Yugoslav Journal of Operations Research
(20##), Number #, #-#
DOI: https://doi.org/10.2298/YJOR240315003K

Researh Article

AVAILABILITY BASED MAINTENANCE ANALYSIS FOR SYSTEM WITH REPAIR TIME THRESHOLD

Nataša KONTREC

Faculty of Sciences and Mathematics, University of Pristina in Kosovska Mitrovica, Lole Ribara 29, Kosovska Mitrovica, Serbia, natasa.kontrec@pr.ac.rs
ORCID: 0000-0002-6955-9887

Stefan PANIĆ

Faculty of Sciences and Mathematics, University of Pristina in Kosovska Mitrovica, Lole Ribara 29, Kosovska Mitrovica, Serbia, stefan.panic@pr.ac.rs
ORCID: 0000-0002-5868-1764

Biljana PANIĆ*

Faculty of Organizational Sciences, University of Belgrade, Jove Ilića 154, Belgrade, Serbia, biljana.panic@fon.bg.ac.rs
ORCID: 0000-0002-5047-9887

Received: March 2024 / Accepted: November 2024

Abstract: In this paper, we are observing a reparable system comprised of several components. The aim is to determine at which repair rate the system achieves the highest availability. To accomplish that, the procedure for the calculation of the repair rate in function of the desired level of availability for the system with a predetermined repair rate threshold is created. The presented approach is based on the exploration of the probability density function (PDF) of a system's repair time by observing the probability that repair rates of its components surpass a determined threshold. The obtained results can be applied in reliability theory, maintenance planning, and many other fields. To prove the application of the developed model, the results are verified by a numerical example for an unmanned vehicle system.

Keywords: Repair time, maintenance, availability, probability density function, threshold.

MSC: 90B25, 60K10, 62N05.

---

* Corresponding author

1. INTRODUCTION

In reliability theory, systems can be classified into repairable and non-repairable. A non-repairable system is a system that can be removed or just replaced with a completely new one. With repairable systems, we have a completely different situation. After a failure, the system is returned to an operating condition and can be used again after certain maintenance activities. At the same time, it should be taken into account that the occurrence of a failure is not a single process, but rather a recurring event [1]. In such systems, an important performance measure is called the intensity of the failure or failure rate.

If the observed system is renewed through maintenance or repairs, in the case of the repairable systems, the expected time of failure-free operation is known as Mean Time Between Failures (MTBF). In the case of the so-called non-repairable systems, we talk about the mean time to the first failure, or simply the Mean Time To Failure (MTTF). Both functions should always be used when the failure density function is specified because the level of reliability that can be attributed to a certain value of MTBF or MTTF depends on the shape of that function. With repairable systems, we have another variable, which is Mean Time To Repair the system (MTTR) i.e. the time needed to return the system to its operational state. The mean time between two failures is a sum of MTTF and MTTR [2].

The effect of immediate post-failure repair is a key feature for models of repairable systems and it is related to its failure rate. Reviewing the literature, we noticed that a great number of research papers are dealing with repairable systems in which the effect of the repair is minimal. After the repair, such a system returns to the same state it was in before the appearance of failure. As described in the literature [3] [4] [5], this type of repair is denoted as minimal repair.

In addition to minimal repair, the literature often talks about perfect repair. A perfect repair means that after repair, the system is again in an "As Good as New” state. This type of repair can be classified as a restoration process and is discussed in works [6] and [7].

In this paper, we deal only with repairable systems with perfect repair. That means that systems that return to their original state after the fault has been removed. We will model such a system with the alternating renewal process, which in the general case, models a system that alternates between two states over time. For example, we observe a device that is in a functional state until a failure occurs, after which the device is repaired and returned to its original state. The basic assumption is that pairs of random times spent successively in two states form independent, identically distributed sequences. In the beginning, the system is working and remains in that state for some time, then it goes into the failure state, which also has a certain duration.

Let us denote the consecutive times of the system operational states as $X = \{X_1, X2, X_3 \ldots X_N\}$ and the consecutive times of repair rates as $Y = \{Y_1, Y_2, Y_3 \ldots Y_N\}$. In the beginning, the state of the system is operational. The distribution function of X and Y is entirely unidentified. The application of alternated renewal process and its application has been described in papers [8] [9] [10] [11]. Usually, in maintenance theory, authors are emphasizing the importance of failure prediction but the determination of repair time is somehow neglected.

For that reason, in this paper, we are observing a reparable system with an alternating renewal process where MTTF is Rayleigh distributed and we are investigating the repair process in function of availability. Availability is defined as the probability that a system is performing according to instructions in the defined time interval. This research is based on the result of the paper [10] where the authors presented the method for determination of the annual repair rate for system units to attain the requested availability. Considering the system's repair rate PDF calculated as the probability that repair rates of its components surpass the determined threshold, we will predict in which time interval switching, repair, or replacement should be performed. The repair time threshold serves as a decision point for maintenance activities, allowing operators to prioritize repairs based on the impact of repair duration on system availability. The key idea is to prevent prolonged downtimes by ensuring that repairs are completed within a time window that maintains the system's desired availability level.

Also, the following assumption was made:

- The system alternates between operational and failure states;
- MTTF is Rayleigh distributed as in [10];
- After being repaired, the system starts to behave as brand new.

This paper is organized into several sections. Section 2 provides literature review while Section 3 presents the method for maintenance decision making based on the define thresholds. Section 4 presents results and discussion while Section 5 is reserved for conclusion.

2. LITERATURE REVIEW

The optimization of maintenance and system reliability has been extensively researched in both theoretical and practical domains. The primary focus has been on minimizing system downtime, maximizing availability, and reducing maintenance costs. In this section, we will review the latest research on maintenance optimization strategies, specifically focusing on repairable systems and models that consider perfect and imperfect repairs, as well as threshold-based decision-making. Maintenance optimization has been a critical area of research as industries struggle to prolong the operational life of their systems while minimizing repair and failure costs. Availability optimizations has been discussed in several papers [12] [13] [14]. In [15], De Jonge and Scarf provided a review of maintenance optimization methods, emphasizing the importance of balancing preventive maintenance and repair activities to enhance system availability. The concept of imperfect repair is elaborated in [16] where authors analyzed systems with multiple failure modes. Their research introduced a model that accounts for imperfect repairs by considering both preventive and corrective actions, emphasizing the need to optimize maintenance schedules based on failure severity and repair quality. Jain and Singh [17] extended this by focusing on fault-tolerant systems with multi-type failures and imperfect repairs. Their model considered the degradation of components over time and proposed strategies to optimize availability in systems that experience gradual wear and tear. In systems that experience degradation over time, Pedersen et al. [18] introduced a model that combined two-stage degradation processes with hard failures and imperfect repairs. Their research demonstrated the importance of monitoring system degradation and adjusting maintenance strategies accordingly to extend the life of the system. Threshold-based maintenance approaches have been used to optimize maintenance schedules and reduce unnecessary repairs. These methods rely on defining critical thresholds that trigger maintenance actions based on system performance data. Our research expands on this approach by introducing a threshold-based model that calculates the probability of repair rates surpass predetermined thresholds. This ensures that components with the highest repair times are prioritized for maintenance, and therefore the system's overall availability. Additionally, our model incorporates a stochastic repair rate estimation which allows us to account explain the variability and uncertainty in repair times. This makes the model more adaptive to real-world conditions, where the timing of failures and repairs is not always predictable. By focusing on the components with the highest repair times and utilizing probability-based thresholds, our approach offers greater precision in optimizing system availability, reducing both unnecessary maintenance and the risk of unexpected system failures, thus enhancing the overall effectiveness of the maintenance strategy.

3. MAINTAINENCE DECISIONS BASED ON THE THRESHOLD

As mentioned above, one of the most important reparable system parameters is steady availability which can be defined as the probability that in time *t*, the system will be in an operational state. It can be calculated as $\lim_{t\to\infty} A(t)$ but usually is complicated to get explicit results. So, according to the renewal theorem [19] the availability of the system can also be calculated as:

$$A = \frac{E(X)}{E(X)+E(Y)}, \tag{1}$$

where E(*X*) is a mathematical expectation that the system is in an operational state and E(*Y*) denotes the expectation that the system is nonoperational. According to the literature when failure and repair time are exponentially distributed that means that $MTTF = 1/\lambda$ and then, we can express the availability as :

$$A = \frac{MTTF}{MTTF + MTTR} = \frac{\mu}{\mu+\lambda}. \tag{2}$$

The Eq. (2) due to its simplicity is easier to use than Eq. (1).

Since the failure time is exponentially distributed as stated earlier, its square value has Rayleigh distribution:

$$p(t) = \frac{2t}{x}\exp\left(-\frac{t^2}{x}\right), \tag{3}$$

where x is a distribution parameter defined by relation E(t^2)=x. Based on that, MTTF is:

$$MTTF = \int_0^\infty tp(t)dt = \int_0^\infty \frac{2t^2}{x}\exp\left(-\frac{t^2}{x}\right)dt. \tag{4}$$

In the already mentioned paper [10] the authors have calculated the PDF of the repair rate as:

$$p(\mu) = \frac{8A^2}{(1-A^2)\mu^3\pi x_0}\exp\left(-\frac{4A^2}{(1-A^2)\mu^2\pi x_0}\right) \tag{5}$$

where x_0 is the expected value of x for the observed part of the failed system.

The Eq. (5) presents the mathematically characterized random process of repair. The repair process modeled this way provides us with the sample value of the repair process for the corresponding value of MTTF and availability. Based on the above approach, the dynamical characteristics of system performance could be obtained by simulation of the repair rate process by generating its samples.

The above-presented expression delivers the PDF of the repair rate for a system unit. This paper aims to calculate the probability of the system's repair rate when its units surpass the determined threshold, in order to predict at which time interval switching, replacement, or repair should be completed to accomplish the required availability. We will observe a system consisting of two or more units or components. Let $p_1(\mu)$ be the PDF function of the first component given with [20]:

$$p_1(\mu_1) = \frac{8A_1^2}{(1-A_1^2)\mu_1^3 \pi x_{01}} \exp\left(-\frac{4A_1^2}{(1-A_1^2)\mu_1^2 \pi x_{01}}\right), \tag{6}$$

with cumulative distribution function (CDF) of this component expressed as:

$$F_1(\mu_1) = 1 - \exp\left(-\frac{4A_1^2}{(1-A_1^2)\mu_1^2 \pi x_{01}}\right). \tag{7}$$

Similarly, we can express the PDF of the second part $p_2(\mu)$, which belongs to the observed system by:

$$p_2(\mu_2) = \frac{8A_2^2}{(1-A_2^2)\mu_2^3 \pi x_{02}} \exp\left(-\frac{4A_2^2}{(1-A_2^2)\mu_2^2 \pi x_{02}}\right) \tag{8}$$

with CDF of this component expressed as:

$$F_2(\mu_2) = 1 - \exp\left(-\frac{4A_2^2}{(1-A_2^2)\mu_2^2 \pi x_{02}}\right) \tag{9}$$

Based on these expressions authors calculated maximal and minimal repair time [21].

The main contribution of this paper is the definition of a threshold that serves to determine which part of the observed system has the greatest impact on its availability and therefore has priority during the repair.

We will observe two components in this case. We will define μ_T, the repair threshold. If the value of the repair rate is higher than some defined threshold, that part does not affect the availability of the system as much as the time of its repair, i.e. service or replacement takes less time. Such repair times are generally overlooked. In that case, we observed and service the subsystem that has a lower repair rate, which means that the repair time lasts longer and that it has a more significant effect on the duration of the repair, i.e. downtimes and therefore availability. In such case, a PDF of such repair rate could be calculated according to:

$$p(\mu) = \begin{cases} p_1(\mu) & \mu > \mu_T \\ p_1(\mu)\left(1 + F_1(\mu_T)\right) & \mu \le \mu_T \end{cases} \tag{10}$$

while the CDF of such repair rate could be calculated according to:

$$F(\mu) = \begin{cases} F_1(\mu) F_2(\mu_T) & \mu > \mu_T \\ F_1(\mu) - F_2(\mu_T) + F_1(\mu) F_2(\mu_T) & \mu \le \mu_T \end{cases} \tag{11}$$

Based on the obtained CDF we can determine the reliability of the system. After substituting Eqs. (6)-(9) into (9) and (10) we can obtain this system repair rate characteristic based on which we could predict the interval of switching, repairing, or replacement.

4. RESULTS AND DISCUSSION

The proposed repair rate threshold model has several advantages over traditional maintenance strategies. Firstly, it allows for real-time adjustments based on the repair rate of individual components, optimizing maintenance intervals and preventing unnecessary downtime. By incorporating different Mean Time to Failure (MTTF) values for key components, the model tailors maintenance strategies, enhancing overall system availability. It effectively maintains high availability levels by prioritizing repairs based on stochastic behavior, minimizing unexpected failures. By PDFs and CDFs, the model accounts for the random nature of failures, leading to more accurate maintenance predictions.

The model is analyzed using the data from [22]. The focus is on one realization of the unmanned aerial vehicle (UAV). Two critical components, the engine and propeller, have been considered. The insufficient level of reliability of current UAV solutions leads to a high probability of failure occurrence [10]. The following characteristics have been observed: UAV aircraft has 1440 flight hours; MTTF of the engine is 750 flight hours; MTTF of the propeller is 500 flight hours.

Based on the expressions presented in the previous section, a numerical analysis was conducted to calculate the probability that repair rates of its components surpass the determined threshold of an UAV system comprised of three components in order to acquire availability of A=0.75, A=0.95 by emphasizing the stochastic nature of this process.

Figure 1 shows the PDF of the repair rate threshold, representing the likelihood that the repair rate exceeds a specific threshold. The peaks of the curves indicate the most probable thresholds at which maintenance should occur. Higher availability levels (A = 0.95) require more stringent thresholds (lower μ values), indicating more frequent maintenance, whereas lower availability (A = 0.75) allows for more lenient thresholds and less frequent maintenance actions.

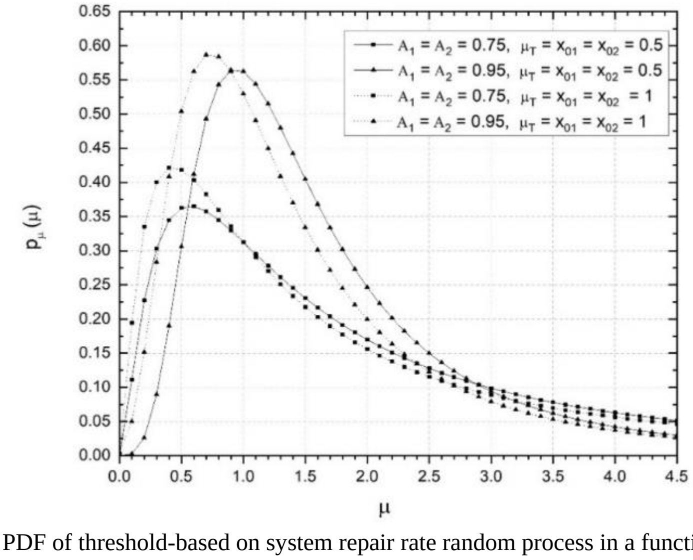

Figure 1: PDF of threshold-based on system repair rate random process in a function of system availability

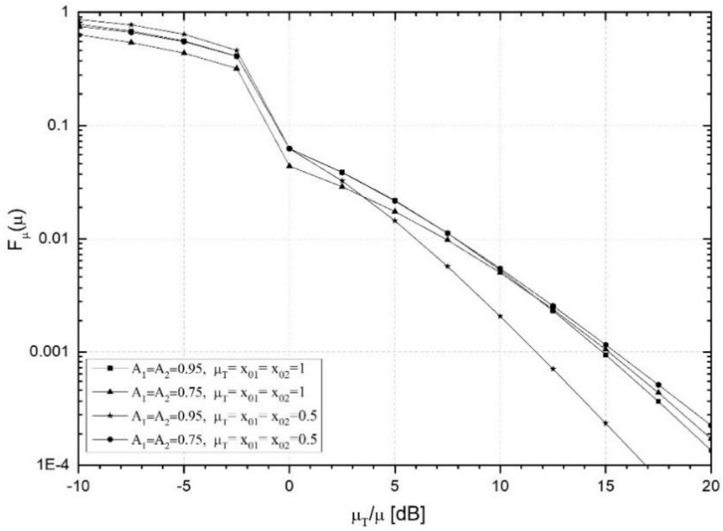

Figure 2: CDF of threshold-based system on repair rate random process in a function of system availability

In Figure 2 CDF values of the threshold based on the system repair rate random process as a function of system availability have been presented.

The presented figures show that repair conducted in a certain time frame will provide the desired level of system availability. By selecting the optimal level of repair rate threshold μ_T, obtained information can be efficiently used for planning of system's maintenance activities.

The repair rate threshold model enables cost-effective, predictive maintenance by ensuring that repairs are performed when needed to maintain the desired level of system availability. This approach optimizes maintenance intervals based on real-time data and stochastic modeling, reducing unnecessary repairs and enhancing system reliability. The model is particularly valuable in complex systems like UAVs, where operational efficiency and high availability are critical.

5. CONCLUSION

In this work capitalizing on the PDF and CDF of repair rate which present the precise mathematical characterization of repair as a stochastic process, we have presented an approach how to determine the probability that repair rates of its components surpass the determined threshold for the system comprised of multiple subsystems or components. Further, we have observed the proposed model by applying it to the UAV system comprised of three major component subsystems and we have graphically presented the PDF and CDF of the threshold-based repair rate system process. Based on this data we can designate and predict the time interval in which maintenance action should be completed in order to reach the requested level of availability. This method can be applied in the same manner to other repairable systems that alternate between operational times and downtimes, and the results can be used in maintenance activities planning, and in the process of dynamical system maintenance.

While the proposed approach offers many advantages, it is not without limitations. First, the model assumes perfect repair after failure, which may not always be realistic in real-world systems where imperfect or partial repairs are more common. Incorporating imperfect repair into the model could provide a more accurate representation of real-world conditions and improve the model's applicability to a broader range of systems.

Despite these limitations, the repair rate threshold model represents a valuable tool for improving the maintenance and reliability of repairable systems, particularly in applications where maximizing system availability is critical.

Funding: This research was funded by the Ministry of Education, Science and Technological Development, Republic of Serbia, grant number 451-03-65/2024-03/ 200123.

REFERENCES

[1] M. P. Almeida, R. S. Paixão, P. L. Ramos, V. Tomazella, F. Louzada and R. S. Ehlers, "Bayesian non-parametric frailty model for dependent competing risks in a repairable systems framework," *Reliability Engineering & System Safety,* vol. 204, pp. 107145, 2020.

[2] A. Fatemeh, Failure Rate Analysis. Failure Analysis and Prevention, InTech, 2017.

[3] M. Brown and F. Prochan, "Imperfect Repair," *Journal of Applied Probability,* vol. 20, no. 4. pp. 851-859, 1983.

[4] L. Knopik and K. Migawa, "Optimal age-replacement policy for non-repairable technical objects with warranty," *Eksploatacja i Niezawodnosc - Maintenance and Reliability,* vol. 19, no. 2. pp. 172-178, 2017.

[5] H. Pham and H. Wang, "Imperfect maintenance," *European Journal of Operational Research,* vol. 94, no. 3. pp. 425-438, 1996.

[6] Q. Yang, N. Zhang and Y. Hong, "Reliability Analysis of Repairable Systems With Dependent Component Failures Under Partially Perfect Repair," *IEEE Transactions on Reliability,* vol. 62, no. 2. pp. 490-498, 2013.

[7] G.-L. Liao, "Production and Maintenance Policies for an EPQ Model With Perfect Repair, Rework, Free-Repair Warranty, and Preventive Maintenance," *IEEE Transactions on Systems, Man, and Cybernetics: Systems,* vol. 46, pp. 1129-1139, 2016.

[8] C.-T. Chen and J. Yuan, "Throughput analysis using alternating renewal process in balanced serial production lines with no interstage buffers," *Journal of the Chinese Institute of Engineers,* vol. 20, no. 5. pp. 522-532, 2003.

[9] A. D. Crescenso, "On random motions with velocities alternating at Erlang-distributed random times," *Advances in Applied Probability,* vol. 33, no. 3. pp. 690-701, 2001.

[10] N. Kontrec, S. Panić, M. Petrović and H. Milošević, "A stochastic model for estimation of repair rate for system operating under performance based logistics," *Eksploatacja i Niezawodnosc – Maintenance and Reliability,* vol. 20, no. 1. pp. 68-72, 2018.

[11] R. E. Mortensen, "Alternating renewal process models for electric power system loads," *IEEE Transactions on Automatic Control,* vol. 35, no. 11. pp. 1245-1249, 1990.

[12] S. Shakuntla, A. Lal, S. Bhatia and J. Singh, "Reliability analysis of polytube industry using supplementary variable technique," *Applied Mathematics and Computation,* vol. 218, no. 8. pp. 3981-3992, 2011.

[13] T. Yang, C. Cui, Y. Shen and Y. Lv, "A novel denitration cost optimization system for power unit boilers," *Applied Thermal Engineering,* vol. 96, pp. 400-410, 2016.

[14] S. Shakuntla, U. M. Modibbo, M. Mijinyawa, S. Malik, S. Verma and P. Khurana, "Mathematical Model for Analysing Availability of Threshing Combine Machine Under Reduced Capacity," *Yugoslav Journal of Operations Research,* vol. 32, no. 4. pp. 425-437, 2022.

[15] B. D. Jonge and P. A. Scarf, "A review on maintenance optimization", *European Journal of Operational Research,* vol. 285, no. 3. pp. 805-824, 2020.

[16] Q. Qiu, B. Liu, C. Lin and J. Wang, "Availability analysis and maintenance optimization for multiple failure mode systems considering imperfect repair," *Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability,* vol. 235, no. 6. pp. 982-997, 2021.

[17] M. Jain and V. P. Singh, "Availability analysis of fault-tolerant degraded machining system with imperfect repair and multi-type failures," *International Journal of Applied and Computational Mathematics,* vol. 9, no. 3. pp. 35, 2023.

[18] T. I. Pedersen, X. Liu and J. Vatn, "Maintenance optimization of a system subject to two-stage degradation, hard failure, and imperfect repair," *Reliability Engineering & System Safety,* vol. 237, pp. 109313, 2023.

[19] S. M. Ross, *Applied probability models with optimization applications*, Reprint edition, Dover Publications, 1992.

[20] N. Kontrec, S. Panić and M. Petrović, "System maintenance decision making based on repair rate statistics," in *Proceedings of XIII Balcan Conference on Operational Research – BALCOR 2018, 25-28. May, Belgrade, Serbia,* 2018.

[21] N. Kontrec, S. Panić, B. Panić, A. Marković and D. Stošović, "Mathematical Approach for System Repair Rate Analysis Used in Maintenance Decision Making," *Axioms,* vol. 10, no. 2. pp. 96, 2021.

[22] K. Kang, K. H. Doerr, M. Boudreau and U. Apte. *A decision support model for valuing proposed improvements in component reliability*. Monterey, California. Naval Postgraduate School, 2005.