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A MULTI-ATTRIBUTE DECISION MAKING CONTEXT FOR SUPPLY CHAIN MANAGEMENT USING POSSIBILITY SINGLE-VALUED NEUTROSOPHIC SOFT SETTINGS

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Abstract: Argued decisions, risk estimation, empirical study, forecasts, apprehension supervision, and productive exposition depend on the degree of possibility. It enables us to assess the degree to which specific results are acceptable and to make more informed decisions by drawing on the information at our disposal and logic. Although some scholars have previously examined hybrid structures resembling fuzzy soft sets with possibility degree settings serving as fuzzy membership grades, the current study introduces an innovative structure that permits more adaptable and comprehensive settings: single-valued neutrosophic grades, to serve as possibility degrees. Thus, this study aims to introduce a new mathematical structure, i.e., possibility single-valued neutrosophic soft set (psv-NSOS), which combines three important theories (i.e., possibility theory, single-valued neutrosophic theory, and soft set theory). The basic notions, and set-theoretic operations, i.e., union and intersection, of psv-NSOS are investigated and manipulated with matrix representations. Considering evaluating suppliers for a real estate construction project as a multi-attribute decision-making issue, an algorithm that utilizes the matrix manipulations of proposed set-theoretic operations is presented. The suggested algorithm's robustness is confirmed by comparison to evaluate its dependability and adaptability for modeling uncertainties related to the supplier selection problem.

Keywords: Possibility degree, single-valued neutrosophic set, soft set, decision making, supply chain management.

MSC: 03B52,90B50,03E72.

1. INTRODUCTION

Any organization must carefully consider its potential choices when choosing a supplier since it immediately affects the reliability, affordability, and quality of its goods and services. However, the uncertainty and vagueness surrounding the supplier selection problem can make it extremely complex [1, 2, 3, 4]. Regarding uncertainty management, Zadeh [5, 6] described the notions of fuzzy sets [7] as a foundation. The possibility theory (POT) refers to the likelihood or probability of an event or statement being true or occurring. It is an important aspect of reasoning and decision-making, as it helps us assess the likelihood of different outcomes and make informed choices. Possibility theory plays an important role in decision-making, risk assessment, scientific inquiry, forecasting and planning, and uncertainty management. When presenting information or arguments, considering the possibility degree (POD) enhances effective communication. It enables clearer communication and promotes rational discourse by distinguishing between speculation, conjecture, and well-supported assertions [8, 9, 10, 11]. The fuzzy set is meant as a foundation for POD. Szmidt & Baldwin [12] applied the concept of POD for studying the connection among histogram, intuitionistic fuzzy set [13], and mass assignment theory. With the introduction of the soft set [14] and its hybrid extensions with applications [15, 16, 17, 18, 19], the POD is used in several fields by considering the notions of soft set hybrids like Alkhazaleh et al. [20] combined fuzzy soft set [21] and POT to develop possibility fuzzy soft set (pFSOS), Bashir et al. [22] developed possibility intuitionistic fuzzy soft set (pIFSOS) by adjoining POT and intuitionistic fuzzy set, and others in [23, 24, 25]. Karaaslan [26] developed the possibility neutrosophic soft set (pNSOS) by considering POT and neutrosophic set [27] collectively, then computed its correlation coefficient in [28]. Researchers such as Bhuvaneshwari & Sweety [29], Khalil et al. [30], Jia-hua et al. [31], Chatterjee et al. [32], and Garg & Arora [33] are prominent regarding the introduction of soft set-hybrids based on POT. The multiple decisive opinions have a vital role in decision-making problems, and they are the basic part of the soft expert set [34]. This concept was further utilized and glued with POT to develop some new structures [35, 36, 37, 38, 39]. A single-valued neutrosophic set (sv-NS) [40] is projected to make neutrosophic sets applicable to real scenarios. This concept was adjoined with SOS to develop a single-valued neutrosophic soft set (sv-NSOS) [41]. Recently, the researchers [42, 43, 44, 45, 46] made rich contributions in the field of soft set extensions with different applications.

1.1. Relevant literature, gap and motivation

The supplier selection problem (SSP) is a multifaceted, intricate decision-making task marked by ambiguity and uncertainty. It necessitates assessing possible suppliers according to various standards. Variations in supplier efficiency, shifting market conditions, and imprecise or missing information are the sources of the inherent uncertainties. Furthermore, the subjective nature of many assessment criteria adds ambiguity, which makes it challenging to determine unambiguous preferences. As a result, companies need to use advanced methods and instruments for decision-making to sort through the complexity and choose the best supplier [47, 48, 49, 50, 51, 52]. Many scholars have previously discussed implementing the SSP to articulate ambiguities and uncertainties related to attribute selection. Xiao et al. [53] used an integrated approach of FCM and fuzzy soft set to SSP by considering risk factors. Patra and Mondal [54] used a balanced solution technique with a soft set to study SSP by assessing risk variables. Chang [55] employed an integrated approach of intuitionistic fuzzy weighted averaging technique and soft set to investigate SSP with imprecise information. Selvachandran and Peng [56] utilized the refined TOPSIS technique to model vagueness associated with SSP using vague parameterized vague soft sets. Chang [57] used an integrated approach of hesitant fuzzy sets and soft sets to model hesitancy attached to SSP by considering linguistic terms. Wen et al. [58] discussed SSP by employing the integrated context of two-tuple linguistic settings and a soft set with vague data. Similarly, the investigations of Agarwal et al. [59], Shahab et al. [60], Asadi et al. [61] and Ihsan et al. [62] are also pertinent regarding the modeling uncertainties and vagueness involved in SSP using generalized intuitionistic fuzzy soft sets, Pythagorean fuzzy soft-max aggregation operators, and hybrid fuzzy sets, respectively. After carefully examining the evaluated literature mentioned above, it can be argued that these are insufficient for the issues that follow:

1. Decision-makers need to be able to act objectively or neutrally by providing well-informed judgments based on appropriate criteria in a dynamic situation. This calls for a certain configuration that offers unbiased, independent membership grades.
2. Acquiring the views of a third-party expert who can offer a possibility grade, a measure of acceptance, is sometimes necessary to evaluate the biases of decision-makers.
3. It is necessary to use possibility degrees in terms of single-valued neutrosophic numbers to deal with indeterminate situations because possibility degrees in terms of fuzzy membership grades are insufficient.

The proposed theoretical model, possibility single-valued neutrosophic soft set (psv-NSOS) can easily manage the above-mentioned issues collectively. It considers the ideas of sv-NSOS and POT collectively while considering sv-NS valued possibility grade. Since it enables the decision convener to specify the acceptance level in terms of the three dimension grades—truth, indeterminacy, and falsity—this concept is more broadly applicable. Its adjustable settings will result in more reliable and consistent selections. The convener's acceptance level was assessed in the aforementioned references using a fuzzy membership grade, but the approach that is being presented here is more flexible because the convener is now free to express his or her opinions about the approximated alternatives' acceptance level by agreeing, disagreeing, or not sure. The salient contributions of the present study are described as:

1. The concepts of psv-NSOS have been introduced, which generalizes the basic concepts of pFSOS, pIFSOS, and pNSOS. The fuzzy possibility grade is replaced with a single-valued neutrosophic possibility grade to preserve novelty.
2. The investigation and example-based illustration of the set-theoretic operations of psv-NSOS are presented, which are necessary for a correct comprehension of the suggested idea.
3. An algorithm is proposed to choose a suitable for real estate building project based on matrix manipulation of psv-NSOS. An example is given to validate the method; it assesses three suppliers using eight different parameters.

The systematic layout of the paper is presented in Figure 1.

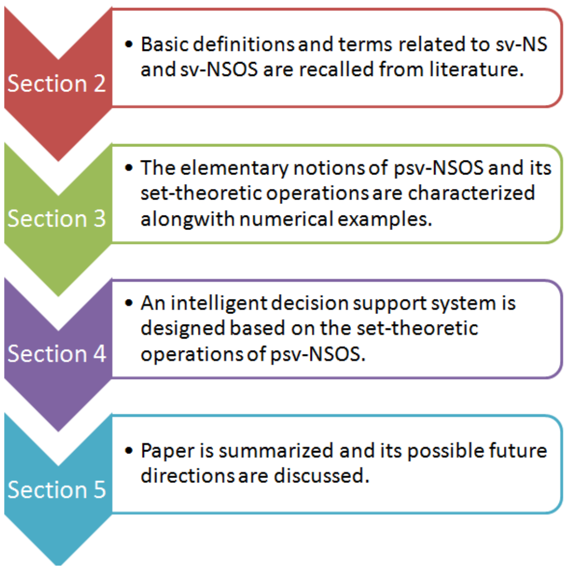

Figure 1: Layout of the paper

2. PRELIMINARIES

The symbol U is referred to as an initial universe, and E is a set of parameters throughout the study. The generalization of NS, known as a svNS, makes the NS applicable to practical applications. The svNS explains the NS from a technical point of view. It uses standard subsets of $[0, 1]$ for describing true, indeterminate, and false memberships instead of nonstandard subsets of $]^{-}0, 1^{+}[$.

Definition 1. *[40] Let a mapping $\tilde{\zeta} : \tilde{U} \to [0, 1] \times [0, 1] \times [0, 1]$ defined by a triad $\tilde{\zeta}(x) = \langle \alpha_{\tilde{\aleph}}(x), \beta_{\tilde{\aleph}}(x), \gamma_{\tilde{\aleph}}(x)\rangle$ for all $x \in \tilde{U}$ such that all three components of triad $\alpha_{\tilde{\aleph}}(x), \beta_{\tilde{\aleph}}(x), \gamma_{\tilde{\aleph}}(x) \in [0, 1]$ with $\alpha_{\tilde{\aleph}}(x) + \beta_{\tilde{\aleph}}(x) + \gamma_{\tilde{\aleph}}(x) \in [0, 3]$. The components $\alpha_{\tilde{\aleph}}(x), \beta_{\tilde{\aleph}}(x)$ and $\gamma_{\tilde{\aleph}}(x)$ are named as truth, indeterminacy and falsity of $x \in \tilde{U}$. An svNS $\tilde{\aleph}$ on $\tilde{U}$ is stated as $\tilde{\aleph} = \left\{ \left(x, \langle \alpha_{\tilde{\aleph}}(x), \beta_{\tilde{\aleph}}(x), \gamma_{\tilde{\aleph}}(x)\rangle\right) : x \in \tilde{U} \right\}$. The family of svNSs defined on $\tilde{U}$ is represented by $2^{(svNS)\tilde{U}}$.*

Consequently, SOS is established, which offers a suitable arrangement for the entitlement of such an approach in the form of approximation mapping. The existing fuzzy set-like models are not anticipated to allow for the parameterization tool, which is an essential mode for many decision-making scenarios.

Definition 2. *[14] Let $\psi : E \to 2^{\tilde{U}}$ be an approximate mapping defined by $\psi(e) \subseteq \tilde{U}$ for all $e \in E$ where E is a set of parameters and $2^{\tilde{U}}$ is the power set of $\tilde{U}$. A SOS $\mathbb{S}$ on $\tilde{U}$ is stated as $\mathbb{S} = \{(e, \psi(e)) : e \in E\}$ where $\psi(e)$ is regarded as e-approximate element of $\mathbb{S}$.*

Example 3. *Let $\tilde{U} = \{x_1, x_2, x_3, x_4, x_5, x_6\}$ be the universal set consisting of some houses and $E = \{e_1, e_2, e_3\}$ be the set of parameters where e_1 is for "beautiful", e_2 is for "costly", and e_3 is for "woody". The approximations of $\tilde{U}$ by approximate mapping $\psi : E \to 2^{\tilde{U}}$ based on E are $\psi(e_1) = \psi(beautiful) = \{x_1, x_5, x_6\}$, $\psi(e_2) = \psi(costly) = \{x_1, x_3, x_5, x_6\}$ and $\psi(e_3) = \psi(woody) = \{x_2, x_4\}$. Thus the SOS $\mathbb{S}$ is constructed as*

$$\mathbb{S} = \{(e_1, \psi(e_1)), (e_2, \psi(e_2)), (e_3, \psi(e_3))\}$$

or

$$\mathbb{S} = \{(beautiful, \{x_1, x_5, x_6\}), (costly, \{x_1, x_3, x_5, x_6\}), (woody, \{x_2, x_4\})\}.$$

Definition 4. *[41] Let an approximate mapping $\varpi : E \to 2^{(svNS)\tilde{U}}$ defined by $\varpi(e) \subseteq \tilde{U}$ for all $e \in E$ where E is a set of parameters and $2^{(svNS)\tilde{U}}$ is the collection of svNSs defined on $\tilde{U}$. A sv-NSOS $\tilde{\aleph}_{\mathbb{S}}$ on $\tilde{U}$ is stated as $\tilde{\aleph}_{\mathbb{S}} = \{(e, \varpi(e)) : e \in E\}$ where $\varpi(e)$ is regarded as e-approximate element of $\mathbb{S}$.*

3. POSSIBILITY SINGLE-VALUED NEUTROSOPHIC SOFT SET

In this section, essential notions of possibility single-valued neutrosophic soft set (psv-NSOS) are characterized. The complete methodology of the paper is depicted

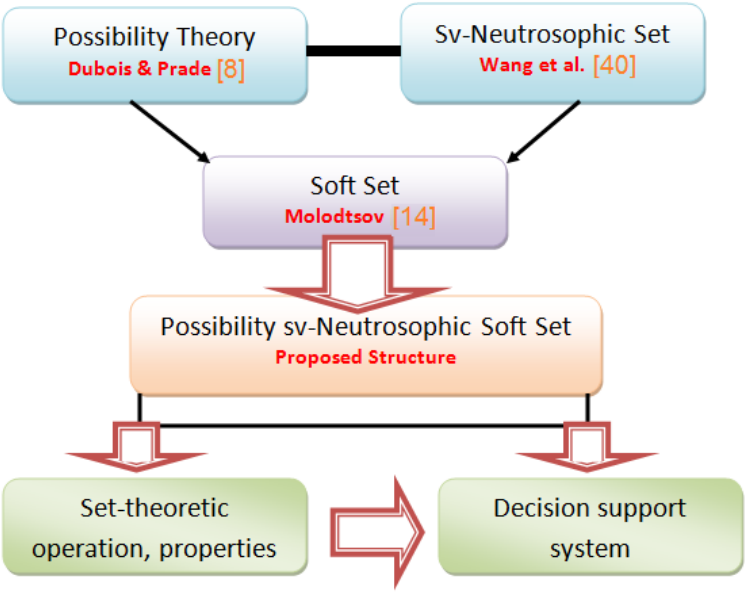

Figure 2: Methodology of the paper

in Figure 2. This figure makes it clear that four stages are there to complete the methodology of the paper. The first stage relates to the literature review, which recalls three theories: possibility theory, single-valued neutrosophic set, and soft set. In the second stage, these theories are essentially integrated with soft set theory to generate the suggested structure, which is a possibility single-valued neutrosophic soft set. In the third stage, the investigations are conducted into set-theoretic operations, properties, and significant consequences of the proposed structure. The last stage presents an intelligent decision-assistance system using the features of the proposed model.

Definition 5. *If the family of all sv-NSs on U is represented by $N_{sv}(U)$ then a psv-NSOS $F_{\psi\mu}$ is the set of objects $(E, \zeta_F(e))$ such that*

$$\zeta_F(e) = \left\{ \left(\frac{\hat{u}}{\psi(e)(\hat{u})}, \mu(e) \right) : \psi(e)(\hat{u}) \in N_{sv}(U), \mu(e) \in N_{sv}(E) \right\}$$

where $\psi(e)(\hat{u}) = \{\langle \alpha_\psi(e)(\hat{u}), \beta_\psi(e)(\hat{u}), \gamma_\psi(e)(\hat{u})\rangle\}$ and $\mu(e) = \{\langle \alpha_\mu(e), \beta_\mu(e), \gamma_\mu(e)\rangle\}$ with $0 \le \alpha_\psi(e)(\hat{u}) + \beta_\psi(e)(\hat{u}) + \gamma_\psi(e)(\hat{u}) \le 3$, $0 \le \alpha_\mu(e) + \beta_\mu(e) + \gamma_\mu(e) \le 3$ and $\alpha_\psi(e)(\hat{u}), \beta_\psi(e)(\hat{u}), \gamma_\psi(e)(\hat{u}), \alpha_\mu(e), \beta_\mu(e), \gamma_\mu(e) \in [0, 1]$. Hence psv-NSOS $F_{\psi\mu}$ can jointly be constructed as

$$F_{\psi\mu} = \left\{ \begin{array}{l} \left(\frac{\hat{u}}{\langle \alpha_\psi(e)(\hat{u}), \beta_\psi(e)(\hat{u}), \gamma_\psi(e)(\hat{u})\rangle}, \langle \alpha_\mu(e), \beta_\mu(e), \gamma_\mu(e)\rangle \right) : \\ \psi(e)(\hat{u}) = \langle \alpha_\psi(e)(\hat{u}), \beta_\psi(e)(\hat{u}), \gamma_\psi(e)(\hat{u})\rangle \in N_{sv}(U), \\ \mu(e) = \langle \alpha_\mu(e), \beta_\mu(e), \gamma_\mu(e)\rangle \in N_{sv}(E) \end{array} \right\}.$$

In short,

$$F_{\psi\mu} = \{\langle \alpha_\psi(e)(\hat{u}), \beta_\psi(e)(\hat{u}), \gamma_\psi(e)(\hat{u})\rangle\langle \alpha_\mu(e), \beta_\mu(e), \gamma_\mu(e)\rangle\}.$$

The family of all psv-NSOSs is represented by $\Omega_{psv-NSOS}$.

Example 6. *Suppose that Mr. Adem wants to buy a car for his domestic use and there are three different models of cars available in the market. Let* $U = \{C_1, C_2, C_3\}$ *be the set of alternatives consisting of these models. This purchase is accomplished based on some parameters that are enclosed in the set* $E = \{e_1 = performance,\ e_2 = security,\ e_3 = comfort\}$. *He is accompanied by his two friends, who are experts in car purchasing and thus play the role of decision makers in this purchase. Based on their opinions, the psv-NSOS* $F_{\psi\mu}$ *is constructed as:*

$$F_{\psi\mu} = \left\{ \begin{array}{l} \zeta_F(e_1) = \left\{ \begin{array}{l} \left(\frac{C_1}{\langle 0.6,0.7,0.9\rangle}\langle 0.5, 0.7, 0.5\rangle\right), \\ \left(\frac{C_2}{\langle 0.5,0.8,0.8\rangle}\langle 0.6, 0.8, 0.6\rangle\right), \\ \left(\frac{C_3}{\langle 0.7,0.9,0.7\rangle}, \langle 0.7, 0.8, 0.6\rangle\right) \end{array} \right\}, \\ \zeta_F(e_2) = \left\{ \begin{array}{l} \left(\frac{C_1}{\langle 0.5,0.8,0.7\rangle}\langle 0.6, 0.7, 0.6\rangle\right), \\ \left(\frac{C_2}{\langle 0.7,0.7,0.8\rangle}\langle 0.7, 0.8, 0.7\rangle\right), \\ \left(\frac{C_3}{\langle 0.7,0.7,0.8\rangle}, \langle 0.8, 0.9, 0.8\rangle\right) \end{array} \right\}, \\ \zeta_F(e_3) = \left\{ \begin{array}{l} \left(\frac{C_1}{\langle 0.8,0.6,0.8\rangle}\langle 0.9, 0.5, 0.7\rangle\right), \\ \left(\frac{C_2}{\langle 0.6,0.9,0.8\rangle}\langle 0.6, 0.6, 0.8\rangle\right), \\ \left(\frac{C_3}{\langle 0.8,0.7,0.6\rangle}\langle 0.7, 0.6, 0.8\rangle\right) \end{array} \right\} \end{array} \right\}$$

and its matrix manipulation is

$$F_{\psi\mu} = \begin{bmatrix} \left\langle \begin{array}{c} 0.6, \\ 0.7, \\ 0.9 \end{array} \right\rangle \left\langle \begin{array}{c} 0.5, \\ 0.7, \\ 0.5 \end{array} \right\rangle & \left\langle \begin{array}{c} 0.5, \\ 0.8, \\ 0.8 \end{array} \right\rangle \left\langle \begin{array}{c} 0.6, \\ 0.8, \\ 0.6 \end{array} \right\rangle & \left\langle \begin{array}{c} 0.7, \\ 0.9, \\ 0.7 \end{array} \right\rangle \left\langle \begin{array}{c} 0.7, \\ 0.8, \\ 0.6 \end{array} \right\rangle \\ \left\langle \begin{array}{c} 0.5, \\ 0.8, \\ 0.7 \end{array} \right\rangle \left\langle \begin{array}{c} 0.6, \\ 0.7, \\ 0.6 \end{array} \right\rangle & \left\langle \begin{array}{c} 0.7, \\ 0.7, \\ 0.8 \end{array} \right\rangle \left\langle \begin{array}{c} 0.7, \\ 0.8, \\ 0.7 \end{array} \right\rangle & \left\langle \begin{array}{c} 0.7, \\ 0.7, \\ 0.8 \end{array} \right\rangle \left\langle \begin{array}{c} 0.8, \\ 0.9, \\ 0.8 \end{array} \right\rangle \\ \left\langle \begin{array}{c} 0.8, \\ 0.6, \\ 0.8 \end{array} \right\rangle \left\langle \begin{array}{c} 0.9, \\ 0.5, \\ 0.7 \end{array} \right\rangle & \left\langle \begin{array}{c} 0.6, \\ 0.9, \\ 0.8 \end{array} \right\rangle \left\langle \begin{array}{c} 0.6, \\ 0.6, \\ 0.8 \end{array} \right\rangle & \left\langle \begin{array}{c} 0.8, \\ 0.7, \\ 0.6 \end{array} \right\rangle \left\langle \begin{array}{c} 0.7, \\ 0.6, \\ 0.8 \end{array} \right\rangle \end{bmatrix}.$$

Definition 7. *Let* $F_{\psi\mu} = \{\langle \alpha_\psi(e)(\hat{u}), \beta_\psi(e)(\hat{u}), \gamma_\psi(e)(\hat{u})\rangle\langle \alpha_\mu(e), \beta_\mu(e), \gamma_\mu(e)\rangle\}$ *and* $G_{\psi\mu} = \{\langle \alpha'_\psi(e)(\hat{u}), \beta'_\psi(e)(\hat{u}), \gamma'_\psi(e)(\hat{u})\rangle\langle \alpha'_\mu(e), \beta'_\mu(e), \gamma'_\mu(e)\rangle\}$ *be psv-NSOSs then* $F_{\psi\mu} \subseteq G_{\psi\mu}$ *if*

1. $\alpha_\psi(e)(\hat{u}) \leq \alpha'_\psi(e)(\hat{u}), \beta_\psi(e)(\hat{u}) \leq \beta'_\psi(e)(\hat{u}), \gamma_\psi(e)(\hat{u}) \geq \gamma'_\psi(e)(\hat{u})$, *and*
2. $\alpha_\mu(e) \leq \alpha'_\mu(e), \beta_\mu(e) \leq \beta'_\mu(e), \gamma_\mu(e) \geq \gamma'_\mu(e)$.

Example 8. *By taking assumptions of Example 6, we have matrix version of psv-NSOS $G_{\psi\mu}$ as*

$$G_{\psi\mu} = \begin{bmatrix} \left\langle \begin{array}{c} 0.7, \\ 0.8, \\ 0.8 \end{array} \right\rangle \left\langle \begin{array}{c} 0.6, \\ 0.8, \\ 0.4 \end{array} \right\rangle & \left\langle \begin{array}{c} 0.6, \\ 0.9, \\ 0.7 \end{array} \right\rangle \left\langle \begin{array}{c} 0.7, \\ 0.9, \\ 0.5 \end{array} \right\rangle & \left\langle \begin{array}{c} 0.8, \\ 0.9, \\ 0.6 \end{array} \right\rangle \left\langle \begin{array}{c} 0.8, \\ 0.9, \\ 0.5 \end{array} \right\rangle \\ \left\langle \begin{array}{c} 0.6, \\ 0.9, \\ 0.6 \end{array} \right\rangle \left\langle \begin{array}{c} 0.7, \\ 0.8, \\ 0.5 \end{array} \right\rangle & \left\langle \begin{array}{c} 0.8, \\ 0.8, \\ 0.7 \end{array} \right\rangle \left\langle \begin{array}{c} 0.8, \\ 0.9, \\ 0.6 \end{array} \right\rangle & \left\langle \begin{array}{c} 0.8, \\ 0.8, \\ 0.7 \end{array} \right\rangle \left\langle \begin{array}{c} 0.9, \\ 0.9, \\ 0.7 \end{array} \right\rangle \\ \left\langle \begin{array}{c} 0.9, \\ 0.7, \\ 0.7 \end{array} \right\rangle \left\langle \begin{array}{c} 0.9, \\ 0.6, \\ 0.6 \end{array} \right\rangle & \left\langle \begin{array}{c} 0.7, \\ 0.9, \\ 0.7 \end{array} \right\rangle \left\langle \begin{array}{c} 0.7, \\ 0.7, \\ 0.7 \end{array} \right\rangle & \left\langle \begin{array}{c} 0.9, \\ 0.8, \\ 0.5 \end{array} \right\rangle \left\langle \begin{array}{c} 0.8, \\ 0.7, \\ 0.7 \end{array} \right\rangle \end{bmatrix}.$$

Clearly, it can be observed that $F_{\psi\mu} \subseteq G_{\psi\mu}$.

Definition 9. *Let $F_{\psi\mu}$ and $G_{\psi\mu}$ be psv-NSOSs then $F_{\psi\mu} = G_{\psi\mu}$ if $F_{\psi\mu} \subseteq G_{\psi\mu}$ and $G_{\psi\mu} \subseteq F_{\psi\mu}$.*

Definition 10. *Let $F_{\psi\mu} = \{\langle \alpha_\psi(e)(\hat{u}), \beta_\psi(e)(\hat{u}), \gamma_\psi(e)(\hat{u})\rangle \langle \alpha_\mu(e), \beta_\mu(e), \gamma_\mu(e)\rangle\}$ and $G_{\psi\mu} = \{\langle \alpha'_\psi(e)(\hat{u}), \beta'_\psi(e)(\hat{u}), \gamma'_\psi(e)(\hat{u})\rangle \langle \alpha'_\mu(e), \beta'_\mu(e), \gamma'_\mu(e)\rangle\}$ be psv-NSOSs defined on parametric subsets E_1 and E_2 respectively then $F_{\psi\mu} \cup G_{\psi\mu} = H_{\psi\mu}$ with $H_{\psi\mu} = \{\langle \alpha''_\psi(e)(\hat{u}), \beta''_\psi(e)(\hat{u}), \gamma''_\psi(e)(\hat{u})\rangle \langle \alpha''_\mu(e), \beta''_\mu(e), \gamma''_\mu(e)\rangle\}$ if*

$$\zeta_H(e) = \begin{cases} \zeta_F(e) & e \in E_1 \setminus E_2 \\ \zeta_G(e) & e \in E_2 \setminus E_1 \\ \zeta_F(e) \cup \zeta_G(e) & e \in E_1 \cap E_2 \end{cases}$$

subject to the following conditions:

1. $\alpha''_\psi(e)(\hat{u}) = max\{\alpha_\psi(e)(\hat{u}), \alpha'_\psi(e)(\hat{u})\}$, $\beta''_\psi(e)(\hat{u}) = min\{\beta_\psi(e)(\hat{u}), \beta'_\psi(e)(\hat{u})\}$, $\gamma'_\psi(e)(\hat{u}) = min\{\gamma_\psi(e)(\hat{u}), \gamma'_\psi(e)(\hat{u})\}$, *and*
2. $\alpha''_\mu(e) = max\{\alpha_\mu(e), \alpha'_\mu(e)\}$, $\beta''_\mu(e) = min\{\beta_\mu(e), \beta'_\mu(e)\}$, $\gamma''_\mu(e) = min\{\gamma_\mu(e), \gamma'_\mu(e)\}$.

Definition 11. *Let $F_{\psi\mu}$ and $G_{\psi\mu}$ be psv-NSOSs as defined in Definition 10 for parametric subsets E_1 and E_2 respectively then $F_{\psi\mu} \cap G_{\psi\mu} = J_{\psi\mu}$ with $J_{\psi\mu} = \{\langle \alpha'''_\psi(e)(\hat{u}), \beta'''_\psi(e)(\hat{u}), \gamma'''_\psi(e)(\hat{u})\rangle \langle \alpha'''_\mu(e), \beta'''_\mu(e), \gamma'''_\mu(e)\rangle\}$ if*

$$\zeta_J(e) = \zeta_F(e) \cap \zeta_G(e) \,\forall\, e \in E_1 \cap E_2$$

subject to the following conditions:

1. $\alpha'''_\psi(e)(\hat{u}) = min\{\alpha_\psi(e)(\hat{u}), \alpha'_\psi(e)(\hat{u})\}$, $\beta'''_\psi(e)(\hat{u}) = max\{\beta_\psi(e)(\hat{u}), \beta'_\psi(e)(\hat{u})\}$, $\gamma'_\psi(e)(\hat{u}) = max\{\gamma_\psi(e)(\hat{u}), \gamma'_\psi(e)(\hat{u})\}$, *and*
2. $\alpha'''_\mu(e) = min\{\alpha_\mu(e), \alpha'_\mu(e)\}$, $\beta'''_\mu(e) = max\{\beta_\mu(e), \beta'_\mu(e)\}$, $\gamma'''_\mu(e) = max\{\gamma_\mu(e), \gamma'_\mu(e)\}$.

Example 12. *Assuming the matrix versions of psv-NSOSs $F_{\psi\mu}$ and $G_{\psi\mu}$ as given in Example 6 and 8 respectively, we have*

$$H_{\psi\mu} = \begin{bmatrix} \left\langle \begin{matrix} 0.7, \\ 0.6, \\ 0.8 \end{matrix} \right\rangle \left\langle \begin{matrix} 0.6, \\ 0.6, \\ 0.4 \end{matrix} \right\rangle & \left\langle \begin{matrix} 0.6, \\ 0.7, \\ 0.7 \end{matrix} \right\rangle \left\langle \begin{matrix} 0.7, \\ 0.7, \\ 0.5 \end{matrix} \right\rangle & \left\langle \begin{matrix} 0.8, \\ 0.7, \\ 0.6 \end{matrix} \right\rangle \left\langle \begin{matrix} 0.8, \\ 0.7, \\ 0.5 \end{matrix} \right\rangle \\ \left\langle \begin{matrix} 0.6, \\ 0.7, \\ 0.6 \end{matrix} \right\rangle \left\langle \begin{matrix} 0.7, \\ 0.6, \\ 0.5 \end{matrix} \right\rangle & \left\langle \begin{matrix} 0.8, \\ 0.6, \\ 0.7 \end{matrix} \right\rangle \left\langle \begin{matrix} 0.8, \\ 0.7, \\ 0.6 \end{matrix} \right\rangle & \left\langle \begin{matrix} 0.8, \\ 0.6, \\ 0.7 \end{matrix} \right\rangle \left\langle \begin{matrix} 0.9, \\ 0.7, \\ 0.7 \end{matrix} \right\rangle \\ \left\langle \begin{matrix} 0.9, \\ 0.5, \\ 0.7 \end{matrix} \right\rangle \left\langle \begin{matrix} 0.9, \\ 0.4, \\ 0.6 \end{matrix} \right\rangle & \left\langle \begin{matrix} 0.7, \\ 0.7, \\ 0.7 \end{matrix} \right\rangle \left\langle \begin{matrix} 0.7, \\ 0.5, \\ 0.7 \end{matrix} \right\rangle & \left\langle \begin{matrix} 0.9, \\ 0.6, \\ 0.5 \end{matrix} \right\rangle \left\langle \begin{matrix} 0.8, \\ 0.5, \\ 0.7 \end{matrix} \right\rangle \end{bmatrix},$$

and

$$J_{\psi\mu} = \begin{bmatrix} \left\langle \begin{matrix} 0.6, \\ 0.9, \\ 0.9 \end{matrix} \right\rangle \left\langle \begin{matrix} 0.5, \\ 0.9, \\ 0.5 \end{matrix} \right\rangle & \left\langle \begin{matrix} 0.5, \\ 1.0, \\ 0.8 \end{matrix} \right\rangle \left\langle \begin{matrix} 0.6, \\ 1.0, \\ 0.6 \end{matrix} \right\rangle & \left\langle \begin{matrix} 0.7, \\ 1.0, \\ 0.7 \end{matrix} \right\rangle \left\langle \begin{matrix} 0.7, \\ 1.0, \\ 0.6 \end{matrix} \right\rangle \\ \left\langle \begin{matrix} 0.5, \\ 1.0, \\ 0.7 \end{matrix} \right\rangle \left\langle \begin{matrix} 0.6, \\ 0.9, \\ 0.6 \end{matrix} \right\rangle & \left\langle \begin{matrix} 0.7, \\ 0.9, \\ 0.8 \end{matrix} \right\rangle \left\langle \begin{matrix} 0.7, \\ 1.0, \\ 0.7 \end{matrix} \right\rangle & \left\langle \begin{matrix} 0.7, \\ 0.9, \\ 0.8 \end{matrix} \right\rangle \left\langle \begin{matrix} 0.8, \\ 1.0, \\ 0.8 \end{matrix} \right\rangle \\ \left\langle \begin{matrix} 0.8, \\ 0.8, \\ 0.8 \end{matrix} \right\rangle \left\langle \begin{matrix} 0.9, \\ 0.7, \\ 0.7 \end{matrix} \right\rangle & \left\langle \begin{matrix} 0.6, \\ 1.0, \\ 0.8 \end{matrix} \right\rangle \left\langle \begin{matrix} 0.6, \\ 0.8, \\ 0.8 \end{matrix} \right\rangle & \left\langle \begin{matrix} 0.8, \\ 0.9, \\ 0.6 \end{matrix} \right\rangle \left\langle \begin{matrix} 0.7, \\ 0.8, \\ 0.8 \end{matrix} \right\rangle \end{bmatrix}.$$

4. DECISION-SUPPORT SYSTEM BASED ON psv-NSOS

This section is meant to propose a robust algorithm first for the optimal selection of suppliers for a real estate project, and then its validation is assessed by presenting a case study.

4.1. Policy Implication

The issue of uncertain supplier selection highlights the need for efficient supply chain management strategies, which have significant policy implications. To mitigate the risks arising from interruptions, quality issues, and price fluctuations, authorities ought to incentivize companies to expand their supplier base, minimize dependence on a single source, and foster transparency in their supplier relationships. Investments in technology and data analytics are also essential to enhancing the supply chain's resilience and visibility. Furthermore, encouraging trade agreements and international collaboration to provide a strong and diverse global supplier network will help mitigate the adverse effects of cautious supplier selection on the domestic economy and security, providing a stronger foundation for the supply chain industry [51, 52, 63, 64, 65].

4.2. Proposed Algorithm

This part is aimed to present a robust algorithm to select the best supplier while coping with the above mentioned challenges by using set-theoretic operations of psv-NSOS.

Algorithm for optimum supplier selection.

Start

Input
1. Assume U, E, and D as set of alternatives, set of parameters, and committee of decision-makers.

Construction
2. Construct psv-NSOSs $F_{\psi\mu}$ and $G_{\psi\mu}$ by collecting the opinions of decision-makers and represent them in matrix versions Δ_1 and Δ_2 respectively, using Definition 5.
3. Combine the approximations of both decision-makers, i.e., $F_{\psi\mu}$ and $G_{\psi\mu}$, by determining their union, i,e., $F_{\psi\mu} \cup G_{\psi\mu} = H_{\psi\mu}$ with matrix version Δ_3.

Computation
4. Transform the entities of Δ_3 into fuzzy values by using the formula

$$\varpi_{\psi\mu} = \frac{\pi_\psi + \pi_\mu}{3} \tag{1}$$

where

$$\pi_\psi = |\alpha''_\psi(e)(\hat{u}) - \beta''_\psi(e)(\hat{u}) - \gamma''_\psi(e)(\hat{u})| \tag{2}$$

and

$$\pi_\mu = |\alpha''_\mu(e) - \beta''_\mu(e) - \gamma''_\mu(e)|. \tag{3}$$

Hence new matrix Δ_4 is formed.
5. Determine score values for each alternative by taking arithmetic mean of the entries in the respective column of each alternative in Δ_4.

Output
6. Select the alternative with maximum score.

End

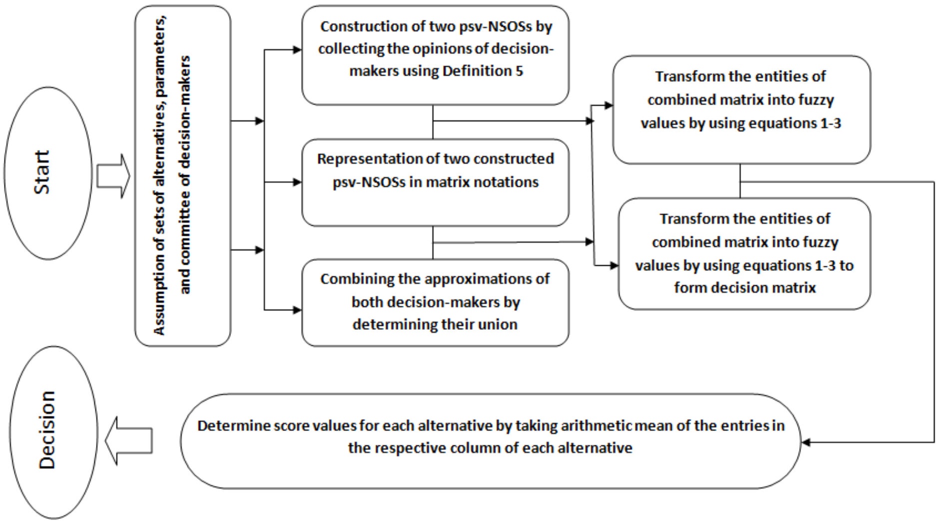

Figure 3: Flowchart of proposed algorithm

4.3. Problem Statement

To ensure the success of the project, it is essential to discover and evaluate potential suppliers when choosing suppliers for the construction project. It may, however, come with several challenges. Here are some typical challenges [63, 64, 65] in choosing suppliers for construction projects:

1. **Limited supplier options:** Finding appropriate suppliers who satisfy the particular needs and standards of the project can be difficult, especially if the market is not overflowing with possibilities. This might be the result of things like geographic restrictions or a limited market.
2. **Quality and reliability:** To prevent delays, cost overruns, and poor workmanship, it is essential to ensure the quality and dependability of suppliers. Nevertheless, evaluating the caliber and dependability of suppliers can be difficult, especially in the absence of historical data or references.
3. **Capacity and resources:** It is crucial to choose suppliers who have the capacity and resources to meet the demands of the project because construction projects frequently have tight deadlines. It is essential to assess a supplier's capacity to manage the scope and volume of the project to avoid delays or disruptions.
4. **Financial stability:** It's critical to evaluate the financial stability of suppliers to make sure they can continue to operate during the project. Construction projects need substantial financial commitments, and if a supplier experiences financial problems or declares bankruptcy while the project is underway, it may cause delays and result in possible losses.

5. **Compatibility and communication:** Smooth project execution depends on the project team and suppliers having good communication and getting along. It might be difficult to make sure that suppliers are responsive, have suitable working styles, and have effective communication routes, especially when working with several providers.
6. **Compliance and regulatory requirements:** Construction projects must adhere to several rules and compliance requirements. To minimize legal problems or project delays, it's critical to choose suppliers who follow these specifications. Assessing supplier compliance can be difficult, though, especially for complicated projects or when dealing with foreign vendors.
7. **Cost considerations:** While price is a crucial consideration when choosing a provider, finding a balance between price and quality can be difficult. Choosing the lowest-cost provider could result in rework or project failure if the quality or dependability of the products or services is compromised.
8. **Supply chain risks:** Multiple levels of suppliers are frequently involved in intricate supply chains for construction projects. Identifying and reducing supply chain risks, such as dependencies on a single supplier, delays, and disruptions, can be difficult but is essential to the success of a project.

Project stakeholders can use a variety of tactics to overcome these difficulties, including in-depth supplier evaluations, consulting with industry professionals for ideas, visiting the project site, creating explicit contractual conditions, and keeping open lines of communication with suppliers. The selection process can be streamlined and made more transparent by utilizing technology and digital platforms for supplier search, evaluation, and cooperation.

Example 13. ***Step-Input:*** *The management of a real-estate agency, "ARAAZI.HUB" (a supposed name), is intended to launch a construction project in some particular cities in Pakistan. The company needs construction raw materials on a large scale. In this regard, bids are called from various supplier firms through a national newspaper. The company receives many proposals, therefore, a domestic committee is constituted for screening and shortlisting proposals based on market reputation and the proposal's creditability. At the screening stage, three supplier firms C_1, C_2, and C_3 are short-listed, which forms the set of alternatives, i.e., $U = \{C_1, C_2, C_3\}$. Two experts (decision makers), $D_1 =$ procurement manager and $D_2 =$ chief financial officer, are hired for the evaluation of the shortlisted firms based on their expert opinions. After keen analysis of literature [66, 67, 68] and mutual understanding, both decision-makers are agreed on some evaluating attributes that are: $e_1 =$ limited supplier options; $e_2 =$ quality and reliability; $e_3 =$ capacity and resources; $e_4 =$ financial stability; $e_5 =$ compatibility and communication; $e_6 =$ compliance and regulatory requirements; $e_7 =$ cost considerations; and $e_8 =$ supply chain risks. These attributes are part of the set of attributes E.*

Step-Construction: *Based on the evaluating attributes, both decision-makers approximate the alternatives in terms of psv-NSOSs $F_{\psi\mu}$ and $G_{\psi\mu}$ respectively. The approximations of $F_{\psi\mu}$ are*

$\zeta_F(e_1) =$

Figure 4: Opted attributes

$$\left\{\left(\frac{C_1}{\left\langle\begin{array}{l}0.6,\\0.7,\\0.6\end{array}\right\rangle}\left\langle\begin{array}{l}0.6,\\0.7,\\0.7\end{array}\right\rangle\right),\left(\frac{C_2}{\left\langle\begin{array}{l}0.6,\\0.7,\\0.8\end{array}\right\rangle}\left\langle\begin{array}{l}0.6,\\0.7,\\0.9\end{array}\right\rangle\right),\left(\frac{C_3}{\left\langle\begin{array}{l}0.6,\\0.7,\\0.6\end{array}\right\rangle}\left\langle\begin{array}{l}0.6,\\0.7,\\0.7\end{array}\right\rangle\right)\right\},$$

$$\zeta_F(e_2) =$$

$$\left\{\left(\frac{C_1}{\left\langle\begin{array}{l}0.7,\\0.8,\\0.6\end{array}\right\rangle}\left\langle\begin{array}{l}0.7,\\0.8,\\0.7\end{array}\right\rangle\right),\left(\frac{C_2}{\left\langle\begin{array}{l}0.7,\\0.8,\\0.8\end{array}\right\rangle}\left\langle\begin{array}{l}0.7,\\0.8,\\0.9\end{array}\right\rangle\right),\left(\frac{C_3}{\left\langle\begin{array}{l}0.7,\\0.8,\\0.6\end{array}\right\rangle}\left\langle\begin{array}{l}0.7,\\0.8,\\0.7\end{array}\right\rangle\right)\right\},$$

$$\zeta_F(e_3) =$$

$$\left\{\left(\frac{C_1}{\left\langle\begin{array}{l}0.8,\\0.9,\\0.6\end{array}\right\rangle}\left\langle\begin{array}{l}0.8,\\0.9,\\0.7\end{array}\right\rangle\right),\left(\frac{C_2}{\left\langle\begin{array}{l}0.8,\\0.9,\\0.8\end{array}\right\rangle}\left\langle\begin{array}{l}0.8,\\0.9,\\0.9\end{array}\right\rangle\right),\left(\frac{C_3}{\left\langle\begin{array}{l}0.8,\\0.9,\\0.6\end{array}\right\rangle}\left\langle\begin{array}{l}0.8,\\0.9,\\0.7\end{array}\right\rangle\right)\right\},$$

$$\zeta_F(e_4) = \left\{ \left(\frac{C_1}{\left\langle \begin{array}{c} 0.5, \\ 0.6, \\ 0.6 \end{array} \right\rangle} \left\langle \begin{array}{c} 0.5, \\ 0.6, \\ 0.7 \end{array} \right\rangle \right), \left(\frac{C_2}{\left\langle \begin{array}{c} 0.5, \\ 0.6, \\ 0.8 \end{array} \right\rangle} \left\langle \begin{array}{c} 0.5, \\ 0.6, \\ 0.9 \end{array} \right\rangle \right), \left(\frac{C_3}{\left\langle \begin{array}{c} 0.5, \\ 0.6, \\ 0.6 \end{array} \right\rangle} \left\langle \begin{array}{c} 0.5, \\ 0.6, \\ 0.7 \end{array} \right\rangle \right) \right\},$$

$$\zeta_F(e_5) = \left\{ \left(\frac{C_1}{\left\langle \begin{array}{c} 0.5, \\ 0.5, \\ 0.6 \end{array} \right\rangle} \left\langle \begin{array}{c} 0.5, \\ 0.5, \\ 0.7 \end{array} \right\rangle \right), \left(\frac{C_2}{\left\langle \begin{array}{c} 0.5, \\ 0.5, \\ 0.8 \end{array} \right\rangle} \left\langle \begin{array}{c} 0.5, \\ 0.5, \\ 0.9 \end{array} \right\rangle \right), \left(\frac{C_3}{\left\langle \begin{array}{c} 0.5, \\ 0.5, \\ 0.6 \end{array} \right\rangle} \left\langle \begin{array}{c} 0.5, \\ 0.5, \\ 0.7 \end{array} \right\rangle \right) \right\},$$

$$\zeta_F(e_6) = \left\{ \left(\frac{C_1}{\left\langle \begin{array}{c} 0.6, \\ 0.6, \\ 0.6 \end{array} \right\rangle} \left\langle \begin{array}{c} 0.6, \\ 0.6, \\ 0.7 \end{array} \right\rangle \right), \left(\frac{C_2}{\left\langle \begin{array}{c} 0.6, \\ 0.6, \\ 0.8 \end{array} \right\rangle} \left\langle \begin{array}{c} 0.6, \\ 0.6, \\ 0.9 \end{array} \right\rangle \right), \left(\frac{C_3}{\left\langle \begin{array}{c} 0.6, \\ 0.6, \\ 0.6 \end{array} \right\rangle} \left\langle \begin{array}{c} 0.6, \\ 0.6, \\ 0.7 \end{array} \right\rangle \right) \right\},$$

$$\zeta_F(e_7) = \left\{ \left(\frac{C_1}{\left\langle \begin{array}{c} 0.7, \\ 0.7, \\ 0.6 \end{array} \right\rangle} \left\langle \begin{array}{c} 0.7, \\ 0.7, \\ 0.7 \end{array} \right\rangle \right), \left(\frac{C_2}{\left\langle \begin{array}{c} 0.7, \\ 0.7, \\ 0.8 \end{array} \right\rangle} \left\langle \begin{array}{c} 0.7, \\ 0.7, \\ 0.9 \end{array} \right\rangle \right), \left(\frac{C_3}{\left\langle \begin{array}{c} 0.7, \\ 0.7, \\ 0.6 \end{array} \right\rangle} \left\langle \begin{array}{c} 0.7, \\ 0.7, \\ 0.7 \end{array} \right\rangle \right) \right\},$$

$$\zeta_F(e_8) = \left\{ \left(\frac{C_1}{\left\langle \begin{array}{c} 0.8, \\ 0.8, \\ 0.6 \end{array} \right\rangle} \left\langle \begin{array}{c} 0.8, \\ 0.8, \\ 0.7 \end{array} \right\rangle \right), \left(\frac{C_2}{\left\langle \begin{array}{c} 0.8, \\ 0.8, \\ 0.8 \end{array} \right\rangle} \left\langle \begin{array}{c} 0.8, \\ 0.8, \\ 0.9 \end{array} \right\rangle \right), \left(\frac{C_3}{\left\langle \begin{array}{c} 0.8, \\ 0.8, \\ 0.6 \end{array} \right\rangle} \left\langle \begin{array}{c} 0.8, \\ 0.8, \\ 0.7 \end{array} \right\rangle \right) \right\}$$

and its matrix manipulation is

$$\Delta_1 = \begin{bmatrix} \begin{pmatrix} \langle 0.6, 0.7, 0.6 \rangle \\ \langle 0.6, 0.7, 0.7 \rangle \end{pmatrix} & \begin{pmatrix} \langle 0.6, 0.7, 0.8 \rangle \\ \langle 0.6, 0.7, 0.9 \rangle \end{pmatrix} & \begin{pmatrix} \langle 0.6, 0.7, 0.6 \rangle \\ \langle 0.6, 0.7, 0.7 \rangle \end{pmatrix} \\ \begin{pmatrix} \langle 0.7, 0.8, 0.6 \rangle \\ \langle 0.7, 0.8, 0.7 \rangle \end{pmatrix} & \begin{pmatrix} \langle 0.7, 0.8, 0.8 \rangle \\ \langle 0.7, 0.8, 0.9 \rangle \end{pmatrix} & \begin{pmatrix} \langle 0.7, 0.8, 0.6 \rangle \\ \langle 0.7, 0.8, 0.7 \rangle \end{pmatrix} \\ \begin{pmatrix} \langle 0.8, 0.9, 0.6 \rangle \\ \langle 0.8, 0.9, 0.7 \rangle \end{pmatrix} & \begin{pmatrix} \langle 0.8, 0.9, 0.8 \rangle \\ \langle 0.8, 0.9, 0.9 \rangle \end{pmatrix} & \begin{pmatrix} \langle 0.8, 0.9, 0.6 \rangle \\ \langle 0.8, 0.9, 0.7 \rangle \end{pmatrix} \\ \begin{pmatrix} \langle 0.5, 0.6, 0.6 \rangle \\ \langle 0.5, 0.6, 0.7 \rangle \end{pmatrix} & \begin{pmatrix} \langle 0.5, 0.6, 0.8 \rangle \\ \langle 0.5, 0.6, 0.9 \rangle \end{pmatrix} & \begin{pmatrix} \langle 0.5, 0.6, 0.6 \rangle \\ \langle 0.5, 0.6, 0.7 \rangle \end{pmatrix} \\ \begin{pmatrix} \langle 0.5, 0.5, 0.6 \rangle \\ \langle 0.5, 0.5, 0.7 \rangle \end{pmatrix} & \begin{pmatrix} \langle 0.5, 0.5, 0.8 \rangle \\ \langle 0.5, 0.5, 0.9 \rangle \end{pmatrix} & \begin{pmatrix} \langle 0.5, 0.5, 0.6 \rangle \\ \langle 0.5, 0.5, 0.7 \rangle \end{pmatrix} \\ \begin{pmatrix} \langle 0.6, 0.6, 0.6 \rangle \\ \langle 0.6, 0.6, 0.7 \rangle \end{pmatrix} & \begin{pmatrix} \langle 0.6, 0.6, 0.8 \rangle \\ \langle 0.6, 0.6, 0.9 \rangle \end{pmatrix} & \begin{pmatrix} \langle 0.6, 0.6, 0.6 \rangle \\ \langle 0.6, 0.6, 0.7 \rangle \end{pmatrix} \\ \begin{pmatrix} \langle 0.7, 0.7, 0.6 \rangle \\ \langle 0.7, 0.7, 0.7 \rangle \end{pmatrix} & \begin{pmatrix} \langle 0.7, 0.7, 0.8 \rangle \\ \langle 0.7, 0.7, 0.9 \rangle \end{pmatrix} & \begin{pmatrix} \langle 0.7, 0.7, 0.6 \rangle \\ \langle 0.7, 0.7, 0.7 \rangle \end{pmatrix} \\ \begin{pmatrix} \langle 0.8, 0.8, 0.6 \rangle \\ \langle 0.8, 0.8, 0.7 \rangle \end{pmatrix} & \begin{pmatrix} \langle 0.8, 0.8, 0.8 \rangle \\ \langle 0.8, 0.8, 0.9 \rangle \end{pmatrix} & \begin{pmatrix} \langle 0.8, 0.8, 0.6 \rangle \\ \langle 0.8, 0.8, 0.7 \rangle \end{pmatrix} \end{bmatrix}.$$

The approximations of $G_{\psi\mu}$ are

$$\zeta_F(e_1) = \left\{ \left(\frac{C_1}{\left\langle \begin{matrix} 0.9, \\ 0.8, \\ 0.5 \end{matrix} \right\rangle} \left\langle \begin{matrix} 0.9, \\ 0.8, \\ 0.6 \end{matrix} \right\rangle \right), \left(\frac{C_2}{\left\langle \begin{matrix} 0.9, \\ 0.8, \\ 0.7 \end{matrix} \right\rangle} \left\langle \begin{matrix} 0.9, \\ 0.8, \\ 0.8 \end{matrix} \right\rangle \right), \left(\frac{C_3}{\left\langle \begin{matrix} 0.9, \\ 0.8, \\ 0.9 \end{matrix} \right\rangle} \left\langle \begin{matrix} 0.9, \\ 0.8, \\ 0.5 \end{matrix} \right\rangle \right) \right\},$$

$$\zeta_F(e_2) = \left\{ \left(\frac{C_1}{\left\langle \begin{matrix} 0.8, \\ 0.7, \\ 0.5 \end{matrix} \right\rangle} \left\langle \begin{matrix} 0.8, \\ 0.7, \\ 0.6 \end{matrix} \right\rangle \right), \left(\frac{C_2}{\left\langle \begin{matrix} 0.8, \\ 0.7, \\ 0.7 \end{matrix} \right\rangle} \left\langle \begin{matrix} 0.8, \\ 0.7, \\ 0.8 \end{matrix} \right\rangle \right), \left(\frac{C_3}{\left\langle \begin{matrix} 0.8, \\ 0.7, \\ 0.9 \end{matrix} \right\rangle} \left\langle \begin{matrix} 0.8, \\ 0.7, \\ 0.5 \end{matrix} \right\rangle \right) \right\},$$

$$\zeta_F(e_3) = \left\{ \left(\frac{C_1}{\left\langle \begin{matrix} 0.7, \\ 0.6, \\ 0.5 \end{matrix} \right\rangle} \left\langle \begin{matrix} 0.7, \\ 0.6, \\ 0.6 \end{matrix} \right\rangle \right), \left(\frac{C_2}{\left\langle \begin{matrix} 0.7, \\ 0.6, \\ 0.7 \end{matrix} \right\rangle} \left\langle \begin{matrix} 0.7, \\ 0.6, \\ 0.8 \end{matrix} \right\rangle \right), \left(\frac{C_3}{\left\langle \begin{matrix} 0.7, \\ 0.6, \\ 0.9 \end{matrix} \right\rangle} \left\langle \begin{matrix} 0.7, \\ 0.6, \\ 0.5 \end{matrix} \right\rangle \right) \right\},$$

$$\zeta_F(e_4) = \left\{ \left(\frac{C_1}{\left\langle \begin{matrix} 0.6, \\ 0.5, \\ 0.5 \end{matrix} \right\rangle} \left\langle \begin{matrix} 0.6, \\ 0.5, \\ 0.6 \end{matrix} \right\rangle \right), \left(\frac{C_2}{\left\langle \begin{matrix} 0.6, \\ 0.5, \\ 0.7 \end{matrix} \right\rangle} \left\langle \begin{matrix} 0.6, \\ 0.5, \\ 0.8 \end{matrix} \right\rangle \right), \left(\frac{C_3}{\left\langle \begin{matrix} 0.6, \\ 0.5, \\ 0.9 \end{matrix} \right\rangle} \left\langle \begin{matrix} 0.6, \\ 0.5, \\ 0.5 \end{matrix} \right\rangle \right) \right\},$$

$\zeta_F(e_5) =$

$$\left\{\left(\frac{C_1}{\left\langle \begin{matrix} 0.9, \\ 0.5, \\ 0.5 \end{matrix} \right\rangle} \left\langle \begin{matrix} 0.8, \\ 0.5, \\ 0.5 \end{matrix} \right\rangle\right), \left(\frac{C_2}{\left\langle \begin{matrix} 0.7, \\ 0.5, \\ 0.5 \end{matrix} \right\rangle} \left\langle \begin{matrix} 0.6, \\ 0.5, \\ 0.5 \end{matrix} \right\rangle\right), \left(\frac{C_3}{\left\langle \begin{matrix} 0.5, \\ 0.5, \\ 0.5 \end{matrix} \right\rangle} \left\langle \begin{matrix} 0.9, \\ 0.5, \\ 0.5 \end{matrix} \right\rangle\right)\right\},$$

$\zeta_F(e_6) =$

$$\left\{\left(\frac{C_1}{\left\langle \begin{matrix} 0.9, \\ 0.6, \\ 0.6 \end{matrix} \right\rangle} \left\langle \begin{matrix} 0.8, \\ 0.6, \\ 0.6 \end{matrix} \right\rangle\right), \left(\frac{C_2}{\left\langle \begin{matrix} 0.7, \\ 0.6, \\ 0.6 \end{matrix} \right\rangle} \left\langle \begin{matrix} 0.6, \\ 0.6, \\ 0.6 \end{matrix} \right\rangle\right), \left(\frac{C_3}{\left\langle \begin{matrix} 0.5, \\ 0.6, \\ 0.6 \end{matrix} \right\rangle} \left\langle \begin{matrix} 0.9, \\ 0.6, \\ 0.6 \end{matrix} \right\rangle\right)\right\},$$

$\zeta_F(e_7) =$

$$\left\{\left(\frac{C_1}{\left\langle \begin{matrix} 0.9, \\ 0.7, \\ 0.7 \end{matrix} \right\rangle} \left\langle \begin{matrix} 0.8, \\ 0.7, \\ 0.7 \end{matrix} \right\rangle\right), \left(\frac{C_2}{\left\langle \begin{matrix} 0.7, \\ 0.7, \\ 0.7 \end{matrix} \right\rangle} \left\langle \begin{matrix} 0.6, \\ 0.7, \\ 0.7 \end{matrix} \right\rangle\right), \left(\frac{C_3}{\left\langle \begin{matrix} 0.5, \\ 0.7, \\ 0.7 \end{matrix} \right\rangle} \left\langle \begin{matrix} 0.9, \\ 0.7, \\ 0.7 \end{matrix} \right\rangle\right)\right\},$$

$\zeta_F(e_8) =$

$$\left\{\left(\frac{C_1}{\left\langle \begin{matrix} 0.9, \\ 0.8, \\ 0.8 \end{matrix} \right\rangle} \left\langle \begin{matrix} 0.8, \\ 0.8, \\ 0.8 \end{matrix} \right\rangle\right), \left(\frac{C_2}{\left\langle \begin{matrix} 0.7, \\ 0.8, \\ 0.8 \end{matrix} \right\rangle} \left\langle \begin{matrix} 0.6, \\ 0.8, \\ 0.8 \end{matrix} \right\rangle\right), \left(\frac{C_3}{\left\langle \begin{matrix} 0.5, \\ 0.8, \\ 0.8 \end{matrix} \right\rangle} \left\langle \begin{matrix} 0.9, \\ 0.8, \\ 0.8 \end{matrix} \right\rangle\right)\right\}$$

and its matrix manipulation is

$$\Delta_2 = \begin{bmatrix}
\begin{pmatrix} \langle 0.9, 0.8, 0.5\rangle \\ \langle 0.9, 0.8, 0.6\rangle \end{pmatrix} & \begin{pmatrix} \langle 0.9, 0.8, 0.7\rangle \\ \langle 0.9, 0.8, 0.8\rangle \end{pmatrix} & \begin{pmatrix} \langle 0.9, 0.8, 0.9\rangle \\ \langle 0.9, 0.8, 0.5\rangle \end{pmatrix} \\
\begin{pmatrix} \langle 0.8, 0.7, 0.5\rangle \\ \langle 0.8, 0.7, 0.6\rangle \end{pmatrix} & \begin{pmatrix} \langle 0.8, 0.7, 0.7\rangle \\ \langle 0.8, 0.7, 0.8\rangle \end{pmatrix} & \begin{pmatrix} \langle 0.8, 0.7, 0.9\rangle \\ \langle 0.8, 0.7, 0.5\rangle \end{pmatrix} \\
\begin{pmatrix} \langle 0.7, 0.6, 0.5\rangle \\ \langle 0.7, 0.6, 0.6\rangle \end{pmatrix} & \begin{pmatrix} \langle 0.7, 0.6, 0.7\rangle \\ \langle 0.7, 0.6, 0.8\rangle \end{pmatrix} & \begin{pmatrix} \langle 0.7, 0.6, 0.9\rangle \\ \langle 0.7, 0.6, 0.5\rangle \end{pmatrix} \\
\begin{pmatrix} \langle 0.6, 0.5, 0.5\rangle \\ \langle 0.6, 0.5, 0.6\rangle \end{pmatrix} & \begin{pmatrix} \langle 0.6, 0.5, 0.7\rangle \\ \langle 0.6, 0.5, 0.8\rangle \end{pmatrix} & \begin{pmatrix} \langle 0.6, 0.5, 0.9\rangle \\ \langle 0.6, 0.6, 0.5\rangle \end{pmatrix} \\
\begin{pmatrix} \langle 0.9, 0.5, 0.5\rangle \\ \langle 0.8, 0.5, 0.5\rangle \end{pmatrix} & \begin{pmatrix} \langle 0.7, 0.5, 0.5\rangle \\ \langle 0.6, 0.5, 0.5\rangle \end{pmatrix} & \begin{pmatrix} \langle 0.5, 0.5, 0.5\rangle \\ \langle 0.9, 0.5, 0.5\rangle \end{pmatrix} \\
\begin{pmatrix} \langle 0.9, 0.6, 0.6\rangle \\ \langle 0.8, 0.6, 0.6\rangle \end{pmatrix} & \begin{pmatrix} \langle 0.7, 0.6, 0.6\rangle \\ \langle 0.6, 0.6, 0.6\rangle \end{pmatrix} & \begin{pmatrix} \langle 0.5, 0.6, 0.6\rangle \\ \langle 0.9, 0.6, 0.6\rangle \end{pmatrix} \\
\begin{pmatrix} \langle 0.9, 0.7, 0.7\rangle \\ \langle 0.8, 0.7, 0.7\rangle \end{pmatrix} & \begin{pmatrix} \langle 0.7, 0.7, 0.7\rangle \\ \langle 0.6, 0.7, 0.7\rangle \end{pmatrix} & \begin{pmatrix} \langle 0.5, 0.7, 0.7\rangle \\ \langle 0.9, 0.7, 0.7\rangle \end{pmatrix} \\
\begin{pmatrix} \langle 0.9, 0.8, 0.8\rangle \\ \langle 0.8, 0.8, 0.8\rangle \end{pmatrix} & \begin{pmatrix} \langle 0.7, 0.8, 0.8\rangle \\ \langle 0.6, 0.8, 0.8\rangle \end{pmatrix} & \begin{pmatrix} \langle 0.5, 0.8, 0.8\rangle \\ \langle 0.9, 0.8, 0.8\rangle \end{pmatrix}
\end{bmatrix}.$$

Combining the approximations of both decision-makers and representing in the following matrix:

$$\Delta_3 = \begin{bmatrix} \begin{pmatrix} \langle 0.9,0.7,0.5\rangle \\ \langle 0.9,0.7,0.6\rangle \end{pmatrix} & \begin{pmatrix} \langle 0.9,0.7,0.7\rangle \\ \langle 0.9,0.7,0.8\rangle \end{pmatrix} & \begin{pmatrix} \langle 0.9,0.7,0.6\rangle \\ \langle 0.9,0.7,0.5\rangle \end{pmatrix} \\ \begin{pmatrix} \langle 0.8,0.7,0.5\rangle \\ \langle 0.8,0.7,0.6\rangle \end{pmatrix} & \begin{pmatrix} \langle 0.8,0.7,0.7\rangle \\ \langle 0.8,0.7,0.8\rangle \end{pmatrix} & \begin{pmatrix} \langle 0.8,0.7,0.6\rangle \\ \langle 0.8,0.7,0.5\rangle \end{pmatrix} \\ \begin{pmatrix} \langle 0.8,0.6,0.5\rangle \\ \langle 0.8,0.6,0.6\rangle \end{pmatrix} & \begin{pmatrix} \langle 0.8,0.6,0.7\rangle \\ \langle 0.8,0.6,0.8\rangle \end{pmatrix} & \begin{pmatrix} \langle 0.8,0.6,0.6\rangle \\ \langle 0.8,0.6,0.5\rangle \end{pmatrix} \\ \begin{pmatrix} \langle 0.6,0.5,0.5\rangle \\ \langle 0.6,0.5,0.6\rangle \end{pmatrix} & \begin{pmatrix} \langle 0.6,0.5,0.7\rangle \\ \langle 0.6,0.5,0.8\rangle \end{pmatrix} & \begin{pmatrix} \langle 0.6,0.5,0.6\rangle \\ \langle 0.6,0.6,0.5\rangle \end{pmatrix} \\ \begin{pmatrix} \langle 0.9,0.5,0.5\rangle \\ \langle 0.8,0.5,0.5\rangle \end{pmatrix} & \begin{pmatrix} \langle 0.7,0.5,0.5\rangle \\ \langle 0.6,0.5,0.5\rangle \end{pmatrix} & \begin{pmatrix} \langle 0.5,0.5,0.5\rangle \\ \langle 0.9,0.5,0.5\rangle \end{pmatrix} \\ \begin{pmatrix} \langle 0.9,0.6,0.6\rangle \\ \langle 0.8,0.6,0.6\rangle \end{pmatrix} & \begin{pmatrix} \langle 0.7,0.6,0.6\rangle \\ \langle 0.6,0.6,0.6\rangle \end{pmatrix} & \begin{pmatrix} \langle 0.6,0.6,0.6\rangle \\ \langle 0.9,0.6,0.6\rangle \end{pmatrix} \\ \begin{pmatrix} \langle 0.9,0.7,0.6\rangle \\ \langle 0.8,0.7,0.7\rangle \end{pmatrix} & \begin{pmatrix} \langle 0.7,0.7,0.7\rangle \\ \langle 0.7,0.7,0.7\rangle \end{pmatrix} & \begin{pmatrix} \langle 0.7,0.7,0.6\rangle \\ \langle 0.9,0.7,0.7\rangle \end{pmatrix} \\ \begin{pmatrix} \langle 0.9,0.8,0.6\rangle \\ \langle 0.8,0.8,0.7\rangle \end{pmatrix} & \begin{pmatrix} \langle 0.8,0.8,0.8\rangle \\ \langle 0.8,0.8,0.8\rangle \end{pmatrix} & \begin{pmatrix} \langle 0.8,0.8,0.6\rangle \\ \langle 0.9,0.8,0.7\rangle \end{pmatrix} \end{bmatrix}.$$

Stage-Computation: *By using Equation 2 and Equation 3, the entries of Δ_3 are transformed to fuzzy values and given in the following matrices.*

$$\Delta_4 = \begin{bmatrix} \langle 0.3\rangle\langle 0.4\rangle & \langle 0.5\rangle\langle 0.6\rangle & \langle 0.4\rangle,\langle 0.3\rangle \\ \langle 0.4\rangle\langle 0.5\rangle & \langle 0.6\rangle\langle 0.7\rangle & \langle 0.5\rangle,\langle 0.4\rangle \\ \langle 0.3\rangle\langle 0.4\rangle & \langle 0.5\rangle\langle 0.6\rangle & \langle 0.4\rangle\langle 0.3\rangle \\ \langle 0.4\rangle\langle 0.5\rangle & \langle 0.6\rangle\langle 0.7\rangle & \langle 0.5\rangle\langle 0.5\rangle \\ \langle 0.1\rangle\langle 0.2\rangle & \langle 0.3\rangle\langle 0.4\rangle & \langle 0.5\rangle\langle 0.1\rangle \\ \langle 0.3\rangle\langle 0.4\rangle & \langle 0.5\rangle\langle 0.6\rangle & \langle 0.6\rangle\langle 0.3\rangle \\ \langle 0.4\rangle\langle 0.6\rangle & \langle 0.7\rangle\langle 0.7\rangle & \langle 0.6\rangle\langle 0.5\rangle \\ \langle 0.5\rangle\langle 0.7\rangle & \langle 0.8\rangle\langle 0.8\rangle & \langle 0.6\rangle\langle 0.6\rangle \end{bmatrix}.$$

By using Equation 1, we get

$$\Delta_4 = \begin{bmatrix} 0.233 & 0.367 & 0.233 \\ 0.300 & 0.433 & 0.300 \\ 0.233 & 0.367 & 0.233 \\ 0.300 & 0.433 & 0.333 \\ 0.100 & 0.233 & 0.200 \\ 0.233 & 0.367 & 0.300 \\ 0.333 & 0.467 & 0.367 \\ 0.400 & 0.533 & 0.400 \end{bmatrix}.$$

Stage-Output: *Now compute the score values by taking arithmetic mean of the entries in the respective columns of alternatives.*
Score value of C_1 = 0.2665
Score value of C_2 = 0.4000
Score value of C_3 = 0.2958.

The ranking is $C_2 > C_3 > C_1$. As maximum score is achieved by C_2, therefore, its proposal is recommended for final selection.

4.4. Comparison and Sensitivity Analysis

In the existing literature on fuzzy soft set-like structures, numerous researchers have explored the supplier selection problem through various decision-making techniques and methods. However, the contributions of Chatterjee et al. [69], Dai & Bai [70] and Keshavarz-Ghorabaee et al. [71] are particularly pertinent for comparison in this study, as they have specifically addressed the supplier selection problem within fuzzy soft settings. Their work is more directly relevant due to the similar application of fuzzy methodologies in evaluating and selecting suppliers, thus providing a more aligned basis for comparative analysis. The number of alternatives (suppliers) is limited to three to ensure consistency and computational fairness in comparisons. Previous structures have overlooked indeterminacy and possibility degrees when approximating alternatives, highlighting their limitations in scenarios involving uncertainty and potentiality. Consequently, the proposed approach offers greater reliability and flexibility, enabling decision-makers to express indeterminate opinions and incorporate possibility settings to evaluate the acceptance levels of approximations by conveners. This inclusively enhances the robustness of the decision-making process in complex and uncertain environments. Table 1 demonstrates that the proposed algorithm produces consistent score values for the alternatives, highlighting its reliability when compared to existing relevant models. This consistency indicates that the proposed method can systematically evaluate and rank suppliers more effectively, reducing variability and ensuring stable results across different scenarios. Such reliability underscores the algorithm's robustness and potential superiority over traditional models in supplier selection processes. Although the score values for the alternatives are initially calculated

Table 1: Comparison Analysis

References	C_1	C_2	C_3	Ranking
Chatterjee et al. [69]	56.25	54.61	64.79	$C_3 > C_1 > C_2$
Dai & Bai [70]	0.9707	0.9080	0.9634	$C_3 > C_1 > C_2$
Keshavarz-Ghorabaee et al. [71]	0.5807	0.9076	0.4604	$C_2 > C_1 > C_3$
Proposed approach	0.2665	0.4000	0.2958	$C_2 > C_3 > C_1$

using the arithmetic mean of the entries in their respective columns, the sensitivity of these scores is further assessed using other Pythagorean means. This additional analysis reveals that the ranking of the alternatives remains consistent, with no changes observed, regardless of the statistical method employed. This consistency across different averaging techniques highlights the robustness and reliability of the proposed scoring method for accurately ranking the alternatives. Thus, keep-

Table 2: Sensitivity of score values

$\downarrow$ Pythagorean Means \ Alternatives $\rightarrow$	C_1	C_2	C_3	Ranking
Arithmetic Mean	0.2665	0.4000	0.2958	$C_2 > C_3 > C_1$
Geometric Mean	0.2498	0.3903	0.2883	$C_2 > C_3 > C_1$
Harmonic Mean	0.2283	0.3791	0.2809	$C_2 > C_3 > C_1$

ing in view the results of Table 1 and Table 2, some advantageous features of the proposed context are outlined as:

1. It provides the decision-makers with reliable and flexible settings where they act objectively or neutrally by providing well-informed judgments based on appropriate criteria in a dynamic situation. Thus, it establishes a certain configuration that offers unbiased, independent membership grades.
2. It can easily evaluate the biases of decision-makers by acquiring the views of a third-party expert who can offer a possibility grade that is meant to measure the acceptance level of opinions given by decision-makers.
3. It uses possibility degrees in terms of single-valued neutrosophic numbers to deal with indeterminate situations that overview the limitations of existing literature in which possibility degrees are used in terms of fuzzy membership grades.

Despite its beneficial aspects, it also has some limitations that are outlined as:

1. In some situations, the uncertainty associated with the choice of decisive parameters is encountered, so the concept of fuzzy set-like parameterization is necessary to quantify the uncertain nature of these parameters. The proposed context, psv-NSOS, is inadequate with such settings.
2. Sometimes parameters are multifaceted, i.e., they have corresponding subvalues. If these are ignored, the decision-making process can become questionable. This means that multisoft or hypersoft settings are required to control subvalues as multi-argument tuples.

5. CONCLUSIONS

The present work proposes a unique framework that permits single-valued neutrosophic grades as possibility degrees to be determined in more flexible and generalized settings. The proposed theoretical model, possibility single-valued neutrosophic soft set (psv-NSOS), combines three significant theories: possibility theory, single-valued neutrosophic theory, and soft set theory, to quantify the uncertainties and vagueness. For its use in other fields of study, the basic notions, and set-theoretic operations, i.e., union and intersection, of psv-NSOS are investigated and explained by matrix manipulations. A dependable approach based on the set-theoretic operations of psv-NSOS is suggested for the assessment of suppliers for real estate development projects. The primary benefits of this research lie in its wider applicability, as it provides the decision-maker with the capacity to express the level of acceptance for the three dimension grades: truth, indeterminacy, and falsity. Its changeable settings will result in more consistent and dependable conclusions. The convener can now freely indicate agreement, disagreement, or indecision regarding the predicted alternatives' acceptability level, making the proposed strategy more flexible. Other extensions of fuzzy sets can be taken into consideration as possibility-degrees in this study. Likewise, the suggested research can be improved even further for structures like multisoft sets and hypersoft sets. Regarding the study's future scope and directions, it should be integrated with expert, rough, complex, interval-valued, fuzzy parameterized, and other settings that will produce more generalized structures if more accurate and dependable results are expected from this suggested strategy. A broad variety of fuzzy logic and other theoretical fields of computer science may also fall under its purview.

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