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**Research article**

# THE EFFECT OF BOOTSTRAP DEA IN SUSTAINABLE INVESTMENT ANALYSIS USING EXIT TIME

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**Abstract**: The primary objective of this research is to develop more reliable portfolios by accurately calculating risk and return, emphasizing a secure asset weighting strategy. We employ the DEA bootstrap method and the SPP-CVaR (Stop-Profit Point-Conditional Value at Risk) methodology to achieve this objective. Previous scholarly research often lacks a robust statistical foundation for evaluating asset performance, particularly regarding sustainability, as traditional approaches rely on single data samples. Additionally, many studies fail to account for the relevance of risk and return until the investor exits the market. We introduce a new approach focusing on exit time to evaluate sustainable investments to address this gap. We employ data envelopment analysis (DEA) to assess the performance of these assets, comparing results from both the DEA bootstrap method and traditional DEA models. Our DEA models incorporate SPP-CVaR (Conditional Value at Risk) as a measure of risk and mean return as the output variable, both calculated until the investor exits the market. Traditional DEA models have limitations in statistical interpretation, so we enhance our analysis with the DEA bootstrap method. This method involves resampling data to create multiple samples, offering a distribution of performance measures for each asset and providing a more comprehensive understanding of asset performance and uncertainty. By comparing the bootstrap and results of conventional methods, we demonstrate the advantages of using statistical

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techniques to evaluate and compare financial assets. The SPP-CVaR is calculated by deriving and converting the risk-neutral density, simulating price paths, and identifying stop-profit points. We then analyze the exit time and price distributions to compute the SPP-CVaR for each stop-profit point. The value of this study lies in its integration of sustainability analysis with risk measures, helping investors build profitable and ethically aligned portfolios. By providing a detailed assessment of an asset's sustainability profile, our approach assists investors in making informed decisions that align with their financial and ethical goals.

**Keywords**: Performance evaluation, Stop-Profit Point methodology, Data Envelopment Analysis (DEA), Bootstrap DEA, risk measure, efficiency.

**MSC:** 91G10, 91G70, 90C08, 62F40.

## 1. INTRODUCTION

Sustainable investment has become integral to modern financial strategies, driven by the increasing importance of Environmental, Social, and Governance (ESG) criteria. Investors are increasingly focused on balancing profitability with sustainability, making it crucial to measure these investments' efficiency and performance accurately. Investments prioritizing sustainability can address environmental and social challenges by holding financial markets accountable for their impacts [1]. However, sustainable investment in the stock market extends beyond merely considering ESG criteria; it requires sophisticated tools and methods to assess risk and return effectively.

Data Envelopment Analysis (DEA), first introduced in the literature in 1978 [2], is a widely used non-parametric method for measuring efficiency in various contexts, such as real-property maintenance activities [3], the efficiency of Army recruiting districts [4], and nursing home care in the Netherlands [5]. It is particularly effective in comparing the relative efficiencies of decision-making units (DMUs) that perform similar functions by transforming multiple inputs into outputs. This method allows for a detailed and quantitative analysis of how efficiently resources are utilized across these comparable entities.

DEA has become vital in portfolio and asset pricing research, which helps assess relative efficiencies and enhance portfolio performance [6]. Classic DEA models provide an effective and practical approach to approximate portfolio efficiency by sampling portfolios and incorporating market frictions, demonstrating convergence to the efficient portfolio frontier with increasing sample size [7]. By redefining the financial production process and treating risk as an input, DEA's ability to rank portfolios is significantly improved, aligning efficiency measures with risk-return theory and offering practical advantages in financial analysis [8].

Furthermore, integrating DEA with multi-source data and machine learning techniques has proven to optimize stock selection schemes, substantially improving the out-of-sample performance of portfolio strategies and outperforming traditional diversification methods [9]. Beyond financial applications, DEA is also valuable in evaluating sustainability performance, providing insights into eco-efficiency, and setting benchmarks to mitigate environmental degradation. However, challenges remain in integrating social and institutional dimensions [10].

In the energy sector, DEA has effectively evaluated dynamic investment performance. Using window analysis and directional distance functions, energy portfolios are found to achieve higher efficiency than single-energy investments, particularly when employing risk-sensitive portfolio methods [11]. The growing use of DEA in sustainability research further underscores the need for unified definitions and methodologies to capture the multi-dimensional nature of sustainability, addressing gaps in social dimensions that remain underrepresented in current practices [12]. While widely used, traditional DEA models have drawbacks, especially regarding sensitivity to data variability. Bootstrap techniques have been integrated into DEA to overcome these limitations, resulting in what we call Bootstrap DEA. This approach enhances the robustness and reliability of efficiency scores by providing confidence intervals and correcting biases. It allows for a more refined analysis, crucial for making well-informed investment decisions.

Another crucial element in investment analysis is exit time—the period an investment stays in a portfolio before being sold off. Exit time offers valuable insights into investments' sustainability and long-term performance, making it an essential factor in assessing sustainable investments. Traditional approaches often overlook the timing of an investor's exit, which is a key factor that can significantly impact an asset's perceived risk and return.

Existing literature has extensively explored sustainability indices' risks, returns, and overall market impact. For instance, studies such as those by Tularam et al. [13], De Souza Cunha & Samanez [14], and López et al. [15] have conducted comprehensive analyses on sustainability, highlighting its potential for global diversification gains in conventional stock portfolios [16]. Integrating ESG factors is the most widely embraced and rapidly expanding strategy in sustainable investment [17]. Investing in sustainable assets can enhance the diversification of conventional stock portfolios worldwide [18]. According to Pástor et al. [19], sustainable investing generates a positive social impact by encouraging firms to adopt greener practices and shifting more real investment toward environmentally friendly firms while reducing investment in less sustainable firms. Sustainable investing is now regarded as a mainstream strategy, as the integration of ESG factors into investment processes has been proven to create value, enhance risk management, and align with stakeholder priorities [20]. Furthermore, pension fund participants demonstrate strong support for increased sustainability efforts, even at the expense of financial returns, highlighting the significant influence of social preferences in shaping investment policies [21].

Additionally, sustainable investing influences societal goals through shareholder engagement, capital allocation, and indirect impacts, underscoring the importance of policy measures to drive transformative changes beyond promoting good business practices [22]. Empirical studies reveal that ESG indices perform comparably to conventional indices, offering investors viable substitutes for portfolio diversification and risk hedging while providing additional ESG benefits [23]. These findings collectively reinforce the dual value of sustainable investing, presenting it as a strategy that benefits both companies and investors by establishing long-term value and aligning performance with broader societal objectives [24]. However, the relationship between ESG ratings and stock returns remains ambiguous, as evidenced by findings in specific markets, such as the Norwegian stock market [25]. Sustainable investment has taken on a broader significance when selecting assets for investment or building a portfolio.

To address the evolving nature of sustainable investment, which increasingly emphasizes the balance of risk and return over other factors, our research explores the multifaceted nature of sustainability in financial markets. Many risk-measuring approaches are used in the field of financial risk management. These complex statistical methods are essential for assessing the possible level of financial risk present in a company or investment portfolio over a given time horizon. In the mid-1990s, Konno & Shirakawa [26] introduced an optimization approach for constructing an optimal stock portfolio based on minimizing semi-variance. Concurrently, the concept of "value at risk" (VaR) emerged as a risk measure, initially proposed by Baumol in 1963 but gaining prominence later. Artzner et al. [27] criticized the VaR for lacking subadditivity and convexity. Recognizing the limitations of the VaR, Rockafellar & Uryasev [28] proposed the conditional value at risk (CVaR) as a more robust risk measure, addressing suboptimality issues associated with local minima in the optimization process. CVaR considers the magnitude of losses beyond a certain threshold and tail risk, enhancing risk management in securities markets. Subsequent contributions by Pflug & Swietanowski [29], Ogryczak & Ruszczyn´ski [30], and others further developed the CVaR method. Chekhlov, Uryasev, & Zabarankin [31] applied CVaR minimization to portfolio optimization problems.

Our study builds on these advancements by introducing the Stop-Profit Point (SPP-CVaR) measure, which incorporates exit time as a critical factor in portfolio optimization. This measure, developed by Bin [32], provides a more comprehensive risk assessment by capturing the risks associated with stop strategies, such as price volatility and exit time uncertainty. Despite the progress in risk measurement, previous studies have focused mainly on the entire investment period, neglecting the importance of exit timing. Moreover, past research often utilized DEA to evaluate asset performance based on a single sample, limiting the results' reliability.

To overcome these limitations, we propose a novel approach that considers the timing of an investor's exit from the market, using the DEA bootstrap method to generate 2000 samples for a more robust and statistically significant analysis. Our methodology integrates the SPP-CVaR risk measure, where risk is calculated based on the exit time determined by an investor's stop-profit point. Our study aimed to establish a sustainable ecosystem within the financial market and compare it with an unsustainable environment. To estimate the parameters $\mu$ and $\sigma$ of the stochastic differential equation (SDE), we use Maximum Likelihood Estimation (MLE), which involves maximizing the likelihood function based on the observed log returns. To calculate the SPP-CVaR, we estimate the risk-neutral density using kernel density estimation. We then transform this density into the real-world density using a beta distribution and calibrate the parameters accordingly.

To simulate price paths, we consider actual market probabilities and risks. From these simulations, we derive the real and SPP densities for both prices and exit times, incorporating predefined stop-profit points. We simulate random entry times to improve the exit time distribution accuracy. Finally, by applying the density transfer function, we obtain the SPP of the density of exit time. For high precision, we simulate 100,000 or more price paths. These metrics are applied in a two-step sustainable investment approach. The initial step focuses on calculating efficiency scores using a DEA model, based on single risk and return measures. In the second step, multiple samples of efficiency scores are generated from the initial step, employing Bootstrap DEA to produce more reliable and accurate efficiency scores. This approach aims to support investors in selecting the most suitable assets for their portfolios

We also consider the risk and mean return for the entire time horizon, allowing us to present the unsustainable investment case where the efficiency of each asset is calculated for the entire period. We compare both cases regarding risk, mean return, and efficiency scores. We used graphs to illustrate both sustainable and unsustainable investments, demonstrating how each asset can be chosen strategically to create a profitable portfolio that aims to satisfy investors. This comprehensive approach allows us to offer valuable insights into sustainable financial market ecosystems, risk management, and asset optimization. The rest of this paper is structured as follows. Section 2 presents the mathematical definition and formulation. The methodology is explained in section 3. Section 4 includes experimental testing of the methodology.

## 2. PRELIMINARY

This section concentrates on the risk evaluation tools employed in our investigation. Beyond traditional measures such as the VaR and CVaR, which assess risk for the entire time horizon, we have introduced the SPP-CVaR into our analysis. SPP-CVaR uniquely accounts for the investor's exit time, providing a risk assessment until the investor exits the market. This approach enables us to amplify the degree of sustainable investment.

**Definition 1.** *For a specified portfolio comprised of N assets, wherein a position vector characterizes each asset,* $\Lambda = \{\lambda_1 \lambda_{,2}, \lambda_3, \ldots . \lambda_n\}^T$ *and respective returns denoted as* $Y = \{y_1, y_2, y_{3,} \ldots ., y_n\}$*, the risk measure is defined as follows:*

*For any* $\boldsymbol{\lambda} \in \Lambda$ *, considering a probability distribution linked with y, the probability that the loss is less than or equal to a threshold* $\alpha$ *is defined as:*

$$\boldsymbol{\psi}(\lambda, \alpha) = p(f(\Lambda, Y) \leq \alpha) = \int_{f(\Lambda,Y) \leq \alpha} \pi(\lambda, \alpha) dy \tag{1}$$

*VaR associated with the portfolio can be defined as:*

$$VaR_\beta(\lambda, \pi) = min\{\alpha \in R: \boldsymbol{\psi}(\lambda, \alpha) \geq \boldsymbol{\beta}\} \tag{2}$$

*Where* $\boldsymbol{\beta}$ *is the confidence level and* $f(\Lambda, Y) = -\Lambda^T Y$ *is a loss function.*

*CVaR, which was proposed by Rockafellar & Uryasev [28], is defined as follows:*

$$CVaR_\beta(\lambda) = min\, \alpha + \frac{1}{1-\beta} \int_{y \in R^N} [f(\Lambda, Y) - \alpha]^+ \pi(y) dy \tag{3}$$

*Where* $\pi(y)$ *is the notation for the probability distribution associated with Y.*

**Definition 2.** *One of the key variables in conducting research is the stop-profit point (*$m$*).* $m$ *is the price at which an investor decides to exit the market. In practical terms, the stop-profit point can be defined as the difference between the buying and selling prices. However, it is important to note that this point is influenced by various factors, such as transaction costs and the trader's risk tolerance.*

**Definition 3.** *The stochastic differential equation (SDE) describes the price process is based on geometric Brownian motion and is given by:*

$$dS(t) = S(t)(\mu dt + \sigma dW(t)) \tag{4}$$

*W(t) is the Winner process or Brownian motion, where* $\mu$ *and* $\sigma$ *are constants.*

*The following formula gives the exit time for the stop-profit point:*

$$\gamma_m = min\{t \geq 0 \, ; W(t) = m\} \tag{5}$$

$\gamma_m$ *is the first time that the price reaches the stop-profit point.*

*The density function of the time at which the price first reaches the stop-profit point was given by [32]:*

$$f_{\gamma_m}(t) = \frac{|m|}{t\sqrt{2\pi t}} e^{-\frac{m^2}{2t}} \tag{6}$$

*The density function of the price process with a stop-profit point can also be obtained as follows [32]:*

$$\pi(y) = \sum_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2(t-s)}} e^{-\frac{y_i^2}{2\sigma^2(t-s)}} \tag{7}$$

*where* $0 \le s \le t$ .

**Definition 4.** *The risk measure known as SPP-CVaR considers the risk up to the point when an investor exits the market, which contrasts with other risk measures that assess risk over time. We can solve the portfolio optimization problem by minimizing SPP-CvaR defined by Bin [32] as follows:*

$$min\ SPP - CVaR_\beta(\lambda) = \min_{(x,\alpha)\in\chi * R} \alpha + \frac{1}{1-\beta}\int_{y\in R^N} [f(\lambda, y) - \alpha]^+ \ \nu(.)dy \tag{8}$$

*Where*

$$\nu(.) = \int_0^T \pi(.) f_{\gamma_m}(.)dt = \sum_{i=1}^{k} \frac{1}{\sqrt{2\pi\sigma^2(t-s)}} e^{-\frac{y^2}{2\sigma^2(t-s)}} \ \frac{|m|}{t\sqrt{2\pi t}} \ e^{-\frac{m^2}{2t}} \tag{9}$$

$\pi(.)$ *is the density function of the price process with a stop-profit point and* $f_{\gamma_m}(.)$ *is the cumulative distribution function of the time the price first reaches the stop-profit point.*

*The corresponding unconditional density function* $\nu(.)$ *can be obtained from the conditional density function of the price process* $\pi(.)$ *and the density function of the time that first crosses the stop-profit point.* $f_{\gamma_m}(.)$.

**Definition 5.** *The RDM model was proposed by Portela et al. [33] and inspired by the Directional Distance Function model by Chambers et al. [34], which can be applied for computing efficiency in the presence of negative data. The present paper uses the RDM model since some mean returns are negative.*

*For DMU* $_j$ , $j = 1,2,\dots,n$ *with inputs* $x_{ij}$ , $i = 1,2,\dots,m$ *and outputs* $y_{rj}$ , $r = 1,2,\dots,s$ *in* $R^{m+s}$ *and the unit* $o \in \{1,2,\dots,n\}$ *which is under assessment. The generic directional distance model is represented as:*

$$max \begin{Bmatrix} \theta \mid \sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{ro} + \theta R_{ro} , r = 1,2,\dots\dots,s \\ \sum_{j=1}^{n} \lambda_j x_{ij} \le x_{io} - \theta R_{io} , i = 1,2,\dots\dots, m \\ \sum_{j=1}^{n} \lambda_j = 1 , \theta, \lambda_j, R_{ro}, R_{io} \ge 0 \end{Bmatrix} \tag{10}$$

*The above model is a non-oriented case, where the input contraction and output expansion improve simultaneously. For a given data set, when some of them are negative, an ideal point is defined as* $I = (\max_j y_j, r = 1,2,\dots\dots,s\ ,\ \min_j x_j\ , i = 1,2,\dots,m)$ *.The vectors* $R_{ro}$ *and* $R_{io}$*which refers to the range of possible improvement of DMU* $_o$ *are:*

$$R_{io} = x_{io} - \min_j\{x_{ij}\} \ \ , \ i = 1,2,\dots,m \tag{11}$$

$$R_{ro} = \max_{j}\{y_{rj}\} - y_{ro} \quad, r = 1,2, \dots, s \tag{12}$$

*At the ideal point, I the range of possible improvement can be seen as a surrogate for the maximum improvement that $DMU_o$ could achieve on each input and output. Such an improvement can never be negative* [33].

## 3. SUSTAINABLE EFFICIENCY SCORE

In this section, our primary focus is on sustainable investment. To carry out our analysis, we employ the Data Envelopment Analysis (DEA) model to evaluate asset efficiency, considering the exit time based on negative data. Subsequently, we utilize the bootstrap DEA method to compute the bootstrap score, which serves as a component of sustainable investment. To create more sustainable investments, we calculate risk and return until the investment exits the market, as this approach provides more accurate results for investment. Our risk and return calculations are based on the exit time, where the price reaches the stop-profit point.

### 3.1. Initial step in sustainable investing

In the first step, we calculate the exit time based on the stop-profit point. Subsequently, corresponding to the exit time, we calculate the risk and return until the exit time. Following the aims and financial goals, investors select the optimal exit time, intending to create a portfolio. With risk and mean return until exit, we used the following model to calculate the efficiency score when our return was positive.

$$\begin{aligned}
&\min \quad \theta \\
&S.t. \;\; E(Y(\lambda)) \geq E(Y_o) \\
&SPP - CVaR\big(Y(\lambda)\big) \leq \; \theta \,(SPP - CVaR_\beta^o) \\
&e^T\lambda = 1 \\
&\text{where } \lambda \geq 0, \theta \geq 0.
\end{aligned} \tag{13}$$

The optimal solution θ indicates the efficiency score of the asset under evaluation. $SPP - CVaR_\beta^o$ is the value of the risk, and $E(Y_o)$ is the mean return of the asset under assessing and $o \in \{1,2, \dots, n\}$. The vector $\boldsymbol{\Lambda} = \{\boldsymbol{\lambda_1}\boldsymbol{\lambda_{,2}}, \boldsymbol{\lambda_3}, \dots . \boldsymbol{\lambda_n}\}^{\boldsymbol{T}}$ represents the policy of investing in different proportions of assets in a portfolio and $e$ is a vector where all the elements are one. To illustrate the first constraint, the return of a portfolio is defined by $Y(\lambda) = \sum_{j=1}^{n} \lambda_j Y^j$, and the portfolio's mean return is computed as $\big(Y(\lambda)\big) = \sum_{j=1}^{n} \lambda_j E(Y^j)$.

As introduced by Ghasemi Doudkanlou et al. [35], the model is utilized to compute the efficiency score when the return can take negative value. It yields precise results, owing to its consideration of both risk and return factors until the time of investment exit from the market.

Assuming that $Y_{day1}, Y_{day2}, Y_{day3}, \dots\dots, Y_{exit\ time}$ are the log of the returns of a specific asset up to the exit time, and regarding the negative return, we defined the vector $g^T$ such that

$$g^T = \Big(R_{SPP-CVaR_\beta^o} \,, R_{E(Y_o)}\Big) \tag{14}$$

Where

$$\begin{pmatrix} R_{SPP-CVaR_\beta^o} & = \left[SPP - CVaR_\beta^o - \min\left(SPP - CVaR_\beta^j : j = 1,2,\dots,n\right)\right] \\ R_{E(Y_o)} & = \left[\max\left(E\left(Y_j\right) : j = 1,2,3,\dots,n\right) - E(Y_o)\right] \end{pmatrix}$$

$g^T$ is a vector that shows the direction in which $\theta$ is to be maximaized. This vector is a range of possible improvements in the input and output and β is the confidence level. $SPP - CVaR_\beta^o$ is the value of the risk, and $E(Y_o)$ is the mean return of the asset. Then, we solve the following nonlinear model:

$$\begin{aligned} &\max \quad \theta \\ &S.t. \;\; E(Y(\lambda)) \geq E(Y_o) + \theta R_{E(Y_o)} \\ &SPP - CVaR\left(Y(\lambda)\right) \leq \; SPP - CVaR_\beta^o - \; \theta R_{SPP-CVaR_\beta^o} \\ &e^T\lambda = 1 \\ &\text{where } \lambda \geq 0, \theta \geq 0. \end{aligned} \tag{15}$$

This model is based on the RDM model in DEA with negative data. It should be noted that the optimal value of the model reflects the inefficiency score of each asset, and it measures the distance between the asset under evaluation and the efficient frontier. The optimal solution $\theta$ indicates the inefficiency score of the asset under evaluation, and the asset is efficient when the inefficiency score is zero. In other words, $1 - \theta = \hat{\theta}$ shows the efficiency score of the asset under evaluation.

Using the efficiency scores derived from the first step in sustainable investing, which are more reliable than those generated by models that calculate risk and return over the entire time horizon, we incorporate these scores into the second step for several reasons. First, to address the inherent unpredictability of financial markets, and second, to mitigate uncertainty and reduce risk levels. Consequently, we aim to utilize Bootstrap DEA, which relies on a multitude of resampled efficiency scores rather than a single set, ensuring greater robustness in our analysis.

### 3.2. Second step in sustainable investing

We used the bootstrap DEA method, which enhances the reliability of the efficiency score, fostering a more robust investment analysis. Many researchers, such as Alonso et al. [36], George Assaf et al. [37], Halkos & Tzeremes [38], Tu & Zhang [39], have emphasized the importance of bootstrap methods as an alternative way to perform statistical inference, especially when the sample size is small or when the sampling distributions are difficult to calculate analytically. That is often the case when dealing with nonlinear models or when pretesting is involved.

The following eight steps must be undertaken to collect bootstrap estimates to execute the homogeneous bootstrap algorithm. $\left\{\hat{\theta}^*(x,y) | b = 1,\dots,B\right\}$ at a predetermined point $(x,y)$. Our pseudo data consist of pairs $(x_i, y_i)$, where $x_i$ represents positive risk and $y_i$ represents return, which can be both positive and negative.

(1) From the original dataset, we compute $\hat{\theta}$ from model (15) and (13).

(2) We apply the rule of thumb [40] to obtain the bandwidth parameter $h$

$$h = \left(\frac{4\hat{\sigma}^5}{3n}\right)^{\frac{1}{5}} \approx 1.06\hat{\sigma}n^{-\frac{1}{5}} \tag{16}$$

Where $n$ is the number of assets and $\hat{\sigma}$ is obtained from the original efficiency scores. The bandwidth parameter $h$ controls the smoothness of the kernel density estimate. It is critical to balance bias and variance in the estimation.

(3) We generate $\beta_1^*, \ldots, \beta_n^*$ by drawing on the replacement from the set $\{\hat{\theta}_1, \ldots, \hat{\theta}_n, \ldots, 2-\hat{\theta}_1, \ldots, 2-\hat{\theta}_n\}$.

(4) Then, we draw $\varepsilon_i{}^*, i = 1, \ldots, n$ independently from the kernel function $k(.)$ and compute $\beta_i{}^{**} = \beta_i^* + h\varepsilon_i{}^*$ for each $i = 1, \ldots, n$.

Kernel Density Estimation (KDE) relies on several key assumptions: the underlying data must be independently and identically distributed (i.i.d), and the probability density function (pdf) being estimated should be continuous. The kernel function $k\,(\cdot)$ used in KDE must be a symmetric, non-negative function that integrates with one.

(5) for each $i = 1, \ldots, n$ we compute $\beta_i{}^{***}$ as follows:

$$\beta_i{}^{***} = \bar{\beta}^* + \frac{\beta_i{}^{**} - \bar{\beta}^*}{(1+h^2\sigma_k^2\sigma_\beta^2)^{1/2}} \tag{17}$$

where $\bar{\beta}^* = \frac{\sum_{i=1}^n \beta_i^*}{n}$, $\sigma_\beta^2 = \frac{\sum_{i=1}^n (\beta_i^* - \bar{\beta}^*)}{n}$ and $\sigma_k^2$ is the variance of the probability density function used for the kernel function. In addition, $\theta_i^*$, can be computed as

$$\theta_i^* = \begin{cases} 2-\beta_i{}^{***}, & \forall\, \beta_i{}^{***} < 1 \\ \beta_i{}^{***} & otherwise \end{cases} \tag{18}$$

(6) The bootstrap sample is created as $X_n^* = \{(x_i^*, y_i) \mid i = 1, \ldots n\}$ where $x_i^* = \theta_i^* \hat{x}^{\partial}$ , and $(y_i) = \theta_i^* \hat{\theta}_i{}^{-1} x_i$

(7) We computed the DEA efficiency estimates $\hat{\theta}_i{}^*(x_i, y_i)$ for each of the original sample observations using the reference set $X_n^*$ to obtain a set of bootstrap estimates.

(8) Finally, we repeat steps 3-7 B times (at least 2000 times) to obtain a set of bootstrap estimates $\{\hat{\theta}_b{}^*(x, y) \mid b = 1, \ldots B\}$

The bootstrap bias estimate for the original DEA estimator $\hat{\theta}_{DEA}(x, y)$ can be calculated as follows:

$$\widehat{BIAS}(\hat{\theta}_{DEA}(x, y)) = \mathrm{B}^{-1} \sum_{b=1}^{B} \hat{\theta}_{DEA,b}{}^*(x, y) - \hat{\theta}_{DEA}(x, y)) \tag{19}$$

Furthermore, $\hat{\theta}_{DEA}{}^*(x, y)$ are the bootstrap values, and B is the number of bootstrap replications (2000 replications in our case). Then, a biased corrected estimator of $\theta$(x, y) can be calculated as follows:

$$\hat{\hat{\theta}}_{DEA}(\mathrm{x}, \mathrm{y}) = 2\hat{\theta}_{DEA}(x, y) - \mathrm{B}^{-1} \sum_{b=1}^{B} \hat{\theta}_{DEA,b}{}^*(x, y) \tag{20}$$

However, according to Simar & Wilson [41], this bias correction can create additional noise, and the sample variance of the bootstrap $\hat{\theta}_{DEA}{}^*\,(x, y)$ values need to be calculated. The calculation of the variance of the bootstrap values is illustrated below:

$$\hat{\sigma}^2 = \mathrm{B}^{-1} \sum_{\mathrm{b}=1}^{\mathrm{B}} \left[\hat{\theta}_{DEA,b}{}^*(x, y) - \mathrm{B}^{-1} \sum_{\mathrm{b}=1}^{\mathrm{B}} \hat{\theta}_{DEA,b}{}^*(x, y)\right]^2 \tag{21}$$

In addition, bias correction should be avoided unless

$$\frac{\left|\widehat{BIAS}(\hat{\theta}_{DEA}(x, y))\right|}{\hat{\sigma}} > \frac{1}{\sqrt{3}} \tag{22}$$

Finally, according to Daraio & Simar [42], when the bias is larger than the standard deviation ($\sigma$), bias-corrected estimates must be preferred to the original values.

# 4. EMPIRICAL APPLICATION

This section encompasses the presentation of the data, the execution of empirical analysis, and the subsequent interpretation of the results.

## 4.1. Data

In the course of our research, we employed a varied selection of assets encompassing stocks, cryptocurrencies, metals, and commodities. This approach was adopted to elucidate the nuances of sustainable investment across diverse asset classes. Furthermore, we aimed to analyze the performance of these companies at various stop-profit points. We selected Gold due to its pivotal role in the global economy and Bitcoin's burgeoning significance in the prospective economic landscape. Additionally, we chose Oil since it is a fundamental commodity that significantly influences various sectors, showcasing dynamic interplay with broader economic trends and developments.

## 4.2. Results

This section analyzes and calculates the bootstrap DEA score while evaluating risk and return metrics. To support this analysis, we took two steps. First, we establish stop-profit points at the 2%, 4%, 6%, 8%, 10%, and 12% levels. Based on the exit time, we calculate the risk and return. Second, we calculate the efficiency score for each asset based on risk and return, estimated until the exit time. However, we also calculate the risk (CVaR) and return for the entire time horizon, and this approach is referred to as the unsustainable investment method.' Finally, we compare these results with those of sustainable investments. In the case of unsustainable investments, we utilized only the original efficiency score. However, in sustainable investments, we applied resampling techniques to assess the stability and reliability of the DEA efficiency scores. Bootstrap DEA involves generating multiple samples (bootstrap samples) from the original dataset by randomly drawing observations with replacement. Each bootstrap sample is then used to re-estimate the DEA efficiency scores. Our sustainable investment analysis is illustrated in Figures 1 to 7, which explore how each asset is defined in terms of sustainable investment at these specific stop-profit points. The efficiency scores are also presented in Tables 1 to 7 for a comprehensive understanding of asset performance. $\hat{\theta}$ is the efficiency score, and $\hat{\hat{\theta}}$ is the biased corrected efficiency score.

**Table 1:** Key metrics for unsustainable investment (risk and return are calculated for the entire time horizon)

| Asset Name | CVaR | Mean return | $\hat{\theta}^*$ |
|---|---|---|---|
| Coca-Cola | 0.03745 | 0.00018 | 0.79 |
| Amazon | 0.05365 | 0.00018 | 0.68 |
| Pfizer | 0.03969 | 0.0000865 | 0.75 |
| Oil | 0.11004 | 0.00038 | 1 |
| Meta | 0.000888 | 0.00007469 | 0.47 |
| Tesla | 0.09969 | 0.00229 | 1 |
| Gold | 0.02453 | 0.000555 | 0.58 |
| Bitcoin | 0.11175 | 0.00178 | 0.7 |

**Table 2:** Key metrics at the 2% stop-profit point

| Asset Name | SPP-CVaR | Mean return | Exit time | $\hat{\theta}^*$ | $\widehat{BIAS}$ | $\hat{\hat{\theta}}$ |
|---|---|---|---|---|---|---|
| Coca-Cola | 0.00023 | 0.004944 | 2 | 1 | 0 | 1 |
| Amazon | 0.000916 | -0.0001808 | 8 | 0.13 | -0.13 | 0.19 |
| Pfizer | 0.00016 | 0.004614 | 1 | 1 | -0.669 | 1 |
| Oil | 0.0006405 | -0.022883 | 2 | 0.09 | -0.09 | 0.11 |
| Meta | 0.000888 | -0.00033 | 6 | 0.13 | -0.13 | 0.18 |
| Tesla | 0.001274 | -0.002181 | 26 | 0.09 | -0.09 | 0.11 |
| Gold | 0.000849 | 0.000555 | 28 | 0.14 | -0.14 | 0.21 |
| Bitcoin | 0.0000577 | 0.000683005 | 17 | 1 | 0 | 1 |

**Table 3:** Key metrics at the 4% stop-profit point

| Asset Name | SPP-CVaR | Mean return | Exit time | $\hat{\theta}^*$ | $\widehat{BIAS}$ | $\hat{\hat{\theta}}$ |
|---|---|---|---|---|---|---|
| Coca-Cola | 0.0007 | 0.003102 | 8 | 0.21 | -0.21 | 0.26 |
| Amazon | 0.001032939 | -0.000180802 | 8 | 0.16 | -0.16 | 0.21 |
| Pfizer | 0.0001615 | 0.0189 | 2 | 1 | 0 | 1 |
| Oil | 0.0012 | 0.01119 | 3 | 0.37 | -0.37 | 0.54 |
| Meta | 0.000297 | 0.0045272 | 7 | 0.6 | -0.6 | 0.71 |
| Tesla | 0.00147078 | 0.000823773 | 27 | 0.16 | -0.16 | 0.19 |
| Gold | 0.00088579 | 0.001130188 | 29 | 0.18 | -0.18 | 0.21 |
| Bitcoin | 0.000807539 | 0.000683005 | 17 | 0.18 | -0.18 | 0.21 |

**Table 4:** Key metrics at the 6% stop-profit point

| Asset Name | SPP-CVaR | Mean return | Exit time | $\hat{\theta}^*$ | $\widehat{BIAS}$ | $\hat{\hat{\theta}}$ |
|---|---|---|---|---|---|---|
| Coca-Cola | 0.001613 | 0.002759 | 16 | 0.05 | -0.048 | 0.098 |
| Amazon | 0.000966 | 0.005295 | 9 | 0.44 | -0.2939 | 0.73 |
| Pfizer | 0.01018 | 0.0007945 | 74 | 0.01 | -0.00996 | 0.019 |
| Oil | 0.0014067 | 0.011193274 | 3 | 1 | 0 | 1 |
| Meta | 0.0002974 | 0.004527294 | 7 | 1 | 0 | 1 |
| Tesla | 0.0011657 | 0.000823773 | 27 | 0.05 | -0.048 | 0.098 |
| Gold | 0.0009282 | 0.0011301 | 29 | 0.07 | -0.067 | 0.1371 |
| Bitcoin | 0.0000622 | 0.00267 | 21 | 1 | 0 | 1 |

**Table 5:** Key metrics at the 8% stop-profit point

| Asset Name | SPP-CVaR | Mean return | Exit time | $\hat{\theta}^*$ | $\widehat{BIAS}$ | $\hat{\hat{\theta}}$ |
|---|---|---|---|---|---|---|
| Coca-Cola | 0.003711 | 0.002831 | 22 | 0.22 | -0.1877 | 0.4077 |
| Amazon | 0.001048 | 0.00777 | 10 | 1 | 0 | 1 |
| Pfizer | 0.01262 | 0.0007677 | 79 | 0.06 | -0.05766 | 0.117658 |
| Oil | 0.024909 | 0.0001518 | 218 | 0.03 | -0.02942 | 0.059417 |
| Meta | 0.001178 | 0.003991 | 14 | 0.74 | -0.2271 | 0.9671 |
| Tesla | 0.001165 | 0.000823 | 27 | 0.7 | -0.2538 | 0.9538 |
| Gold | 0.001068 | 0.001377 | 43 | 0.76 | -0.2111 | 0.9711 |
| Bitcoin | 0.000813 | 0.0002675 | 21 | 1 | 0 | 1 |

**Table 6:** Key metrics at the 10% stop-profit point

| Asset Name | SPP-CVaR | Mean return | Exit time | $\hat{\theta}^*$ | $\widehat{BIAS}$ | $\hat{\hat{\theta}}$ |
|---|---|---|---|---|---|---|
| Coca-Cola | 0.003532 | 0.003301 | 24 | 0.53 | -0.53 | 1 |
| Amazon | 0.002868 | 0.00758 | 12 | 1 | 0 | 1 |
| Pfizer | 0.021695 | 0.0007059 | 140 | 0.18 | -0.18 | 0.21 |
| Oil | 0.01765 | 0.00042815 | 219 | 0.17 | -0.17 | 0.21 |
| Meta | 0.00351 | 0.002751276 | 36 | 0.51 | -0.51 | 1 |
| Tesla | 0.003332 | -0.000459 | 74 | 0.43 | -0.43 | 0.56 |
| Gold | 0.00106 | 0.0009165 | 98 | 1 | -0.138041 | 1 |
| Bitcoin | 0.000845 | 0.000205 | 177 | 1 | -0.050333 | 1 |

**Table 7:** Key metrics at the 12% stop-profit point

| Asset Name | SPP-CVaR | Mean return | Exit time | $\hat{\theta}^*$ | $\widehat{BIAS}$ | $\hat{\hat{\theta}}$ |
|---|---|---|---|---|---|---|
| Coca-Cola | 0.004888 | 0.003093076 | 34 | 1 | 0 | 1 |
| Amazon | 0.013947 | 0.00114 | 96 | 0.13 | -0.1192 | 0.2492 |
| Pfizer | 0.02233 | 0.0006801 | 142 | 0.05 | -0.048 | 0.098 |
| Oil | 0.013085 | 0.0003295 | 222 | 0.07 | -0.067 | 0.1372 |
| Meta | 0.004394 | 0.002012282 | 43 | 0.73 | -0.23 | 0.967 |
| Tesla | 0.006018 | 0.0012 | 76 | 0.32 | -0.24 | 0.5677 |
| Gold | 0.000915 | 0.0003239 | 363 | 0.97 | -0.059 | 0.97 |
| Bitcoin | 0.000886 | 0.000542 | 179 | 1 | 0 | 1 |

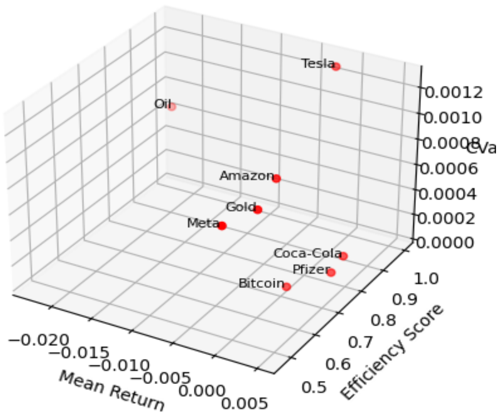


**Figure 1:** Unsustainable investment for all assets (risk and return are calculated for the entire time horizon)

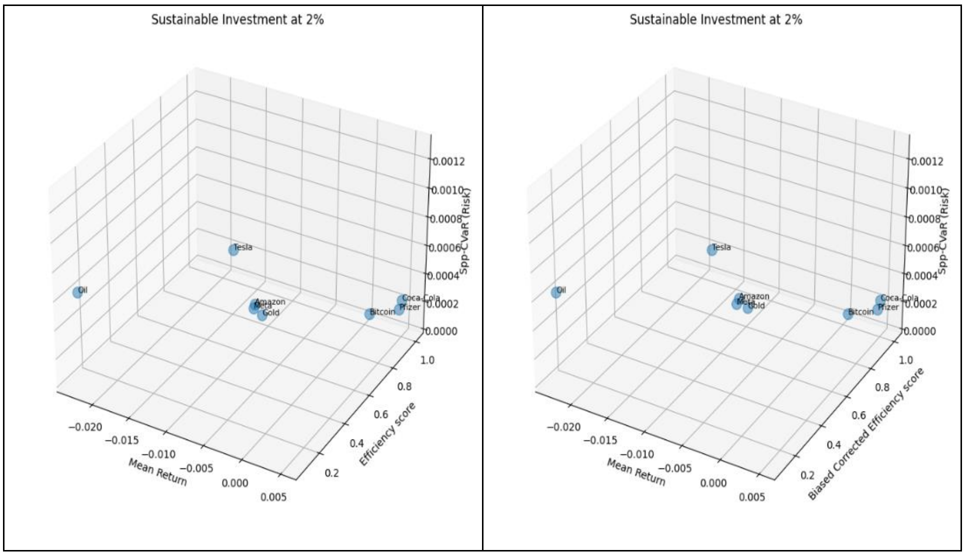


**Figure 2:** (a) Sustainable investment with efficiency score & (b) Sustainable investment with biased corrected efficiency score

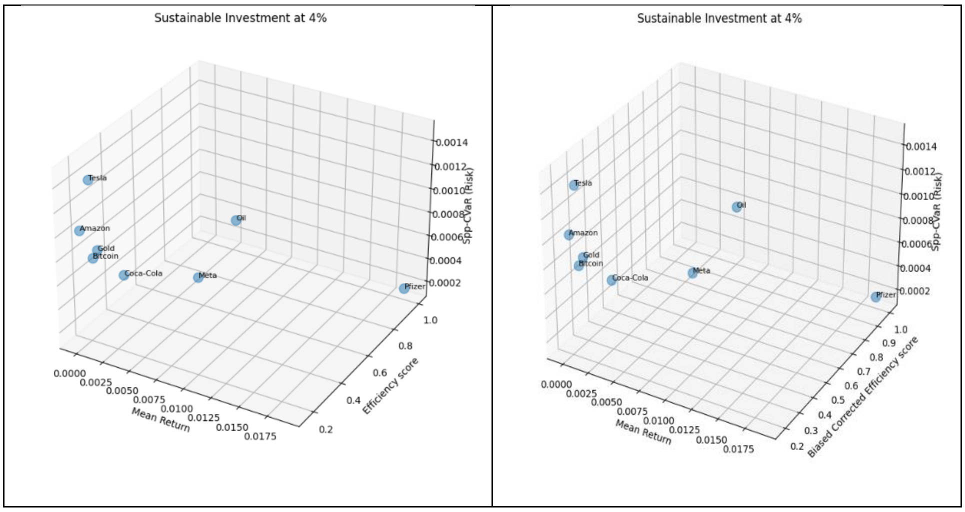


**Figure 3:** (a) Sustainable investment with efficiency score & (b) Sustainable investment with biased corrected efficiency score

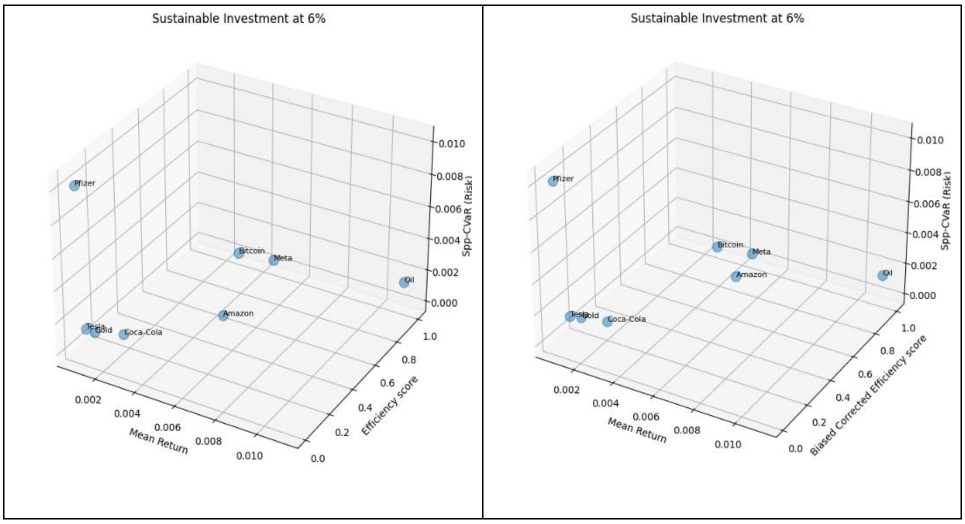


**Figure 4:** (a) Sustainable investment with efficiency score & (b) Sustainable investment with biased corrected efficiency score

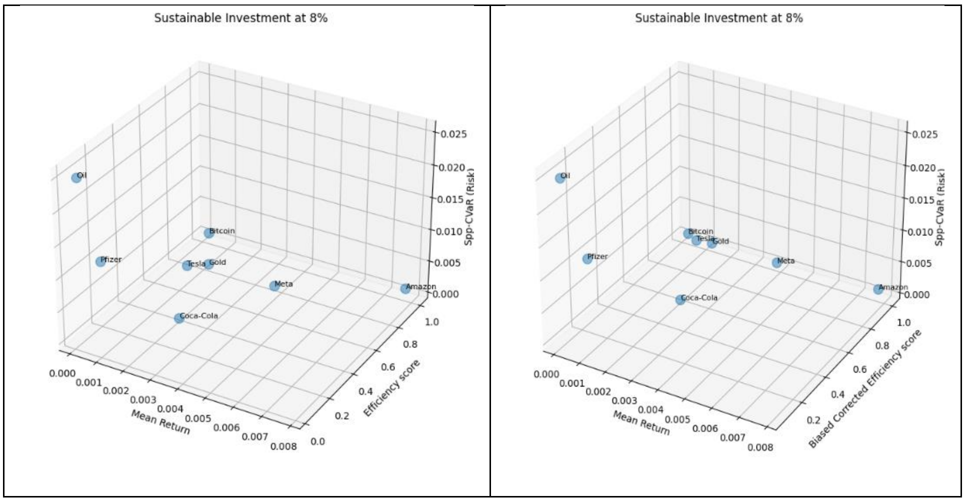


**Figure 5:** (a) Sustainable investment with efficiency score & (b) Sustainable investment with biased corrected efficiency score

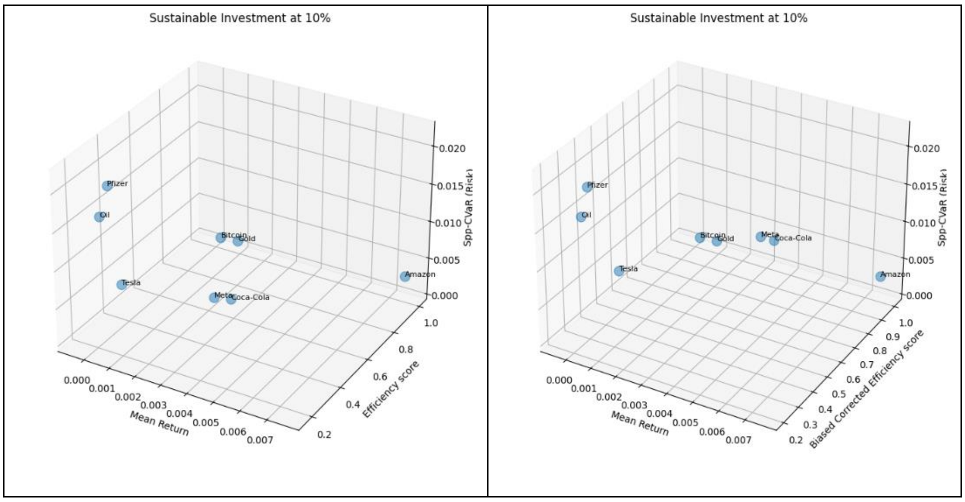


**Figure 6:** (a) Sustainable investment with efficiency score & (b) Sustainable investment with biased corrected efficiency score

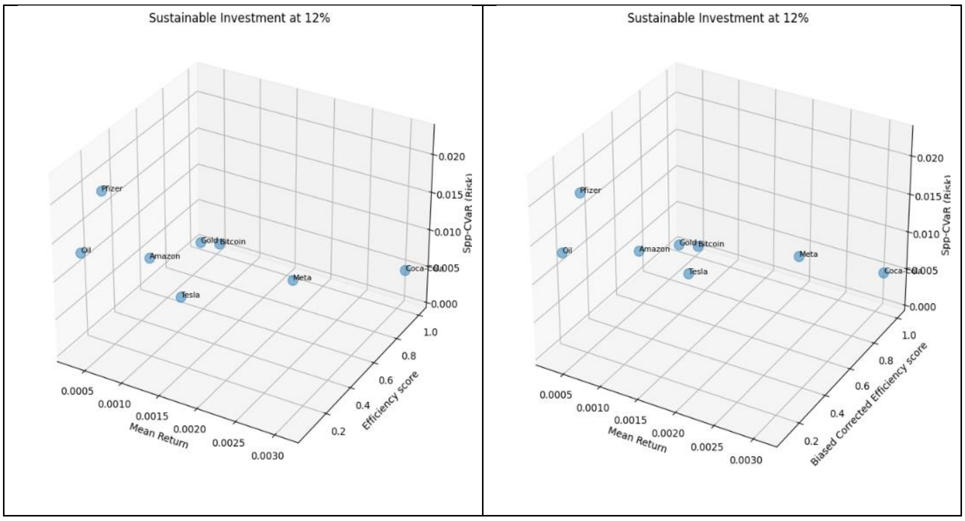


**Figure 7:** (a) Sustainable investment with efficiency score & (b) Sustainable investment with biased corrected efficiency score

Improving the availability of key metrics is essential for fostering a more sustainable investment landscape. Investors can make more informed choices by considering a range of criteria that align with their objectives. Table 1 shows the efficiency scores for each asset, where risk and return are calculated for the entire time horizon. Oil and Tesla perform well compared to other assets, indicating their potential as high-return investments. However, this method can be misleading for investors because the efficiency scores are calculated without considering the investor's exit time. Since after exiting the market, risk and return become irrelevant for the investor, it is crucial to account for the exit time in the efficiency calculations. For example, as demonstrated in Table 2, Amazon and Meta have an efficiency score of 0.13 at a 2% stop-profit point. However, when adjusted for biased corrected efficiency scores, Amazon's efficiency score outperforms Meta's. That indicates that Amazon is a more efficient and potentially more profitable investment after accounting for potential biases in the data than Meta at this stop-profit level.

The bias-corrected efficiency scores provide a more accurate assessment of each asset's performance by mitigating the effects of statistical anomalies or data irregularities. Investors can use these adjusted scores to make more informed decisions. In this case, despite the initial similarity in raw efficiency scores, the adjusted scores reveal Amazon as the better investment choice over Meta at the 2% stop-profit point, highlighting the importance of considering corrected data in investment strategies. By incorporating bias-corrected efficiency scores, investors gain a clearer understanding of asset performance, leading to more effective and strategic decision-making aligned with both financial objectives and sustainability goals. These results are more reliable than those reported by Ghasemi Doudkanlou et al. [35], who did not consider bias correction and only used raw efficiency scores.

Investors should not rely solely on efficiency scores for a well-rounded investment strategy focused on sustainability. It is crucial to consider additional factors such as risk, exit time, and average return on investment. According to Table 2, a risk-averse investor might opt for Bitcoin even though 17 days are required to achieve a 10% profit. This comprehensive approach allows investors to better align their asset choices with financial goals and sustainability benchmarks.

Exit time is a critical component of sustainable investing because it directly impacts other variables, such as SPP-CVaR and mean returns, which are calculated up to that point. Investors obtain a more accurate efficiency score for the underlying asset by considering exit time. This detailed analysis provides valuable insights into how long they may need to wait to achieve a specific profit target, helping them assess the associated risk and expected return during this period.

Understanding the relationship between exit time and investment performance helps investors make more innovative, sustainable choices. For instance, if an asset takes longer to exit but offers higher returns, it might be better suited for someone with a long-term investment horizon. On the other hand, an asset with a shorter exit time and moderate returns could be ideal for those looking to gain quicker profits. This insight allows investors to tailor their strategies to match their financial goals and timelines.

The scatter plots illustrate the sustainable investment analysis at a specific stop-profit point, evaluating investments based on biased corrected efficiency scores, risk, and mean returns. These 3D scatter plots provide a comprehensive view, surpassing traditional 2D graphs in conveying crucial investment decision factors. For instance, at the 12% stop-profit point, the left plot reveals that Pfizer has a low-efficiency score and high risk. Bitcoin and Gold exhibit low risk with relatively similar efficiency scores but differ slightly in their risk levels. The right plot introduces a biased corrected efficiency score for a more refined analysis. Amazon, Tesla, and Meta are clustered closely, indicating similar risk-return profiles, yet Meta has a slightly lower risk compared to Amazon and Tesla. Coca-Cola stands out with a high-efficiency score but moderate risk. These visualizations highlight the intricate balance between risk, return, and efficiency, aiding investors in making more informed and sustainable investment decisions.

## 5. CONCLUSION

This paper focuses on enhancing sustainability in financial markets to establish a more secure environment for investors. Our approach uniquely calculates risk and returns up to the point of exit, resulting in a more accurate efficiency score than traditional models, considering the entire time horizon without factoring in exit times. This method provides a more precise and more practical assessment of asset performance. One of the notable advantages of embracing sustainable investment practices is the ability to make informed trade-offs among various factors, including risk, return, and exit timing. By optimizing these elements, investors can achieve better efficiency scores and align their investment choices with financial goals and sustainability benchmarks. Our methodology's versatility makes it applicable across different asset classes and investment portfolios for short-term and long-term strategies. Sustainable investment practices offer more accurate and reliable data, allowing investors to tailor their strategies based on a comprehensive risk and return analysis.

In contrast, unsustainable investments often limit investors to long-term commitments due to less precise efficiency assessments. By integrating bias-corrected efficiency scores and considering exit times, our approach empowers investors to make more strategic and sustainable investment decisions. Overall, this methodology enhances the accuracy and reliability of investment performance evaluations and promotes a more sustainable and informed investment landscape. That eventually allows investors to navigate the complexities of financial markets effectively and contribute to a more sustainable economic future.

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## REFERENCES

[1] G. Talan and G. D. Sharma, "Doing well by doing good: A systematic review and research agenda for sustainable investment," *Sustainability,* vol. 11, no. 2, p. 353, 2019, doi: 10.3390/su11020353.

[2] A. Charnes, W. W. Cooper, and E. Rhodes, "Measuring the efficiency of decision making units," *European Journal of Operational Research,* vol. 3, no. 1, pp. 429-444, 1978, doi: 10.1016/0377-2217(78)90138-8.

[3] W. F. Bowlin, "Evaluating the efficiency of US Air Force real-property maintenance activities," *Journal of the Operational Research Society,* vol. 38, no. 2, pp. 127-135, 1987, doi: 10.1057/jors.1987.25.

[4] A. Charnes, W. Cooper, D. Divine, G. Klopp, and J. Stutz, "An application of data envelopment analysis to management of US Army recruitment districts," in *Applications of Management Science*, R. L. Schultz Ed. Greenwich Conn: JAI Press, Inc, 1986.

[5] P. Kooreman, "Nursing home care in The Netherlands: a nonparametric efficiency analysis," *Journal of Health Economics,* vol. 13, no. 3, pp. 301-316, 1994, doi: 10.1016/0167-6296(94)90029-9.

[6] E. Eberlein and U. Keller, "Hyperbolic distributions in finance," *Bernoulli,* vol. 1, no. 3, pp. 281-299, 1995, doi: 10.2307/3318481.

[7] W. Liu, Z. Zhou, D. Liu, and H. Xiao, "Estimation of portfolio efficiency via DEA," *Omega,* vol. 52, pp. 107-118, 2015, doi: 10.1016/j.omega.2014.11.006.

[8] A. C. Tarnaud and H. Leleu, "Portfolio analysis with DEA: Prior to choosing a model," *Omega,* vol. 75, pp. 57-76, 2018, doi: 10.1016/j.omega.2017.02.003.

[9] Z. Zhou, M. Gao, H. Xiao, R. Wang, and W. Liu, "Big data and portfolio optimization: A novel approach integrating DEA with multiple data sources," *Omega,* vol. 104, p. 102479, 2021, doi: 10.1016/j.omega.2021.102479.
[10] H. Zhou, Y. Yang, Y. Chen, and J. Zhu, "Data envelopment analysis application in sustainability: The origins, development and future directions," *European Journal of Operational Research,* vol. 264, no. 1, pp. 1-16, 2018, doi: 10.1016/j.ejor.2017.06.023.
[11] Y. J. Zhang and M. Y. Chen, "Evaluating the dynamic performance of energy portfolios: Empirical evidence from the DEA directional distance function," *European Journal of Operational Research,* vol. 269, no. 1, pp. 64-78, 2018, doi: 10.1016/j.ejor.2017.08.008.
[12] G. Tsaples and J. Papathanasiou, "Data envelopment analysis and the concept of sustainability: A review and analysis of the literature," *Renewable and Sustainable Energy Reviews,* vol. 138, p. 110664, 2021, doi: 10.1016/j.rser.2020.110664.
[13] G. A. Tularam, E. Roca, and S. H. W. Victor, "Investigation of Socially Responsible Investment Markets (SRI) Using Dynamic Conditional Correlation (DCC) Method: Implications for Diversification," *Journal of Mathematics and Statistics,* vol. 6, no. 4, pp. 385–394, 2010, doi: 10.3844/jmssp.2010.385.394.
[14] F. A. F. de Souza Cunha and C. P. Samanez "Performance analysis of sustainable investments in the Brazilian stock market: a study about the corporate sustainability index (ISE)," *Journal of Business Ethics,* vol. 117, no. 1, pp. 19-36, 2013, doi:10.1007/s10551-012-1484-2.
[15] M. V. López, A. Garcia, and L. Rodriguez, "Sustainable development and corporate performance: A study based on the Dow Jones sustainability index," *Journal of Business Ethics,* vol. 75, no. 3, pp. 285-300, 2007, doi: 10.1007/s10551-006-9253-8.
[16] M. Balcilar, R. Demirer, and R. Gupta, "Do sustainable stocks offer diversification benefits for conventional portfolios? An empirical analysis of risk spillovers and dynamic correlations," *Sustainability,* vol. 9, no. 10, p. 1799, 2017, doi: 10.3390/su9101799.
[17] Global Sustainable Investment Alliance (GSIA), "Global sustainable investment review," Global Sustainable Investment Alliance, Washington, DC, USA, 2014.
[18] J. Brzeszczyński and G. McIntosh, "Performance of portfolios composed of British SRI stocks," *Journal of Business Ethics,* vol. 120, pp. 335-362, 2014, doi: 10.1007/s10551-012-1541-x.
[19] Ľ. Pástor, R. F. Stambaugh, and L. A. Taylor, "Sustainable investing in equilibrium," *Journal of Financial Economics,* vol. 142, no. 2, pp. 550-571, 2021, doi: 10.1016/j.jfineco.2020.12.011.
[20] S. Bernow, B. Klempner, and C. Magnin, "From 'why'to 'why not': Sustainable investing as the new normal," 2017.
[21] R. Bauer, T. Ruof, and P. Smeets, "Get real! Individuals prefer more sustainable investments," *The Review of Financial Studies,* vol. 34, no. 8, pp. 3976-4043, 2021, doi: 10.1093/rfs/hhab037.
[22] J. F. Kölbel, F. Heeb, F. Paetzold, and T. Busch, "Can sustainable investing save the world? Reviewing the mechanisms of investor impact," *Organization & Environment,* vol. 33, no. 4, pp. 554-574, 2020, doi: 10.1177/1086026620919202.
[23] M. Jain, G. D. Sharma, and M. Srivastava, "Can sustainable investment yield better financial returns: A comparative study of ESG indices and MSCI indices," *Risks,* vol. 7, no. 1, p. 15, 2019, doi: 10.3390/risks7010015.
[24] M. Fulton, B. Kahn, and C. Sharples, "Sustainable investing: Establishing long-term value and performance," *Available at SSRN 2222740,* 2012, doi: 10.2139/ssrn.2222740.
[25] S. R. Fiskerstrand, S. Fjeldavli, T. Leirvik, Y. Antoniuk, and O. Nenadić, "Sustainable investments in the Norwegian stock market," *Journal of Sustainable Finance & Investment,* vol. 10, no. 3, pp. 294-310, 2020, doi: 10.1080/20430795.2019.1677441.
[26] H. Konno and H. Shirakawa, "Existence Of A Nonnegative Equilibrium Price Vector In The Mean-Variance Capital Market," *Mathematical Finance,* vol. 5, no. 3, pp. 233-246, 1995, doi: 10.1111/j.1467-9965.1995.tb00066.x.
[27] P. Artzner, F. Delbaen, J. M. Eber, and D. Heath, "Coherent measures of risk," *Mathematical Finance,* vol. 9, no. 2, pp. 203-228, 1999, doi: 10.1111/1467-9965.00068.

[28] R. T. Rockafellar and S. Uryasev, "Conditional value-at-risk for general loss distributions," *Journal of Banking & Finance,* vol. 26, no. 7, pp. 1443-1471, 2002, doi: 10.1016/S0378-4266(02)00271-6.

[29] G. C. Pflug and A. Świętanowski, "Selected parallel optimization methods for financial management under uncertainty," *Parallel Computing,* vol. 26, no. 1, pp. 3-25, 2000, doi: 10.1016/S0167-8191(99)00093-9.

[30] W. Ogryczak and A. Ruszczyński, "Dual stochastic dominance and quantile risk measures," *International Transactions in Operational Research,* vol. 9, no. 5, pp. 661-680, 2002, doi: 10.1111/1475-3995.00380.

[31] A. Chekhlov, S. Uryasev, and M. Zabarankin, "Portfolio optimization with drawdown constraints," *Supply Chain and Finance*: *World Scientific*, 2004, pp. 209-228.

[32] L. Bin, "A new risk measure and its application in portfolio optimization: The SPP–CVaR approach," *Economic Modelling,* vol. 51, pp. 383-390, 2015, doi: 10.1016/j.econmod.2015.08.013.

[33] M. Portela, E. Thanassoulis, and G. Simpson, "A directional distance approach to deal with negative data in DEA: An application to bank branches," *Journal of Operational Research Society,* vol. 55, no. 10, pp. 1111-1121, 2004.

[34] R. G. Chambers, Y. Chung, and R. Färe, "Profit, directional distance functions, and Nerlovian efficiency," *Journal of Optimization Theory and Applications,* vol. 98, no. 2, pp. 351-364, 1998, doi: 10.1023/A:1022637501082.

[35] M. Ghasemi Doudkanlou, P. Chandro, and S. Banihashemi, "The Effect of Exit Time and Entropy on Asset Performance Evaluation," *Entropy,* vol. 25, no. 9, p. 1252, 2023, doi: 10.3390/e25091252.

[36] A. M. Alonso, D. Pena, and J. Romo, "Introducing model uncertainty by moving blocks bootstrap," *Statistical Papers,* vol. 47, no. 2, pp. 167-179, 2006, doi: 10.1007/s00362-005-0282-7.

[37] A. G. Assaf, C. P. Barros, and R. Matousek, "Technical efficiency in Saudi banks," *Expert Systems with Applications,* vol. 38, no. 5, pp. 5781-5786, 2011, doi: 10.1016/j.eswa.2010.10.054.

[38] G. E. Halkos and N. G. Tzeremes, "The effect of foreign ownership on SMEs performance: An efficiency analysis perspective," *Journal of Productivity Analysis,* vol. 34, pp. 167-180, 2010, doi: 10.1007/s11123-010-0174-2.

[39] D. Tu and L. Zhang, "On the estimation of skewness of a statistic using the jackknife and the bootstrap," *Statistical Papers,* vol. 66, no. 1, pp. 39–56, 1992, doi: 10.1007/BF02925311.

[40] B. W. Silverman, *Density estimation for statistics and data analysis*. Routledge, 2018.

[41] L. Simar and P. W. Wilson, "Statistical inference in nonparametric frontier models: recent developments and perspectives," in *The measurement of productive efficiency and productivity growth*, H. O. Fried, C. A. K. Lovell, and S. S. Schmidt Eds.: Oxford University Press, 2008, pp. 421-521.

[42] C. Daraio and L. Simar, *Advanced robust and nonparametric methods in efficiency analysis: Methodology and applications*. US: Springer Science & Business Media, 2007.