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Research Article

R-RAM: A NOVEL HYBRID MODEL FOR OPTION RANKING

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Abstract: Ranking alternatives considering multiple criteria is a complex task, requiring the selection of both a weight calculation method and a ranking method. This study proposes a hybrid method, R-RAM, combining the strengths of the R and RAM methods. R-RAM calculates criteria weights considering both subjective and objective factors and then ranks alternatives using the RAM method. The performance of R-RAM was evaluated in four case studies: ranking metal cutting experiments, electric bicycles, automotive protective materials, and 3D printers. Results showed that R-RAM consistently performed comparably to existing MCDM methods and outperformed the R method. The average Spearman correlation coefficients between R-RAM and other methods were significantly higher in all cases (0.8833, 0.9167, 1, and 0.9429), compared to those of R (0.6214, 0.8839, 0.7, and 0.7714). Sensitivity analysis demonstrated the stability of rankings produced by R-RAM under various weight scenarios. Using R-RAM eliminates the need for users to spend time and effort thinking about which weighting method or solution ranking method to choose, as R-RAM operates by using R to calculate weights for criteria and RAM to rank the alternatives. The introduction of R-RAM is a noteworthy contribution to the field of option ranking.

Keywords: R method, RAM method, R-RAM method

MSC: 90B50, 91B06.

1. **INTRODUCTION**

Ranking options while considering multiple criteria is an action in MCDM (Multi Criteria Decision Making). MCDM is a technique that has been widely accepted and applied in various fields [1]. It has garnered the interest of many scientists across different disciplines, as evidenced by the rapidly increasing number of papers applying MCDM methods in recent times [2]. With over 200 existing methods, each employing different algorithms, choosing the right method to use has become a complex decision [3]. Studies have shown that the ranking of options significantly depends on the method used [4]. Many researchers have suggested that using only one method to solve each problem can lead to erroneous decisions or decisions lacking the decision-maker's confidence [5]. Combining methods leverages the strengths and mitigates the weaknesses of the individual component methods [6, 7, 8]. Furthermore, employing hybrid models can increase the decision-maker's or decision-making group's confidence in their decisions [9, 10]. Thus, the hybridization of methods has gained significant interest in recent years.

Essentially, ranking alternatives requires a combination of methods for weighting criteria and methods for ranking alternatives. However, selecting specific component methods to create a hybrid is complex due to the vast number of existing criteria weighting methods and alternative ranking methods. If a subjective weighting method is used, criterion weights are influenced by the decision-maker's subjective opinions. Conversely, if an objective weighting method is used, criterion weights rely solely on numerical data, disregarding the decision-maker's (or experts') opinions on the relative importance of criteria. The question is, which weighting method can overcome the limitations of both subjective and objective methods? A weighting method combining subjective and objective elements is considered a solution that leverages the strengths and mitigates the weaknesses of both component methods [11]. The R (Ranking of the attributes and alternatives) method seamlessly integrates subjective and objective elements in determining criterion weights. Subjectivity is reflected in the decision-maker's perspective on the importance of each criterion. Simultaneously, objectivity is ensured through the application of mathematical formulas to calculate criterion weights based on the decision-maker's predetermined priority ranking. The combination of subjective and objective elements guarantees the accuracy of criterion weight values [12]. The R method is built upon the ranking of attributes and alternatives [13]. To our knowledge, R has been implemented differently from most other MCDM methods. R has calculated weights for criterion levels based on the preference level between criteria. Additionally, R also calculates weights for alternative levels when alternative levels are determined through the internal ranking of alternatives, i.e., ranking alternatives for each criterion. This implies that when using R for overall alternative ranking (i.e., ranking alternatives based on all criteria), no additional method is needed to calculate criterion weights. This is a significant difference between R and most other methods. These characteristics of R will be further clarified in the subsequent sections of this paper. This method has been applied in various cases, such as supplier selection, industrial robot selection, industrial material selection, and flexible manufacturing system selection [13]; ranking car models in the Vietnamese market [14], ranking robots, ranking metal cutting methods, and ranking bridge construction alternatives [15], ranking conveyor belts, ranking autonomous vehicles, ranking sorting machines, ranking excavators [16], etc. However, to date, criterion weights calculated using the R method have not been used to rank alternatives using other MCDM methods. In other words, in studies that have applied the R method, the criterion weights calculated using the R method have only been used to rank alternatives using the R method itself, and criterion weights calculated using the R method have not been used to rank alternatives using other MCDM methods. We believe this is a waste that needs to be addressed. This is why the R method was chosen for this study.

In this study, the R method is hybridized with an MCDM method. The MCDM method mentioned in this paper is the RAM (Root Assessment Method) method. The RAM method is used in this study because it is a relatively new method, having emerged only in September 2023, and has the advantage of ranking alternatives considering the balance between criteria [17]. This method has also been proven to work effectively with various data normalization methods [18]. Despite its recent emergence, RAM has been applied in several studies in different fields, such as selecting fire-resistant materials [19], selecting mushroom cultivation [20], ranking universities [21], and ranking the financial health of banks [22], etc.

Based on the above analyses, R will be used in this study to hybridize with RAM. This means that R will be used for two purposes: one is to rank alternatives, and the other is to calculate criterion weights to combine with RAM in ranking alternatives. The hybridization of the R and RAM methods creates a new method, R-RAM, to leverage the combined subjective and objective nature of the R method in calculating criterion weights and to take advantage of the two outstanding advantages of the RAM method as declared in the literature: considering the balance between criteria and working effectively with various data normalization methods.

This study makes significant contributions. First, we propose a new hybrid method, R-RAM, that harmoniously combines the advantages of the R and RAM methods. This method not only provides a more effective tool for ranking alternatives but also considers the balance between criteria comprehensively. Moreover, R-RAM also expands the application scope of criterion weights calculated using the R method and contributes to enriching MCDM theory. Most notably, when using the R-RAM method, the method itself will calculate criterion weights using the R method and rank alternatives using the RAM method without the need for any additional methods.

The structure of the following sections of this paper is as follows. Chapter 2 provides a brief overview of the hybridization of methods in published literature to highlight the gap that this research aims to fill. Chapter 3 summarizes the steps for applying the R and RAM methods. Chapter 4 presents the hybrid model of R and RAM to create the R-RAM method. Chapter 5 discusses the tests conducted to evaluate the accuracy of R-RAM. A sensitivity analysis of the R-RAM method when the weights of the criteria change is presented in section 6. The conclusion of this research is the final content of this paper.

2. LITERATURE REVIEW

As discussed above, hybridizing methods leverages the strengths and mitigates the weaknesses of individual methods, thereby enhancing the decision-maker's confidence. This approach has been widely applied in various fields in recent times. Listing all the studies on this topic is impossible, but some research on the hybridization of methods applied in different fields can be summarized as follows.

TOPSIS **(**Technique for Order Preference by Similarity to Ideal Solution**)** has been hybridized with FUCOM (FUll COnsistency Method) for selecting transportation vehicles. In this study, TOPSIS was used to rank the vehicles, while FUCOM was used to calculate the weights of the criteria for these vehicles [23]. Five methods: ROC (Rank Order Centroid), RS (Rank Sum), FUCA (Faire Un Choix Adéquat), SRP (Simple Ranking Process), and OPARA (Objective Pairwise Adjusted Ratio Analysis) were combined to evaluate the scientific research capacity of lecturers at a university in one academic year. In this study, the first two methods were used to calculate two different sets of scores for the criteria, while the remaining three methods were used to rank the options [24]. BWM (Best-Worst Method), AHP, and TOPSIS were hybridized for warehouse management option selection; in this study, BWM was used to calculate the weights of the criteria, and the other two methods were used to rank the options [25]. The three methods ARAS (Additive Ratio ASsessment), TOPSIS, and GRA (Grey Relational Analysis) were combined to rank materials used in the technology for making freezing tanks; in this study, all three methods were used to rank the materials, while the weights of the criteria were taken from another study [26]. AHP, DEMATEL (Decision Making Experiment and Evaluation Laboratory Method), and SAW (Simple Additive Weighting) were hybridized to evaluate circular supply chains, with AHP used to calculate the weights of the criteria, DEMATEL to identify the relationships between criteria, and SAW to rank the supply chains [27]. The methods FARE (FActor RElationship), Delphi, and VIKOR (VIsekriterijumska optimizacijaI KOmpromisno Resenje) were combined to rank options in the final stage of logistics services. Here, FARE was used to calculate the weights of the criteria, Delphi to evaluate the decision-makers' agreement on the relationships between criteria, and VIKOR to rank the options [28]. MEREC (MEthod based on the Removal Effects of Criteria) and RATMI (Ranking the Alternatives based on the Trace to Median Index) were hybridized to rank forklifts used for transporting goods in warehouses, with MEREC used to calculate the weights of the criteria and RATMI to rank the options [29]. For ranking cities based on smart city and sustainable city trends, ANP (Analytic Network Process) and TOPSIS were combined; ANP was used to calculate the weights of the criteria, and TOPSIS to rank the options [30]. To select a location to build a restaurant next to a highway, a hybrid of the AHP method for calculating criterion weights and the TOPSIS method for ranking alternatives has been conducted [31]. The Borda count, CIMAS (Criteria Importance Assessment), MPSI (Modified Preference Selection Index), Bayesian logic, and RAM have been integrated to rank online shopping platforms [32]. In this study, Borda count is used to rank criteria based on decision-maker preferences, CIMAS is used to assess the importance of criteria by asking decision-makers to rank them according to their importance level, MPSI is used to calculate criterion weights based on the decision matrix values, then Bayesian logic is used to obtain the final criterion weights, while RAM is used to rank alternatives. Three methods, MEREC, entropy, and TOPSIS, have been hybridized to analyze the viewpoints of stakeholders on air pollution in Pakistan, in which the first two methods are used to calculate weights for criteria, and the remaining method is used to rank alternatives [33], etc. In these studies, the hybridization of methods generally involves combining a MCDM method with a method for calculating the weights of criteria. In other words, the hybridization often consists of using one method to determine the weights of the criteria and then using one or several methods to rank the options. This creates a sense that these hybrid approaches have not fully leveraged the inherent capabilities of MCDM methods, which is to not only rank alternatives but also to assign weights to criteria, as exemplified by the R method employed in this study.

In addition to hybridizing methods for calculating criteria weights with methods for ranking options, as described in the studies listed above, hybridizing weighting methods with each other has also been undertaken. The purpose of combining weighting methods is to reduce the influence of a decision-maker's lack of knowledge and subjective opinions on the criteria weights. For instance, a new weighting method has been proposed by hybridizing Entropy and the ANP [34], or by creating a new method for weighting through the hybridization of Delphi, ANP, and Entropy [6]. However, even when users employ hybrid methods for calculating criteria weights, they still need to use MCDM methods to rank the options. Thus, the essence of these efforts remains the combination of a weighting method for criteria with a method for ranking options.

Recently, a new hybridization has been performed by combining PSI (Preference Selection Index) with two methods, TOPSIS and MABAC (Multi Attributive Border Approximation area Comparison), resulting in the creation of new methods called PSI-TOPSIS and PSI- MABAC [35]. In this study, PSI was used both for ranking options and for calculating the weights of criteria. PSI-TOPSIS ranks options using TOPSIS with the criteria weights calculated by PSI, and similarly, PSI- MABAC ranks options using MABAC with the criteria weights calculated by PSI. The results demonstrated that PSI-TOPSIS and PSI-MABAC achieved higher performance compared to using PSI alone [35]. This study perhaps represents a different kind of hybridization compared to the ones previously mentioned. Specifically, the hybridization involves three MCDM methods PSI, TOPSIS, and MARCOS without needing any additional weighting method to calculate the criteria weights. This impressive work has inspired our direction to hybridize R and RAM, as both R and RAM are methods for ranking options, and R also has the function of calculating criteria weights. This motivation has led to the conduct of this research.

3. MATERIALS AND METHODS

To rank options, a decision matrix must first be established with *m* as the number of options to be ranked and *n* as the number of criteria. Let x_{ij} be the value of criterion *j* for option *i*. The letter *B* represents the criterion where higher values are better, and *C* represents the criterion where lower values are better.

Ranking options using the R method is done as follows [13]:

Step 1: Rank the criteria in descending order of their importance.

Step 2: Calculate the weights for the ranks of the criteria using formula (1), where r_j is the rank value of rank *j*.

$$w^{(j)} = \frac{1}{1 + \frac{1}{2} + \cdots + \frac{1}{r_j}}, j = 1 \div n \tag{1}$$

Step 3: Calculate the weights for the criteria w_j using formula (2).

$$w_j = \frac{w^{(j)}}{\sum_{j=1}^{n} w^{(j)}}, j = 1 \div n \tag{2}$$

Step 4: Rank the options for each criterion.

Step 5: Calculate the weights for the ranks of the options using formula (3), where r_t is the rank value of rank *t*.

$$\vartheta^{(t)} = \frac{1}{1 + \frac{1}{2} + \cdots + \frac{1}{r_t}}, t = 1 \div m \tag{3}$$

Step 6: Calculate the weights for the ranks of the options using formula (4).

$$\vartheta_t = \frac{w^{(t)}}{\sum_{k=1}^{m} w^{(t)}}, t = 1 \div m \tag{4}$$

Step 7: Calculate the score S_i for each option using formula (5) and rank the options in descending order of their scores.

$$S_i = \sum_{j=1}^{n} w_k^j * \vartheta_t^i , i = 1 \div m \tag{5}$$

It can be observed that the first three steps of the R method primarily focus on calculating weights for alternatives. Step 1 considers the decision-maker's perspective (expert opinion) on the importance of criteria, while steps 2 and 3 involve calculations to determine criterion weights. Thus, these three steps combine both subjective and objective elements in calculating criterion weights. Subjectivity lies in the decision-maker's determination of preference levels among criteria, while objectivity is reflected in the calculation of criterion weights using mathematical formulas (1) and (2) based on the decision-maker's subjective opinions regarding criterion importance. This approach has been shown to be highly effective, as discussed in the introduction.

Steps 4 to 7 of the R method concentrate on ranking alternatives. However, the ranking of alternatives using the R method relies on the results of ranking alternatives for each criterion (refer to step 4) and does not involve data normalization, which may compromise the objectivity of the evaluation and comparison of alternatives [36, 37]. This issue suggests an idea of focusing solely on exploiting the R method for calculating criterion weights, while using a different MCDM method for ranking alternatives. The MCDM method considered in this study is the RAM method. Nevertheless, in the examples presented in the subsequent section of this paper, the R method is still exploited for both calculating criterion weights and ranking alternatives to provide more in-depth insights into this method.

Ranking options using the RAM method follows this sequence [17]:

Step 1: Normalize the data using formula (6).

$$n_{ij} = \frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}} \tag{6}$$

Step 2: Calculate the normalized values considering the weights of the criteria y_{ij} using formula (7).

$$y_{ij} = w_j \cdot n_{ij} \tag{7}$$

Step 3: Calculate the total normalized score considering the weights of the criteria using formulas (8) and (9).

$$S_{+i} = \sum_{j=1}^{n} y_{+ij} \quad if \quad j \in B \tag{8}$$

$$S_{-i} = \sum_{j=1}^{n} y_{-ij} \quad if \quad j \in C \tag{9}$$

Step 4: Calculate the score RI_i for each option using formula (10). The best option is the one with the highest score.

$$RI_i = \sqrt[2+S_{-i}]{2 + S_{+i}} \tag{10}$$

4. PROPOSED HYBRID MODEL

Based on the steps involved in applying both the R and RAM methods, as previously discussed, the hybridization of these two methods is illustrated in Figure 1. For each problem of ranking alternatives, this hybrid between R and RAM will generate two sets of alternative rankings: one set produced by using R and another set produced by using RAM with criterion weights calculated by the R method. Specifically, if we sequentially apply the steps within the purple rectangle, we will obtain a set of alternative rankings using the R method. In the case of applying the steps within the red polygon, which means using a portion of the R method to calculate criterion weights and then ranking alternatives using the RAM method, we will generate a set of alternative rankings using the RAM method. This ranking is the result of ranking using the RAM method when weights are calculated using the R method, referred to as the ranking results when using the R-RAM method.

By using the R-RAM method to rank alternatives, users will not need to employ any additional supplementary methods, as R-RAM operates on the principle of using R to calculate criterion weights and RAM to rank alternatives. This offers users convenience and simplicity, but it is also considered a limitation of the method, meaning that when using R-RAM to rank alternatives, only criterion weights calculated by the R method can be used, and weights calculated by other methods cannot be utilized.

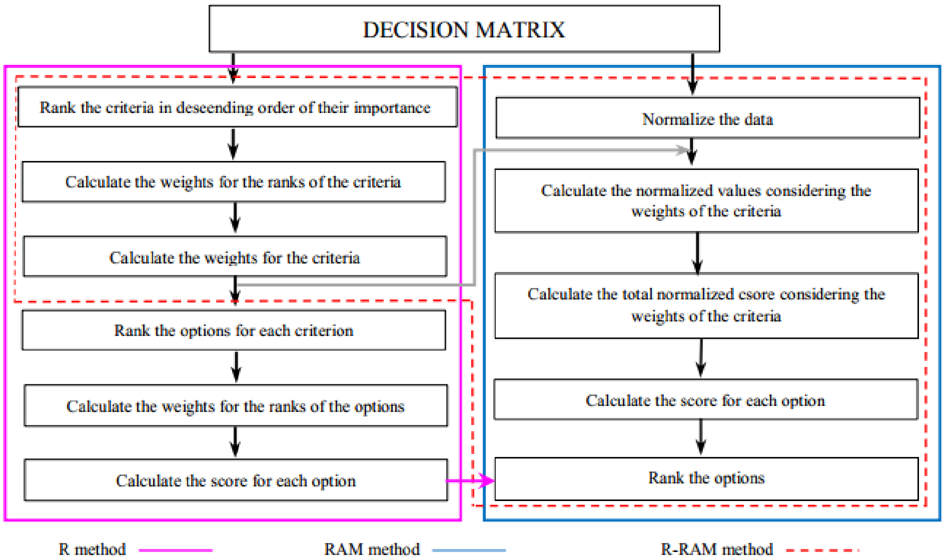

Figure 1: The R-RAM hybrid model

5. EVALUATING THE R-RAM HYBRID MODEL

In this section, the evaluation of the proposed R-RAM hybrid model will be carried out across four different cases in diverse fields. Moreover, each case varies in terms of the number of options, criteria, and types of criteria chosen to ensure the most objective results. Specifically:

Case 1: Ranking 9 options for convenience processes, each option characterized by one type *B* criterion and three type *C* criteria.

Case 2: Ranking 7 types of electric bicycles with 8 type *B* criteria and 2 type *C* criteria.

Case 3: Ranking various materials for automobile protective panel manufacturing with 5 options, 4 type *B* criteria, and 2 type *C* criteria.

Case 4: Ranking 6 types of 3D printers with 5 criteria each for type *B* and type *C*.

These cases illustrate different scenarios to ensure a comprehensive evaluation of the R-RAM hybrid model, considering varying complexities and types of decision-making criteria in each field.

5.1. Ranking of metal cutting options

Nine experiments of metal cutting processes needing ranking have been synthesized in Table 1, with F_x, F_y, F_z, and *MRR* as criteria, and their types (*B* or *C*) are also shown in this table. F_x, F_y, and F_z are the shear force components along the X, Y, and Z axes, respectively, in the OXYZ coordinate system. *MRR* stands for the material removal rate, which is the volume of metal removed per second. This data table was extracted from a previous study where the ranking of experiments was also assessed using SAW, WASPAS (Weighted Aggregates Sum Product ASsessment), TOPSIS, VIKOR, MOORA (Multiobjective Optimization On the basis of Ratio Analysis), COPRAS (COmplex PRroportional Assessment), and PIV (Proximity Indexed Value) methods [38].

Table 1: Data from example 1 [38]

Trial.	F_x	F_y	F_z	*MRR*
Type	*C*	*C*	*C*	*B*
Unit	*N*	*N*	*N*	mm^3/s
#1	59.844	187.437	44.165	11.561
#2	87.943	199.762	99.125	49.062
#3	78.913	127.456	69.874	109.108
#4	54.816	172.714	60.19	28.588
#5	63.117	180.361	68.869	99.039
#6	68.79	113.951	70.694	61.669
#7	46.654	116.88	92.222	57.177
#8	44.989	162.337	63.25	55.462
#9	54.846	167.837	74.165	151.09

Ranking of metal cutting experiments using method *R* is conducted as follows:

Because there are 4 criteria, the weights of the 1st, 2nd, 3rd, and 4th ranks of the criteria are calculated according to formula (1) as follows:

$$w^{(1)} = 1$$

$$w^{(2)} = \frac{1}{1+\frac{1}{2}} = 0.6667$$

$$w^{(3)} = \frac{1}{1+\frac{1}{2}+\frac{1}{3}} = 0.5454$$

$$w^{(4)} = \frac{1}{1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}} = 0.4800$$

According to some references, the four criteria are prioritized in descending order as F_y, *MRR*, F_z, and F_x [39, 40]. Therefore, the weight of each criterion is calculated according to formula (2) as follows:

$$w_{F_y} = \frac{w^{(1)}}{w^{(1)}+w^{(2)}+w^{(3)}+w^{(4)}} = 0.3715$$

$$w_{MRR} = \frac{w^{(2)}}{w^{(1)}+w^{(2)}+w^{(3)}+w^{(4)}} = 0.2476$$

$$w_{F_z} = \frac{w^{(3)}}{w^{(1)}+w^{(2)}+w^{(3)}+w^{(4)}} = 0.2026$$

$$w_{F_x} = \frac{w^{(4)}}{w^{(1)}+w^{(2)}+w^{(3)}+w^{(4)}} = 0.1783$$

Ranking results of options for each criterion are synthesized in Table 2.

Table 2: Rankings of options for each criterion

Trial.	F_x	F_y	F_z	*MRR*
#1	5	8	1	9
#2	9	9	9	7
#3	8	3	5	2
#4	3	6	2	8
#5	6	7	4	3
#6	7	1	6	4
#7	2	2	8	5
#8	1	4	3	6
#9	4	5	7	1

Since there are 9 alternatives, the weights of the ranks from 1 to 9 of the alternatives are calculated according to formula (3) as follows:

$$\vartheta^{(1)} = 1.$$

$$\vartheta^{(2)} = \frac{1}{1+\frac{1}{2}} = 0.6667$$

$$\vartheta^{(3)} = \frac{1}{1+\frac{1}{2}+\frac{1}{3}} = 0.5454$$

$$\vartheta^{(4)} = \frac{1}{1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}} = 0.4800$$

$$\vartheta^{(5)} = \frac{1}{1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}} = 0.4379$$

$$\vartheta^{(6)} = \frac{1}{1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}} = 0.4081$$

$$\vartheta^{(7)} = \frac{1}{1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}} = 0.3856$$

$$\vartheta^{(8)} = \frac{1}{1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}} = 0.3679$$

$$\vartheta^{(9)} = \frac{1}{1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}+\frac{1}{9}} = 0.3534$$

The weights of the ranks of the alternatives are calculated according to formula (4) as follows:

$$\vartheta_1 = \frac{\vartheta^{(1)}}{\vartheta^{(1)}+\vartheta^{(2)}+\vartheta^{(3)}+\vartheta^{(4)}+\vartheta^{(5)}+\vartheta^{(6)}+\vartheta^{(7)}+\vartheta^{(8)}+\vartheta^{(9)}} = 0.2153$$

Similarly ϑ_2, ϑ_3, ϑ_4, ϑ_5, ϑ_6, ϑ_7, ϑ_8, ϑ_9 are calculated as 0.1435, 0.1174, 0.1033, 0.0943, 0.0879, 0.0830, 0.0792, and 0.0761 respectively.

The scores for the experiments are calculated according to formula (5), for example, for the first experiment #1.

$$S_1 = \vartheta_5 \cdot w_{F_x} + \vartheta_8 \cdot w_{F_y} + \vartheta_1 \cdot w_{F_z} + \vartheta_9 \cdot w_{MRR} = 0.10869$$

Similar steps are taken to calculate scores for experiments from #2 to #9. Table 3 summarizes scores and rankings of experiments.

Table 3: Scores and rankings of experiments

Trial.	S_i	*Rank*
#1	0.10869	6
#2	0.07781	9
#3	0.11238	5
#4	0.10227	7
#5	0.09652	8
#6	0.13816	1
#7	0.11829	4
#8	0.12231	3
#9	0.12357	2

Thus, ranking of experiments using method R is completed. The following content presents the ranking of experiments using the RAM method. The normalized values are calculated according to formula (6) and are summarized in Table 4.

Table 4: Normalized values in RAM

Trial.	F_x	F_y	F_z	*MRR*
#1	0.1069	0.1312	0.0687	0.0186
#2	0.1571	0.1398	0.1543	0.0788
#3	0.1409	0.0892	0.1087	0.1752
#4	0.0979	0.1209	0.0937	0.0459
#5	0.1127	0.1262	0.1072	0.1590
#6	0.1229	0.0798	0.1100	0.0990
#7	0.0833	0.0818	0.1435	0.0918
#8	0.0804	0.1136	0.0984	0.0891
#9	0.0980	0.1175	0.1154	0.2426

The normalized values, considering the weights of the criteria, are calculated according to formula (7) and are summarized in Table 5. Note that criteria weights were calculated using method R.

Table 5: Normalized values considering criteria weights

Trial.	F_x	F_y	F_z	*MRR*
#1	0.0191	0.0487	0.0139	0.0046
#2	0.0280	0.0519	0.0313	0.0195
#3	0.0251	0.0331	0.0220	0.0434
#4	0.0175	0.0449	0.0190	0.0114
#5	0.0201	0.0469	0.0217	0.0394
#6	0.0219	0.0296	0.0223	0.0245
#7	0.0149	0.0304	0.0291	0.0227
#8	0.0143	0.0422	0.0199	0.0221
#9	0.0175	0.0436	0.0234	0.0601

The parameters S_{+i}, S_{-i}, and RI_i of each experiment are also calculated according to the corresponding formulas (8), (9), and (10), and the results are summarized in Table 6. This table also lists rankings of experiments by their RI_i scores.

Table 6: Some parameters in RAM method and rankings of experiments

Trial.	S_{+i}	S_{-i}	RI_i	*Rank*
#1	0.0046	0.0817	1.3966	8
#2	0.0195	0.1112	1.3950	9
#3	0.0434	0.0803	1.4099	2
#4	0.0114	0.0813	1.3990	7
#5	0.0394	0.0887	1.4066	3
#6	0.0245	0.0738	1.4051	4
#7	0.0227	0.0743	1.4044	5
#8	0.0221	0.0765	1.4037	6
#9	0.0601	0.0845	1.4144	1

Thus, ranking of experiments using the RAM method is also completed. Figure 2 illustrates the ranking results of experiments using both *R* and *RAM* methods, and other methods (SAW, WASPAS, TOPSIS, VIKOR, MOORA, COPRAS, and PIV) [38]).

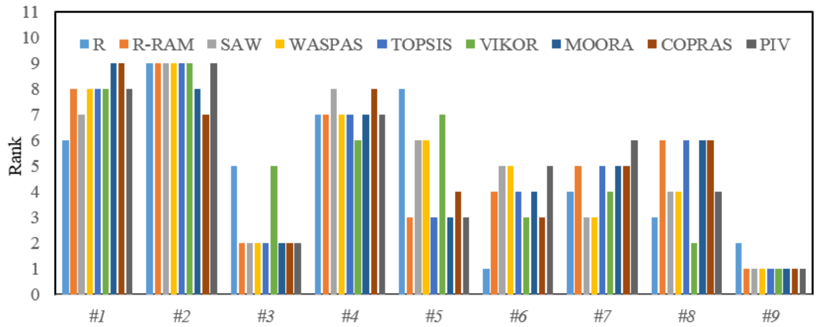

Figure 2: Ranking of experiments using various methods

Table 7 shows the Spearman correlation coefficients ranking among methods. These values are calculated using formula (11), where D_i is the difference in rankings of option i when ranked by different methods [41].

$$S = 1 - \frac{6\sum_{i=1}^{m} D_i^2}{m(m^2-1)} \tag{11}$$

Table 7: Spearman correlation coefficients of Case 1

	R	R-RAM	SAW	WASPAS	TOPSIS	VIKOR	MOORA	COPRAS	PIV
R	1	0.5167	0.7167	0.7000	0.5167	0.9000	0.4667	0.5500	0.5000
R-RAM		1	0.8333	0.8500	1.0000	0.6333	0.9833	0.9333	0.9500
SAW			1	0.9833	0.8333	0.8000	0.8000	0.8000	0.8333
WASPAS				1	0.8500	0.8333	0.8333	0.8167	0.8500
TOPSIS					1	0.6333	0.9833	0.9333	0.9500
VIKOR						1	0.6167	0.6333	0.6833
MOORA							1	0.9667	0.9333
COPRAS								1	0.8667
PIV									1

Figure 2 illustrates that rankings of experiments using the R-RAM method perfectly match those using the TOPSIS method. Using the R-RAM method identifies experiment #9 as the best, consistent with SAW, WASPAS, TOPSIS, VIKOR, MOORA, COPRAS, and PIV methods. Therefore, regarding the task of identifying the best experiment, the R-RAM method is equivalent to SAW, WASPAS, TOPSIS, VIKOR, MOORA, COPRAS, and PIV methods. Conversely, using method R ranks experiment #6 as the best, which differs entirely from other methods. Spearman correlation coefficients between R and SAW, WASPAS, TOPSIS, VIKOR, MOORA, COPRAS, PIV methods are 0.7167, 0.7000, 0.5167, 0.9000, 0.4667, 0.5500, 0.5000, averaging to 0.6214; between R-RAM and SAW, WASPAS, TOPSIS, VIKOR, MOORA, COPRAS, PIV methods are 0.8333, 0.8500, 1.0000, 0.6333, 0.9833, 0.9333, 0.9500, averaging to 0.8833. Hence, the similarity in rankings of experiments using the R-RAM method compared to other methods is significantly higher than when using the R method. The reason for this is that if only the original R method is employed, following the calculation of criteria weights in the first three steps, the subsequent ranking of alternatives is conducted using the remaining four steps (steps 4 to 7). It's worth noting that when implementing steps 4 to 7 of the R method, the ranking of alternatives for each criterion is utilized without any data normalization, unlike most other MCDM methods such as RAM. Conversely, in the R-RAM method, only the initial three steps of the R method are employed for weight calculation, and the RAM method is used for ranking alternatives. In the RAM method, data normalization is performed as per formula (6). It is crucial to highlight that data normalization is an indispensable step in multi-criteria decision-making. By standardizing data to a common scale, it ensures the comparability and objectivity of the evaluation and decision-making process [36, 37]. This also indicates the superiority of the R-RAM method over the R method both in finding the best experiment and in similarity to other methods. Therefore, in this case, the R-RAM method stands out compared to the R method both in finding the best experiment and in its similarity to other methods.

5.2. Ranking of electric bicycle types

In this case, ranking of seven types of electric bicycles, designated as *A1* to *A7*, was conducted. Ten criteria labeled from *C1* to *C10* were used to evaluate each option, where *C1* and *C3* are of type *C*, and the remaining eight criteria are of type *B*. The meanings of criteria from *C1* to *C10* are respectively price, distance per charge, charging time, maximum speed, weight of the bicycle, payload capacity, saddle height from the ground, overall length of the bicycle, overall width of the bicycle, and overall height of the bicycle. Data on electric bicycle types were synthesized in Table 8, extracted from a recent study where rankings of options were also assessed using SAW, MARCOS (Measurement Alternatives and Ranking according to COmpromise Solution), and PSI methods [42].

Based on feedback from some users, a consistent opinion was received regarding the prioritization order among criteria in descending order: *C2* > *C4* > *C3* > *C1* > *C5* > *C6* > *C7* > *C8* > *C9* > *C10*. Based on this prioritization, using method R calculated weights of criteria C1 to C10 respectively as 0.0963, 0.2005, 0.1094, 0.1337, 0.0878, 0.0818, 0.0773, 0.0738, 0.0709, and 0.0685.

Similar to Case 1, electric bicycle types were ranked using both R and R-RAM methods. Figure 3 illustrates the rankings of options when ranked by R, R-RAM, and SAW, MARCOS, and PSI methods in [42].

The ranking results of electric bicycle types illustrated in Figure 3 show that all four methods, R-RAM, SAW, MARCOS, and PSI, identify *A3* as the best option, while method R identifies *A5* as the best option. Thus, in determining the best option, R-RAM demonstrates superiority over R in this case.

Table 8: Data from Example 2 [42]

Alternatives	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
Type	C	B	C	B	B	B	B	B	B	B
Unit	VNĐ	km/1 charge	hour	km/h	kg	kg	mm	mm	mm	mm
A1	7500000	45	7	35	56	160	750	1593	635	1015
A2	7900000	45	7	35	50	150	750	1640	640	1200
A3	9900000	50	7	35	50	180	750	1640	640	1200
A4	9900000	50	7	35	46	180	750	1640	640	1200
A5	11500000	50	7	35	52	180	750	1640	640	1200
A6	13990000	45	7	30	45	75	550	1550	650	1040
A7	13990000	45	8	30	45	75	600	1530	750	1000

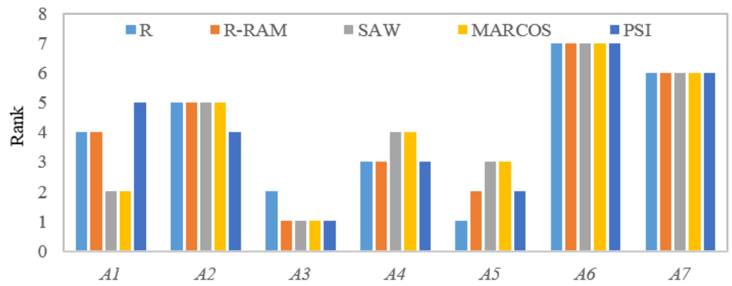

Figure 3: Ranking of electric bicycles using various methods

Spearman correlation coefficients for rankings have also been calculated and synthesized in Table 9. These coefficients between R-RAM and SAW, MARCOS, PSI methods are higher than those between R and SAW, MARCOS, PSI methods. The average Spearman correlation coefficient between R-RAM and other MCDM methods is 0.9167, while the average Spearman correlation coefficient between R and other MCDM methods is 0.8839. This demonstrates that the similarity in ranking results of options using R-RAM with other methods is higher compared to using R. In conclusion, regarding both finding the best electric bicycle type and similarity to other methods, R-RAM method shows superiority over the R method in this case. The reason why the R method is less efficient than the R-RAM method and other MCDM methods, as explained in Example 1, is due to the lack of data normalization when using R to rank alternatives, unlike in other methods.

Table 9: Spearman correlation coefficients of Case 2

	R	R-RAM	SAW	MARCOS	PSI
R	1	0.9643	0.8214	0.8214	0.9286
R-RAM		1	0.8929	0.8929	0.9643
SAW			1	1	0.7857
MARCOS				1	0.7857
PSI					1

5.3. Ranking of materials for automobile protective panels

In this case, the ranking of five types of materials for automobile protective panels was conducted. Each material is characterized by six criteria from *C1* to *C6*, where the meanings of the criteria are compressive strength, flexural modulus, hardness, Charpy impact toughness, elongation, and cost. Among these six criteria, the first four belong to type *B*, while the last two belong to type *C* (Table 10). Data on the five types of materials were cited from another study where rankings of materials were determined using CURLI (Collaborative Unbiased Rank List Integratio), PROMETHEE (Preference Ranking Organization Method for Enrichment Evaluation), and EDAS (Evaluation Based on Distance from Average Solution) methods [43].

Since there is no information indicating which criteria from *C1* to *C6* are more important than others, for simplicity, the prioritization order among criteria in descending order is assumed as *C1* > *C2* > *C3* > *C4* > *C5* > *C6*. Method *R* was used to calculate the weights of criteria from *C1* to *C6*, resulting in corresponding weights of 0.2826, 0.1884, 0.1542, 0.1357, 0.1238, and 0.1154.

Table 10: Data from Example 3 [43]

Alt.	*C1*	*C2*	*C3*	*C4*	*C5*	*C6*
Type	*B*	*B*	*B*	*B*	*C*	*C*
A1	20	700	92	1	500	78
A2	40	1500	92	1	100	84
A3	65	2500	105	2.18	30	114
A4	130	3100	93	3	50	153
A5	70	2500	90	0.6	7	1300

Similar to Case 1, materials were ranked using both R and R-RAM methods. Figure 4 illustrates the rankings of options when ranked by R, R-RAM, CURLI, PROMETHEE, and EDAS methods in [43].

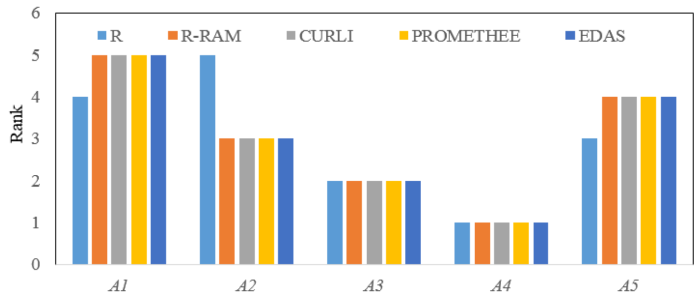

Figure 4: Ranking of materials using various methods

In this case, material *A4* is consistently ranked 1st, and material *A3* is consistently ranked 2nd across all five methods: R, R-RAM, CURLI, PROMETHEE, and EDAS. Thus, in determining the best option, both R and R-RAM methods are considered equally effective and equivalent to other methods. Spearman correlation coefficients for rankings have been calculated (see Table 11) to compare the similarity in ranking results among different methods. It is evident that the correlation coefficient between R-RAM and CURLI, PROMETHEE, EDAS methods is consistently 1, indicating no difference in rankings of options when ranked by these four methods. Reviewing Figure 4 also confirms this clarity. For method R, its Spearman correlation coefficients with CURLI, PROMETHEE, and EDAS methods are all 0.7, showing that the similarity in ranking results of R compared to CURLI, PROMETHEE, and EDAS methods is lower than that of R-RAM. This can also be explained by the fact that when using the R method to rank alternatives, data normalization, a common practice in other methods, was not performed. In conclusion, once again in this case, R-RAM demonstrates superiority over R.

Table 11: Spearman correlation coefficients of Case 3

	R	R-RAM	CURLI	PROMETHEE	EDAS
R	1	0.7	0.7	0.7	0.7
R-RAM		1	1	1	1
CURLI			1	1	1
PROMETHEE				1	1
EDAS					1

5.4. Ranking of 3D printers

In this case, the ranking of six types of 3D printers was conducted, denoted as *Pr1* to *Pr6*. Each option is characterized by ten criteria from *C1* to *C10*. These criteria are named as layer thickness, maximum printing speed, power consumption, maximum extruder temperature, accuracy in X and Y axes, accuracy in Z axis, maximum object size in X direction, maximum object size in Y direction, maximum object size in Z direction, and cost. Among these ten criteria, *C1*, *C3*, *C5*, *C6*, and *C10* are of type *C*, while the remaining criteria are of type *B*. Table 12 summarizes the data on the six types of 3D printers, extracted from another study where rankings of printer types were determined using the PSI method [44].

Table 12: Data from Example 4 [44]

Alt.	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
	C	B	C	B	C	C	B	B	B	C
	mm	mm/s	W	^{0}C	µm	µm	mm	mm	mm	$
Pr1	0.05	200	360	350	12.7	1.25	400	300	520	4290
Pr2	0.05	150	500	300	11	2.5	300	250	200	2999
Pr3	0.1	150	600	260	3.125	1.25	220	220	220	3200
Pr4	0.1	500	600	260	6.25	1.25	240	190	200	2560
Pr5	0.05	200	500	290	10	1	280	280	285	4950
Pr6	0.05	200	500	300	12.5	5	300	200	250	2699

For simplicity, in this case, the prioritization order among criteria is assumed to be in descending order as follows*: C1 > C2 > C3 > C4 > C5 > C6 > C7 > C8 > C9 > C10*. The weights of criteria from *C1* to *C10* were calculated using the R method, resulting in corresponding values of 0.2005, 0.1337, 0.1094, 0.0963, 0.0878, 0.0818, 0.0773, 0.0738, 0.0709, and 0.0685. Figure 5 illustrates the rankings of options when ranked by the R and R-RAM methods in this study, as well as the PSI method in [44].

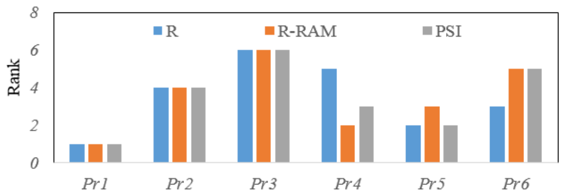

Figure 5: Ranking of 3D printers using various methods

Spearman correlation coefficients for rankings have been calculated and summarized in Table 13. All three methods R, R-RAM, and PSI indicate that *Pr1* is the best option. Therefore, in terms of finding the best option, in this case, R and R-RAM are evaluated to be equally effective and similar to the PSI method. According to Table 13, the Spearman correlation coefficient between R-RAM and PSI is 0.9429, significantly higher than the Spearman coefficient between R and PSI, which is 0.7714. This indicates that in terms of the ability to generate similar rankings among methods, R-RAM outperforms R. This can also be explained by the fact that when using the R method to rank alternatives, data normalization, a common practice in other methods, was not performed. In conclusion, similar to the first three cases, R-RAM shows higher performance than R in this case as well.

Table 13: Spearman correlation coefficients of Case 4

	R	R-RAM	PSI
R	1	0.6000	0.7714
R-RAM		1	0.9429
PSI			1

All four examples have demonstrated that the R-RAM method performs comparably to other MCDM methods and outperforms the R method. To provide a more comprehensive evaluation of the R-RAM method, a sensitivity analysis is presented in the following section.

6. SENSITIVITY ANALYSIS

To confidently apply the R-RAM method to other domains, sensitivity analysis is crucial. Various approaches have been employed for sensitivity analysis, such as increasing or decreasing the number of alternatives to be ranked, changing the weights of criteria, or altering the criteria type [45, 46]. In this study, sensitivity analysis was conducted by varying the weights of criteria [47].

For Example 1 presented in Section 5.1, different weight scenarios were created by considering changes in the priority levels of the criteria. For instance, the first scenario (S1) assumed that the priority levels of the criteria decrease in the order of F_y, *MRR*, F_z, F_x, which is the scenario implemented in Section 5.1. In the second scenario (S2), the priority levels decrease in the order of F_y, F_z, *MRR*, F_x; in the third scenario (S3), they decrease in the order of F_z, *MRR*, F_y, F_x, and so on. In this manner, a total of 24 scenarios were generated, which is the factorial of 4. Table 14 summarizes the weights of the criteria in these 24 different scenarios.

Figure 6 illustrates the ranking of experiments using the R-RAM method in 24 different weight scenarios. Despite significant variations in criteria weights across these scenarios, it is observed that Experiment #9 consistently ranks first, Experiment #3 consistently ranks second, Experiment #5 consistently ranks third, and Experiment #4 consistently ranks seventh. The rankings of other experiments also exhibit minimal changes across different scenarios. These results demonstrate the high stability of the R-RAM method in determining the ranking of alternatives, even when criteria weights are varied.

For Example 2 in Section 5.2, due to the presence of 10 criteria, generating weight scenarios using the same approach as in Example 1 would result in a massive number of scenarios (10! = 3628800). Therefore, a different approach was adopted for generating weight scenarios in this example. In this approach, each criterion is assigned the highest priority level once. After a criterion is selected as the highest priority, the remaining criteria are ranked in descending order of priority. To illustrate this further, consider Scenario 4 (S4) where C4 is assigned the highest priority. For the remaining criteria, the priority levels decrease in the order of C1, C2, C3, C5, C6, C7, C8, C9, C10. Using this approach, ten different weight scenarios were generated as shown in Table 15. Note that the diagonal elements of this table have the highest weight value of 0.2005.

Table 14: Weights of criteria in different scenarios (Example 1)

	F_x	F_y	F_z	*MRR*
S1	0.1783	0.3715	0.2026	0.2476
S2	0.1783	0.3715	0.2476	0.2026
S3	0.1783	0.2026	0.3715	0.2476
S4	0.1783	0.2026	0.2476	0.3715
S5	0.1783	0.2476	0.3715	0.2026
S6	0.1783	0.2476	0.2026	0.3715
S7	0.3715	0.1783	0.2026	0.2476
S8	0.3715	0.1783	0.2476	0.2026
S9	0.3715	0.2026	0.1783	0.2476
S10	0.3715	0.2026	0.2476	0.1783
S11	0.3715	0.2476	0.1783	0.2026
S12	0.3715	0.2476	0.2026	0.1783
S13	0.2026	0.1783	0.3715	0.2476
S14	0.2026	0.1783	0.2476	0.3715
S15	0.2026	0.3715	0.1783	0.2476
S16	0.2026	0.3715	0.2476	0.1783
S17	0.2026	0.2476	0.1783	0.3715
S18	0.2026	0.2476	0.3715	0.1783
S19	0.2476	0.1783	0.3715	0.2026
S20	0.2476	0.1783	0.2026	0.3715
S21	0.2476	0.3715	0.1783	0.2026
S22	0.2476	0.3715	0.2026	0.1783
S23	0.2476	0.2026	0.1783	0.3715
S24	0.2476	0.2026	0.3715	0.1783

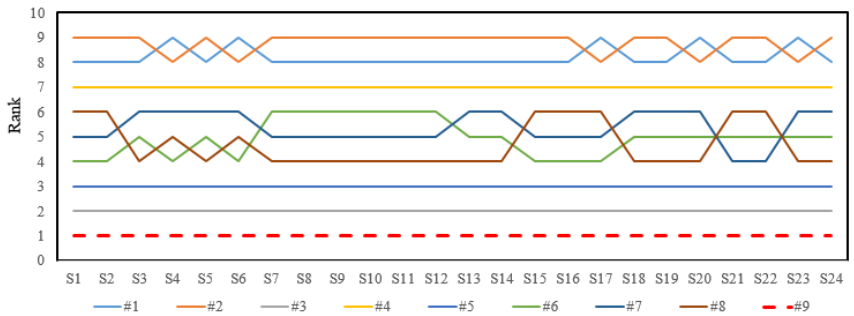

Figure 6: Ranking of alternatives in Example 1 under different scenarios.

The ranking of alternatives using the R-RAM method in these ten weight scenarios is illustrated in Figure 7. It is observed that despite significant variations in criteria weights across scenarios, the ranking of alternatives remains relatively stable. Notably, alternative A3 consistently ranks first in 8 out of 10 scenarios (including S2, S3, S4, S6, S7, S8, S9, S10), while in the remaining 2 scenarios (S1, S5), A3 ranks second. Even for the two alternatives consistently ranked as the worst, they are always either A6 or A7. This further demonstrates the high stability of the alternative rankings in this case.

Table 15: Weights of criteria in different scenarios (Example 2).

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
S1	0.2005	0.1337	0.1094	0.0963	0.0878	0.0818	0.0773	0.0738	0.0709	0.0685
S2	0.1337	0.2005	0.1094	0.0963	0.0878	0.0818	0.0773	0.0738	0.0709	0.0685
S3	0.1337	0.1094	0.2005	0.0963	0.0878	0.0818	0.0773	0.0738	0.0709	0.0685
S4	0.1337	0.1094	0.0963	0.2005	0.0878	0.0818	0.0773	0.0738	0.0709	0.0685
S5	0.1337	0.1094	0.0963	0.0878	0.2005	0.0818	0.0773	0.0738	0.0709	0.0685
S6	0.1337	0.1094	0.0963	0.0878	0.0818	0.2005	0.0773	0.0738	0.0709	0.0685
S7	0.1337	0.1094	0.0963	0.0878	0.0818	0.0773	0.2005	0.0738	0.0709	0.0685
S8	0.1337	0.1094	0.0963	0.0878	0.0818	0.0773	0.0738	0.2005	0.0709	0.0685
S9	0.1337	0.1094	0.0963	0.0878	0.0818	0.0773	0.0738	0.0709	0.2005	0.0685
S10	0.1337	0.1094	0.0963	0.0878	0.0818	0.0773	0.0738	0.0709	0.0685	0.2005

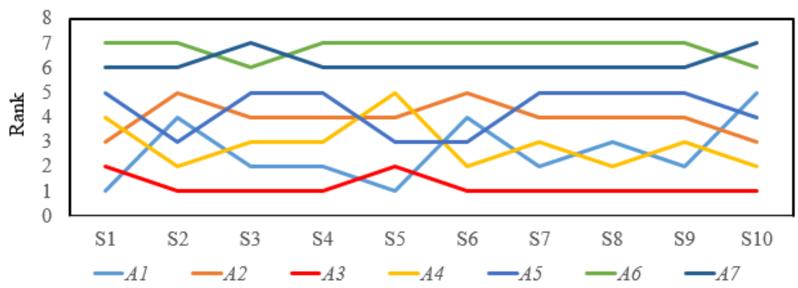

Figure 7: Ranking of alternatives in Example 2 under different scenarios.

For Example 3, the generation of different weight scenarios followed a similar approach as in Example 2. Table 16 summarizes the weights of criteria in six different weight scenarios. In each scenario, one criterion was assigned the highest priority level. For instance, in Scenario S1, C1 was assigned the highest priority, in Scenario S2, C2 was assigned the highest priority, and so on.

Table 16: Weights of criteria in different scenarios (Example 3).

	C1	*C2*	*C3*	*C4*	*C5*	*C6*
S1	0.2826	0.1884	0.1542	0.1357	0.1238	0.1154
S2	0.1884	0.2826	0.1542	0.1357	0.1238	0.1154
S3	0.1542	0.1884	0.2826	0.1357	0.1238	0.1154
S4	0.1357	0.1884	0.1542	0.2826	0.1238	0.1154
S5	0.1238	0.1884	0.1542	0.1357	0.2826	0.1154
S6	0.1154	0.1884	0.1542	0.1357	0.1238	0.2826

The ranking of alternatives using the R-RAM method in these six weight scenarios is illustrated in Figure 8. It is observed that there is almost no change in the ranking of alternatives across different scenarios, except for A1 and A5 which switched ranks in Scenarios S5 and S6. This result strongly confirms the stability of the R-RAM method in determining the ranking of alternatives when criteria weights are varied in these six scenarios.

For Example 4, weight scenarios were generated following the same approach as in Examples 2 and 3. Table 17 presents the weights of criteria in ten different scenarios.

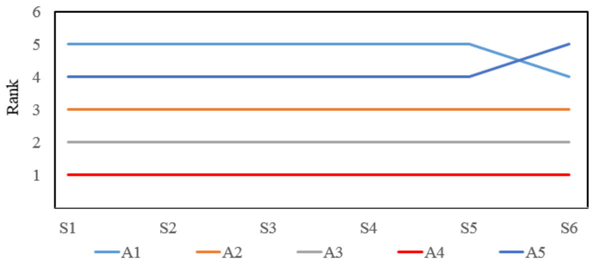

Figure 8: Ranking of alternatives in Example 3 under different scenarios.

Table 17: Weights of criteria in different scenarios (Example 4).

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
S1	0.2005	0.1337	0.1094	0.0963	0.0878	0.0818	0.0773	0.0738	0.0709	0.0685
S2	0.1337	0.2005	0.1094	0.0963	0.0878	0.0818	0.0773	0.0738	0.0709	0.0685
S3	0.1337	0.1094	0.2005	0.0963	0.0878	0.0818	0.0773	0.0738	0.0709	0.0685
S4	0.1337	0.1094	0.0963	0.2005	0.0878	0.0818	0.0773	0.0738	0.0709	0.0685
S5	0.1337	0.1094	0.0963	0.0878	0.2005	0.0818	0.0773	0.0738	0.0709	0.0685
S6	0.1337	0.1094	0.0963	0.0878	0.0818	0.2005	0.0773	0.0738	0.0709	0.0685
S7	0.1337	0.1094	0.0963	0.0878	0.0818	0.0773	0.2005	0.0738	0.0709	0.0685
S8	0.1337	0.1094	0.0963	0.0878	0.0818	0.0773	0.0738	0.2005	0.0709	0.0685
S9	0.1337	0.1094	0.0963	0.0878	0.0818	0.0773	0.0738	0.0709	0.2005	0.0685
S10	0.1337	0.1094	0.0963	0.0878	0.0818	0.0773	0.0738	0.0709	0.0685	0.2005

The ranking of alternatives using the R-RAM method in these ten weight scenarios is illustrated in Figure 9. Once again, it is observed that the ranking of alternatives is quite stable when criteria weights are varied. Notably, Pr1 is consistently ranked first in 9 out of 10 scenarios, and in the remaining scenario (S2), Pr1 ranks second. This result further confirms the stability of the R-RAM method in determining the ranking of alternatives when criteria weights are varied in these ten scenarios.

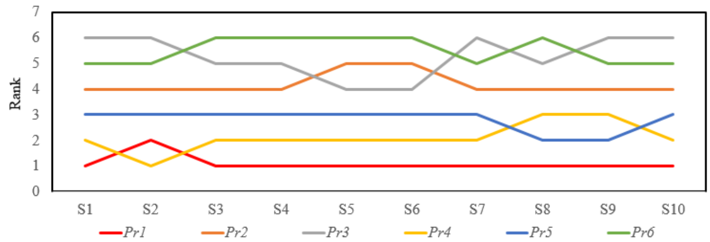

Figure 9: Ranking of alternatives in Example 3 under different scenarios.

The four cases presented above exhibit significant differences. Case 1 involves ranking nine metal cutting tools with one type B criterion and three type C criteria. Case 2 ranks seven electric bicycles based on eight type B criteria and two type C criteria. Case 3 ranks five types of automotive protective plate materials using four type B criteria and two type C criteria. Case 4 pertains to ranking six types of 3D printers with five type B criteria and five type C criteria. Despite these variations, the best alternative identified by R-RAM consistently aligns with the results of other MCDM methods such as SAW, WASPAS, TOPSIS, VIKOR, MOORA, COPRAS, and PIV in Example 1, SAW, MARCOS, and PSI in Example 2, CURLI, PROMETHEE, and EDAS in Example 3, and PSI in Example 4 as reported in the literature. Moreover, the Spearman rank correlation coefficients between R-RAM and other MCDM methods are also very high. The average Spearman correlation coefficients between R-RAM and other methods in Examples 1 to 4 are 0.8833, 0.9167, 1, and 0.9429 respectively, indicating that not only does R-RAM identify the same best alternative as other methods, but the rankings of the remaining alternatives obtained by R-RAM also exhibit a high degree of similarity to those obtained by other methods [48]. Sensitivity analysis of the R-RAM method has demonstrated a high level of stability in the ranking of alternatives across various domains, even when criteria weights are varied.

The achievement of these promising results can be attributed to the unique combination of the R method and RAM in the R-RAM method. This combination not only leverages the ability to incorporate both subjective and objective factors in determining criteria weights, a characteristic inherent in the R method, but also inherits the outstanding advantages of RAM, such as the ability to balance criteria and the efficiency in data normalization.

These aspects provide sufficient grounds to assert that the hybridization of R and RAM to create R-RAM is a scientific approach. R-RAM has been validated as a novel and highly accurate method in the field of MCDM. Moreover, in all the examined cases, R-RAM has consistently outperformed the R method. This does not imply a criticism of the R method but rather serves as a recommendation to adopt R-RAM as an MCDM method if one wishes to avoid the effort of selecting a suitable MCDM method or a weight determination method.

CONCLUSION

The R method is not only an MCDM method but also a method for calculating criteria weights. The criteria weights calculated using the R method consider both the decision-maker's subjective preferences regarding the priority order of criteria and objective factors through the use of mathematical expressions to calculate the weights of criteria according to their priority order. RAM, on the other hand, has the advantage of considering the balance between criteria and operating effectively with various data normalization methods. The hybridization of R and RAM has created a new method called R-RAM, which aims to leverage the strengths of both component methods. R-RAM works by using R to calculate criteria weights and RAM to rank alternatives. R-RAM has been shown to perform comparably to other MCDM methods. Tests to evaluate the performance of R-RAM have shown that the Spearman rank correlation coefficient between R-RAM and other MCDM methods is very high, with a minimum of 0.8833 and even cases where this coefficient is 1. The performance of R-RAM has also been confirmed through sensitivity analysis when used to rank alternatives in various scenarios of criteria weights. These results provide a strong indication that R-RAM is recommended for use without the need for significant effort in selecting another MCDM method for ranking alternatives, nor the need for additional methods to calculate criteria weights.

The R-RAM method is restricted to ranking alternatives based on criteria weights exclusively calculated using the R method. This limitation prevents decision-makers from employing R-RAM when using alternative weight determination methods. Future developments should focus on enhancing R-RAM's flexibility to accommodate various weight assignment approaches.

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REFERENCES

[1] D. A. A. Malik, Y. Yusof, and K. M. N. K. Khalif, "A view of MCDM application in education," *Journal of Physics: Conference Series*, vol. 1988, no. 012063, 2021. doi: 10.1088/1742-6596/1988/1/012063

[2] H. Taherdoost, and M. Madanchian, "Multi-Criteria Decision Making (MCDM) Methods and Concepts," *Encyclopedia*, vol. 3, pp. 77–87, 2023. doi: 10.3390/encyclopedia3010006

[3] M. Baydaş, T. Eren, Ž. Stević, V. Starčević, and R. Parlakkay, "Proposal for an objective binary benchmarking framework that validates each other for comparing MCDM methods through data analytics," *PeerJ Computer Science*, vol. 9, no. e1350, pp. 1-24, 2023. doi: 10.7717/peerj-cs.1350

[4] S. Goyal, S. Agarwal, N. S. S. Singh, T. Mathur, and N. Mathur, "Analysis of Hybrid MCDM Methods for the Performance Assessment and Ranking Public Transport Sector: A Case Study," *Sustainability*, vol. 14, no. 15110, 2022. doi: 10.3390/su142215110

[5] M. Bennani, F. Jawab, Y. Hani, A. ElMhamedi, and D. Amegouz, "A Hybrid MCDM for the Location of Urban Distribution Centers under Uncertainty: A Case Study of Casablanca, Morocco," *Sustainability*, vol. 14, no. 9544, 2022. doi: 10.3390/su14159544

[6] S. Zha, Y. Guo, S. Huang, and S. Wang, "A Hybrid MCDM Method Using Combination Weight for the Selection of Facility Layout in the Manufacturing System: A Case Study," *Mathematical Problems in Engineering*, vol. 2020, no. 1320173, 2020. doi: 10.1155/2020/1320173

[7] A. E. Youssef, and K. Saleem, "A Hybrid MCDM Approach for Evaluating Web-Based E-Learning Platforms," *IEEE Access*, vol. 11, 2023. doi: 10.1109/ACCESS.2023.3294798

[8] H. Cui, S. Dong, J. Hu, M. Chen, B. Hou, J. Zhang, B. Zhang, J. Xian, and F.Chen, "A hybrid MCDM model with Monte Carlo simulation to improve decision-making stability and reliability," *Information Sciences*, vol. 647, no. 119439, 2023. doi: 10.1016/j.ins.2023.119439

[9] E. Ayyildiz, M. Murat, and G. Imamoglu, "A novel hybrid MCDM approach to evaluate universities based on student perspective," *Scientometrics*, vol. 128, pp. 55–86, 2023. doi: 10.1007/s11192-022-04534-z

[10] R. Chaurasiya, and D. Jain, "Hybrid MCDM method on pythagorean fuzzy set and its application," *Decision Making: Applications in Management and Engineering*, vol. 6, no. 1, pp. 379-398, 2023. doi: 10.31181/dmame0306102022c

[11] T. V. Dua, D. V. Duc, N. C. Bao, and D. D. Trung, D. D, "Integration of objective weighting methods for criteria and MCDM methods: application in material selection," *EUREKA: Physics and Engineering*, vol. 2024, no. 2, pp. 131–148, 2024. doi: 10.21303/2461-4262.2024.003171

[12] B. Kizielewicz, and W. Sałabun, "SITW method: a new approach to re-identifying multi-criteria weights in complex decision analysis," *Spectrum of Mechanical Engineering and Operational Research*, vol. 1, no. 1, pp. 215-226, 2024. doi: 10.31181/smeor11202419

[13] R. V. Rao, and J. Lakshmi, "R-method: A simple ranking method for multi-attribute decision-making in the industrial environment," *Journal of Project Management*, vol. 6, pp. 223–230, 2021. doi: 10.5267/j.jpm.2021.5.001

[14] D. D. Trung, D. H. Tien, and N. H. Son, "Decısıon makıng for car selectıon ın Vıetnam," *EUREKA: Physics and Engineering*, vol. 2022, no. 6, pp. 139–150, 2022. doi: 10.21303/2461-4262.2022.002505

[15] D. D. Trung, "Comparison R and CURLI methods for multi-criteria decision making," *Advanced Engineering Letters*, vol. 1, no. 2, pp. 46-56, 2022. doi: 10.46793/adeletters.2022.1.2.3

[16] S. Chatterjee, and S. Chakraborty, "Application of the R method in solving material handling equipment selection problems," *Decision Making: Applications in Management and Engineering*, vol. 6, no. 2, pp. 74-94, 2023. doi: 10.31181/dmame622023391

[17] A. Sotoudeh-Anvari, “Root Assessment Method (RAM): A novel multi-criteria decision making method and its applications in sustainability challenges,” *Journal of Cleaner Production*, vol. 423, no. 138695, 2023. doi: 10.1016/j.jclepro.2023.138695

[18] D. D. Trung, B. Dudić, N.-T. Nguyen, and A. Ašonja, “Data Normalization for Root Assessment Methodology,” *International Journal of Industrial Engineering and Management*, vol. 15, no. 2, pp. 156–168, 2024. doi: 10.24867/IJIEM-2024-2-354

[19] D. D.Trung, “Using RAM method for optimal selection of flame retardant nanocomposite material fabrication solution,” *EPJ Applied Metamaterials*, vol. 11, no. 4, 2024. doi: 10.1051/epjam/2024005

[20] D. D.Trung, D. V. Duc, N. C. Bao, and D. T. T. Thuy, “Using the root assessment method to choose the optimal solution for mushroom cultivation,” *Yugoslav Journal of Operations Research*, 2024. doi: 10.2298/YJOR240115007T

[21] D. T. Do, “Assessing the Impact of Criterion Weights on the Ranking of the Top Ten Universities in Vietnam,” *Engineering, Technology & Applied Science Research*, vol. 14, no. 4, pp. 14899-14903, 2024. doi: 10.48084/etasr.7607

[22] D. D. Trung, B. Dudić, H. T. Dung, and N. X. Truong, “Innovation in financial health assessment: applying MCDM techniques to banks in VIETNAM,” *ECONOMICS - Innovative and Economics Research Journal*, vol.12, no. 2, 2024, doi: 10.2478/eoik-2024-0011

[23] Y. Ali, B. Mehmood, M.Huzaifa, U. Yasir, and A. U. Khan, “Development of a new hybrid multi criteria decision-making method for a car selection scenario,” *FACTA Universitatis, Series: Mechanical Engineering*, vol. 18, no. 3, pp. 357-373, 2020. doi: 10.22190/FUME200305031A

[24] T. N. U. Vo, “Integrating FUCA, SRP, and OPARA Methods to Assess Faculty's Scientific Research Capacity,” *Engineering, Technology & Applied Science Research*, vol. 14, no. 6, pp. 17870-17875, 2024. doi: 10.48084/etasr.8659

[25] I. Ajripour, “Applying a hybrid MCDM technique in warehouse management,” *Studies and articles*, vol. 11, pp. 55-68, 2022. doi: 10.14267/ VEZTUD.2022.11.05

[26] A. S. Eisa, and M. Fattouh, “Hybrid MCDM Model of ARAS -TOPSIS - GRA for Materials Selection Problem,” *Journal of Engineering Research*, vol. 7, no. 2, 2023. doi: 10.21608/ERJENG.2023.200188.1164

[27] A. Gol, I. Shahsavani, F. Fazli, A. –M. Golmohammadi, and R. Tavakkoli-Moghaddam, “A Comprehensive Approach to Evaluating the Effective Factors in Implementing a Circular Supply Chain by A Hybrid MCDM Method,” *International Journal of Supply and Operations Management*, vol. 10, no. 4, pp. 545-563, 2023. doi: 10.22034/IJSOM.2023.109683.2578

[28] M. Krstic, S. Tadic, M. Kovac, V. Roso, and S. Zecevic, “A Novel Hybrid MCDM Model for the Evaluation of Sustainable Last Mile Solutions,” *Mathematical Problems in Engineering*, vol. 2021, no. 5969788, 2021. doi: 10.1155/2021/5969788

[29] A. A. Makki, and R. M. S. Abdulaal, “A Hybrid MCDM Approach Based on Fuzzy MEREC-G and Fuzzy RATMI,” *Mathematics*, vol. 11, no. 3773, 2023. doi: 10.3390/math11173773

[30] G. Ozkaya, and C. Erdin, “Evaluation of smart and sustainable cities through a hybrid MCDM approach based on ANP and TOPSIS technique,” *Heliyon*, vol. 6, no. e05052, 2020. doi: 10.1016/j.heliyon.2020.e05052

[31] A. Biswas, K. H. Gazi, S. P. Mondal, and A. Ghosh, “A Decision-Making Framework for Sustainable Highway Restaurant Site Selecton: AHP-TOPSIS Approach based on the Fuzzy Numbers,” *Spectrum of operatonal research*, vol. 2, no. 1, pp. 1-26, 2025. doi: 10.31181/sor2120256

[32] S. Biswas, B. Biswas, and K. Mitra, “A Novel Group Decision Making Model to Compare Online Shopping Platforms,” *Spectrum of decision making and applications*, vol. 2, no. 1, pp. 1-27, 2025. doi: 10.31181/sdmap2120259

[33] S. Kousar, A. Ansar, N. Kausar, and G. Freen, “Multi-Criteria Decision-Making for Smog Mitgaton: A Comprehensive Analysis of Health, Economic, and Ecological Impacts,” *Spectrum of decision making and applicatons*, vol. 2, no. 1, pp. 53-67, 2025. doi: 10.31181/sdmap2120258

[34] C. –H. Chen, “A Hybrid Multi-Criteria Decision-Making Approach Based on ANP-Entropy TOPSIS for Building Materials Supplier Selection,” *Entropy*, vol. 23, no. 1597, 2021. doi: 10.3390/e23121597

[35] R. N. Wardany, and Zahedi, “A study comparative of PSI, PSI-TOPSIS, and PSI-MABAC methods in analyzing the financial performance of state-owned enterprises companies listed on the Indonesia stock exchange,” *Yugoslav Journal of Operations Research*, 2024. doi: 10.2298/YJOR240115017W

[36] S. T. Mhlanga, and M. Lall, “Influence of Normalization Techniques on Multi-criteria Decision-making Methods,” *Journal of Physics: Conference Series*, vol. 2224, 2021. doi: 6596/2224/1/012076

[37] N. Vafaei, “Data Normalization in Decision Making Processes,” Thesis of MSc in Defense Technologies, Universidade Nova de Lisboa, 2020.

[38] D. D. Trung, “A combination method for multi-criteria decision making problem in turning process,” *Manufacturing review*, vol. 8, no. 26, 2021. doi: 10.1051/mfreview/2021024

[39] V. –H. Nguyen, T. –T. Le, A. –T. Nguyen, X. –T. Hoang, N. –T. Nguyen,and N. –K. Nguyen, “Optimization of milling conditions for AISI 4140 steel using an integrated machine learning-multi objective optimization-multi criteria decision making framework,” *Measurement*, vol. 242, no. 115837, 2025. doi: 10.1016/j.measurement.2024.115837

[40] Hernández-González, W. Luis, Curra-Sosa, A. Dagnier A, R. Pérez-Rodríguez, and P. D. C. Zambrano-Robledo, “Modeling Cutting Forces in High-Speed Turning using Artificial Neural Networks,” *TecnoLógicas*, vol. 24, no. 51, 2021. doi: 10.22430/22565337.1671

[41] N. V. Thien, and N. H. Son, “Material Selection for PMEDM Process,” *International Journal of Mechanical Engineering and Robotics Research*, vol. 13, no. 3, pp. 315-324, 2024. doi: 10.18178/ijmerr.13.3.315-324

[42] T. V. Huy, N. Q. Quyet, V. H. Binh, T. M. Hoang, N. T. T. Tien, L. T. Anh, D. T. Nga, N. Q. Doan, P. H. Tu, and D. D.Trung, “Multi-criteria decision-making for electric bicycle selection,” *Advanced Engineering Letters*, vol. 1, no. 4, pp. 126-135, 2022. doi: 10.46793/adeletters.2022.1.4.2

[43] T. V. Dua, “Application of the collaborative unbiased rank list integration method to select the materials,” *Applied Engineering Letters*, vol. 7, no. 4, pp. 133-142, 2022. doi: 10.18485/aeletters.2022.7.4.1

[44] N. T. P. Giang, and V. Q. Oai, “Application of preference selection index (PSI) method for selecting a 3D printer,” *Journal of Science & Technology*, vol. 58, no. 6A, pp. 47-50, 2022. doi: 10.57001/huih5804.67

[45] L. J. Muhammad, I. Badi, A. A. Haruna, and I. A. Mohammed, “Selecting the best municipal solid waste management techniques in nigeria using multi criteria decision making techniques,” *Rubber Injection Molding Machines*, vol 2, pp. 180-189, 2021. doi: 10.31181/rme2001021801b

[46] D. Bozanic, A. Milic, D. Tesic, W. Sałabun, and D. Pamucar, “D numbers – fucom – fuzzy rafsi model for selecting the group of construction machines for enabling mobility,” *Facta Univ Ser: Mech Eng*, vol. 19, no. 3, pp. 447-471, 2021. doi: 10.22190/FUME210318047B

[47] M. Radovanović, A. Petrovski, E. Cirkin, A. Behlić, Jokić, Željko, D. Chemezov, E. G. Hashimov, M. B. Bouraima, and C. Jana, “Application of the new hybrid model LMAW-G-EDAS multi-criteria decision-making when choosing an assault rifle for the needs of the army,” *Journal of Decision Analytics and Intelligent Computing*, vol. 4, no. 1, pp. 16–31, 2024. doi: 10.31181/jdaic10021012024r

[48] M. Keshavarz-Ghorabaee, A. Rastegar, M. Amiri, E. K.Zavadskas, and J. Antucheviciene, “Multi-Criteria Personnel Evaluation and Selection Using an Objective Pairwise Adjusted Ratio Analysis (OPARA),” *Economic Computation and Economic Cybernetics Studies and Research*, vol. 58, no. 2, pp. 23-45, 2024. doi: 10.24818/18423264/58.2.24.02