject to:

$$9500\lambda \leq 78000 - (4x_{11} + 5x_{12} + 7x_{13} + 8x_{14} + 6x_{21} + 7x_{22} + 10x_{23} + 15x_{24} + 8x_{31} + 10x_{32} + 16x_{33} + 8x_{34})$$
$$(4x_{11} + 5x_{12} + 7x_{13} + 8x_{14} + 6x_{21} + 7x_{22} + 10x_{23} + 15x_{24} + 8x_{31} + 10x_{32} + 16x_{33} + 8x_{34}) \leq 78000$$
$$(2x_{11} + 1x_{12} + 2x_{13} + 2x_{14} + 1x_{21} + 2x_{22} + 2x_{23} + 2x_{24} + 1x_{31} + 2x_{32} + 2x_{33} + 2x_{34}) \leq 367000$$
$$62500\lambda \leq 367000 - (x_{11} + 1x_{12} + 2x_{13} + 2x_{14} + 1x_{21} + 2x_{22} + 2x_{23} + 2x_{24} + 1x_{31} + 2x_{32} + 2x_{33} + 2x_{34})$$
$x_{11} + x_{12} + x_{13} + x_{14} = 15000$
$x_{21} + x_{22} + x_{23} + x_{24} = 18000$
$x_{31} + x_{32} + x_{33} + x_{34} = 10000$
$x_{11} + x_{21} + x_{31} = 8000$
$x_{12} + x_{22} + x_{32} = 10000$
$x_{13} + x_{23} + x_{33} = 11000$
$x_{14} + x_{24} + x_{34} = 14000$
$x_{11} + x_{12} + x_{13} + x_{14} = 15000$
$x_{21} + x_{22} + x_{23} + x_{24} = 18000$
$x_{31} + x_{32} + x_{33} + x_{34} = 10000$
$x_{11} + x_{21} + x_{31} = 8000$
$x_{12} + x_{22} + x_{32} = 10000$
$x_{13} + x_{23} + x_{33} = 11000$
$x_{14} + x_{24} + x_{34} = 14000$
$x_{ij} \geq 0 \qquad i = 1,2,3 \quad \& j = 1,2,3,4$
$0 < \lambda < 1$

The above model is a LPP. We get the following compromise optimal solution for lower bound by using lingo 18.0 software:

$[x^*]$ = [0,0,1100,4000,8000,10000,0,0,0,0,0,10000].

The minimum value of the objective functions of the lower bound (λ^L) is 68000.

Now, the solution to the given FIIMOTP is [68000, 581000].

Appendix B

The above model is a LPP. We get the following compromise optimal solution for lower bound by using lingo 18.0 software:

Table 9: Information on the triangular fuzzy unit transportation cost and delivery time

	D1	D2	D3	Supply
S1	(1,2,3) (2,4,6)	(10,11,12) (1,3,5)	(4,7,10) (5,10,15)	(1,6,11)
S2	(0,1,2) (7,9,11)	(1,6,11) (2,3,4)	(0,1,2) (1,2,3)	(2,3,4)
S3	(1,5,9) (2,9,16)	(5,15,25) (3,9,15)	(3,9,15) (1,5,9)	(3,4,5)
Demand	(3,7,11)	(1,3,5)	(2,3,4)	(6,13,20)

Now, by Step A, using the relation (15), the given FFIMOTP can be made into FIIMOTP

Table 10: Information on the unit transportation cost and delivery time

	D1	D2	D3	Supply
S1	[1+α, 3-α] [2+2α,6-2α]	[10+α,12-α] [1+2α,5-2α]	[4+3α,10-3α] [5+5α,15-5α]	[1+5α, 11-5α]
S2	[α, 2-α] [7+2α,11-2α]	[1+5α,11-5α] [2+α,4-α]	[α, 2-α] [1+2α,3-α]	[2+α, 4-α]
S3	[1+4α, 9-4α] [2+7α,16-7α]	[5+10α,25-10α] [3+6α,15-6α]	[3+6α, 15-6α] [1+4α,9-4α]	[3+α, 5-α]
Demand	[3+4α,11-4α]	[1+2α, 5-2α]	[2+α, 4-α]	[6+7α, 20-7α]

Now, by Step B,

Table 11: Information about the unit transportation cost (Z^1)

	D1	D2	D3	Supply
S1	[1+α, 3-α]	[10+α, 12-α]	[4+3α, 10-3α]	[1+5α, 11-5α]
S2	[α, 2-α]	[1+5α, 11-5α]	[α, 2-α]	[2+α, 4-α]
S3	[1+4α, 9-4α]	[5+10α, 25-10α]	[3+6α, 15-6α]	[3+α, 5-α]
Demand	[3+4α, 11-4α]	[1+2α, 5-2α]	[2+α, 4-α]	[6+7α, 20-7α]

Table 12: The mid-value TP of Z^1

	D1	D2	D3	Supply
S1	2	11	7	6
S2	1	6	1	3
S3	5	15	9	4
Demand	7	3	3	13

The optimal solution to the problem (M) is $m_{11}^{0} = 3, m_{12}^{0} = 3, m_{23}^{0} = 3 \; and \; m_{31}^{0} = 4$

Table 13: The half-width TP of Z^1

	D1	D2	D3	Supply
S1	1-α	1-α	3-3α	5-5α
S2	1-α	5-5α	1-α	1-α
S3	4-4α	10-10α	6-6α	1-α
Demand	4-4α	2-2α	1-α	7-7α

The optimal solution to the problem (W) is $w_{11}^0 = 3 - 3\alpha, w_{12}^0 = 2 - 2\alpha, w_{23}^0 = 1 - \alpha \; and \; w_{31}^0 = 1 - \alpha$.

The optimal solution to the given problem of the unit transportation cost is $[x_{11}^0, y_{11}^0] = [3\alpha, 6 - 3\alpha], [x_{12}^0, y_{12}^0] = [1 + 2\alpha, 5 - 2\alpha], [x_{23}^0, y_{23}^0] = [2 + \alpha, 4 - \alpha]$ and $[x_{31}^0, y_{31}^0] = [3 + \alpha, 5 - \alpha]$.

Now, by using the relation (15), the optimal solution to the problem of unit transportation cost is $\tilde{x}_{11} = (0,3,6), \tilde{x}_{12} = (1,3,5), \tilde{x}_{23} = (2,3,4) \; and \; \tilde{x}_{31} = (3,4,5)$ and the minimum transportation cost is (19, 86,183).

Table 14: Information on the delivery time (Z^2)

	D1	D2	D3	Supply
S1	[2+2α,6-2α]	[1+2α,5-2α]	[5+5α,15-5α]	[1+5α, 11-5α]
S2	[7+2α,11-2α]	[2+α,4-α]	[1+2α,3-α]	[2+α, 4-α]
S3	[2+7α,16-7α]	[3+6α,15-6α]	[1+4α,9-4α]	[3+α, 5-α]
Demand	[3+4α,11-4α]	[1+2α, 5-2α]	[2+α, 4-α]	[6+7α, 20-7α]

Table 15: The mid-value TP of the Z^2

	D1	D2	D3	Supply
S1	4	3	10	6
S2	9	3	2	3
S3	9	9	5	4
Demand	7	3	3	13

The optimal solution to the problem (M) is $m_{11}^0 = 6, m_{22}^0 = 3, m_{31}^0 = 1 \; and \; m_{33}^0 = 3$

Table 16: The half-width TP of Z^2

	D1	D2	D3	Supply
S1	2-2α	2-2α	5-5α	5-5α
S2	2-2α	1-α	1-α	1-α
S3	7-7α	6-6α	4-4α	1-α
Demand	4-4α	2-2α	1-α	7-7α

The optimal solution to the problem (W) is $w_{11}^0 = 4 - 4\alpha, w_{12}^0 = 1 - \alpha, m_{22}^0 = 1 - \alpha, m_{31}^0 = 0 \; and \; m_{33}^0 = 1 - \alpha$

The optimal solution to the given problem of the unit transportation cost is $[x_{11}^0, y_{11}^0] = [2 + 4\alpha, 10 - 4\alpha], [x_{12}^0, y_{12}^0] = [-1 + \alpha, 1 - \alpha], [x_{22}^0, y_{22}^0] = [2 + \alpha, 4 - \alpha]$, $[x_{31}^0, y_{31}^0] = [10 - 10\alpha, 10 - 10\alpha]$ and $[x_{33}^0, y_{33}^0] = [2 + 4\alpha, 10 - 4\alpha]$.

Now, by using the relation (15), the optimal solution to the problem of unit transportation cost is $\tilde{x}_{11} = (2,6,10), \tilde{x}_{12} = (-1,0,1), \tilde{x}_{22} = (2,3,4), \tilde{x}_{31} = (10,0,10) \; and \; \tilde{x}_{33} = (2,6,10)$ and minimum transportation cost is (19, 86,183).

Now, by Step 2, the payoff matrix is given below:

Table17: Payoff matrix of a given problem

		Objectives	
		Z^1	Z^2
Solutions	x_1^*, y_1^*	[56.2,68]	[58.59,70.38]
	x_2^*, y_2^*	[38.5,69.86]	[36.51,63.25]

Now, by Step 3, consider all lower limit values of all intervals from the above payoff matrix

$$Z_U = \begin{bmatrix} 68 & 70.38 \\ 69.86 & 63.25 \end{bmatrix}$$

By using this upper limit payoff matrix, we get

$$\mu(Z_U^1(y_1)) = \begin{cases} 0, & if\ Z_U^1(y_1) \geq U_1 \\ \dfrac{69.86 - Z_U^1(y_1)}{69.86 - 68}, & if\ L_1 < Z_U^1(y_1) < U_1 \\ 1, & if\ Z_U^1(y_1) \leq L_1 \end{cases}$$

$$\mu(Z_U^2(y_2)) = \begin{cases} 0, & if\ Z_U^2(y_2) \geq U_2 \\ \dfrac{70.38 - Z_U^1(y_1)}{70.38 - 63.25}, & if\ L_2 < Z_U^2(y_2) < U_2 \\ 1, & if\ Z_U^2(y_2) \leq L_2 \end{cases}$$

Now, by Step 3, consider all lower limit values of all intervals from the above payoff matrix

$$Z_L = \begin{bmatrix} 56.2 & 58.59 \\ 38.4 & 36.51 \end{bmatrix}$$

By using this lower limit payoff matrix, we get

$$\mu(Z_L^1(x_1)) = \begin{cases} 0, & if\ Z_L^1(x_1) \geq u_1 \\ \dfrac{56.2 - Z_L^1(x_1)}{56.2 - 38.5}, & if\ L_1 < Z_L^1(x_1) < u_1 \\ 1, & if\ Z_L^1(x_1) \leq 1 \end{cases}$$

$$\mu(Z_L^2(x_2)) = \begin{cases} 0, & if\ Z_L^2(x_2) \geq u_2 \\ \dfrac{58.59 - Z_L^2(x_2)}{58.59 - 36.51}, & if\ L_2 < Z_L^2(x_2) < u_2 \\ 1, & if\ Z_L^2(x_2) \leq 1 \end{cases}$$

Now, by Step 4, the corresponding integer LPPs are as follows for upper limit values.

$Maximum\ \lambda$

Subject to:

$$1.86\lambda \leq 69.86 - (2.1y_{11} + 11.1y_{12} + 7.3y_{13} + 1.1y_{21} + 6.5y_{22} + 1.1y_{23} + 5.4y_{31} + 16y_{32} + 9.6y_{33})$$

$$2.1y_{11} + 11.1y_{12} + 7.3y_{13} + 1.1y_{21} + 6.5y_{22} + 1.1y_{23} + 5.4y_{31} + 16y_{32} + 9.6y_{33} \leq 69.86$$

$$7.13\lambda \leq 70.38 - (4.2y_{11} + 3.2y_{12} + 10.5y_{13} + 9.2y_{21} + 3.1y_{22} + 2.1y_{23} + 9.7y_{31} + 9.6y_{32} + 5.4y_{33})$$

$$4.2y_{11} + 3.2y_{12} + 10.5y_{13} + 9.2y_{21} + 3.1y_{22} + 2.1y_{23} + 9.7y_{31} + 9.6y_{32} + 5.4y_{33} \leq 70.38$$

$y_{11} + y_{12} + y_{13} = 6.5$
$y_{21} + y_{22} + y_{23} = 3.1$

$y_{31} + y_{32} + y_{33} = 4.1$
$y_{11} + y_{21} + y_{31} = 7.4$
$y_{12} + y_{22} + y_{32} = 3.2$
$y_{13} + y_{23} + y_{33} = 3.1$
$y_{11} + y_{12} + y_{13} = 6.5$
$y_{21} + y_{22} + y_{23} = 3.1$
$y_{31} + y_{32} + y_{33} = 4.1$
$y_{11} + y_{21} + y_{31} = 7.4$
$y_{12} + y_{22} + y_{32} = 3.2$
$y_{13} + y_{23} + y_{33} = 3.1$
$y_{ij} \geq 0 \qquad i = 1,2,3 \quad \& \quad j = 1,2,3$
$0 < \lambda < 1.$

The above model is a LPP. We get the following compromise optimal solution by using lingo 18.0 software:

$[y^*] = [4.85, 1.65, 0, 0, 1.55, 1.55, 2.55, 0, 1.55]$

The minimum value of the objective functions of the upper bound (λ^U) is 68.93.

Now, by Step 4, the corresponding LPPs are as follows for lower limit values.

$Maximum\ \lambda$

Subject to:

$$17.7\lambda \leq 56.2 - (1.9x_{11} + 10.9x_{12} + 6.7x_{13} + 0.9x_{21} + 5.5x_{22} + 0.9x_{23} + 4.6x_{31} + 15x_{32} + 8.4x_{33})$$

$$1.9x_{11} + 10.9x_{12} + 6.7x_{13} + 0.9x_{21} + 5.5x_{22} + 0.9x_{23} + 4.6x_{31} + 15x_{32} + 8.4x_{33} \leq 56.2$$

$$22.08\lambda \leq 58.59 - (3.8x_{11} + 2.8x_{12} + 9.5x_{13} + 8.8x_{21} + 0.9x_{22} + 2.8x_{23} + 8.3x_{31} + 8.4x_{32} + 4.6x_{33})$$

$$3.8x_{11} + 2.8x_{12} + 9.5x_{13} + 8.8x_{21} + 0.9x_{22} + 2.8x_{23} + 8.3x_{31} + 8.4x_{32} + 4.6x_{33} \leq 58.59$$

$x_{11} + x_{12} + x_{13} = 5.5$
$x_{21} + x_{22} + x_{23} = 2.9$
$x_{31} + x_{32} + x_{33} = 3.9$
$x_{11} + x_{21} + x_{31} = 6.6$
$x_{12} + x_{22} + x_{32} = 2.8$
$x_{13} + x_{23} + x_{33} = 2.9$
$x_{11} + x_{12} + x_{13} = 5.5$
$x_{21} + x_{22} + x_{23} = 2.9$
$x_{31} + x_{32} + x_{33} = 3.9$
$x_{11} + x_{21} + x_{31} = 6.6$
$x_{12} + x_{22} + x_{32} = 2.8$
$x_{13} + x_{23} + x_{33} = 2.9$
$x_{ij} \geq 0 \qquad i = 1,2,3 \quad \& j = 1,2,3$
$0 < \lambda < 1$

The above model is a LPP. We get the following compromise optimal solution for lower bound by using lingo 18.0 software:

$[x^*] = [5.5, 0, 0, 0, 2.8, 0.1, 1.1, 0, 2.8]$.

The minimum value of the objective functions of the lower bound (λ^L) is 45.71.

Now, the solution to the given fully interval integer multi-objective TP is [45.71, 68.93].

Now, by step 3, (by using the relation (15)) we obtain the optimal fuzzy solution to FFIMOTP.

APPENDIX C

Table 18: Comparison of fuzzy and non-fuzzy instances of single and multi-objectives by different approaches

Solution Procedures	**Single-Objective**		**Multi-Objective**	
	Non-Fuzzy (ITP)	**Fuzzy (FTP)**	**Non-Fuzzy (FIIMOTP)**	**Fuzzy (FFIMOTP)**
Annie Christi and Kalpana [3]	×	×	√	×
Akilbasha et al. [2]	√	√	×	×
Ebrahimnejad [7]	×	√	×	×
Hosseinzade and Hassanpour [9]	√	×	×	×
Ishibuchi and Tanaka [10]	√	×	×	×
Oliveira and Antunes [14]	√	×	×	×
Panda and Das [15]	√	×	×	×
Pandian and Natarajan [16]	√	×	×	×
Patel and Dhodiya [20]	×	×	√	×
Roy and Mahapatra [22]	×	×	√	×
Sengupta and Pal [23]	×	×	√	×
Vincent Yu et al. [26]	×	×	√	×
This Paper	√	√	√	√