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Research Article

AN INTERVAL ARITHMETIC APPROACH IN SOLVING INTERVAL-VALUED TRAPEZOIDAL NEUTROSOPHIC FUZZY MULTI-OBJECTIVE MULTI-ITEM SOLID TRANSPORTATION PROBLEM

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Abstract: Unpredictability and uncertainty occur worldwide in various aspects of real life. We cannot predict some specific outcomes or events precisely due to multiple factors, randomness, complexity and limited information. These situations can be handled efficiently in a systematic way by using neutrosophic sets. In real-world applications, transportation is essential in all sorts of movement of goods, services and people to meet various needs and demands efficiently. This study concentrated on the multiple objectives, multiple choice transportation problem in interval-valued trapezoidal neutrosophic contexts. The conversion procedure employs a de-neutrosophication process that relies on interval numbers rather than crisp numbers. By using an interval-valued trapezoidal neutrosophic fuzzy programming method based on interval number, the identified uncertain transportation problem is then solved. Additionally, an illustrative instance is presented to showcase the successful implementation of the proposed methodology.

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Keywords: Trapezoidal fuzzy number, trapezoidal neutrosophic fuzzy number, multi-objective solid transportation problem, interval-based de-neutrosophication, interval number.

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1. INTRODUCTION

The transportation problem is a versatile concept applied in the field of operational research, logistics and supply chain management due to its practical relevance and impact on various industries. It helps to find the most economical means of transporting goods from numerous suppliers to various consumers to satisfy supply and demand constraints. As globalization evolves, classic transportation problems lack in fulfilling the transportation criteria due to the condition of roads, climate changes, types of vehicles, etc. Hence, Haley [1] extended his transportation problem into solid transportation with three dimensions: supply, demand, and conveyance, and the application is majorly in the public distribution sectors. Products from the industries must be delivered from the source to the destination with various modes of transport, including trucks, trains, aircraft or ships. Initially, the researcher's primary objective in solving the solid transportation problem is to minimize the overall cost associated with transportation goods. Addressing multiple conflicting goals in decision-making is necessary in the current real-life scenarios. Hence, in order to develop optimization theory and techniques, a multi-objective transportation problem is essential. Different objective functions may include minimizing transportation costs, time, carbon emissions, inventory costs etc., and maximizing profit, resource utilization, customer satisfaction, service level etc. Lee and Moore [2] studied the transportation problem with multiple objectives. In addition, multi-objective multi-item transportation problem addresses the transportation of various types of items or products concurrently. Each item may have different characteristics, such as size and weight, which cannot affect transport decisions. Environmental concerns, transportation cost per unit, and fuel consumption rate are increasingly significant today. One or more routes are essential to reduce the ecological impact, lower operating costs, and meet sustainability goals.

Moreover, in current real-life circumstances, uncertainty arises everywhere for various reasons, such as financial instability, climate variability, road conditions, traffic scenarios, metropolitan works, etc. Fuzzy set theory by Zadeh [3] can efficiently handle these uncertainties. Jimenez and Verdegay [4] addresses transportation problem by managing ambiguous datasets using interval and fuzzy approaches. Pramanik and Roy [5] addressed a multiobjective transportation model utilizing a priority based fuzzy goal programming approach through fuzzy parameters. For handling fuzzy variables, Kundu et al. [6] presented a defuzzification method in the multi-objective multi-item solid transportation problem (MOMISTP). Chakraborty et al. [7] modeled fuzzy inequality constraint and obtained optimal solution by using three methods in solving fuzzy MOMISTP. Rani et al. [8] suggested converting an unbalanced MOMISTP to a balanced one using trapezoidal fuzzy as parameters numbers. Giri et al. [9] discussed fuzzy MOMISTP with fuzzy fixed-charge to minimize the overall fuzzy cost for balanced and unbalanced problems. Kar et al. [10] formulated a chance constraint model with the credibility of fuzzy variables in solving fuzzy MOMISTP. A multi-objective transportation problem with type-2 trapezoidal fuzzy numbers as parameters, together with goodness of fit and parameter estimation, was given by Kamal et al. [11]. Using the weighting Tchebycheff approach, Khalifa et al. [12] solve fuzzy MOMISTP in a fuzzy environment. Mardanya and Roy [13] derived a novel rule using interval, rough interval and expected value operator for trapezoidal fuzzy MOMISTP. Rekabi et al. [14] formulated an innovative multi-objective model to design a responsive and sustainable pharmaceutical supply chain network, incorporating Cap-and-Trade policies, various manufacturing technology options, and diverse transportation mode selections.

Handling vagueness and hesitation is not appropriate when using a fuzzy set. Attanasov [15] expanded the concept fuzzy set with a negation part (non-membership) along with membership, terming it an intuitionistic set. Pramanik and Roy [16] introduced an intuitionistic fuzzy goal programming approach to address vector optimization problem by resolving an unbalanced transportation problem with multiple objectives. Chakraborty et al. [17] considered multiple choices in solving intuitionistic MOMISTP and obtained the optimal solution using three models: interactive satisfied, global criteria and goal programming. Roy et al. [18] converts the multi-objective intuitionistic fuzzy transportation problem's objective function into an interval form and the optimal solutions are attained by using the techniques such as intuitionistic fuzzy and goal programming. In order to tackle MOMI 4D transportation in an LR-type intuitionistic fuzzy environment, Samanta et al. [19] proposed a unique and convex combination approach. Using weighted Tchebychef metrics and min-max programming, Midya et al. [20] concentrated on intuitionistic MOMI fixed charge STP with parameters represented as trapezoidal fuzzy integers. Shivani and Deepika [21] investigated the impact of proper and incorrect driving styles on carbon emissions, handling three programming strategies to handle MOMI 4D transportation in an interval-valued intuitionistic settings.

Smarandache [22] added one more parameter (indeterminacy) to handle the neutral thoughts that suit even more efficiently than fuzzy and intuitionistic sets. Biswas et al. [23] formulated interval trapezoidal neutrosophic numbers and established several arithmetic operations to address multiple attribute decision making problem. In a neutrosophic environment, Rizk et al. [24] developed a unique algorithm to find the best solution for multi-objective transportation problem. By addressing a neutrosophic multi-objective transportation p-facility location problem, Das et al. [25] collectively describe the impact of carbon emission into the atmosphere. Giri and Roy [26] adapted single-valued trapezoidal neutrosophic number in four-dimensional MOMISTP to lessen the overall transit cost, time and release of carbon. Khalil et al. [27] discussed real-life applications in interval-valued neutrosophic environments. Through the solution of MOTP in a neutrosophic context, Revathi et al. [28] examined uncertain variables. By applying linear programming techniques in real life, Hosseinzadeh and Tayyebi [29] obtained an effective solution in a multi-objective neutrosophic transportation problem. Kumar et al. [30] developed neutrosophic programming approach for multi-objective transportation problem based on non-linear hyperbolic functions. Gupta et al. [31] developed a neutrosophic goal programming method for the multi-objective fixed-charge transportation problem which incorporates neutrosophic characteristics.

1.1. Aim of the article

Contemporary research has focused more on interval numbers than crisp ones due to the intricate nature of real-world environments. Interval numbers offer several advantages when dealing with uncertain data, insufficient information, and modeling fuzziness. Interval numbers are useful for representing data from different sources with different levels of uncertainty, making them well-suited for sensitivity analysis. Representing numbers in a specified range is more practical and flexible. Avoiding the complexity of membership functions, intervals offer a clear and concise means of expressing uncertainty. During literature survey, many experts have gone through their research in MOMISTP using interval numbers. Nagarajan et al. [32] presented a procedure to solve a multi-objective solid transportation problem with interval based parameters. Using the nearest approximation technique, Dalman et al. [33] created an interval programming method to deal trapezoidal fuzzy MOMISTP. Considering the MOMISTP with budget constraints and safety measures, Sifaoui and Aider [34] formulated two models, namely the expected value as well as the chance-constrained model. This leads us to prefer interval numbers instead of crisp ones in interval-valued trapezoidal neutrosophic contexts.

1.2. Structure of the article

This article addresses the subject of solving a multi-objective multi-item solid transportation problem using interval-valued trapezoidal neutrosophic fuzzy approach. An algorithm for interval-valued trapezoidal neutrosophic programming is introduced, employing interval numbers. The efficiency is demonstrated by solving an example and thereafter comparing it with the reference [26]. This article follows a well-defined format, with an introduction in section 1, background information on the topic in section 2, the mathematical formulation of the interval-valued trapezoidal neutrosophic fuzzy MOMISTP and its conversion to an interval problem using an interval-based de-neutrosophication technique elaborated in section 3, a detailed explanation of the proposed algorithm for interval-valued trapezoidal neutrosophic fuzzy programming using interval numbers provided in section 4, a discussion of the problem and numerical illustrations in section 5, sensitivity analysis is done in section 6, the research's positive aspects and drawbacks are highlighted in section 7, and section 8 concludes with a summary of the study's future research and recommendations.

2. PRELIMINARIES

2.1. Interval number

An interval number on $\mathbf{R}$ is defined as $\widetilde{f} = \left[f^L, f^R\right] = \left\{f : f^L \leq f \leq f^R, f \in \mathbf{R}\right\}$. Here f^L and f^R are the left and the right limits of $\widetilde{f}$ respectively. Also, $\widetilde{f}_m = \frac{f^L+f^R}{2}$ and $\widetilde{f}_w = \frac{f^R-f^L}{2}$ are the mid-point and width (half-width) of an interval. Then, the interval number can also be written in the form of a mid-point and width as

$$\widetilde{f} = \langle \widetilde{f}_m, \widetilde{f}_w \rangle = \left\{f : \widetilde{f}_m - \widetilde{f}_w \leq k \leq \widetilde{f}_m + \widetilde{f}_w, f \in \mathbf{R}\right\}.$$

2.2. Arithmetic operations on Interval number

The interval arithmetic operations provided by [35] are given below.
If $\widetilde{f} = \left[f^L, f^R\right]$ and $\widetilde{g} = \left[g^L, g^R\right]$ and for $* \in \{+,-,\times,\div\}$, then

$$\widetilde{f} * \widetilde{g} = \left\langle \widetilde{f}_m, \widetilde{f}_w \right\rangle * \left\langle \widetilde{g}_m, \widetilde{g}_w \right\rangle = \left\langle \widetilde{f}_m * \widetilde{g}_m, \max\{\widetilde{f}_w, \widetilde{g}_w\} \right\rangle.$$

2.3. Trapezoidal fuzzy number

The membership function of a trapezoidal fuzzy number $T = (f_1, f_2, f_3, f_4)$, where $f_1, f_2, f_3, f_4 \in \mathbf{R}$ is given as

$$\mu_T(x) = \begin{cases} \frac{x-f_1}{f_2-f_1}, & f_1 \leq x \leq f_2 \\ 1, & f_2 \leq x \leq f_3 \\ \frac{f_4-x}{f_4-f_3}, & f_3 \leq x \leq f_4 \\ 0, & otherwise \end{cases}$$

2.4. Interval-valued trapezoidal neutrosophic fuzzy number [36]

. Let $f_1, f_2, f_3, f_4 \in \mathbf{R}$ such that $f_1 \leq f_2 \leq f_3 \leq f_4$. An interval-valued trapezoidal neutrosophic fuzzy number can be expressed as $\widetilde{T_n} = \langle (f_1, f_2, f_3, f_4); [\rho^L, \rho^R]$, $[\kappa^L, \kappa^R], [\nu^L, \nu^R] \rangle$, where $\rho^L : X \to [0,1]$, $\rho^R : X \to [0,1]$ are the lower truth and upper truth degrees, $\kappa^L : X \to [0,1]$, $\kappa^R : X \to [0,1]$ are the lower indeterminacy and upper indeterminacy degrees, $\nu^L : X \to [0,1]$, and $\nu^R : X \to [0,1]$ are the lower falsity and upper falsity degrees whose functions are defined as

$$\rho^L_{\widetilde{T_n}}(x) = \begin{cases} \rho^L \frac{x-f_1}{f_2-f_1}, & x \in [f_1, f_2] \\ \rho^L, & x \in [f_2, f_3] \\ \rho^L \frac{f_4-x}{f_4-f_3}, & x \in [f_3, f_4] \\ 0, & otherwise \end{cases} \quad \rho^R_{\widetilde{T_n}}(x) = \begin{cases} \rho^R \frac{x-f_1}{f_2-f_1}, & x \in [f_1, f_2] \\ \rho^R, & x \in [f_2, f_3] \\ \rho^R \frac{f_4-x}{f_4-f_3}, & x \in [f_3, f_4] \\ 0, & otherwise \end{cases}$$

$$\kappa^L_{\widetilde{T_n}}(x) = \begin{cases} \frac{(f_2-x)+\kappa^L(x-f_1)}{f_2-f_1}, & x \in [f_1, f_2] \\ \kappa^L, & x \in [f_2, f_3] \\ \frac{(x-f_3)+\kappa^L(f_4-x)}{f_4-f_3}, & x \in [f_3, f_4] \\ 1, & otherwise \end{cases}$$

$$\kappa^R_{\widetilde{T_n}}(x) = \begin{cases} \frac{(f_2-x)+\kappa^R(x-f_1)}{f_2-f_1}, & x \in [f_1, f_2] \\ \kappa^R, & x \in [f_2, f_3] \\ \frac{(x-f_3)+\kappa^R(f_4-x)}{f_4-f_3}, & x \in [f_3, f_4] \\ 1, & otherwise \end{cases}$$

$$\nu^L_{\widetilde{T}_n}(x) = \begin{cases} \frac{(f_2-x)+\nu^L(x-f_1)}{f_2-f_1}, & x \in [f_1, f_2] \\ \nu^L, & x \in [f_2, f_3] \\ \frac{(x-f_3)+\nu^L(f_4-x)}{f_4-f_3}, & x \in [f_3, f_4] \\ 1, & otherwise \end{cases}$$

$$\nu^R_{\widetilde{T}_n}(x) = \begin{cases} \frac{(f_2-x)+\nu^R(x-f_1)}{f_2-f_1}, & x \in [f_1, f_2] \\ \nu^R, & x \in [f_2, f_3] \\ \frac{(x-f_3)+\nu^R(f_4-x)}{f_4-f_3}, & x \in [f_3, f_4] \\ 1, & otherwise \end{cases}$$

3. STATEMENT AND FORMULATION OF THE PROBLEM

3.1. Notations

m,n: Number of origins and destinations respectively $(i = 1,2,...,m,\ j = 1,2,...,n)$

c: Number of conveyances (vehicles used for transportation) $(k = 1,2,...,c)$

r: The number of routes or paths connecting each origin and each destination $(p = 1,2,...,r)$

x^N_{ijkp} & x^I_{ijkp}: The interval-valued trapezoidal neutrosophic fuzzy number and interval number respectively represents the volume of commodities (products) transported by the k^{th} transportation vehicle (conveyance) along the p^{th} route, beginning from the i^{th} source and concluding at the j^{th} destination.

L_{ijp} & L^I_{ijp}: The distance between the i^{th} starting point and the j^{th} destination utilizing the p^{th} route in the form of a real value and an interval number respectively.

F^N_{ijkp} & F^I_{ijkp}: Interval-valued trapezoidal neutrosophic fuzzy fixed charge and interval fixed charge when some volume of goods (products) moved by the k^{th} transportation vehicle (conveyance) along the p^{th} route, begins at the source denoted as i^{th} and concludes at the destination designated as j^{th} respectively.

T^N_{ijkp} & T^I_{ijkp}: Unit interval-valued trapezoidal neutrosophic fuzzy transportation time and interval transportation time per kilometre through p^{th} route by k^{th} transportation vehicle (conveyance) from the i^{th} origin and reaching the j^{th} destination respectively.

P^N_{ijkp} & P^I_{ijkp}: Unit interval-valued trapezoidal neutrosophic fuzzy carbon emission and interval carbon emission per kilometre via k^{th} transportation vehicle (conveyance) on a p^{th} route, starting from the i^{th} origin and ending at the j^{th} destination, respectively.

y_{ijkp}: The binary element with a value of 1, if there is a transportation of certain quantities by k^{th} transportation vehicle (conveyance) through p^{th} route from the i^{th} source to the j^{th} destination and 0, otherwise.

A^N_i & A^I_i : The interval-valued trapezoidal neutrosophic fuzzy number and interval number indicating the products availability at the i^{th} origin respectively.

D^N_j & D^I_j : Demand for the products expressed as an interval-valued trapezoidal neutrosophic fuzzy number and interval number at the j^{th} destination respectively.

$\boldsymbol{E_{kp}^N}$ & $\boldsymbol{E_{kp}^I}$: Transport vehicle capacity (conveyance) for the k^{th} vehicle along the p^{th} route, expressed as an interval-valued trapezoidal neutrosophic fuzzy number and interval number.

$\boldsymbol{Z_k^N}$ & $\boldsymbol{Z_k^I}$: The objective function expressed as interval number and interval-valued trapezoidal neutrosophic fuzzy number respectively.

3.2. Assumptions

- All interval-valued trapezoidal neutrosophic fuzzy number are non-negative.
- The routes and the driving style are smooth and good.
- No penalty and tax for carbon emission.

3.3. Mathematical Formulation

3.3.1. Model 1: Interval-valued trapezoidal neutrosophic fuzzy multi-objective multi-item solid transportation problem:

Given optimization problems are formulated in consideration of economic conditions and the desire to develop a cleaner environment. Our primary goal is to reduce carbon emissions, transportation expenses, and travel time. Model 1 is developed in accordance with the objective function and constraints established by [26]. In this study, we examine various factors including transportation cost, transportation time, fixed cost, carbon emissions, transportation vehicles, availability, and demand. All of these factors are represented by interval-valued trapezoidal neutrosophic fuzzy numbers. By representing in this manner, one can effectively manage risks and address uncertainty in projects or situations involving precise or uncertain parameter estimations in a methodical fashion. In addition, the exact distance between the starting and ending points is specified in real numbers. It is assumed in this context that the products are conveyed via c conveyances along r routes from m sources to destinations n.

Minimizing transportation cost

$$Minimize\ Z_1^N = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{c}\sum_{p=1}^{r} C_{ijkp}^N\, x_{ijkp}^N\, L_{ijp} + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{c}\sum_{p=1}^{r} F_{ijkp}^N\, y_{ijkp} \tag{1}$$

Minimizing transportation time

$$Minimize\ Z_2^N = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{c}\sum_{p=1}^{r} T_{ijkp}^N\, x_{ijkp}^N\, L_{ijp} \tag{2}$$

Minimizing carbon emission

$$Minimize\ Z_3^N = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{c}\sum_{p=1}^{r} P_{ijkp}^N\, x_{ijkp}^N\, L_{ijp} \tag{3}$$

subject to

$$\sum_{j=1}^{n}\sum_{k=1}^{c}\sum_{p=1}^{r} x_{ijkp}^N \leq A_i^N, \quad i = 1,2,...,m \tag{4}$$

$$\sum_{i=1}^{m}\sum_{k=1}^{c}\sum_{p=1}^{r} x_{ijkp}^N \geq B_j^N, \quad j = 1,2,...,n \tag{5}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijkp}^N \leq E_{kp}^N, \qquad k = 1,2,...,c\ \&\ p = 1,2,...,r \tag{6}$$

$$x_{ijkp}^N \geq 0^N\ \forall\ i,j,k,p \tag{7}$$

3.3.2. A technique to de-neutrosophication based on intervals

The transformation of the interval-valued trapezoidal neutrosophic fuzzy number into an interval number, rather than a crisp number, provides greater flexibility in analyzing the results and helps to avoid the complexity in real-life circumstances. Therefore, we transform Model 1 into Model 2 using the interval-based de-neutrosophication technique proposed by [36]. Specifically, we transform an interval-valued trapezoidal neutrosophic fuzzy number into an interval number by assigning arbitrary values for the α, β, and γ-cut, represented as the midpoint and width (half-width).

The transportation cost C_{ijkp}^N is calculated interms of interval number as follows.

Let $C_{ijkp}^N = \langle (c_1, c_2, c_3, c_4); [\rho^L, \rho^R], [\kappa^L, \kappa^R], [\nu^L, \nu^R] \rangle$ be any transportation cost in interval-valued trapezoidal neutrosophic fuzzy number.

Let α, β, γ, r and $s \in$ [0,1] and assume β & γ as $1-\alpha$ then the $(\alpha, \beta\ \&\ \gamma)$ - cut of an interval-valued trapezoidal neutrosophic fuzzy transportation cost is given as

$$\left[\alpha^L_{C^N_{ijkp}}, \alpha^R_{C^N_{ijkp}}\right]; \left[\beta^L_{C^N_{ijkp}}, \beta^R_{C^N_{ijkp}}\right]; \left[\gamma^L_{C^N_{ijkp}}, \gamma^R_{C^N_{ijkp}}\right]. \tag{8}$$

Let $\mathscr{M}(\alpha^L_{C^N_{ijkp}}), \mathscr{W}(\alpha^L_{C^N_{ijkp}}), \mathscr{M}(\alpha^R_{C^N_{ijkp}}), \mathscr{W}(\alpha^R_{C^N_{ijkp}}), \mathscr{M}(\beta^L_{C^N_{ijkp}}), \mathscr{W}(\beta^L_{C^N_{ijkp}}), \mathscr{M}(\beta^R_{C^N_{ijkp}}),$ $\mathscr{W}(\beta^R_{C^N_{ijkp}}), \mathscr{M}(\gamma^L_{C^N_{ijkp}}), \mathscr{W}(\gamma^L_{C^N_{ijkp}}), \mathscr{M}(\gamma^R_{C^N_{ijkp}})$ and $\mathscr{W}(\gamma^R_{C^N_{ijkp}})$ are the mid-point and width (half-width) form of $(\alpha, \beta\ \&\ \gamma)$ - cut of an interval-valued trapezoidal neutrosophic transportation cost respectively. Here,

$$\mathscr{M}(\alpha^L_{C^N_{ijkp}}) = \frac{1}{2}\{(c_1+c_4)+\frac{\alpha}{\rho^L}(c_2-c_1-c_4+c_3)\},$$
$$\mathscr{W}(\alpha^L_{C^N_{ijkp}}) = \frac{1}{2}\{(c_4-c_1)-\frac{\alpha}{\rho^L}(c_4-c_3+c_2-c_1)\}$$
$$\mathscr{M}(\alpha^R_{C^N_{ijkp}}) = \frac{1}{2}\{(c_1+c_4)+\frac{\alpha}{\rho^R}(c_2-c_1-c_4+c_3)\}$$
$$\mathscr{W}(\alpha^R_{C^N_{ijkp}}) = \frac{1}{2}\{(c_4-c_1)-\frac{\alpha}{\rho^R}(c_4-c_3+c_2-c_1)\}$$
$$\mathscr{M}(\beta^L_{C^N_{ijkp}}) = \frac{1}{2}\{(c_1+c_4)+\frac{\alpha}{1-\kappa^L}(c_2-c_1-c_4+c_3)\}$$
$$\mathscr{W}(\beta^L_{C^N_{ijkp}}) = \frac{1}{2}\{(c_4-c_1)-\frac{\alpha}{1-\kappa^L}(c_4-c_3+c_2-c_1)\}$$
$$\mathscr{M}(\beta^R_{C^N_{ijkp}}) = \frac{1}{2}\{(c_1+c_4)+\frac{\alpha}{1-\kappa^R}(c_2-c_1-c_4+c_3)\}$$
$$\mathscr{W}(\beta^R_{C^N_{ijkp}}) = \frac{1}{2}\{(c_4-c_1)-\frac{\alpha}{1-\kappa^R}(c_4-c_3+c_2-c_1)\}$$
$$\mathscr{M}(\gamma^L_{C^N_{ijkp}}) = \frac{1}{2}\{(c_1+c_4)+\frac{\alpha}{1-\nu^L}(c_2-c_1-c_4+c_3)\}$$
$$\mathscr{W}(\gamma^L_{C^N_{ijkp}}) = \frac{1}{2}\{(c_4-c_1)-\frac{\alpha}{1-\nu^L}(c_4-c_3+c_2-c_1)\}$$
$$\mathscr{M}(\gamma^R_{C^N_{ijkp}}) = \frac{1}{2}\{(c_1+c_4)+\frac{\alpha}{1-\nu^R}(c_2-c_1-c_4+c_3)\}$$
$$\mathscr{W}(\gamma^R_{C^N_{ijkp}}) = \frac{1}{2}\{(c_4-c_1)-\frac{\alpha}{1-\nu^R}(c_4-c_3+c_2-c_1)\}$$

Then, we can write equation (8) as

$$\big[\big\langle \mathscr{M}(\alpha^L_{C^N_{ijkp}}), \mathscr{W}(\alpha^L_{C^N_{ijkp}})\big\rangle, \big\langle \mathscr{M}(\alpha^R_{C^N_{ijkp}}), \mathscr{W}(\alpha^R_{C^N_{ijkp}})\big\rangle\big];$$
$$\big[\big\langle \mathscr{M}(\beta^L_{C^N_{ijkp}}), \mathscr{W}(\beta^L_{C^N_{ijkp}})\big\rangle, \big\langle \mathscr{M}(\beta^R_{C^N_{ijkp}}), \mathscr{W}(\beta^R_{C^N_{ijkp}})\big\rangle\big];$$
$$\big[\big\langle \mathscr{M}(\gamma^L_{C^N_{ijkp}}), \mathscr{W}(\gamma^L_{C^N_{ijkp}})\big\rangle, \big\langle \mathscr{M}(\gamma^R_{C^N_{ijkp}}), \mathscr{W}(\gamma^R_{C^N_{ijkp}})\big\rangle\big]$$

Now, apply convex combination to combine the lower and upper bound of each membership functions and it is written as

$$\big[\text{r.}\big\langle \mathscr{M}(\alpha^L_{C^N_{ijkp}}), \mathscr{W}(\alpha^L_{C^N_{ijkp}})\big\rangle + \text{(1-r).}\big\langle \mathscr{M}(\alpha^R_{C^N_{ijkp}}), \mathscr{W}(\alpha^R_{C^N_{ijkp}})\big\rangle\big];$$
$$\big[\text{r.}\big\langle \mathscr{M}(\beta^L_{C^N_{ijkp}}), \mathscr{W}(\beta^L_{C^N_{ijkp}})\big\rangle + \text{(1-r).}\big\langle \mathscr{M}(\beta^R_{C^N_{ijkp}}), \mathscr{W}(\beta^R_{C^N_{ijkp}})\big\rangle\big];$$
$$\big[\text{r.}\big\langle \mathscr{M}(\gamma^L_{C^N_{ijkp}}), \mathscr{W}(\gamma^L_{C^N_{ijkp}})\big\rangle + \text{(1-r).}\big\langle \mathscr{M}(\gamma^R_{C^N_{ijkp}}), \mathscr{W}(\gamma^R_{C^N_{ijkp}})\big\rangle\big]$$

Now, using the arithmetic operations provided in section 2.2, we obtained

$$\left[\text{r}\left\langle \mathscr{M}(\alpha^{L}_{C^{N}_{ijkp}}) + \text{(1-r)}..\mathscr{M}(\alpha^{R}_{C^{N}_{ijkp}}), \max\{\mathscr{W}(\alpha^{L}_{C^{N}_{ijkp}}), \mathscr{W}(\alpha^{R}_{C^{N}_{ijkp}})\}\right\rangle\right];$$
$$\left[\text{r.}\left\langle \mathscr{M}(\beta^{L}_{C^{N}_{ijkp}}) + \text{(1-r)}..\mathscr{M}(\beta^{R}_{C^{N}_{ijkp}}), \max\{\mathscr{W}(\beta^{L}_{C^{N}_{ijkp}}), \mathscr{W}(\beta^{R}_{C^{N}_{ijkp}})\}\right\rangle\right];$$
$$\left[\text{r.}\left\langle \mathscr{M}(\gamma^{L}_{C^{N}_{ijkp}}) + \text{(1-r)}..\mathscr{M}(\gamma^{R}_{C^{N}_{ijkp}}), \max\{\mathscr{W}(\gamma^{L}_{C^{N}_{ijkp}}), \mathscr{W}(\gamma^{R}_{C^{N}_{ijkp}})\}\right\rangle\right]$$

Here, we choose r=1 as the highest membership grade, then

$$\left\langle \mathscr{M}(\alpha^{L}_{C^{N}_{ijkp}}), \max\{\mathscr{W}(\alpha^{L}_{C^{N}_{ijkp}}), \mathscr{W}(\alpha^{R}_{C^{N}_{ijkp}})\}\right\rangle;$$
$$\left\langle \mathscr{M}(\beta^{L}_{C^{N}_{ijkp}}), \max\{\mathscr{W}(\beta^{L}_{C^{N}_{ijkp}}), \mathscr{W}(\beta^{R}_{C^{N}_{ijkp}})\}\right\rangle;$$
$$\left\langle \mathscr{M}(\gamma^{L}_{C^{N}_{ijkp}}), \max\{\mathscr{W}(\gamma^{L}_{C^{N}_{ijkp}}), \mathscr{W}(\gamma^{R}_{C^{N}_{ijkp}})\}\right\rangle$$

For converting into an interval number, it is written as

$$s.\left\langle \mathscr{M}(\alpha^{L}_{C^{N}_{ijkp}}), \max\{\mathscr{W}(\alpha^{L}_{C^{N}_{ijkp}}), \mathscr{W}(\alpha^{R}_{C^{N}_{ijkp}})\}\right\rangle + (1-s).\Big\{\left\langle \mathscr{M}(\beta^{L}_{C^{N}_{ijkp}}),\right.$$
$$\left.\max\{\mathscr{W}(\beta^{L}_{C^{N}_{ijkp}}), \mathscr{W}(\beta^{R}_{C^{N}_{ijkp}})\}\right\rangle + \left\langle \mathscr{M}(\gamma^{L}_{C^{N}_{ijkp}}), \max\{\mathscr{W}(\gamma^{L}_{C^{N}_{ijkp}}), \mathscr{W}(\gamma^{R}_{C^{N}_{ijkp}})\}\right\rangle\Big\}$$

Again by section 2.2,

$$\Big\langle \text{s}..\mathscr{M}(\alpha^{L}_{C^{N}_{ijkp}}) + \text{(1-s)}..\mathscr{M}(\beta^{L}_{C^{N}_{ijkp}}) + \text{(1-s)}..\mathscr{M}(\gamma^{L}_{C^{N}_{ijkp}}), \max\Big\{\mathscr{W}(\alpha^{L}_{C^{N}_{ijkp}}),$$
$$\mathscr{W}(\alpha^{R}_{C^{N}_{ijkp}}), \mathscr{W}(\beta^{L}_{C^{N}_{ijkp}}), \mathscr{W}(\beta^{R}_{C^{N}_{ijkp}}), \mathscr{W}(\gamma^{L}_{C^{N}_{ijkp}}), \mathscr{W}(\gamma^{R}_{C^{N}_{ijkp}})\Big\}\Big\rangle$$

As a result, we obtain the interval form of an interval-valued trapezoidal neutrosophic fuzzy number by choosing the highest membership grade for ‘s’ (s=1) and is attained as

$$\mathscr{R}(C^{N}_{ijkp}) = \Big\langle \mathscr{M}(\alpha^{L}_{C^{N}_{ijkp}}), \max\Big\{\mathscr{W}(\alpha^{L}_{C^{N}_{ijkp}}), \mathscr{W}(\alpha^{R}_{C^{N}_{ijkp}}), \mathscr{W}(\beta^{L}_{C^{N}_{ijkp}}), \mathscr{W}(\beta^{R}_{C^{N}_{ijkp}}),$$
$$\mathscr{W}(\gamma^{L}_{C^{N}_{ijkp}}), \mathscr{W}(\gamma^{R}_{C^{N}_{ijkp}})\Big\}\Big\rangle$$

Similarly, we can convert the interval-valued trapezoidal neutrosophic fuzzy fixed cost, transportation time and carbon emission.

3.4. Ranking of interval-valued trapezoidal neutrosophic fuzzy numbers

The mathematical connection between two interval-valued trapezoidal neutrosophic fuzzy numbers utilizing interval numbers is provided in the reference [36]. Let’s consider two interval-valued trapezoidal neutrosophic fuzzy numbers as

$$\widetilde{T_n}1 = \left\langle (a_1, a_2, a_3, a_4); [\rho 1^L, \rho 1^R], [\kappa 1^L, \kappa 1^R], [\nu 1^L, \nu 1^R]\right\rangle$$

$$\widetilde{T_n}2 = \left\langle (b_1, b_2, b_3, b_4); [\rho 2^L, \rho 2^R], [\kappa 2^L, \kappa 2^R], [\nu 2^L, \nu 2^R]\right\rangle.$$

The interval form of the two interval-valued trapezoidal neutrosophic fuzzy numbers is assumed using the section 3.3.2, with the representation of mid-point and width (half-width) as $R(\widetilde{T_n 1}) = \langle m(\widetilde{T_n 1}), w(\widetilde{T_n 1})\rangle$ and $R(\widetilde{T_n 2}) = \langle m(\widetilde{T_n 2}), w(\widetilde{T_n 2})\rangle$.

Then, the acceptibility grade $A\left(R(\widetilde{T_n 1}) \lessdot R(\widetilde{T_n 2})\right)$ of the first interval is considered to be inferior to the second interval is given as

$$A\left(\widetilde{T_n 1} \lessdot \widetilde{T_n 2}\right) = A\left(R(\widetilde{T_n 1}) \lessdot R(\widetilde{T_n 2})\right) = \frac{m(\widetilde{T_n 2}) - m(\widetilde{T_n 1})}{w(\widetilde{T_n 2}) + w(\widetilde{T_n 1})},$$

where $w(\widetilde{T_n 2}) + w(\widetilde{T_n 1}) \neq 0$.
And it is explained in the following manner:

(i) If $A(\widetilde{T_n 1} \lessdot \widetilde{T_n 2}) = 0$, then the acceptance of $\widetilde{T_n 1}$ inferior to $\widetilde{T_n 2}$ is not allowed.

(ii) If $0 < A(\widetilde{T_n 1} \lessdot \widetilde{T_n 2}) < 1$ & $A(\widetilde{T_n 1} \lessdot \widetilde{T_n 2}) \geq 1$, then then the acceptance of $\widetilde{T_n 1}$ inferior to $\widetilde{T_n 2}$ is allowed.

3.4.1. Model 2: Interval multi-objective multi-item solid transportation problem

Minimizing transportation cost

$$Minimize\ Z_1^I = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{c}\sum_{p=1}^{r} C_{ijkp}^I\, x_{ijkp}^I\, L_{ijp}^I + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{c}\sum_{p=1}^{r} F_{ijkp}^I\, y_{ijkp} \tag{9}$$

Minimizing transportation time

$$Minimize\ Z_2^I = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{c}\sum_{p=1}^{r} T_{ijkp}^I\, x_{ijkp}^I\, L_{ijp}^I \tag{10}$$

Minimizing carbon emission

$$Minimize\ Z_3^I = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{c}\sum_{p=1}^{r} P_{ijkp}^I\, x_{ijkp}^I\, L_{ijp}^I \tag{11}$$

subject to

$$\sum_{j=1}^{n}\sum_{k=1}^{c}\sum_{p=1}^{r} x^I_{ijkp} \leq A^I_i, \quad i = 1,2,...,m \tag{12}$$

$$\sum_{i=1}^{m}\sum_{k=1}^{c}\sum_{p=1}^{r} x^I_{ijkp} \geq B^I_j, \quad j = 1,2,...,n \tag{13}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x^I_{ijkp} \leq E^I_{kp}, \qquad k = 1,2,...,c \ \& \ p = 1,2,...,r \tag{14}$$

$$x^I_{ijkp} \geq 0^I \ \forall \ i,j,k,p \tag{15}$$

where, $C^I_{ijkp} = \langle \mathscr{M}(C^I_{ijkp}), \mathscr{W}(C^I_{ijkp}) \rangle$, $x^I_{ijkp} = \langle \mathscr{M}(x^I_{ijkp}), \mathscr{W}(x^I_{ijkp}) \rangle$,

$L^I_{ijp} = \langle \mathscr{M}(L^I_{ijp}), \mathscr{W}(L^I_{ijp}) \rangle, F^I_{ijkp} = \langle \mathscr{M}(F^I_{ijkp}), \mathscr{W}(F^I_{ijkp}) \rangle$,

$T^I_{ijkp} = \langle \mathscr{M}(T^I_{ijkp}), \mathscr{W}(T^I_{ijkp}) \rangle$, $P^I_{ijkp} = \langle \mathscr{M}(P^I_{ijkp}), \mathscr{W}(P^I_{ijkp}) \rangle$

$A^I_i = \langle \mathscr{M}(A^I_i), \mathscr{W}(A^I_i) \rangle$, $B^I_j = \langle \mathscr{M}(B^I_j), \mathscr{W}(B^I_j) \rangle$,

$E^I_{kp} = \langle \mathscr{M}(E^I_{kp}), \mathscr{W}(E^I_{kp}) \rangle$, and $Z^I_R = \langle \mathscr{M}(Z^I_R), \mathscr{W}(Z^I_R) \rangle$, $R = 1,2,3$

4. PROPOSED INTERVAL-VALUED TRAPEZOIDAL NEUTROSOPHIC FUZZY PROGRAMMING APPROACH USING INTERVAL NUMBERS

This section presents the proposed interval-valued trapezoidal neutrosophic fuzzy (IVTNF) programming approach using interval numbers.

Step (1) Convert Model 1 into Model 2 using section 3.3.2.

Step (2) Check whether the problem considered in Model 2 is balanced or not.

i.e., $\mathscr{R}(\sum_{i=1}^{m} A^N_i) = \mathscr{R}(\sum_{j=1}^{n} B^N_j) = \mathscr{R}(\sum_{k=1}^{c}\sum_{p=1}^{r} E^N_{kp})$.

i.e., $\sum_{i=1}^{m} A^I_i = \sum_{j=1}^{n} B^I_j = \sum_{k=1}^{c}\sum_{p=1}^{r} E^I_{kp}$.

Now, check whether $\sum_{i=1}^{m} A^I_i = \sum_{j=1}^{n} B^I_j$ holds.

Let $\sum_{i=1}^{m} A^I_i = \langle \mathscr{M}(A^I_i), \mathscr{W}(A^I_i) \rangle$ and $\sum_{j=1}^{n} B^I_j = \langle \mathscr{M}(B^I_j), \mathscr{W}(B^I_j) \rangle$

Case (i) If $\sum_{i=1}^{m} A^I_i = \sum_{j=1}^{n} B^I_j$, then move to step (3).

Case (ii) If $\sum_{i=1}^{m} A^I_i < \sum_{j=1}^{n} B^I_j$, then introduce dummy interval availability equal to

$$\sum_{j=1}^{n} B^I_i - \sum_{i=1}^{m} A^I_i = \langle \mathscr{M}(B^I_j), \mathscr{W}(B^I_j) \rangle - \langle \mathscr{M}(A^I_i), \mathscr{W}(A^I_i) \rangle$$

$$= \langle \mathscr{M}(B^I_j) - \mathscr{M}(A^I_i), \max\{\mathscr{W}(A^I_i), \mathscr{W}(B^I_j)\} \rangle$$

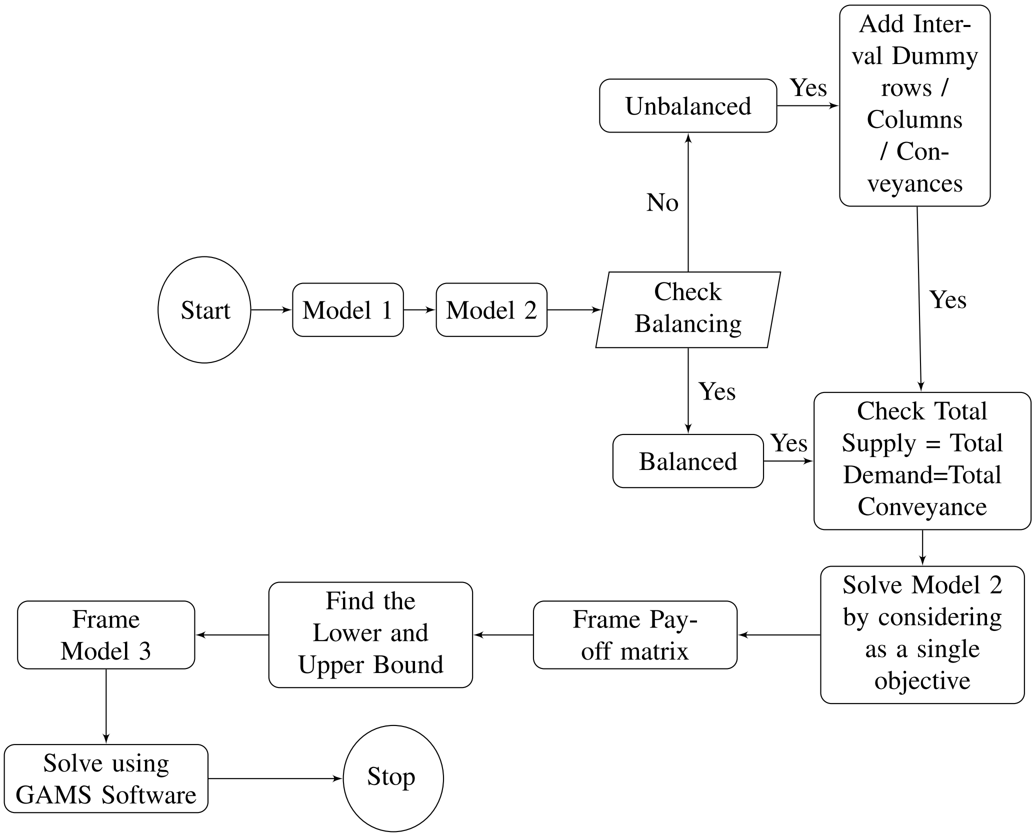

Figure 1: Flowchart of Proposed Algorithm

Case (iii) If $\sum_{i=1}^{m} A_i^I > \sum_{j=1}^{n} B_j^I$, then introduce dummy interval demand equal to

$$\begin{aligned}\sum_{i=1}^{m} A_i^I - \sum_{j=1}^{n} B_j^I &= \langle \mathscr{M}(A_i^I), \mathscr{W}(A_i^I) \rangle - \langle \mathscr{M}(B_j^I), \mathscr{W}(B_j^I) \rangle \\ &= \langle \mathscr{M}(A_i^I) - \mathscr{M}(B_j^I), \max\{\mathscr{W}(A_i^I), \mathscr{W}(B_j^I)\} \rangle\end{aligned}$$

Step (3) Check whether $\sum_{i=1}^{m} A_i^I = \sum_{j=1}^{n} B_j^I = \sum_{k=1}^{c} \sum_{p=1}^{r} E_{kp}^I$ holds.
Let us consider

$$\sum_{i=1}^{m} A_i^I = \sum_{j=1}^{n} B_j^I = T = \langle \mathscr{M}(T_q^I), \mathscr{W}(T_q^I) \rangle \text{ and } \sum_{k=1}^{c} \sum_{p=1}^{r} E_{kp}^I = \langle \mathscr{M}(E_{kp}^I), \mathscr{W}(E_{kp}^I) \rangle$$

Case (i) If $T = \sum_{k=1}^{c} \sum_{p=1}^{r} E_{kp}^I$, then proceed with step (4).

Case (ii) If $T < \sum_{k=1}^{c} \sum_{p=1}^{r} E_{kp}^I$, then check whether interval dummy availability / demand was added in step (2). Then go through with the following steps.

Subcase (ii(a)) If any one of interval dummy source / destination was already added, then increase that corresponding interval dummy source / destination by using the formula

$$\begin{aligned}\sum_{k=1}^{c}\sum_{p=1}^{r} E_{kp}^{I} - T &= \left\langle \mathscr{M}(E_{kp}^{I}), \mathscr{W}(E_{kp}^{I}) \right\rangle - \left\langle \mathscr{M}(T_{q}^{I}), \mathscr{W}(T_{q}^{I}) \right\rangle \\ &= \left\langle \mathscr{M}(E_{kp}^{I}) - \mathscr{M}(T_{r}^{I}),\ \max\{\mathscr{M}(E_{kp}^{I}), \mathscr{W}(T_{r}^{I})\} \right\rangle \\ &= \left\langle \mathscr{M}(E1_{kp}^{I}), \mathscr{W}(E1_{kp}^{I}) \right\rangle \text{ (say).}\end{aligned}$$

Subcase (ii(b)) If neither an interval dummy source nor an interval dummy destination were introduced in Step 2, then add an interval dummy source and an interval dummy destination with availability and demand equal to $\left\langle \mathscr{M}(E1_{kp}^{I}), \mathscr{W}(E1_{kp}^{I}) \right\rangle$

Case (iii) If $T > \sum_{k=1}^{c}\sum_{p=1}^{r} E_{kp}^{I}$, then check whether interval dummy availability / demand was added in step (2). Then go through with the following steps.

Subcase (iii(a)) If any one of interval dummy source / destination was already added, then increase that corresponding interval dummy source / destination by using the formula

$$\begin{aligned}T - \sum_{k=1}^{c}\sum_{p=1}^{r} E_{kp}^{I} &= \left\langle \mathscr{M}(T_{q}^{I}), \mathscr{W}(T_{q}^{I}) \right\rangle - \left\langle \mathscr{M}(E_{kp}^{I}), \mathscr{W}(E_{kp}^{I}) \right\rangle \\ &= \left\langle \mathscr{M}(T_{q}^{I}) - \mathscr{M}(E_{kp}^{I}),\ \max\{\mathscr{M}(E_{kp}^{I}), \mathscr{W}(T_{q}^{I})\} \right\rangle \\ &= \left\langle \mathscr{M}(E1_{kp}^{I}), \mathscr{W}(E1_{kp}^{I}) \right\rangle \text{(say).}\end{aligned}$$

And also, introduce an dummy interval destination / interval source having the interval demand / interval availability as $\left\langle \mathscr{M}(E1_{kp}^{I}), \mathscr{W}(E1_{kp}^{I}) \right\rangle$.

Subcase (iii(b)) If neither an dummy interval source nor an dummy interval destination was introduced in Step 2, then add an dummy interval source as well as an dummy interval destination having interval availability and interval demand equal to $\left\langle \mathscr{M}(E1_{kp}^{I}), \mathscr{W}(E1_{kp}^{I}) \right\rangle$.

The transportation cost from the newly constructed source to all destinations through any conveyances is assumed to be zero. Model 2 has now transformed into a balanced multi-objective multi-item interval transportation problem.

Step (4) Determine the most efficient solution $X_{R}^{I},\ R = 1,2,3$ for each specific case. The values of X_{R}^{I} for model 2 can be obtained by treating the multi-objective as a single goal. That is, focus on one objective function at a time and disregard the others.

Solve Model 2 using the following two steps.

(i) Use GAMS software to find $\mathscr{M}(Z_{R}^{I}),\ R = 1,2,3$

(ii) Calculation for the width (half-width) $\mathscr{W}(Z_{R}^{I}),\ R = 1,2,3$ is given as follows.

$$\mathscr{W}(Z_{1}^{I}) = Max\{\mathscr{W}(C_{ijkp}^{I}), \mathscr{W}(L_{ijp}^{I}), \mathscr{W}(F_{ijkp}^{I}), \mathscr{W}(A_{i}^{I}), \mathscr{W}(B_{j}^{I}), \mathscr{W}(E_{kp}^{I})\}$$
$$\mathscr{W}(Z_{2}^{I}) = Max\{\mathscr{W}(T_{ijkp}^{I}), \mathscr{W}(L_{ijp}^{I}), \mathscr{W}(A_{i}^{I}), \mathscr{W}(B_{j}^{I}), \mathscr{W}(E_{kp}^{I})\}$$
$$\mathscr{W}(Z_{3}^{I}) = Max\{\mathscr{W}(P_{ijkp}^{I}), \mathscr{W}(L_{ijp}^{I}), \mathscr{W}(A_{i}^{I}), \mathscr{W}(B_{j}^{I}), \mathscr{W}(E_{kp}^{I})\},$$

where $i = 1,2,...,m,\ j = 1,2,...,n,\ k = 1,2,...,c\ \&\ p = 1,2,...,r.$

Therefore, the respective optimal solutions X_R^I are obtained for different R objectives.

Step (5) Find out the objective values using the above calculated optimal solutions and frame out a pay-off matrix using the objective values as

$$\begin{array}{c} \\ X_1^I \\ X_2^I \\ \vdots \\ X_R^I \end{array} \begin{array}{c} \begin{array}{cccc} Z_1^I & Z_2^I & \cdots & Z_R^I \end{array} \\ \begin{pmatrix} Z_1^I(X_1^I) & Z_2^I(X_1^I) & \cdots & Z_R^I(X_1^I) \\ Z_1^I(X_2^I) & Z_2^I(X_2^I) & \cdots & Z_R^I(X_2^I) \\ \vdots & \vdots & \ddots & \vdots \\ Z_1^I(X_R^I) & Z_2^I(X_R^I) & \cdots & Z_R^I(X_R^I) \end{pmatrix} \end{array}$$

Step (6) Determine the lower and upper bounds for each objective function by the following instructions.

(i) The lower bound (L_R^{IT}) and upper bound (U_R^{IT}) of truth membership function in the form of interval is calculated using $L_R^{IT} = \min[Z_R^I(X_R)]$ and $U_R^{IT} = \max[Z_R^I(X_R)],\ R = 1,2,3$ respectively.

(ii) The lower bound (L_R^{II}) and upper bound (U_R^{II}) of indeterminacy membership function in the form of interval is calculated using $L_R^{II} = L_R^{IT}$ and $U_R^{II} = L_R^{IT} + s(U_R^{IT} - L_R^{IT}),\ R = 1,2,3$ respectively.

(iii) The lower bound (L_R^{IF}) and upper bound (U_R^{IF}) of falsity membership function in the form of interval is calculated using $L_R^{IF} = L_R^{IT} + t(U_R^{IT} - L_R^{IT})$ and $U_R^{IF} = U_R^{IT}$, $R = 1,2,3$ respectively, where s and t are chosen by the decision maker between $(0,1)$ for indeterminacy and falsity.

Then formulate the lower $\mu^L(Z_R^I(x))$ and upper $\mu^U(Z_R^I(x))$ truth membership functions as follows.

$$\mu^L(Z_R^I(x)) = \begin{cases} 1, & Z_R^I(x) < L_R^{IT} \\ \rho\,\frac{U_R^{IT} - Z_R^I(x)}{U_R^{IT} - L_R^{IT}}, & L_R^{IT} \le Z_R^I(x) \le U_R^{IT} \\ 0, & Z_R^I(x) > U_R^{IT} \end{cases}$$

$$\mu^U(Z_R^I(x)) = \begin{cases} 1, & Z_R^I(x) < L_R^{IT} \\ \frac{U_R^{IT} - Z_R^I(x)}{U_R^{IT} - L_R^{IT}}, & L_R^{IT} \le Z_R^I(x) \le U_R^{IT} \\ 0, & Z_R^I(x) > U_R^{IT} \end{cases}$$

Similarly the lower $\pi^L(Z_R^I(x))$ and upper $\pi^U(Z_R^I(x))$ indeterminacy membership functions as follows.

$$\pi^L(Z_R^I(x)) = \begin{cases} 1, & Z_R^I(x) < L_R^{II} \\ \rho\,\frac{U_R^{II} - Z_R^I(x)}{U_R^{II} - L_R^{II}}, & L_R^{II} \le Z_R^I(x) \le U_R^{II} \\ 0, & Z_R^I(x) > U_R^{II} \end{cases}$$

$$\pi^U(Z_R^I(x)) = \begin{cases} 1, & Z_R^I(x) < L_R^{II} \\ \frac{U_R^{II} - Z_R^I(x)}{U_R^{II} - L_R^{II}}, & L_R^{II} \le Z_R^I(x) \le U_R^{II} \\ 0, & \text{if } Z_R^I(x) > U_R^{II} \end{cases}$$

Formulate the lower $\nu^L(Z_R^I(x))$ and upper $\nu^U(Z_R^I(x))$ falsity membership functions as follows.

$$\nu^L(Z_R^I(x)) = \begin{cases} 0, & Z_R^I(x) < L_R^{IF} \\ \rho\,\frac{Z_R^I(x) - L_R^{IF}}{U_R^{IF} - L_R^{IF}}, & L_R^{IF} \le Z_R^I(x) \le U_R^{IF} \\ 1, & Z_R^I(x) > U_R^{IF} \end{cases}$$

$$\nu^U(Z_R^I(x)) = \begin{cases} 0, & Z_R^I(x) < L_R^{IF} \\ \frac{Z_R^I(x) - L_R^{IF}}{U_R^{IF} - L_R^{IF}}, & L_R^{IF} \le Z_R^I(x) \le U_R^{IF} \\ 1, & Z_R^I(x) > U_R^{IF} \end{cases}$$

where $0 \le \rho \le 1,\ U_R^{IT} \ne L_R^{IT},\ U_R^{II} \ne L_R^{II}$ and $U_R^{IF} \ne L_R^{IF}$.

Step (7) By employing the membership functions mentioned above, Model 2 can be formulated as Model 3 and is presented as follows.

Model 3

$$\begin{aligned} &\text{Maximize } \gamma - \zeta - \tau \\ &\text{subject to } \mu^L(Z_R^I(x)) \ge \gamma,\ \ \mu^U(Z_R^I(x)) \ge \gamma, \\ &\qquad \pi^L(Z_R^I(x)) \le \zeta,\ \ \pi^U(Z_R^I(x)) \le \zeta, \\ &\qquad \nu^L(Z_R^I(x)) \le \tau,\ \ \nu^U(Z_R^I(x)) \le \tau \\ &\qquad \gamma \ge \zeta,\ \gamma \ge \tau,\ 0 \le \gamma \le 1,\ 0 \le \zeta \le 1,\ 0 \le \tau \le 1 \\ &\qquad \text{constraints (12) to (15)} \end{aligned}$$

Model 3 again written as

$$\begin{aligned} &\text{Maximize } \gamma - \zeta - \tau \\ &\text{subject to } Z_R^I(x) + \gamma(U_R^{IT} - L_R^{IT}) \le U_R^{IT},\ \ Z_R^I(x) + \frac{\gamma}{\rho}(U_R^{IT} - L_R^{IT}) \le U_R^{IT}, \\ &\qquad Z_R^I(x) + \zeta(U_R^{II} - L_R^{II}) \ge U_R^{II},\ \ Z_R^I(x) + \frac{\zeta}{\rho}(U_R^{II} - L_R^{II}) \ge U_R^{II}, \\ &\qquad Z_R^I(x) - \tau(U_R^{IF} - L_R^{IF}) \le L_R^{IF},\ \ Z_R^I(x) - \frac{\tau}{\rho}(U_R^{IF} - L_R^{IF}) \le U_R^{IF} \\ &\qquad \gamma \ge \zeta,\ \gamma \ge \tau,\ 0 \le \gamma \le 1,\ 0 \le \zeta \le 1,\ 0 \le \tau \le 1 \\ &\qquad \text{constraints (12) to (15).} \end{aligned}$$

Here R=1,2,3 and γ, ζ, τ represents the degree of truth, indeterminacy and falsity respectively.

Step (8) Use GAMS software to solve Model 3 to find the efficient solution.

5. NUMERICAL ILLUSTRATION

The construction company intends to convey the materials from two distinct factories to two separate building sites by employing two distinct types of vehicles (V) via two distinct routes (Paths(P)). The objective of the decision makers is to determine the quantity of product transportation by minimizing transportation expenses, duration, and carbon emissions. The problem's foundation is determined by referencing [26], and we analyze it inside a framework of interval-valued trapezoidal neutrosophic conditions. Approach the topic from a neutrosophic perspective, allowing decision makers to thoroughly assess their ideas from all potential angles, encompassing positive, negative and neutral aspects. The interval-valued trapezoidal neutrosophic fuzzy (IVTNF) transportation cost are given in Table 1. The IVTNF fixed cost, IVTNF carbon emission, IVTNF transportation time, IVTNF supply and IVTNF demand are displayed in Table 2, 3 and 4 and 5 respectively.

Table 1: Interval-valued trapezoidal neutrosophic fuzzy transportation cost

j →	V c	IVTNF tranportation cost (C^N_{ijkp})	
i ↓	- P r	D_1	D_2
Sources 1	1-1	(3,5,6,8); [0.8,1.0]; [0.2,0.4]; [0.1,0.3]	(5,6,7,8); [0.8,1.0]; [0.0,0.2]; [0.1,0.3]
	1-2	(2,4,6,9); [0.6,0.8]; [0.1,0.3]; [0.0,0.2]	(4,6,9,11); [0.8,1.0]; [0.1,0.3]; [0,0.2]
	2-1	(4,5,6,7); [0.7,0.9]; [0.1,0.3]; [0.0,0.2]	(2,5,6,9); [0.6,0.8]; [0.2,0.4]; [0.0,0.2]
	2-2	(5,7,9,11); [0.7,0.9]; [0.2,0.4]; [0.1,0.3]	(4,6,9,11); [0.8,1.0]; [0.1,0.3]; [0,0.2]
Sources 2	1-1	(5,9,11,13); [0.7,0.9]; [0.1,0.3]; [0,0.2]	(7,9,11,13); [0.8,1.0]; [0,0.2]; [0.2,0.4]
	1-2	(6,7,8,9); [0.7,0.9]; [0.2,0.4]; [0,0.2]	(6,8,10,12); [0.6,0.8]; [0.2,0.4]; [0.1,.3]
	2-1	(2,3,5,7); [0.6,0.8]; [0.1,0.3]; [0,0.2]	(3,4,5,6); [0.6,0.8]; [0,0.2]; [0.2,0.4]
	2-2	(4,6,8,10); [0.7,0.9]; [0.0,0.2]; [0.1,0.3]	(4,5,6,8); [0.7,0.9]; [0.0,0.2]; [0.2,0.4]

Table 2: Interval-valued trapezoidal neutrosophic fuzzy fixed cost

j →	V c	IVTNF fixed cost (F^N_{ijkp})	
i ↓	- P r	D_1	D_2
Sources 1	1-1	(2,3,4,5); [0.8,1.0]; [0.1,0.3]; [0.0,0.2]	(5,6,7,8); [0.6,0.8]; [0.2,0.4]; [0.0,0.2]
	1-2	(3,5,7,8); [0.6,0.8]; [0.1,0.3]; [0.2,0.4]	(6,7,8,9); [0.6,0.8]; [0.1,0.3]; [0.1,0.3]
	2-1	(4,5,6,8); [0.7,0.9]; [0.1,0.3]; [0.1,0.3]	(3,4,5,6); [0.6,0.8]; [0.0,0.2]; [0.0,0.2]
	2-2	(3,4,5,6); [0.7,0.9]; [0.0,0.2]; [0.1,0.3]	(4,5,6,7); [0.7,0.9]; [0.2,0.4]; [0.0,0.2]
Sources 2	1-1	(6,7,8,10); [0.7,0.9]; [0.0,0.2]; [0.0,0.2]	(3,5,7,10); [0.8,1.0]; [0.0,0.2]; [0.0,0.2]
	1-2	(5,7,8,10); [0.6,0.8]; [0.1,0.3]; [0.0,0.2]	(5,6,7,9); [0.8,1.0]; [0.1,0.3]; [0.0,0.2]
	2-1	(4,5,7,9); [0.6,0.8]; [0.1,0.3]; [0,0.2]	(4,5,7,9); [0.7,0.9]; [0.1,0.3]; [0.0,0.2]
	2-2	(4,6,7,8); [0.7,0.9]; [0.1,0.3]; [0.2,0.4]	(5,6,7,9); [0.7,0.9]; [0.2,0.4]; [0.1,0.3]

Table 3: Interval-valued trapezoidal neutrosophic fuzzy carbon emission

j →	V c	IVTNF carbon emission (P^N_{ijkp})	
i ↓	- P r	D_1	D_2
Sources 1	1-1	(7,8,9,10); [0.7,0.9]; [0.1,0.3]; [0.0,0.2]	(4,5,6,7); [0.6,0.8]; [0.1,0.3]; [0.1,0.3]
	1-2	(6,7,8,9); [0.6,0.8]; [0.0,0.2; [0.1,0.3]	(5,6,8,9); [0.7,0.9]; [0.1,0.3]; [0.1,0.3]
	2-1	(5,6,7,8); [0.8,1.0]; [0.2,0.4]; [0.1,0.3]	(3,5,7,9); [0.7,0.9]; [0.1,0.3]; [0.2,0.4]
	2-2	(6,7,9,10); [0.7,0.9]; [0.2,0.4]; [0.1,0.3]	(5,6,7,10); [0.6,0.8]; [0.0,0.2]; [0.0,0.2]
Sources 2	1-1	(7,9,10,11); [0.7,0.9]; [0.1,0.3]; [0.2,0.4]	(3,4,6,7); [0.7,0.9]; [0.0,0.2]; [0.1,0.3]
	1-2	(3,5,7,8); [0.7,0.9]; [0.0,0.2]; [0.0,0.2]	(4,5,7,10); [0.7,0.9]; [0.1,0.3]; [0.2,0.4]
	2-1	(6,7,8,9); [0.8,1.0]; [0.1,0.3]; [0.1,0.3]	(5,6,9,10); [0.6,0.8]; [0.1,0.3]; [0.1,0.3]
	2-2	(3,4,6,9); [0.8,1.0]; [0.0,0.2]; [0.0,0.2]	(4,6,8,10); [0.6,0.8]; [0,0.2]; [0,0.2]

Table 4: Interval-valued trapezoidal neutrosophic fuzzy transportation time

j →	V c	IVTNF transportation time (T^N_{ijkp})	
i ↓	- P r	D_1	D_2
Sources 1	1-1	(3,5,7,11);[0.6,0.8];[0.2,0.4];[0.0,0.2]	(3,4,6,8);[0.6,0.8];[0.0,0.2];[0.0,0.2]
	1-2	(4,5,6,8);[0.7,0.9];[0.2,0.4;[0.2,0.4]	(7,9,10,11);[0.6,0.8];[0.2,0.4;[0.2,0.4]
	2-1	(5,7,9,12);[0.7,0.9];[0.1,0.3];[0.1,0.3]	(5,6,7,8);[0.7,0.9];[0.1,0.3];[0.1,0.3]
	2-2	(6,7,9,10);[0.7,0.9];[0.2,0.4];[0.1,0.3]	(7,9,10,12);[0.7,0.9];[0.0,0.2];[0.1,0.3]
Sources 2	1-1	(4,6,8,10);[0.6,0.8];[0.2,0.4];[0.1,0.3]	(2,5,7,9);[0.8,1.0];[0.1,0.3];[0.1,0.3]
	1-2	(3,5,7,9);[0.7,0.9];[0.0,0.2];[0.1,0.3]	(3,6,8,9);[0.8,1.0];[0.0,0.2];[0.0,0.2]
	2-1	(3,5,7,8);[0.7,0.9];[0.1,0.3];[0.0,0.2]	(3,4,7,9);[0.7,0.9];[0,0.2];[0,0.2]
	2-2	(4,6,7,9);[0.6,0.8];[0.2,0.4];[0.1,0.3]	(4,5,6,7);[0.6,0.8];[0.1,0.3];[0.1,0.3]

Table 5: Interval-valued trapezoidal neutrosophic availability and demand and its corresponding interval form

i	A^N_i	A^I_i	j	D^N_j	D^I_j
1	(5,7,9,11);[0.8,1.0];[0.0,0.2];[0.0,0.2]	⟨8,1⟩	1	(4,6,8,9);[0.6,0.8];[0.0,0.2];[0.1,0.3]	⟨7.33,1⟩
2	(6,8,10,12);[0.7,0.9];[0.0,0.2];[0.1,0.3]	⟨9,1⟩	2	(5,6,7,8);[0.8,1.0];[0.1,0.3];[0.0,0.2]	⟨6.5,0.5⟩

The IVTNF conveyance and its corresponding conversion to interval numbers are given in Table 6. The distances of each routes in real form and its interval form are available in Table 7. Using section 3.3.2, Model 1 is transformed into Model 2 by converting the IVTNF parameters into interval numbers and it is given in Table 8.

Table 6: Interval-valued trapezoidal neutrosophic conveyance (Vehicle k) and its corresponding interval form

k	E^N_{k1}	E^I_{k1}	k	E^N_{k2}	E^I_{k2}
1	(3,5,6,8);[0.6,0.8];[0.2,0.4];;[0.1,0.3]	⟨5.5,0.3⟩	1	(4,5,6,7);[0.8,1.0];[0.0,0.2];;[0.1,0.3]	⟨5.5,0.5⟩
2	(4,6,7,9);[0.7,0.9];[0.1,0.3];[0.1,0.3]	⟨6.5,0.3⟩	2	(5,7,8,10);[0.7,0.9];[0.0,0.2];[0.1,0.3]	⟨7.5,0.5⟩

Table 7: Distance of routes in real and its corresponding interval form

Availability (i)	Demand (j)	L_{ij1}	L^I_{ij1}	L_{ij2}	L^I_{ij2}
1	1	12	⟨12,0⟩	34	⟨34,0⟩
	2	18	⟨18,0⟩	56	⟨56,0⟩
2	1	45	⟨45,0⟩	16	⟨16,0⟩
	2	37	⟨37,0⟩	48	⟨48,0⟩

Table 8: Transportation cost(C^I_{ijkp}), fixed cost(F^I_{ijkp}), carbon emission(P^I_{ijkp}) and transportation time(T^I_{ijkp}) in interval form

j → i ↓	Vehicle c - Path r	C^I_{ijkp} D_1	C^I_{ijkp} D_2	F^I_{ijkp} D_1	F^I_{ijkp} D_2
Sources 1	V1-P1	$\langle 5.50, 0.50\rangle$	$\langle 6.50, 0.50\rangle$	$\langle 3.50, 0.50\rangle$	$\langle 6.50, 0.50\rangle$
	V1-P2	$\langle 4.67, 1\rangle$	$\langle 7.50, 1.50\rangle$	$\langle 6.33, 0.83\rangle$	$\langle 7.50, 0.39\rangle$
	V2-P1	$\langle 5.50, 0.50\rangle$	$\langle 5.50, 0.50\rangle$	$\langle 5.29, 0.33\rangle$	$\langle 4.50, 0.50\rangle$
	V2-P2	$\langle 8, 0.78\rangle$	$\langle 7.50, 1.50\rangle$	$\langle 4.50, 0.50\rangle$	$\langle 5.50, 0.50\rangle$
Sources 2	V1-P1	$\langle 10.43, 1\rangle$	$\langle 10, 1\rangle$	$\langle 7.29, 0.50\rangle$	$\langle 5.88, 1\rangle$
	V1-P2	$\langle 7.50, 0.50\rangle$	$\langle 9, 0.78\rangle$	$\langle 7.50, 0.50\rangle$	$\langle 6.38, 050\rangle$
	V2-P1	$\langle 3.67, 1\rangle$	$\langle 4.50, 0.50\rangle$	$\langle 5.67, 1\rangle$	$\langle 5.79, 1\rangle$
	V2-P2	$\langle 7, 1\rangle$	$\langle 5.29, 0.50\rangle$	$\langle 6.71, 0.33\rangle$	$\langle 6.29, 0.33\rangle$
j → i ↓	Vehicle c - Path r	(P^I_{ijkp}) D_1	(P^I_{ijkp}) D_2	(T^I_{ijkp}) D_1	(T^I_{ijkp}) D_2
Sources 1	V1-P1	$\langle 8.50, 0.50\rangle$	$\langle 5.50, 0.39\rangle$	$\langle 5.33, 1\rangle$	$\langle 4.67, 1\rangle$
	V1-P2	$\langle 7.50, 0.50\rangle$	$\langle 7, 0.89\rangle$	$\langle 5.29, 0.33\rangle$	$\langle 9.83, 0.13\rangle$
	V2-P1	$\langle 6.50, 0.50\rangle$	$\langle 6, 0.78\rangle$	$\langle 7.79, 0.72\rangle$	$\langle 6.50, 0.39\rangle$
	V2-P2	$\langle 8, 0.89\rangle$	$\langle 5.83, 0.50\rangle$	$\langle 8, 1\rangle$	$\langle 9.50, 0.50\rangle$
Sources 2	V1-P1	$\langle 9.71, 0.33\rangle$	$\langle 5, 1\rangle$	$\langle 7, 0.78\rangle$	$\langle 6.13, 1\rangle$
	V1-P2	$\langle 6.21, 1\rangle$	$\langle 5.57, 0.78\rangle$	$\langle 6, 1\rangle$	$\langle 7.25, 1\rangle$
	V2-P1	$\langle 7.50, 0.50\rangle$	$\langle 7.5, 1.39\rangle$	$\langle 6.21, 1\rangle$	$\langle 5.29, 1.50\rangle$
	V2-P2	$\langle 4.75, 1\rangle$	$\langle 7, 1\rangle$	$\langle 6.50, 0.28\rangle$	$\langle 5.50, 0.39\rangle$

5.1. Problem Discussion

First check whether the problem is balanced or not i.e., $\sum_{i=1}^{m} A_i^I = \sum_{j=1}^{n} B_j^I = \sum_{k=1}^{c}\sum_{p=1}^{r} E_{kp}^I$. From Table 6 & 7, $\sum_{i=1}^{m} A_i^I = \langle 17, 1\rangle$ and $\sum_{j=1}^{n} B_j^I = \langle 13.83, 1\rangle$. Hence $\sum_{i=1}^{m} A_i^I > \sum_{j=1}^{n} B_j^I$. Then by step 2 (case (iii)), increase the total interval demand by $\langle 3.17, 1\rangle$. Therefore, $\sum_{i=1}^{m} A_i^I = \langle 17, 1\rangle = \sum_{j=1}^{n} B_j^I$. Also, the total conveyances is given as $\sum_{k=1}^{c}\sum_{p=1}^{r} E_{kp}^I = \langle 25, 0.5\rangle$. Hence by step (3), $T = \langle 17, 1\rangle$. Therefore, $\sum_{i=1}^{m} A_i^I = \sum_{j=1}^{n} B_j^I < \sum_{k=1}^{c}\sum_{p=1}^{r} E_{kp}^I$. Hence by step 3, subcase (ii (a)), increase the total interval demand that already added by $\langle 8, 1\rangle$. And introduce dummy interval source and its interval availability as $\langle 8, 1\rangle$. Hence the problem is balanced. i.e., $\sum_{i=1}^{m} A_i^I = \sum_{j=1}^{n} B_j^I = \sum_{k=1}^{c}\sum_{p=1}^{r} E_{kp}^I = \langle 25, 1\rangle$.

Since new interval dummy source and dummy destination are introduced, assume C^I_{3jkp} and C^I_{i3kp}, where $i, j = 1, 2, 3$ and $k, p = 1, 2$.

Now the problem can be written as

$$\begin{aligned}
&\text{Minimize } Z_1^I = \langle 66, 0.5\rangle x^I_{1111} + \langle 66, 0.5\rangle\, x^I_{1121} + \langle 158.78, 1\rangle x^I_{1112} \\
&+ \langle 272, 0.78\rangle x^I_{1122} + \langle 117, 0.5\rangle x^I_{1211} + \langle 99, 0.50\rangle x^I_{1221} + \langle 420, 1.50\rangle x^I_{1212} \\
&+ \langle 420, 1.50\rangle x^I_{1222} + \langle 469.35, 1\rangle x^I_{2111} + \langle 165.15, 1\rangle x^I_{2121} + \langle 120, 0.50\rangle x^I_{2112} \\
&+ \langle 112, 1\rangle x^I_{2122} + \langle 370, 1\rangle x^I_{2211} + \langle 166.5, 0.5\rangle x^I_{2221} + \langle 432, 0.78\rangle x^I_{2212} \\
&+ \langle 258.9, 0.5\rangle x^I_{2222} + \langle 95.13, 1\rangle
\end{aligned}$$

$$\text{Minimize } Z_2^I = \langle 63.96, 1\rangle x_{1111}^I + \langle 93.48, 0.72\rangle x_{1121}^I + \langle 179.86, 0.33\rangle x_{1112}^I$$
$$+ \langle 272, 1\rangle x_{1122}^I + \langle 84.06, 1\rangle x_{1211}^I + \langle 117, 0.39\rangle x_{1221}^I + \langle 550.48, 0.13\rangle x_{1212}^I$$
$$+ \langle 532, 0.5\rangle x_{1222}^I + \langle 315, 0.78\rangle x_{2111}^I + \langle 279.45, 1\rangle x_{2121}^I + \langle 96, 1\rangle x_{2112}^I$$
$$+ \langle 104, 0.28\rangle x_{2122}^I + \langle 226.81, 1\rangle x_{2211}^I + \langle 195.73, 1.50\rangle x_{2221}^I + \langle 348, 1\rangle x_{2212}^I$$
$$+ \langle 264, 0.39\rangle x_{2222}^I$$
$$\text{Minimize } Z_3^I = \langle 102, 0.5\rangle x_{1111}^I + \langle 78, 0.5\rangle x_{1121}^I + \langle 255, 0.5\rangle x_{1112}^I$$
$$+ \langle 272, 0.89\rangle x_{1122}^I + \langle 99, 0.39\rangle x_{1211}^I + \langle 108, 0.78\rangle x_{1221}^I + \langle 392, 0.89\rangle x_{1212}^I$$
$$+ \langle 326.5, 0.5\rangle x_{1222}^I + \langle 436.95, 0.3\rangle x_{2111}^I + \langle 337.5, 0.5\rangle x_{2121}^I + \langle 99.36, 1\rangle x_{2112}^I$$
$$+ \langle 76, 1\rangle x_{2122}^I + \langle 185, 1\rangle x_{2211}^I + \langle 277.5, 1.39\rangle x_{2221}^I + \langle 267.36, 0.78\rangle x_{2212}^I$$
$$+ \langle 336, 1\rangle x_{2222}^I$$

subject to

$$x_{1111}^I + x_{1112}^I + x_{1121}^I + x_{1122}^I + x_{1211}^I + x_{1212}^I + x_{1221}^I + x_{1222}^I + x_{1311}^I$$
$$+ x_{1312}^I + x_{1321}^I + x_{1322}^I = \langle 8, 1\rangle$$
$$x_{2111}^I + x_{2112}^I + x_{2121}^I + x_{2122}^I + x_{2211}^I + x_{2212}^I + x_{2221}^I + x_{2222}^I + x_{2311}^I$$
$$+ x_{2312}^I + x_{2321}^I + x_{2322}^I = \langle 9, 1\rangle$$
$$x_{3111}^I + x_{3112}^I + x_{3121}^I + x_{3122}^I + x_{3211}^I + x_{3212}^I + x_{3221}^I + x_{3222}^I + x_{3311}^I$$
$$+ x_{3312}^I + x_{3321}^I + x_{3322}^I = \langle 8, 1\rangle$$
$$x_{1111}^I + x_{1112}^I + x_{1121}^I + x_{1122}^I + x_{2111}^I + x_{2112}^I + x_{2121}^I + x_{2122}^I + x_{3111}^I$$
$$+ x_{3112}^I + x_{3121}^I + x_{3122}^I = \langle 7.33, 1\rangle$$
$$x_{1211}^I + x_{1212}^I + x_{1221}^I + x_{1222}^I + x_{2211}^I + x_{2212}^I + x_{2221}^I + x_{2222}^I + x_{3211}^I$$
$$+ x_{3212}^I + x_{3221}^I + x_{3222}^I = \langle 6.5, 0.5\rangle$$
$$x_{1311}^I + x_{1312}^I + x_{1321}^I + x_{1322}^I + x_{2311}^I + x_{2312}^I + x_{2321}^I + x_{2322}^I + x_{3311}^I + x_{3312}^I$$
$$+ x_{3321}^I + x_{3322}^I = \langle 11.17, 1\rangle$$
$$x_{1111}^I + x_{1211}^I + x_{1311}^I + x_{2111}^I + x_{2211}^I + x_{2311}^I + x_{3111}^I + x_{3211}^I + x_{3311}^I = \langle 5.5, 0.3\rangle$$
$$x_{1112}^I + x_{1212}^I + x_{1312}^I + x_{2112}^I + x_{2212}^I + x_{2312}^I + x_{3112}^I + x_{3212}^I + x_{3312}^I = \langle 5.5, 0.5\rangle$$
$$x_{1121}^I + x_{1221}^I + x_{1321}^I + x_{2121}^I + x_{2221}^I + x_{2321}^I + x_{3121}^I + x_{3221}^I + x_{3321}^I = \langle 6.5, 0.3\rangle$$
$$x_{1122}^I + x_{1222}^I + x_{1322}^I + x_{2122}^I + x_{2222}^I + x_{2322}^I + x_{3122}^I + x_{3222}^I + x_{3322}^I = \langle 7.5, 0.5\rangle$$
$$x_{ijkp}^I \geq 0^I \ \forall\, i, j = 1,2,3 \ \&\ k, p = 1,2.$$

Use step 4, to solve the problem individually and obtain the solution as

$$X_1^I \implies \begin{cases} x_{1121}^I = \langle 5.83, 1.50\rangle, & x_{1322}^I = \langle 2.17, 1.50\rangle,\ x_{2312}^I = \langle 3, 1.50\rangle, \\ x_{2321}^I = \langle 0.67, 1.50\rangle, & x_{2322}^I = \langle 5.33, 1.50\rangle,\ x_{3112}^I = \langle 1.5, 1.50\rangle, \\ x_{3211}^I = \langle 5.5, 1.50\rangle, & x_{3212}^I = \langle 1, 1.50\rangle \text{ and } Z_1^I = \langle 479.91, 1.50\rangle. \end{cases}$$

$$X_2^I \implies \begin{cases} x_{1111}^I = \langle 5.5, 1.50\rangle, & x_{1121}^I = \langle 0.33, 1.50\rangle,\ x_{1322}^I = \langle 2.17, 1.50\rangle, \\ x_{2321}^I = \langle 3.67, 1.50\rangle, & x_{2322}^I = \langle 5.33, 1.50\rangle,\ x_{3121}^I = \langle 1.5, 1.50\rangle, \\ x_{3212}^I = \langle 5.5, 1.50\rangle, & x_{3221}^I = \langle 1, 1.50\rangle \text{ and } Z_2^I = \langle 382.63, 1.50\rangle \end{cases}$$

$$X_3^I \implies \begin{cases} x_{2122}^I = \langle 5.83, 1.39\rangle, & x_{1321}^I = \langle 6.33, 1.39\rangle,\ x_{1322}^I = \langle 1.67, 1.39\rangle, \\ x_{2312}^I = \langle 3, 1.39\rangle, & x_{2321}^I = \langle 0.17, 1.39\rangle,\ x_{3112}^I = \langle 1.5, 1.39\rangle, \\ x_{3211}^I = \langle 5.5, 1.39\rangle, & x_{3212}^I = \langle 1, 1.39\rangle \text{ and } Z_3^I = \langle 443.08, 1.39\rangle \end{cases}$$

By step 5, the pay-off matrix is framed as follows.

$$\begin{array}{c} \\ X_1^I \\ X_2^I \\ X_3^I \end{array} \begin{array}{c} \begin{array}{ccc} \quad Z_1^I \quad & \quad Z_2^I \quad & \quad Z_3^I \quad \end{array} \\ \begin{pmatrix} \langle 479.91, 1.50\rangle & \langle 544.99, 1.50\rangle & \langle 454.74, 1.50\rangle \\ \langle 479.91, 1.50\rangle & \langle 382.63, 1.50\rangle & \langle 586.74, 1.50\rangle \\ \langle 748.09, 1.50\rangle & \langle 606.32, 1.50\rangle & \langle 443.08, 1.39\rangle \end{pmatrix} \end{array}$$

Based on step 5, For each objective function, find the lower and upper bounds of all the three membership functions. Assume the value for s & t as 0.2 and it is shown below.

$$\begin{array}{c} \\ L_R^{IT} \\ U_R^{IT} \\ L_R^{II} \\ U_R^{II} \\ L_R^{IF} \\ U_R^{IF} \end{array} \begin{array}{c} \begin{array}{ccc} \quad R=1 \quad & \quad R=2 \quad & \quad R=3 \quad \end{array} \\ \begin{pmatrix} \langle 479.91, 1.50\rangle & \langle 382.63, 1.50\rangle & \langle 443.08, 1.39\rangle \\ \langle 748.09, 1.50\rangle & \langle 606.32, 1.50\rangle & \langle 586.74, 1.50\rangle \\ \langle 479.91, 1.50\rangle & \langle 382.63, 1.50\rangle & \langle 443.08, 1.39\rangle \\ \langle 533.55, 1.50\rangle & \langle 427.37, 1.50\rangle & \langle 471.81, 1.50\rangle \\ \langle 533.55, 1.50\rangle & \langle 427.37, 1.50\rangle & \langle 471.81, 1.50\rangle \\ \langle 748.09, 1.50\rangle & \langle 606.32, 1.50\rangle & \langle 586.74, 1.50\rangle \end{pmatrix} \end{array}$$

Now using step 6, Model 2 can be converted into Model 3 for the problem is given as follows.

Maximize $\gamma - \zeta - \tau$
subject to

$$Z_1^I(x) + \gamma\langle 268.18, 1.50\rangle \leq \langle 748.09, 1.50\rangle,\ Z_1^I(x) + \frac{\gamma}{\rho}\langle 268.18, 1.50\rangle \leq \langle 748.09, 1.50\rangle,$$

$$Z_1^I(x) + \zeta\langle 53.64, 1.50\rangle \geq \langle 533.55, 1.50\rangle,\ Z_1^I(x) + \frac{\zeta}{\rho}\langle 53.64, 1.50\rangle \geq \langle 533.55, 1.50\rangle$$

$$Z_1^I(x) - \tau\langle 214.54, 1.50\rangle \leq \langle 533.55, 1.50\rangle,\ Z_1^I(x) - \frac{\tau}{\rho}\langle 214.54, 1.50\rangle \leq \langle 533.55, 1.50\rangle$$

$$Z_2^I(x) + \gamma\langle 223.69, 1.50\rangle \leq \langle 606.32, 1.50\rangle,\ Z_2^I(x) + \frac{\gamma}{\rho}\langle 223.69, 1.50\rangle \leq \langle 606.32, 1.50\rangle$$

$$Z_2^I(x) + \zeta\langle 44.74, 1.50\rangle \geq \langle 427.37, 1.50\rangle,\ Z_2^I(x) + \frac{\zeta}{\rho}\langle 44.74, 1.50\rangle \geq \langle 427.37, 1.50\rangle$$

$$Z_2^I(x) - \tau\langle 178.95, 1.50\rangle \leq \langle 427.37, 1.50\rangle,\ Z_2^I(x) - \frac{\tau}{\rho}\langle 178.95, 1.50\rangle \leq \langle 427.37, 1.50\rangle$$

$$Z_3^I(x)+\gamma\langle 143.66, 1.50\rangle \leq \langle 586.74, 1.50\rangle,\ Z_3^I(x)+\frac{\gamma}{\rho}\langle 143.66, 1.50\rangle \leq \langle 586.74, 1.50\rangle$$
$$Z_3^I(x)+\zeta\langle 28.73, 1.50\rangle \geq \langle 471.81, 1.50\rangle,\ Z_3^I(x)+\frac{\zeta}{\rho}\langle 28.73, 1.50\rangle \geq \langle 471.81, 1.50\rangle$$
$$Z_3^I(x)-\tau\langle 114.93, 1.50\rangle \leq \langle 471.81, 1.50\rangle,\ Z_3^I(x)-\frac{\tau}{\rho}\langle 114.93, 1.50\rangle \leq \langle 471.81, 1.50\rangle$$
$$\gamma \geq \zeta,\ \gamma \geq \tau,\ 0 \leq \gamma \leq 1,\ 0 \leq \zeta \leq 1,\ 0 \leq \tau \leq 1$$
constraints (12) to (15)

According to step 7, we addressed the above problem using GAMS software and achieved the effective solution, which is presented in Table 9.

Table 9: Optimal solution in inteval form(Mid-point and width)

Optimal solution (Model 3)	Transportation cost	Transportation time	Carbon emission
$x^I_{1111} = \langle 2.3935, 1.50\rangle$ $x^I_{1121} = \langle 2.2705, 1.50\rangle$ $x^I_{2122} = \langle 1.1661, 1.50\rangle$	$\langle 533.73, 1.50\rangle$	$\langle 486.74, 1.50\rangle$	$\langle 509.76, 1.50\rangle$

Table 10: Comparison Table

	Transportation cost	Transportation time	Carbon emission
Proposed method	[532.23, 535.23]	[485.24, 488.24]	[508.26, 511.26]
[26] [Interval-valued NP]	785.55	823.14	734.43
[26] [Interval-valued PHFP]	969.97	951.21	848.55
[24] [NCPA]	948.07	892.97	872.46

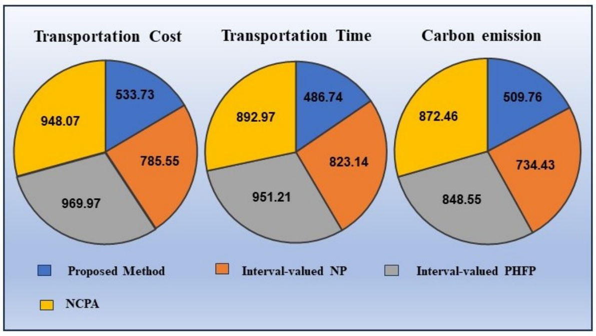

Figure 2: Comparison of the optimal solution between the proposed method and the existing methods

6. SENSITIVITY ANALYSIS

Sensitivity analysis is a technique employed to analyze how the input values impact the output of the proposed model. It is used to understand the robustness of the model and its results produced by the suggested method. In the proposed interval-valued trapezoidal neutrosophic fuzzy programming approach using interval numbers, we have used three decision variables $s,t \in (0,1)$ & $\rho \in [0,1]$. And these decision variables are analyzed in three ways as given below.

(i) Both s & t took same values in $(0,1)$ and $\rho = 0.9$ (fixed), for example, $s,\ t \in (0,1)$ & $\rho = 0.9$.

(ii) Both s,t took same values in $(0,1)$ and $\rho \in [0,1]$, for example, $s,\ t = 0.1$ & $\rho \in [0,1]$.

(iii) s & t took different values in $(0,1)$ and $\rho \in [0,1]$. For example, $s = 0.1, t = 0.8$ & $\rho \in [0,1]$.

In case (i), we have obtained the following interval transportation cost, time and carbon emission.

If $s,t = 0.1$ & $\rho = 0.9$, then $Z_1^I = \langle 506.74, 1.50\rangle$, $Z_2^I = \langle 483.98, 1.50\rangle$ & $Z_3^I = \langle 508.19, 1.50\rangle$.
If $s,t = 0.2$ & $\rho = 0.9$, then $Z_1^I = \langle 533.73, 1.50\rangle$, $Z_2^I = \langle 486.74, 1.50\rangle$ & $Z_3^I = \langle 509.76, 1.50\rangle$.
If $s,t = 0.3$ & $\rho = 0.9$, then $Z_1^I = \langle 560.36, 1.50\rangle$, $Z_2^I = \langle 489.22, 1.50\rangle$ & $Z_3^I = \langle 511.54, 1.50\rangle$.
If $s,t = 0.4$ & $\rho = 0.9$, then $Z_1^I = \langle 587.18, 1.50\rangle$, $Z_2^I = \langle 491.85, 1.50\rangle$ & $Z_3^I = \langle 513.22, 1.50\rangle$.
If $s,t = 0.5$ & $\rho = 0.9$, then $Z_1^I = \langle 614.01, 1.50\rangle$, $Z_2^I = \langle 494.48, 1.50\rangle$ & $Z_3^I = \langle 514.91, 1.50\rangle$.
If $s,t = 0.6$ & $\rho = 0.9$, then $Z_1^I = \langle 640.82, 1.50\rangle$, $Z_2^I = \langle 516.85, 1.50\rangle$ & $Z_3^I = \langle 529.28, 1.50\rangle$.
If $s,t = 0.7$ & $\rho = 0.9$, then $Z_1^I = \langle 667.63, 1.50\rangle$, $Z_2^I = \langle 539.20, 1.50\rangle$ & $Z_3^I = \langle 543.63, 1.50\rangle$.
If $s,t = 0.8$ & $\rho = 0.9$, then $Z_1^I = \langle 694.47, 1.50\rangle$, $Z_2^I = \langle 561.59, 1.50\rangle$ & $Z_3^I = \langle 558.02, 1.50\rangle$.
If $s,t = 0.9$ & $\rho = 0.9$, then $Z_1^I = \langle 721.27, 1.50\rangle$, $Z_2^I = \langle 583.96, 1.50\rangle$ & $Z_3^I = \langle 572.38, 1.50\rangle$.
The above effective solutions are displayed graphically in Figure 3, 4 and 5.

In case (ii), we attained the solutions are as same as case (i) for all $\rho \in [0,1]$. From this, we observed that, if s & t both chosen as same value, we got the same solution for all $\rho \in [0,1]$.

For case (iii), different values for s & t are chosen randomly and checked with all $\rho \in [0,1]$.

For example, if $s = 0.1,\ t = 0.8$, for $\rho = 0$ we obtained $Z_1^I = \langle 506.73, 1.50\rangle$, $Z_2^I = \langle 561.6, 1.50\rangle$ & $Z_3^I = \langle 483.6, 1.50\rangle$ and for the remaining ρ values, the solutions are $Z_1^I = \langle 506.73, 1.50\rangle$, $Z_2^I = \langle 483.94, 1.50\rangle$ & $Z_3^I = \langle 508.20, 1.50\rangle$.
If $s = 0.6,\ t = 0.8$, then the solution for $\rho = 0$ is $Z_1^I = \langle 640.84, 1.50\rangle$, $Z_2^I = \langle 561.6, 1.50\rangle$ & $Z_3^I = \langle 551.20, 1.50\rangle$ and the remaining solutions for $\rho \in (0,1]$ are $Z_1^I = \langle 640.83, 1.50\rangle$, $Z_2^I = \langle 516.85, 1.50\rangle$ & $Z_3^I = \langle 529.29, 1.50\rangle$.
If $s = 0.8,\ t = 0.1$, for $\rho = 0, 0.1$ we have $Z_1^I = \langle 694.5, 1.50\rangle$, $Z_2^I = \langle 561.59, 1.50\rangle$ & $Z_3^I = \langle 558, 1.50\rangle$. And for remaining ρ values, the solution is $Z_1^I = \langle 614, 1.50\rangle$, $Z_2^I = \langle 494.47, 1.50\rangle$ & $Z_3^I = \langle 514.91, 1.50\rangle$
If $s = 0.8,\ t = 0.6$, for $\rho = 0.1$ we have $Z_1^I = \langle 640.8, 1.50\rangle$, $Z_2^I = \langle 514.1, 1.50\rangle$ & $Z_3^I = \langle 525, 1.50\rangle$. And for remaining ρ=0,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9,1, the solution is $Z_1^I = \langle 640.8, 1.50\rangle$, $Z_2^I = \langle 516.9, 1.50\rangle$ & $Z_3^I = \langle 529.3, 1.50\rangle$. The above obtained solutions are displayed graphically in Figure 6, 7 & 8. From the above analysis, we observed that for all combination of parameters s,t & ρ , the interval transportation time, interval transportation cost and interval carbon emission are obtained minimum when compared to the existing methods [26] & [24].

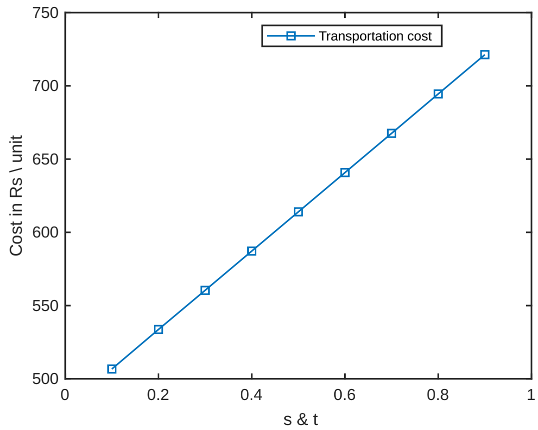

Figure 3: Transportation cost obtained using same values of $s,t \in (0,1)$ and $\rho \in [0,1]$

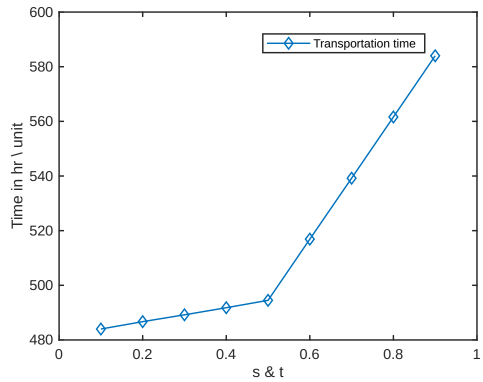

Figure 4: Transportation time obtained using same values of $s,t \in (0,1)$ and $\rho \in [0,1]$

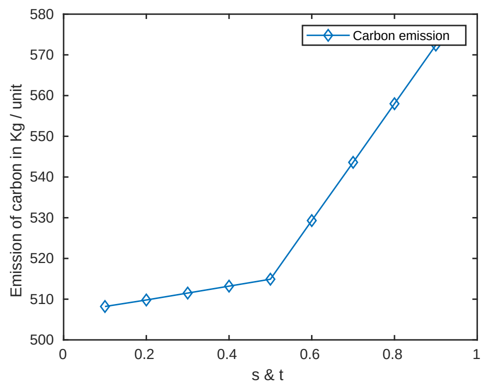

Figure 5: Carbon emission obtained using same values of $s,t \in (0,1)$ and $\rho \in [0,1]$

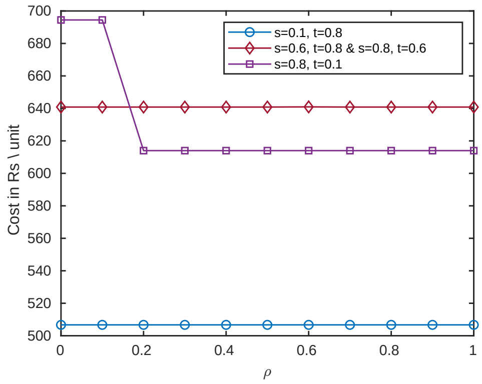

Figure 6: Transportation cost obtained using different values of $s,t \in (0,1)$ and $\rho \in [0,1]$

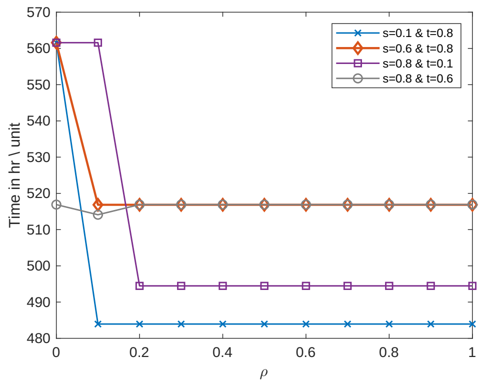

Figure 7: Transportation time obtained using different values of $s,t \in (0,1)$ and $\rho \in [0,1]$

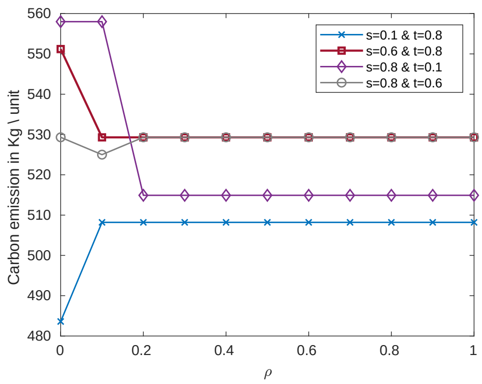

Figure 8: Carbon emission obtained using different values of $s,t \in (0,1)$ and $\rho \in [0,1]$

7. ADVANTAGES OF THE PROPOSED APPROACH

- The authors in [26] define the parameters of the multi-objective multi-item transportation problem as single-valued trapezoidal neutrosophic fuzzy numbers.

- They classified the neutrosophic transportation problem into five models in order to achieve the optimal solution through the application of neutrosophic programming approach.
- They employ three models to transform the supplied neutrosophic problem into an interval problem, also they assumed the situation to be imbalanced.
- The parameters analyzed in our article are represented as interval-valued trapezoidal neutrosophic fuzzy numbers. Analyzing trapezoidal fuzzy numbers with interval truth, interval indeterminacy and interval falsity are desirable and efficient.
- Our proposed solution instantly transforms the unbalanced problem into a balanced one. Balanced transportation problems can depict situations where resources are appropriately aligned with demand, resulting in potentially enhanced stability and efficiency in logistics.
- Furthermore, our proposed solution utilizes a single model (Model 2) to transform the original problem into an interval problem.
- In addition, we achieve a more effective solution compared to the one provided by [26] as seen in Table 10. Figure 2 provides a graphical representation of it.
- Moreover, sensitivity analysis provides more valid to the proposed programming based on interval numbers in neutrosophic environment with interval-valued trapezoidal numbers as parameters.

8. CONCLUSION AND PROSPECTS FOR FURTHER RESEARCH

This article discusses the multi-objective multi-item solid transportation issue in a neutrosophic environment, using interval-valued trapezoidal neutrosophic numbers as parameters. We utilize the interval-valued trapezoidal neutrosophic fuzzy programming approach to tackle the problem. The originality of this study is in the utilization of interval numbers, as alternative to crisp numbers, to address the situation at hand. We conducted a validation of our suggested approach by solving an example, and we achieved a very efficient solution when comparing it to the solutions presented in the references [26] and [24]. The comparison of transportation cost, time, and carbon emission with the existing techniques is clearly presented in Table 10 using interval numbers. Comparing the mid-point of the transportation cost, time, and carbon emission of the suggested approach with interval-valued pythagorean hesitant fuzzy programming, interval-valued neutrosophic programming, and neutrosophic compromise programming approach is also shown in Figure 2. Finally, the problem is analyzed for the parameters s, t & ρ and results are represented graphically in Figure 3 to 8. In future study, metaheuristic methodologies such as genetic algorithm and simulated annealing will be employed for constructing and developing algorithms to solve problems involving interval numbers. The purpose of this study is to improve the algorithmic efficacy, practical applicability, and theoretical foundation of solutions for the multi-objective, multi-item interval-valued trapezoidal neutrosophic transportation problem by employing interval numbers.

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REFERENCES

[1] K. Haley, "New methods in mathematical programming—the solid transportation problem," *Operations research*, vol. 10, no. 4, pp. 448–463, 1962.

[2] S. M. Lee and L. J. Moore, "Optimizing transportation problems with multiple objectives," *AIIE transactions*, vol. 5, no. 4, pp. 333–338, 1973.

[3] L. A. Zadeh, "Fuzzy sets," *Information and Control*, 1965.

[4] F. Jiménez and J. L. Verdegay, "Uncertain solid transportation problems," *Fuzzy sets and systems*, vol. 100, no. 1-3, pp. 45–57, 1998.

[5] S. Pramanik and T. K. Roy, "Multiobjective transportation model with fuzzy parameters: priority based fuzzy goal programming approach," *Journal of Transportation Systems Engineering and Information Technology*, vol. 8, no. 3, pp. 40–48, 2008.

[6] P. Kundu, S. Kar, and M. Maiti, "Multi-objective multi-item solid transportation problem in fuzzy environment," *Applied mathematical modelling*, vol. 37, no. 4, pp. 2028–2038, 2013.

[7] D. Chakraborty, D. K. Jana, and T. K. Roy, "Multi-objective multi-item solid transportation problem with fuzzy inequality constraints," *Journal of Inequalities and Applications*, vol. 2014, pp. 1–21, 2014.

[8] D. Rani, T. Gulati, and A. Kumar, "On fuzzy multiobjective multi-item solid transportation problem," *Journal of Optimization*, vol. 2015, no. 1, p. 787050, 2015.

[9] P. K. Giri, M. K. Maiti, and M. Maiti, "Fully fuzzy fixed charge multi-item solid transportation problem," *Applied Soft Computing*, vol. 27, pp. 77–91, 2015.

[10] M. B. Kar, P. Kundu, S. Kar, and T. Pal, "A multi-objective multi-item solid transportation problem with vehicle cost, volume and weight capacity under fuzzy environment," *Journal of Intelligent & Fuzzy Systems*, vol. 35, no. 2, pp. 1991–1999, 2018.

[11] M. Kamal, A. Alarjani, A. Haq, F. N. K. Yusufi, and I. Ali, "Multi-objective transportation problem under type-2 trapezoidal fuzzy numbers with parameters estimation and goodness of fit," *Transport*, vol. 36, no. 4, pp. 317–338, 2021.

[12] H. Khalifa, M. Elhenawy, M. Masoud, H. Bhuiyan, and N. R. Sabar, "On multi-objective multi-item solid transportation problem in fuzzy environment," *International journal of applied and computational mathematics*, vol. 7, no. 1, p. 24, 2021.

[13] D. Mardanya and S. K. Roy, "New approach to solve fuzzy multi-objective multi-item solid transportation problem," *RAIRO-Operations Research*, vol. 57, no. 1, pp. 99–120, 2023.

[14] S. Rekabi, F. Goodarzian, H. S. Garjan, F. Zare, J. Muñuzuri, and I. Ali, "A data-driven mathematical model to design a responsive-sustainable pharmaceutical supply chain network: a benders decomposition approach," *Annals of Operations Research*, pp. 1–42, 2023.

[15] K. T. Atanassov, *On intuitionistic fuzzy sets theory*. Springer, 2012, vol. 283.

[16] S. Pramanik, T. K. Roy, and P. Garden, "An intuitionistic fuzzy goal programming approach to vector optimization problem," *NIFS11*, vol. 5, 2005.

[17] D. Chakraborty, D. K. Jana, and T. K. Roy, "A new approach to solve multi-objective multi-choice multi-item atanassov's intuitionistic fuzzy transportation problem using chance operator," *Journal of intelligent & fuzzy systems*, vol. 28, no. 2, pp. 843–865, 2015.

[18] S. K. Roy, A. Ebrahimnejad, J. L. Verdegay, and S. Das, "New approach for solving intuitionistic fuzzy multi-objective transportation problem," *Sādhanā*, vol. 43, pp. 1–12, 2018.

[19] S. Samanta, D. K. Jana, G. Panigrahi, and M. Maiti, "Novel multi-objective, multi-item and four-dimensional transportation problem with vehicle speed in lr-type intuitionistic fuzzy environment," *Neural computing and Applications*, vol. 32, pp. 11 937–11 955, 2020.

[20] S. Midya, S. K. Roy, and V. F. Yu, "Intuitionistic fuzzy multi-stage multi-objective fixed-charge solid transportation problem in a green supply chain," *International Journal of Machine Learning and Cybernetics*, vol. 12, pp. 699–717, 2021.

[21] Shivani and D. Rani, "Multi-objective multi-item four dimensional green transportation problem in interval-valued intuitionistic fuzzy environment," *International Journal of System Assurance Engineering and Management*, vol. 15, no. 2, pp. 727–744, 2024.

[22] F. Smarandache, *A unifying field in logics: neutrosophic logic. Neutrosophy, neutrosophic set, neutrosophic probability*. Infinite Study, 2005.

[23] P. Biswas, S. Pramanik, and B. C. Giri, "Distance measure based madm strategy with interval trapezoidal neutrosophic numbers," *Neutrosophic Sets and Systems*, vol. 19, pp. 40–46, 2018.

[24] R. M. Rizk-Allah, A. E. Hassanien, and M. Elhoseny, "A multi-objective transportation model under neutrosophic environment," *Computers & Electrical Engineering*, vol. 69, pp. 705–719, 2018.

[25] S. K. Das and S. K. Roy, "Effect of variable carbon emission in a multi-objective transportation-p-facility location problem under neutrosophic environment," *Computers & Industrial Engineering*, vol. 132, pp. 311–324, 2019.

[26] B. K. Giri and S. K. Roy, "Neutrosophic multi-objective green four-dimensional fixed-charge transportation problem," *International Journal of Machine Learning and Cybernetics*, vol. 13, no. 10, pp. 3089–3112, 2022.

[27] S. Khalil, S. Kousar, G. Freen, and M. Imran, "Multi-objective interval-valued neutrosophic optimization with application," *International Journal of Fuzzy Systems*, pp. 1–13, 2022.

[28] A. Revathi, S. Mohanaselvi, and B. Said, "An efficient neutrosophic technique for uncertain multi objective transportation problem," *Neutrosophic Sets and Systems*, vol. 53, no. 1, p. 27, 2023.

[29] E. Hosseinzadeh and J. Tayyebi, "A compromise solution for the neutrosophic multi-objective linear programming problem and its application in transportation problem," *Journal of applied research on industrial engineering*, vol. 10, no. 1, pp. 1–10, 2023.

[30] A. Kumar, P. Singh, and Y. Kacher, "Neutrosophic hyperbolic programming strategy for uncertain multi-objective transportation problem," *Applied Soft Computing*, vol. 149, p. 110949, 2023.

[31] G. Gupta, Shivani, and D. Rani, "Neutrosophic goal programming approach for multi-objective fixed-charge transportation problem with neutrosophic parameters," *OPSEARCH*, pp. 1–27, 2024.

[32] A. Nagarajan, K. Jeyaraman, and S. K. Prabha, "Multiobjective solid transportation problem with interval cost in source and demand parameters," *International Journal of Computer & Organization Trends*, vol. 8, no. 1, pp. 33–41, 2014.

[33] H. Dalman, N. Güzel, and M. Sivri, "A fuzzy set-based approach to multi-objective multi-item solid transportation problem under uncertainty," *International Journal of Fuzzy Systems*, vol. 18, pp. 716–729, 2016.

[34] T. Sifaoui and M. Aïder, "Uncertain interval programming model for multi-objective multi-item fixed charge solid transportation problem with budget constraint and safety measure," *Soft Computing*, vol. 24, no. 13, pp. 10 123–10 147, 2020.

[35] M. Ma, M. Friedman, and A. Kandel, "A new fuzzy arithmetic," *Fuzzy sets and systems*, vol. 108, no. 1, pp. 83–90, 1999.

[36] S. Sinika and G. Ramesh, "To enhance new interval arithmetic operations in solving linear programming problem using interval-valued trapezoidal neutrosophic numbers," *Mathematics and Statistics*, vol. 11, no. 4, pp. 710–725, 2023.