

Research Article

**DECISION MAKING METHOD TO OPTIMAL
SELECTION OF AREA FOR BUILDING PROJECT
BASED ON FUZZY PARAMETERIZED
NEUTROSOPHIC SOFT EXPERT SET**

Muhammad IHSAN

*University of Management and Technology Lahore 54000, Pakistan
mihkhb@gmail.com, ORCID:0000-0001-6107-5493*

Received: July 2024 / Accepted: January 2025

Abstract: Fuzzy parameterized neutrosophic soft expert sets is a useful extension of fuzzy parameterized intuitionistic fuzzy soft expert and neutrosophic soft sets. This set deals with attributes as well but with many definitive assessments, whereas the soft set only addresses one set of attributes. The development of this model addresses the limitations of existing soft set-based models, particularly their inability to handle multiple experts' opinions in a decision-making context. This paper's major goal is to introduce the theory of fuzzy parameterized neutrosophic soft expert set theory. It also discusses the notion's numerous related ideas and the basic operations on it, such as the complement, union, intersection, AND, and OR. The essential features of this idea are established, as are relevant rules such as De Morgan's laws. A generalised algorithm is then used to apply the suggested idea of fuzzy parameterized neutrosophic soft expert sets in a decision-making situation. Moreover, in order to check the validity of the proposed algorithm, an example related to the choice of the area of building project using the decision-making method is presented. The suggested method provides reliable, reliable decision-making outcomes, and its enhanced capacity and dependability are confirmed by a robust comparison with earlier models.

Keywords: Soft set, soft expert set, neutrosophic soft set, neutrosophic soft expert set, decision-making, optimization.

MSC: 03B52; 03E72; 52A01.

1. INTRODUCTION

Decision making in uncertain environments refers to the process of making choices or decisions when there is a lack of complete information or knowledge about potential outcomes. This can include situations where there is a high degree of risk or ambiguity, and decision makers must weigh the potential outcomes of different choices in order to make the best decision possible. This can be done using techniques such as decision analysis, game theory, or other decision-making models that take into account factors such as risk, probability, and uncertainty. We are aware that the real world is rife with uncertainty, inaccuracy, and ambiguity. In actuality, the majority of the issues we encountered are ambiguous rather than specific. Classical approaches are not always effective when dealing with unclear data since these difficulties contain a wide variety of uncertainty. It is well known that mathematical theories such as the theories of probabilities, fuzzy sets [1], rough sets [2], and others are useful tools for characterising uncertainty. Zare Ahmadabadi et al. [3] used hesitant fuzzy to study factors affecting the supply chain management. Rahadian et al. [4] used decision-making approach for the evaluation of different countries energy investment. The neutrosophic set(NS) was defined by Smarandache [5] as a generalisation of fuzzy sets, intuitionistic fuzzy sets, and classical sets. The NS finds wide-ranging applications in several fields such as databases, control theory, topology, and medical diagnosis. Membership in truth, indeterminacy, and falsehood are completely independent to one another in a NS, where indeterminacy is quantified explicitly. A scientific or technical description of the operators of the set-theoretic view and the NS is required. It will be difficult to implement in real applications if not. For this reason, before demonstrating its numerous properties and set theoretic operations, Wang et al. [6] created a single valued neutrosophic set (SVNS). Ghafariyan et al. [7] used neutrosophic structure to describe the different applications. In order to overcome the shortcomings of decision-making techniques when dealing with ambiguous data, Nafei et al. [8, 9, 10] presented a unique framework for decision-making that combines the TOPSIS with neutrosophic triplets. The NS and their hybrid designs have advanced in theory and application [11, 12].

Molodtsov [13] pointed out that there were drawbacks to each of these theories, though. To circumvent these issues, Molodtsov presented a novel approach to describing uncertainty called a soft set. This theory has since been applied to many other domains, including probability theory, measurement theory, game theory, operations research, and smoothness of functions.

Cybernetics, machine learning, intelligent systems, information sciences, and other disciplines are similarly impacted. Most of these uses are previously illustrated in Molodtsov's book. Vellapandi et al. [14] presented an original method to choose an option for the soft set across all universes using the certainty of a parameter, and they established a new notion of certainty and coverage of a parameter of the soft set. Karatas et al. [15] gave the idea of an effective neutrosophic soft set with its valuable usage. Zhao et al. [16] used the neutrosophic with soft set-like structure in decision making problems. Pethaperumal et al. [17] used a soft set-like structure in the health system. Debnath [18] made an extension in the soft set-like structure. Smarandache et al. [19, 20, 21] gave some characteristics in soft set-like structures.

A few basic operations on soft sets were defined by Maji [22]. In 2009, Ali [23] introduced a number of innovative soft set operations. Furthermore, Maji et al. [24] defined a fuzzy soft set and gave an example of its application in a scenario involving decision-making [25]. Research on applications of soft set theory has increased as a result of these definitions. Çağman [26, 27] applied the soft set theory to decision-making.

Chen [28] discussed the parameterization reduction of soft sets and its applications. Simultaneously, Feng [29] introduced the use of soft sets in interval-valued fuzzy soft set-based decision-making. Jiang [30] introduced a novel technique for applying interval-valued intuitionistic fuzzy sets in decision-making. Later studies by Feng, Maji, and others have looked at the more general features and applications of soft set theory; see, for example, [31, 32]. It is observed that fuzzy soft sets have extended the theory of soft sets; these are cited in [33, 34]. Rough soft sets were recently added to semi-rings for the first time, and Zhan [35] documented several properties of rough soft semi-rings. Muniba et al. [36] introduced a new method for the prediction of climate change by using MCDM technique. Smarandache [37] gave the idea of a super hypersoft set with different applications. Many interesting and contemporary applications of soft set theory have involved the use of intuitionistic fuzzy sets and rough set theory. Using fuzzy soft sets, Jiang et al. [39] extended the customisable technique for decision-making. They explicitly offered an adjustable strategy to intuitionistic fuzzy soft set-based decision-making and provided various pertinent examples by using soft sets of intuitionistic fuzzy soft sets. Different convexity (concavity) structures on soft sets were presented by Rahman et al. [40, 41]. With some different results, they investigated the various concavity and convexity properties in soft set and fuzzy set environments. While soft set structures address the opinions of a single expert, some situations necessitate the opinions of multiple experts, such as in a questionnaire. To address this shortcoming, Alkhazaleh et al. [42] developed the notion of the soft expert set (SE-set) and offered applications in decision-making. Ihsan et al. [43] explored the special properties of the convexity on the FSE-set, which was gestated. Alkhazaleh et al. [44] employed a soft expert set in an uncertain environment to conceptualise fuzzy soft expert set (FSE-set). They discussed their potential application in making decisions. It was theorised that convexity-cum-concavity on SE-set, and Ihsan et al. [45] addressed various features. Intuitionistic fuzzy soft expert sets were developed by Broumi et al. [46, 47], who also presented their use in decision-making.

1.1. Research gap and novelty

Fuzzy parameterized soft set (FPS-set) was developed by Çağman et al. [48], who also applied a significant degree to parameters. A concept of operations on fuzzy parameterized fuzzy soft sets (FPSF-set) was presented by Çağman et al. [49]. Subsequently, the FPSF-set-aggregation operator was established in order to generate the FPSF-set-decision making method, which facilitates the development of more efficient decision processes. The t-norms and t-conorms products of fuzzy parameterized fuzzy soft sets were defined by Zhu and Zhan [50]. The AND-FPSF-set decision making method and the OR-FPSF-set decision making method are built using these products, respectively. Lastly, decision-making strategies are used to solve situations including uncertainties. Fuzzy parameterized fuzzy soft sets (FPFS-sets) are a useful tool for solving decision-making problems, as explored by Riaz and Hashmi [51]. They defined AND and OR operations for FPFS-

sets. They examined the comparison tables, reduct, and aggregation operation for the FPFS-set. They illustrated our processes in the form of a few algorithms with the help of several examples. Sulukan et al. [52] proposed fuzzy parameterized intuitionistic fuzzy soft sets. Ihsan et al. [53] constructed the structure of a fuzzy parameterized single-valued neutrosophic soft set. They next used these sets to the performance-based value assignment (PVA) problem. The architectures of fuzzy parameterized and soft expert sets were combined, according to Bashir et al. [54], to generate hybrid fuzzy parameterized soft expert sets (FPSE-set) with applications in DMPs. As an extension of the fuzzy soft expert set, Hazaymeh et al. [55] introduced the concept of fuzzy parameterized fuzzy soft expert set by giving a membership value to each parameter in a collection of parameters. They also talked about the characteristics of its basic operations, such as complement, union intersection, "AND," and "OR." Lastly, they offered an application of this concept to decision-making issues. The notion of fuzzy parameterized intuitionistic fuzzy soft expert sets was put forth by Ganeshsree and Salleh [56]. They also specified a number of words related to this concept, as well as the fundamental operations on it, such as complement, union, intersection, AND, and OR. This idea was demonstrated by De Morgan's laws and other relevant fundamental characteristics and rules. A generalised technique is then used to apply the proposed concept of fuzzy parameterized neutrosophic soft expert sets to a decision-making problem. A novel mathematical framework, the fuzzy parameterized neutrosophic soft expert set, is developed in response to this literary demand. It is more versatile and flexible because it can handle the following scenarios as a whole:

1. The parameterization of fuzzy set-like structures in soft set environments with fuzzy set-like settings has been the subject of much research. It is clear from the literature assessment of the most pertinent models presented above that the models discussed above used conventional soft approximate mapping to examine the parameterized nature of the parametric domain and concentrated on a single set of attributes. They are unable to handle scenarios involving three-dimensional type data (membership, non-membership, and indeterminacy) when there are several deciding factors.
2. Above described soft sets structured for the necessity of involving the information of single expert opinions, but this literature demands to handle the situation of multiple experts' decisive opinions in a single structure which leads to the development of new models like this one.
3. The FPNSE-set is developed to handle such scenarios since the incapability of soft expert set-like models with neutrosophic settings results in the necessity for new structure. This novel model stresses careful examination of three-dimensional data, making it a flexible and trustworthy model for a fair decision-support system.

1.2. Main contributions

The main contributions of the suggested work have been discussed below:

1. Using vivid numerical examples, the ideas and algebraic characteristics of the FPNSE-set are explained.
2. By identifying their pertinent fuzzy parameterized grades, the ambiguous characteristics of the parameters and sub-parameters under examination are assessed.

3. Using set-based operations of FPNSES-set, a novel robust MADM-based algorithm is provided to evaluate construction project area analyses. The suggested algorithms' processes are simple to comprehend and devoid of computational complexity.
4. In the end, a strong comparison of the suggested structure with the existing structures has been made.

We develop innovative structures of fuzzy parameterized neutrosophic soft expert sets and characterise them with the aid of algorithm-based decision-support systems, inspired by the aforementioned literature in general and [54, 55, 56] in particular. The remainder of the study is structured as follows: For a better understanding of the primary topic, Section 2 reviews some key definitions of elementary nature from the literature. In Sections 3 and 4, respectively, theories of FPNSE-set are developed together with their decision-support system. In order to see the benefits of the proposed study, a comparison with some current relevant models is offered in Section 5, and the discussion of generalisation and the advantages of the proposed work is presented in Section 6. In the final section, the article is summarised along with suggested next steps.

2. PRELIMINARIES

This section provides definitions and explanations of some basic words from the literature that are connected to the primary topic.

Definition 1. [5] The definition of neutrosophic set \mathfrak{M} on $\hat{\mathfrak{X}}$ is given by

$\mathfrak{M} = \{ \langle \chi, (\tau_{\mathfrak{M}}(\chi), l_{\mathfrak{M}}(\chi), F_{\mathfrak{M}}(\chi)) \rangle : \chi \in \mathbb{E}, \tau_{\mathfrak{M}}, l_{\mathfrak{M}}, F_{\mathfrak{M}} \in]^{-0}, 1^{+}[\}$ having the condition $0^{-} \leq \tau_{\mathfrak{M}}(\chi), l_{\mathfrak{M}}(\chi), F_{\mathfrak{M}}(\chi) \leq 3^{+}$, where $\tau_{\mathfrak{M}}, l_{\mathfrak{M}}, F_{\mathfrak{M}}$ are constructed here for membership, indeterminacy, and non-membership functions.

Definition 2. [5] Consider two neutrosophic sets \mathfrak{P} and \mathfrak{T} such that

$\mathfrak{P} = \{ \langle \kappa, (\tau_{\mathfrak{P}}(\kappa), l_{\mathfrak{P}}(\kappa), F_{\mathfrak{P}}(\kappa)) \rangle : \kappa \in \mathbb{E}, \tau_{\mathfrak{P}}, l_{\mathfrak{P}}, F_{\mathfrak{P}} \in]^{-0}, 1^{+}[\}$,
 $\mathfrak{T} = \{ \langle \kappa, (\tau_{\mathfrak{T}}(\kappa), l_{\mathfrak{T}}(\kappa), F_{\mathfrak{T}}(\kappa)) \rangle : \kappa \in \mathbb{E}, \tau_{\mathfrak{T}}, l_{\mathfrak{T}}, F_{\mathfrak{T}} \in]^{-0}, 1^{+}[\}$, then,

1. Neutrosophic set $\mathfrak{M} \subseteq \mathfrak{N}$ if
 $\tau_{\mathfrak{M}}(\bar{\lambda}) \leq \tau_{\mathfrak{N}}(\bar{\lambda}), l_{\mathfrak{M}} \geq l_{\mathfrak{N}}, F_{\mathfrak{M}}(\bar{\lambda}) \leq F_{\mathfrak{N}}(\bar{\lambda})$.
2. The compliment operation is
 $\mathfrak{M}^c = \{ \langle \bar{\lambda}, (\tau_{\mathfrak{M}}(\bar{\lambda}), 1 - l_{\mathfrak{M}}(\bar{\lambda}), F_{\mathfrak{M}}(\bar{\lambda})) \rangle : \bar{\lambda} \in \mathbb{E}, \tau_{\mathfrak{M}}, l_{\mathfrak{M}}, F_{\mathfrak{M}} \in]^{-0}, 1^{+}[\}$.
3. The operation union is
 $\max(\tau_{\mathfrak{M}}(\bar{\lambda}), \tau_{\mathfrak{N}}(\bar{\lambda}), \min(l_{\mathfrak{M}}(\bar{\lambda}), l_{\mathfrak{N}}(\bar{\lambda})), \min(F_{\mathfrak{M}}(\bar{\lambda}), F_{\mathfrak{N}}(\bar{\lambda})),$
4. The operation intersection is
 $\min(\tau_{\mathfrak{M}}(\bar{\lambda}), \tau_{\mathfrak{N}}(\bar{\lambda}), \max(l_{\mathfrak{M}}(\bar{\lambda}), l_{\mathfrak{N}}(\bar{\lambda})), \max(F_{\mathfrak{M}}(\bar{\lambda}), F_{\mathfrak{N}}(\bar{\lambda})).$

Definition 3. [52] Let \mathcal{H} is the cartesian product of \heartsuit , \heartsuit , \diamond and collection of experts and parameters are \heartsuit , \heartsuit respectively. While \diamond is the symbol for the opinions of experts i.e, $\diamond = \{0 = \text{disagree (dag)}, 1 = \text{agree (ag)}\}$ and $\hat{\mathfrak{X}}$ for the universe with $P(\hat{\mathfrak{X}})$ as a power set and $\mathbb{I} = [0, 1]$. For the sake of simplicity, it is assumed that there are only two possible values for the collection of opinions in this study: agree and disagree. It is possible to incorporate more alternatives for the collection of opinions, such as more focused opinions. A pair (I_{ζ}, β) can be considered as a FPIFSE-set with g_{ζ} is $I_{\zeta} : \beta \rightarrow P(\hat{\mathfrak{X}})$ such that β is being used as a subsets of \heartsuit .

Definition 4. [52] The property of being a subset of FPIFSE-set can be considered when certain conditions are fulfilled (i) when the set \mathfrak{R} of FPIFSE-set (I_{ξ}, \mathfrak{R}) is a subset of χ of another FPIFSE-set (F_{η}, χ) and (ii) $I_{\xi}(\xi)$ is being as an intuitionistic fuzzy subset of $F_{\eta}(\xi)$ for all $\xi \in I_{\xi}$.

Definition 5. [52] Another subset property of FPIFSE-set is an agree FPIFSE-set which is \subseteq FPIFSE-set $(g_{\Gamma}, \mp)^1 = \{g_{\Gamma}(\check{\phi}) : \check{\phi} \in \heartsuit \times \varpi \times 1\}$.

Definition 6. [52] Another subset property of FPIFSE-set is a dis-agree FPIFSE-set which is actually \subseteq FPIFSE-set $(g_{\Gamma}, \mp)^0 = \{g_{\Gamma}(\check{\phi}) : \check{\phi} \in \heartsuit \times \varpi \times 0\}$.

3. FUZZY PARAMETERIZED NEUTROSOPHIC SOFT EXPERT SET (FPNSE-SET)

Some fundamental properties are described here. This section describes the development of the FPNSE-set using the concept of the fuzzy parameterized intuitionistic fuzzy soft expert set.

Definition 7. *Fuzzy Parameterized Neutrosophic Soft Expert Set*

A fuzzy parameterized neutrosophic soft expert set $\partial_{\mathcal{F}\mathcal{D}}$ over $\hat{\mathfrak{X}}$ is defined as

$$\partial_{\mathcal{F}\mathcal{D}} = \left\{ \left(\left(\frac{\hat{\Lambda}}{\chi_{\mathcal{F}\mathcal{D}}(\hat{\Lambda})}, \hat{K}_i, \hat{Y}_i \right), \frac{\hat{\delta}}{\phi_{\mathcal{F}\mathcal{D}}(\hat{\delta})} \right); \forall \hat{\Lambda} \in \mathcal{H}, \hat{K}_i \in \check{\zeta}, \hat{Y}_i \in \diamond, \hat{\delta} \in \hat{\mathfrak{X}} \right\}$$

where $\chi_{\mathcal{F}\mathcal{D}} : \check{\zeta} \rightarrow FP(\hat{\mathfrak{X}})$, and $\phi_{\mathcal{F}\mathcal{D}} : \check{\zeta} \rightarrow NP(\hat{\mathfrak{X}})$ is called approximate function of FPNSE-set, $\check{\zeta} \subseteq \mathcal{H} = \heartsuit \times \varpi \times \diamond$.

Example 8. Consider a scenario where a network of universities seeks a construction company to update the campus building in response to globalisation and requires specialised personnel (experts) to evaluate the facility's functionality. Let $\hat{\mathfrak{X}} = \{\nabla_1, \nabla_2\}$ be a set of companies and distinct attributes set is $\{\mathfrak{R}_1 = \text{cheap}, \mathfrak{R}_2 = \text{standard}\}$. Let $\{\mathfrak{R}_1/0.2, \mathfrak{R}_2/0.3\}$ be the fuzzy subset of I^{\heartsuit} (set of fuzzy subsets of \heartsuit). Now $\mathcal{H} = \heartsuit \times \varpi \times \diamond$.

$$\mathcal{H} = \left\{ (\mathfrak{R}_1/0.2, \mathbb{k}_1, 0), (\mathfrak{R}_1/0.2, \mathbb{k}_1, 1), (\mathfrak{R}_1/0.2, \mathbb{k}_2, 0), (\mathfrak{R}_1/0.2, \mathbb{k}_2, 1), \right. \\ \left. , (\mathfrak{R}_1/0.2, \mathbb{k}_3, 0), (\mathfrak{R}_1/0.2, \mathbb{k}_3, 1), (\mathfrak{R}_2/0.3, \mathbb{k}_1, 0), (\mathfrak{R}_2/0.3, \mathbb{k}_1, 1), \right. \\ \left. (\mathfrak{R}_2/0.3, \mathbb{k}_2, 0), (\mathfrak{R}_2/0.3, \mathbb{k}_2, 1), (\mathfrak{R}_2/0.3, \mathbb{k}_3, 0), (\mathfrak{R}_2/0.3, \mathbb{k}_3, 1), \right\}$$

and $\varpi = \{\mathbb{k}_1, \mathbb{k}_2, \mathbb{k}_3, \}$ be a set of specialists. We have:

$$\begin{aligned} \mathfrak{I}_1 &= \mathfrak{I}(\mathfrak{R}_1/0.2, \mathbb{k}_1, 1) = \left\{ \frac{\nabla_1}{\langle 0.3, 0.5, 0.4 \rangle}, \frac{\nabla_2}{\langle 0.7, 0.4, 0.5 \rangle} \right\}, \\ \mathfrak{I}_2 &= \mathfrak{I}(\mathfrak{R}_1/0.2, \mathbb{k}_2, 1) = \left\{ \frac{\nabla_1}{\langle 0.4, 0.6, 0.3 \rangle}, \frac{\nabla_2}{\langle 0.8, 0.4, 0.5 \rangle} \right\}, \\ \mathfrak{I}_3 &= \mathfrak{I}(\mathfrak{R}_1/0.2, \mathbb{k}_3, 1) = \left\{ \frac{\nabla_1}{\langle 0.7, 0.4, 0.3 \rangle}, \frac{\nabla_2}{\langle 0.5, 0.3, 0.6 \rangle} \right\}, \\ \mathfrak{I}_4 &= \mathfrak{I}(\mathfrak{R}_2/0.3, \mathbb{k}_1, 1) = \left\{ \frac{\nabla_1}{\langle 0.9, 0.6, 0.3 \rangle}, \frac{\nabla_2}{\langle 0.4, 0.3, 0.4 \rangle} \right\}, \end{aligned}$$

$$\begin{aligned}
\mathfrak{J}_5 &= \mathfrak{J}(\mathfrak{R}_2/0.3, \mathbb{k}_2, 1) = \left\{ \frac{\nabla_1}{\langle 0.4, 0.7, 0.6 \rangle}, \frac{\nabla_2}{\langle 0.8, 0.9, 0.7 \rangle} \right\}, \\
\mathfrak{J}_6 &= \mathfrak{J}(\mathfrak{R}_2/0.3, \mathbb{k}_3, 1) = \left\{ \frac{\nabla_1}{\langle 0.5, 0.3, 0.7 \rangle}, \frac{\nabla_2}{\langle 0.3, 0.6, 0.4 \rangle} \right\}, \\
\mathfrak{J}_7 &= \mathfrak{J}(\mathfrak{R}_1/0.2, \mathbb{k}_1, 0) = \left\{ \frac{\nabla_1}{\langle 0.3, 0.2, 0.6 \rangle}, \frac{\nabla_2}{\langle 0.7, 0.4, 0.5 \rangle} \right\}, \\
\mathfrak{J}_8 &= \mathfrak{J}(\mathfrak{R}_1/0.2, \mathbb{k}_2, 0) = \left\{ \frac{\nabla_1}{\langle 0.3, 0.8, 0.4 \rangle}, \frac{\nabla_2}{\langle 0.9, 0.2, 0.4 \rangle} \right\}, \\
\mathfrak{J}_9 &= \mathfrak{J}(\mathfrak{R}_1/0.2, \mathbb{k}_3, 0) = \left\{ \frac{\nabla_1}{\langle 0.3, 0.7, 0.5 \rangle}, \frac{\nabla_2}{\langle 0.2, 0.8, 0.6 \rangle} \right\}, \\
\mathfrak{J}_{10} &= \mathfrak{J}(\mathfrak{R}_2/0.3, \mathbb{k}_1, 0) = \left\{ \frac{\nabla_1}{\langle 0.8, 0.7, 0.6 \rangle}, \frac{\nabla_2}{\langle 0.3, 0.6, 0.7 \rangle} \right\}, \\
\mathfrak{J}_{11} &= \mathfrak{J}(\mathfrak{R}_2/0.3, \mathbb{k}_2, 0) = \left\{ \frac{\nabla_1}{\langle 0.7, 0.3, 0.5 \rangle}, \frac{\nabla_2}{\langle 0.7, 0.6, 0.4 \rangle} \right\}, \\
\mathfrak{J}_{12} &= \mathfrak{J}(\mathfrak{R}_2/0.3, \mathbb{k}_3, 0) = \left\{ \frac{\nabla_1}{\langle 0.6, 0.8, 0.5 \rangle}, \frac{\nabla_2}{\langle 0.7, 0.9, 0.4 \rangle} \right\}.
\end{aligned}$$

The FPNSE-set can be described as $(\mathfrak{J}, \mathbb{Q}) =$

$$\left\{ \begin{aligned} & \left(\mathfrak{R}_1/0.2, \mathbb{k}_1, 1 \right), \left\{ \frac{\nabla_1}{\langle 0.3, 0.5, 0.4 \rangle}, \frac{\nabla_2}{\langle 0.7, 0.4, 0.5 \rangle} \right\} \right), \\ & \left(\mathfrak{R}_1/0.2, \mathbb{k}_2, 1 \right), \left\{ \frac{\nabla_1}{\langle 0.4, 0.6, 0.3 \rangle}, \frac{\nabla_2}{\langle 0.8, 0.4, 0.5 \rangle} \right\} \right), \\ & \left(\mathfrak{R}_1/0.2, \mathbb{k}_3, 1 \right), \left\{ \frac{\nabla_1}{\langle 0.7, 0.4, 0.3 \rangle}, \frac{\nabla_2}{\langle 0.5, 0.3, 0.6 \rangle} \right\} \right), \\ & \left(\mathfrak{R}_2/0.3, \mathbb{k}_1, 1 \right), \left\{ \frac{\nabla_1}{\langle 0.9, 0.6, 0.3 \rangle}, \frac{\nabla_2}{\langle 0.4, 0.3, 0.4 \rangle} \right\} \right), \\ & \left(\mathfrak{R}_2/0.3, \mathbb{k}_2, 1 \right), \left\{ \frac{\nabla_1}{\langle 0.4, 0.7, 0.6 \rangle}, \frac{\nabla_2}{\langle 0.8, 0.9, 0.7 \rangle} \right\} \right), \\ & \left(\mathfrak{R}_2/0.3, \mathbb{k}_3, 1 \right), \left\{ \frac{\nabla_1}{\langle 0.5, 0.3, 0.7 \rangle}, \frac{\nabla_2}{\langle 0.3, 0.6, 0.4 \rangle} \right\} \right), \\ & \left(\mathfrak{R}_1/0.2, \mathbb{k}_1, 0 \right), \left\{ \frac{\nabla_1}{\langle 0.3, 0.2, 0.6 \rangle}, \frac{\nabla_2}{\langle 0.7, 0.4, 0.5 \rangle} \right\} \right), \\ & \left(\mathfrak{R}_1/0.2, \mathbb{k}_2, 0 \right), \left\{ \frac{\nabla_1}{\langle 0.3, 0.8, 0.4 \rangle}, \frac{\nabla_2}{\langle 0.9, 0.2, 0.4 \rangle} \right\} \right), \\ & \left(\mathfrak{R}_1/0.2, \mathbb{k}_3, 0 \right), \left\{ \frac{\nabla_1}{\langle 0.3, 0.7, 0.5 \rangle}, \frac{\nabla_2}{\langle 0.2, 0.8, 0.6 \rangle} \right\} \right), \\ & \left(\mathfrak{R}_2/0.3, \mathbb{k}_1, 0 \right), \left\{ \frac{\nabla_1}{\langle 0.8, 0.7, 0.6 \rangle}, \frac{\nabla_2}{\langle 0.3, 0.6, 0.7 \rangle} \right\} \right), \\ & \left(\mathfrak{R}_2/0.3, \mathbb{k}_2, 0 \right), \left\{ \frac{\nabla_1}{\langle 0.7, 0.3, 0.5 \rangle}, \frac{\nabla_2}{\langle 0.7, 0.6, 0.4 \rangle} \right\} \right), \\ & \left(\mathfrak{R}_2/0.3, \mathbb{k}_3, 0 \right), \left\{ \frac{\nabla_1}{\langle 0.6, 0.8, 0.5 \rangle}, \frac{\nabla_2}{\langle 0.7, 0.9, 0.4 \rangle} \right\} \right) \end{aligned} \right\}.$$

Definition 9. A subset property of A FPNSE-set $(\mathfrak{J}_1, \mathbb{Q})$ with another FPNSE-set $(\mathfrak{J}_2, \mathbb{P})$ can be seen as when set $\mathbb{Q} \subseteq \mathbb{P}$, and also fulfills the condition $\forall \gamma \in \mathbb{Q}, \mathfrak{J}_1(\gamma) \subseteq \mathfrak{J}_2(\gamma)$ and symbolized as $(\mathfrak{J}_1, \mathbb{Q}) \subseteq (\mathfrak{J}_2, \mathbb{P})$.

Example 10. Consider the Example 8, suppose

$$\mathbb{Q}_1 = \{ (\mathfrak{R}_1/0.2, \mathbb{k}_1, 1), (\mathfrak{R}_1/0.2, \mathbb{k}_2, 1), (\mathfrak{R}_1/0.2, \mathbb{k}_3, 0) \},$$

$$\mathbb{Q}_2 = \{ (\mathfrak{R}_1/0.2, \mathbb{k}_1, 1), (\mathfrak{R}_1/0.2, \mathbb{k}_2, 1), (\mathfrak{R}_1/0.2, \mathbb{k}_2, 0), (\mathfrak{R}_1/0.2, \mathbb{k}_3, 0), (\mathfrak{R}_1/0.2, \mathbb{k}_3, 1) \}.$$

It is clear that $\mathbb{Q}_1 \subset \mathbb{Q}_2$.

Suppose $(\mathfrak{I}_1, \mathbb{Q}_1)$ and $(\mathfrak{I}_2, \mathbb{Q}_2)$ be defined as following

$$(\mathfrak{I}_1, \mathbb{Q}_1) = \left\{ \begin{array}{l} \left((\mathfrak{R}_1/0.2, \mathbb{k}_1, 1), \left\{ \frac{\nabla_1}{\langle 0.3, 0.5, 0.4 \rangle}, \frac{\nabla_2}{\langle 0.7, 0.4, 0.5 \rangle} \right\} \right), \\ \left((\mathfrak{R}_1/0.2, \mathbb{k}_2, 1), \left\{ \frac{\nabla_1}{\langle 0.3, 0.4, 0.5 \rangle}, \frac{\nabla_2}{\langle 0.6, 0.4, 0.6 \rangle} \right\} \right), \\ \left((\mathfrak{R}_1/0.2, \mathbb{k}_3, 0), \left\{ \frac{\nabla_1}{\langle 0.1, 0.6, 0.3 \rangle}, \frac{\nabla_2}{\langle 0.1, 0.7, 0.4 \rangle} \right\} \right) \end{array} \right\},$$

$$(\mathfrak{I}_2, \mathbb{Q}_2) = \left\{ \begin{array}{l} \left((\mathfrak{R}_1/0.2, \mathbb{k}_1, 1), \left\{ \frac{\nabla_1}{\langle 0.2, 0.3, 0.6 \rangle}, \frac{\nabla_2}{\langle 0.7, 0.4, 0.7 \rangle} \right\} \right), \\ \left((\mathfrak{R}_1/0.2, \mathbb{k}_2, 1), \left\{ \frac{\nabla_1}{\langle 0.4, 0.3, 0.4 \rangle}, \frac{\nabla_2}{\langle 0.8, 0.3, 0.5 \rangle} \right\} \right), \\ \left((\mathfrak{R}_1/0.2, \mathbb{k}_3, 1), \left\{ \frac{\nabla_1}{\langle 0.7, 0.2, 0.4 \rangle}, \frac{\nabla_2}{\langle 0.5, 0.2, 0.6 \rangle} \right\} \right), \\ \left((\mathfrak{R}_1/0.2, \mathbb{k}_3, 0), \left\{ \frac{\nabla_1}{\langle 0.2, 0.5, 0.1 \rangle}, \frac{\nabla_2}{\langle 0.2, 0.6, 0.3 \rangle} \right\} \right) \end{array} \right\}$$

which shows that $(\mathfrak{I}_1, \mathbb{Q}_1) \subseteq (\mathfrak{I}_2, \mathbb{Q}_2)$.

Definition 11. Two FPNSE-sets will be $(\mathfrak{I}_1, \mathbb{Q}_1) = (\mathfrak{I}_2, \mathbb{Q}_2)$ if $(\mathfrak{I}_1, \mathbb{Q}_1) \subseteq (\mathfrak{I}_2, \mathbb{Q}_2)$ and $(\mathfrak{I}_2, \mathbb{Q}_2) \subseteq (\mathfrak{I}_1, \mathbb{Q}_1)$.

Definition 12. The complement of a FPNSE-set $(\mathfrak{I}, \mathbb{Q})$, denoted by $(\mathfrak{I}, \mathbb{Q})^c$, is defined by $(\mathfrak{I}, \mathbb{Q})^c = \tilde{c}(\mathfrak{I}(\varsigma)) \forall \varsigma \in \mathfrak{K}$ while \tilde{c} is a NF complement.

Example 13. Making use of Example 8 for complement, we have

$$(\mathfrak{I}, \mathbb{Q})^c = \left\{ \begin{array}{l} \left((\mathfrak{R}_1/0.2, \mathbb{k}_1, 1), \left\{ \frac{\nabla_1}{\langle 0.4, 0.5, 0.2 \rangle}, \frac{\nabla_2}{\langle 0.5, 0.8, 0.7 \rangle} \right\} \right), \\ \left((\mathfrak{R}_1/0.2, \mathbb{k}_2, 1), \left\{ \frac{\nabla_1}{\langle 0.3, 0.8, 0.6 \rangle}, \frac{\nabla_2}{\langle 0.5, 0.9, 0.8 \rangle} \right\} \right), \\ \left((\mathfrak{R}_1/0.2, \mathbb{k}_3, 1), \left\{ \frac{\nabla_1}{\langle 0.3, 0.8, 0.7 \rangle}, \frac{\nabla_2}{\langle 0.6, 0.7, 0.5 \rangle} \right\} \right), \\ \left((\mathfrak{R}_2/0.3, \mathbb{k}_1, 1), \left\{ \frac{\nabla_1}{\langle 0.3, 0.9, 0.9 \rangle}, \frac{\nabla_2}{\langle 0.4, 0.5, 0.4 \rangle} \right\} \right), \\ \left((\mathfrak{R}_2/0.3, \mathbb{k}_2, 1), \left\{ \frac{\nabla_1}{\langle 0.6, 0.5, 0.4 \rangle}, \frac{\nabla_2}{\langle 0.7, 0.9, 0.8 \rangle} \right\} \right), \\ \left((\mathfrak{R}_2/0.3, \mathbb{k}_3, 1), \left\{ \frac{\nabla_1}{\langle 0.7, 0.6, 0.5 \rangle}, \frac{\nabla_2}{\langle 0.4, 0.4, 0.3 \rangle} \right\} \right) \end{array} \right\}.$$

Definition 14. An agree-FPNSE-set $(\mathfrak{I}, \mathbb{Q})_{ag}$ over \mathfrak{K} , is a FPNSE-subset of $(\mathfrak{I}, \mathbb{Q})$ and is characterized as $(\mathfrak{I}, \mathbb{Q})_{ag} = \{\mathfrak{I}_{ag}(\varsigma) : \varsigma \in G \times D \times \{1\}\}$.

Example 15. Finding agree-FPNSE-set determined in 8, we get $(\mathfrak{I}, \mathbb{Q}) =$

$$\left\{ \begin{array}{l} \left((\mathfrak{R}_1/0.2, \mathbb{k}_1, 1), \left\{ \frac{\nabla_1}{\langle 0.3, 0.5, 0.4 \rangle}, \frac{\nabla_2}{\langle 0.7, 0.4, 0.5 \rangle} \right\} \right), \\ \left((\mathfrak{R}_1/0.2, \mathbb{k}_2, 1), \left\{ \frac{\nabla_1}{\langle 0.4, 0.6, 0.3 \rangle}, \frac{\nabla_2}{\langle 0.8, 0.4, 0.5 \rangle} \right\} \right), \\ \left((\mathfrak{R}_1/0.2, \mathbb{k}_3, 1), \left\{ \frac{\nabla_1}{\langle 0.7, 0.4, 0.3 \rangle}, \frac{\nabla_2}{\langle 0.5, 0.3, 0.6 \rangle} \right\} \right), \\ \left((\mathfrak{R}_2/0.3, \mathbb{k}_1, 1), \left\{ \frac{\nabla_1}{\langle 0.9, 0.6, 0.3 \rangle}, \frac{\nabla_2}{\langle 0.4, 0.3, 0.4 \rangle} \right\} \right), \\ \left((\mathfrak{R}_2/0.3, \mathbb{k}_2, 1), \left\{ \frac{\nabla_1}{\langle 0.4, 0.7, 0.6 \rangle}, \frac{\nabla_2}{\langle 0.8, 0.9, 0.7 \rangle} \right\} \right), \\ \left((\mathfrak{R}_2/0.3, \mathbb{k}_3, 1), \left\{ \frac{\nabla_1}{\langle 0.5, 0.3, 0.7 \rangle}, \frac{\nabla_2}{\langle 0.3, 0.6, 0.4 \rangle} \right\} \right) \end{array} \right\}.$$

Definition 16. A disagree-FPNSE-set $(\mathfrak{J}, \mathbb{Q})_{dag}$ over $\hat{\mathfrak{K}}$, is a FPNSE-subset of $(\mathfrak{J}, \mathbb{Q})$ and is characterized as $(\mathfrak{J}, \mathbb{Q})_{dag} = \{\mathfrak{J}_{dag}(\varsigma) : \varsigma \in \mathfrak{V} \times \mathfrak{W} \times \{0\}\}$.

Example 17. Getting disagree-FPNSE-set determined in example 8, $(\mathfrak{J}, \mathbb{Q}) =$

$$\left\{ \begin{array}{l} \left((\mathfrak{R}_1/0.2, \mathbb{k}_1, 0), \left\{ \frac{\nabla_1}{\langle 0.3, 0.2, 0.6 \rangle}, \frac{\nabla_2}{\langle 0.7, 0.4, 0.5 \rangle}, \right\} \right), \\ \left((\mathfrak{R}_1/0.2, \mathbb{k}_2, 0), \left\{ \frac{\nabla_1}{\langle 0.3, 0.8, 0.4 \rangle}, \frac{\nabla_2}{\langle 0.9, 0.2, 0.4 \rangle}, \right\} \right), \\ \left((\mathfrak{R}_1/0.2, \mathbb{k}_3, 0), \left\{ \frac{\nabla_1}{\langle 0.3, 0.7, 0.5 \rangle}, \frac{\nabla_2}{\langle 0.2, 0.8, 0.6 \rangle}, \right\} \right), \\ \left((\mathfrak{R}_2/0.3, \mathbb{k}_1, 0), \left\{ \frac{\nabla_1}{\langle 0.8, 0.7, 0.6 \rangle}, \frac{\nabla_2}{\langle 0.3, 0.6, 0.7 \rangle}, \right\} \right), \\ \left((\mathfrak{R}_2/0.3, \mathbb{k}_2, 0), \left\{ \frac{\nabla_1}{\langle 0.7, 0.3, 0.5 \rangle}, \frac{\nabla_2}{\langle 0.7, 0.6, 0.4 \rangle}, \right\} \right), \\ \left((\mathfrak{R}_2/0.3, \mathbb{k}_3, 0), \left\{ \frac{\nabla_1}{\langle 0.6, 0.8, 0.5 \rangle}, \frac{\nabla_2}{\langle 0.7, 0.9, 0.4 \rangle}, \right\} \right) \end{array} \right\}.$$

Definition 18. A relative null FPNSE-set of a FPNSE-set $(\mathfrak{J}_1, \mathbb{Q}_1)$ w.r.t $\mathbb{Q}_1 \subset \mathbb{Q}$, denoted by $(\mathfrak{J}_1, \mathbb{Q}_1)_{RN}$, if $\mathfrak{J}_1(g) = \emptyset, \forall g \in \mathbb{Q}_1$.

Example 19. Taking Example 8, if

$$(\mathfrak{J}_1, \mathbb{Q}_1)_{RN} = \{((\mathfrak{R}_1/0.1, \mathbb{k}_1, 1), \emptyset), ((\mathfrak{R}_2/0.3, \mathbb{k}_2, 1), \emptyset), ((\mathfrak{R}_2/0.3, \mathbb{k}_3, 1), \emptyset)\}.$$

Definition 20. A FPNSE-set $(\mathfrak{J}_2, \mathbb{Q}_2)$ is called a relative whole FPNSE-set w.r.t $\mathbb{Q}_2 \subset \mathbb{Q}$, denoted by $(\mathfrak{J}_2, \mathbb{Q}_2)_{\hat{\mathfrak{K}}}$, if $\mathfrak{J}_1(g) = \hat{\mathfrak{K}}, \forall g \in \mathbb{Q}_2$.

Example 21. Taking Example 8, if

$$(\mathfrak{J}_2, \mathbb{Q}_2)_{\hat{\mathfrak{K}}} = \{((\mathfrak{R}_1/0.2, \mathbb{k}_1, 1), \hat{\mathfrak{K}}), ((\mathfrak{R}_2/0.3, \mathbb{k}_2, 1), \hat{\mathfrak{K}}), ((\mathfrak{R}_2/0.3, \mathbb{k}_3, 1), \hat{\mathfrak{K}})\}$$

where $\mathbb{Q}_2 \subseteq \mathbb{Q}$.

Definition 22. The absolute whole FPNSE-set of a FPNSE-set $(\mathfrak{J}, \mathbb{Q})$ denoted by $(\mathfrak{J}, \mathbb{Q})_{AW}$, if $\mathfrak{J}(g) = \hat{\mathfrak{K}}, \forall g \in \mathbb{Q}$.

Example 23. Using Example 8, if

$$(\mathfrak{J}, \mathbb{Q})_{AW} = \left\{ \begin{array}{l} (\mathfrak{R}_1/0.2, \mathbb{k}_1, 1), \hat{\mathfrak{K}}, (\mathfrak{R}_1/0.2, \mathbb{k}_2, 1), \hat{\mathfrak{K}}, (\mathfrak{R}_1/0.2, \mathbb{k}_3, 1), \hat{\mathfrak{K}}, \\ (\mathfrak{R}_2/0.3, \mathbb{k}_1, 1), \hat{\mathfrak{K}}, (\mathfrak{R}_2/0.3, \mathbb{k}_2, 1), \hat{\mathfrak{K}}, (\mathfrak{R}_2/0.3, \mathbb{k}_3, 1), \hat{\mathfrak{K}}, \\ (\mathfrak{R}_2/0.3, \mathbb{k}_1, 1), \hat{\mathfrak{K}}, (\mathfrak{R}_2/0.3, \mathbb{k}_2, 1), \hat{\mathfrak{K}} \end{array} \right\}.$$

4. SET THEORETIC OPERATIONS OF FPNSE-SET

In this portion, some set theoretic operations are presented with detailed examples.

Definition 24. The union of FPNSE-sets $(\mathfrak{J}_1, \mathbb{T})$ and $(\mathfrak{J}_2, \mathbb{L})$ over $\hat{\mathfrak{K}}$ is $(\mathfrak{J}_3, \mathbb{L})$ with $\mathbb{L} = \mathbb{T} \cup \mathbb{L}$, defined as

$$\mathfrak{J}_3(\varsigma) = \begin{cases} \mathfrak{J}_1(\varsigma) & ; \varsigma \in \mathbb{T} \setminus \mathbb{L} \\ \mathfrak{J}_2(\varsigma) & ; \varsigma \in \mathbb{L} \setminus \mathbb{T} \\ \cup(\mathfrak{J}_1(\varsigma), \mathfrak{J}_2(\varsigma)) & ; \varsigma \in \mathbb{T} \cap \mathbb{L} \end{cases}$$

where

$$\cup(\mathfrak{J}_1(\varsigma), \mathfrak{J}_2(\varsigma)) = \{\langle u, \max(\mathfrak{J}_1(\varsigma), \mathfrak{J}_2(\varsigma)), \min(\mathfrak{J}_1(\varsigma), \mathfrak{J}_2(\varsigma)), \min(\mathfrak{J}_1(\varsigma), \mathfrak{J}_2(\varsigma)) \rangle : u \in \hat{\mathfrak{K}}\}.$$

Example 25. Using Example 8, with two sets

$$\mathbb{Q}_1 = \{(\mathfrak{R}_1/0.2, \mathbb{k}_1, 1), (\mathfrak{R}_1/0.2, \mathbb{k}_2, 1), (\mathfrak{R}_2/0.3, \mathbb{k}_2, 1)\}$$

$$\mathbb{Q}_2 = \{(\mathfrak{R}_1/0.2, \mathbb{k}_1, 1), (\mathfrak{R}_2/0.3, \mathbb{k}_1, 1), (\mathfrak{R}_1/0.2, \mathbb{k}_2, 1), (\mathfrak{R}_2/0.3, \mathbb{k}_2, 1)\}.$$

Suppose $(\mathfrak{J}_1, \mathbb{Q}_1)$ and $(\mathfrak{J}_2, \mathbb{Q}_2)$ over $\hat{\mathfrak{K}}$ are two FPNSE-sets such that

$$\begin{aligned} (\mathfrak{J}_1, \mathbb{Q}_1) &= \left\{ \begin{aligned} &\left((\mathfrak{R}_1/0.2, \mathbb{k}_1, 1), \left\{ \frac{\nabla_1}{\langle 0.3, 0.5, 0.4 \rangle}, \frac{\nabla_2}{\langle 0.7, 0.4, 0.5 \rangle} \right\} \right), \\ &\left((\mathfrak{R}_1/0.2, \mathbb{k}_2, 1), \left\{ \frac{\nabla_1}{\langle 0.4, 0.6, 0.3 \rangle}, \frac{\nabla_2}{\langle 0.8, 0.4, 0.5 \rangle} \right\} \right), \\ &\left((\mathfrak{R}_2/0.3, \mathbb{k}_2, 1), \left\{ \frac{\nabla_1}{\langle 0.4, 0.7, 0.6 \rangle}, \frac{\nabla_2}{\langle 0.8, 0.9, 0.7 \rangle} \right\} \right), \end{aligned} \right\}, \\ (\mathfrak{J}_2, \mathbb{Q}_2) &= \left\{ \begin{aligned} &\left((\mathfrak{R}_1/0.2, \mathbb{k}_1, 1), \left\{ \frac{\nabla_1}{\langle 0.2, 0.3, 0.4 \rangle}, \frac{\nabla_2}{\langle 0.7, 0.3, 0.2 \rangle} \right\} \right), \\ &\left((\mathfrak{R}_1/0.2, \mathbb{k}_2, 1), \left\{ \frac{\nabla_1}{\langle 0.4, 0.3, 0.8 \rangle}, \frac{\nabla_2}{\langle 0.8, 0.3, 0.5 \rangle} \right\} \right), \\ &\left((\mathfrak{R}_2/0.3, \mathbb{k}_1, 1), \left\{ \frac{\nabla_1}{\langle 0.1, 0.3, 0.6 \rangle}, \frac{\nabla_2}{\langle 0.9, 0.1, 0.7 \rangle} \right\} \right), \\ &\left((\mathfrak{R}_2/0.3, \mathbb{k}_2, 1), \left\{ \frac{\nabla_1}{\langle 0.4, 0.2, 0.3 \rangle}, \frac{\nabla_2}{\langle 0.6, 0.3, 0.5 \rangle} \right\} \right) \end{aligned} \right\}. \end{aligned}$$

Then $(\mathfrak{J}_1, \mathbb{Q}_1) \cup (\mathfrak{J}_2, \mathbb{Q}_2) = (\mathfrak{J}_3, \mathbb{Q}_3)$

$$(\mathfrak{J}_3, \mathbb{Q}_3) = \left\{ \begin{aligned} &\left((\mathfrak{R}_1/0.2, \mathbb{k}_1, 1), \left\{ \frac{\nabla_1}{\langle 0.3, 0.3, 0.4 \rangle}, \frac{\nabla_2}{\langle 0.7, 0.3, 0.2 \rangle} \right\} \right), \\ &\left((\mathfrak{R}_1/0.2, \mathbb{k}_2, 1), \left\{ \frac{\nabla_1}{\langle 0.4, 0.3, 0.3 \rangle}, \frac{\nabla_2}{\langle 0.8, 0.3, 0.5 \rangle} \right\} \right), \\ &\left((\mathfrak{R}_2/0.3, \mathbb{k}_1, 1), \left\{ \frac{\nabla_1}{\langle 0.1, 0.3, 0.6 \rangle}, \frac{\nabla_2}{\langle 0.9, 0.1, 0.7 \rangle} \right\} \right), \\ &\left((\mathfrak{R}_2/0.3, \mathbb{k}_2, 1), \left\{ \frac{\nabla_1}{\langle 0.4, 0.2, 0.3 \rangle}, \frac{\nabla_2}{\langle 0.6, 0.3, 0.5 \rangle} \right\} \right) \end{aligned} \right\}.$$

Definition 26. Restricted Union of two FPNSE-sets $(\mathfrak{J}_1, \mathbb{Q}_1)$, $(\mathfrak{J}_2, \mathbb{Q}_2)$ over $\hat{\mathfrak{K}}$ is $(\mathfrak{J}_3, \mathbb{L})$ with $\mathbb{L} = \mathbb{Q}_1 \cap \mathbb{Q}_2$, defined as $\mathfrak{J}_3(\varsigma) = \mathfrak{J}_1(\varsigma) \cup_{\mathbb{R}} \mathfrak{J}_2(\varsigma)$ for $\varsigma \in \mathbb{Q}_1 \cap \mathbb{Q}_2$.

Example 27. Consider the above Example, we have

Then $(\mathfrak{J}_1, \mathbb{Q}_1) \cup_{\mathbb{R}} (\mathfrak{J}_2, \mathbb{Q}_2) = (\mathfrak{J}_3, \mathbb{L})$

$$(\mathfrak{J}_3, \mathbb{L}) = \left\{ \begin{aligned} &\left((\mathfrak{R}_1/0.2, \mathbb{k}_1, 1), \left\{ \frac{\nabla_1}{\langle 0.3, 0.3, 0.4 \rangle}, \frac{\nabla_2}{\langle 0.7, 0.3, 0.2 \rangle} \right\} \right), \\ &\left((\mathfrak{R}_1/0.2, \mathbb{k}_2, 1), \left\{ \frac{\nabla_1}{\langle 0.4, 0.3, 0.3 \rangle}, \frac{\nabla_2}{\langle 0.8, 0.3, 0.5 \rangle} \right\} \right), \\ &\left((\mathfrak{R}_2/0.3, \mathbb{k}_2, 1), \left\{ \frac{\nabla_1}{\langle 0.4, 0.2, 0.3 \rangle}, \frac{\nabla_2}{\langle 0.6, 0.3, 0.5 \rangle} \right\} \right) \end{aligned} \right\}.$$

Proposition 28. If $(\mathfrak{J}_1, \mathbb{Q}_1)$, $(\mathfrak{J}_2, \mathbb{Q}_2)$ and $(\mathfrak{J}_3, \mathbb{Q}_3)$ are three FPNSE-sets over $\hat{\mathfrak{K}}$, then

1. $(\mathfrak{J}_1, \mathbb{Q}_1) \cup (\mathfrak{J}_2, \mathbb{Q}_2) = (\mathfrak{J}_2, \mathbb{Q}_2) \cup (\mathfrak{J}_1, \mathbb{Q}_1)$,
2. $((\mathfrak{J}_1, \mathbb{Q}_1) \cup (\mathfrak{J}_2, \mathbb{Q}_2)) \cup (\mathfrak{J}_3, \mathbb{Q}_3) = (\mathfrak{J}_1, \mathbb{Q}_1) \cup ((\mathfrak{J}_2, \mathbb{Q}_2) \cup (\mathfrak{J}_3, \mathbb{Q}_3))$,
3. $(\mathfrak{J}, \mathbb{Q}) \cup \Phi = (\mathfrak{J}, \mathbb{Q})$.

Definition 29. The intersection of FPNSE-sets (\mathfrak{J}_1, \top) and (\mathfrak{J}_2, \perp) over $\hat{\mathfrak{K}}$ is $(\mathfrak{J}_3, \mathbb{L})$ with $\mathbb{L} = \top \cap \perp$, defined as

$$\mathfrak{J}_3(\varsigma) = \begin{cases} \mathfrak{J}_1(\varsigma) & ; \varsigma \in \top - \perp \\ \mathfrak{J}_2(\varsigma) & ; \varsigma \in \perp - \top \\ \cap(\mathfrak{J}_1(\varsigma), \mathfrak{J}_2(\varsigma)) & ; \varsigma \in \top \cap \perp \end{cases}$$

$$\cap(\mathfrak{J}_1(\varsigma), \mathfrak{J}_2(\varsigma)) = \{ \langle u, \min(\mathfrak{J}_1(\varsigma), \mathfrak{J}_2(\varsigma)), \max(\mathfrak{J}_1(\varsigma), \mathfrak{J}_2(\varsigma)), \max(\mathfrak{J}_1(\varsigma), \mathfrak{J}_2(\varsigma)) \rangle : u \in \hat{\mathfrak{K}} \}.$$

Example 30. Using data of Example 25, we have

Then $(\mathfrak{J}_1, \mathbb{Q}_1) \cap (\mathfrak{J}_2, \mathbb{Q}_2) = (\mathfrak{J}_3, \mathbb{Q}_3)$

$$(\mathfrak{J}_3, \mathbb{Q}_3) = \left\{ \begin{aligned} & \left((\mathfrak{R}_1/0.2, \mathbb{k}_1, 1), \left\{ \frac{\nabla_1}{\langle 0.2, 0.5, 0.4 \rangle}, \frac{\nabla_2}{\langle 0.7, 0.4, 0.5 \rangle} \right\} \right), \\ & \left((\mathfrak{R}_1/0.2, \mathbb{k}_2, 1), \left\{ \frac{\nabla_1}{\langle 0.4, 0.6, 0.8 \rangle}, \frac{\nabla_2}{\langle 0.8, 0.4, 0.5 \rangle} \right\} \right), \\ & \left((\mathfrak{R}_2/0.3, \mathbb{k}_2, 1), \left\{ \frac{\nabla_1}{\langle 0.4, 0.7, 0.6 \rangle}, \frac{\nabla_2}{\langle 0.6, 0.9, 0.7 \rangle} \right\} \right) \end{aligned} \right\}.$$

Definition 31. Extended intersection of $(\mathfrak{J}_1, \mathbb{S})$ and $(\mathfrak{J}_2, \mathbb{R})$ over $\hat{\mathfrak{K}}$ is $(\mathfrak{J}_3, \mathbb{L})$ with $\mathbb{L} = \mathbb{S} \cup \mathbb{R}$, defined as

$$\mathfrak{J}_3(\varsigma) = \begin{cases} \mathfrak{J}_1(\varsigma) & ; \varsigma \in \mathbb{S} - \mathbb{R} \\ \mathfrak{J}_2(\varsigma) & ; \varsigma \in \mathbb{R} - \mathbb{S} \\ \mathfrak{J}_1(\varsigma) \cap \mathfrak{J}_2(\varsigma) & ; \varsigma \in \mathbb{S} \cap \mathbb{R}. \end{cases}$$

Example 32. Reconsidering data from Example 25, we have

Then $(\mathfrak{J}_1, \mathbb{Q}_1) \cap_{\mathbb{E}} (\mathfrak{J}_2, \mathbb{Q}_2) = (\mathfrak{J}_3, \mathbb{L})$

$$(\mathfrak{J}_3, \mathbb{L}) = \left\{ \begin{aligned} & \left((\mathfrak{R}_1/0.2, \mathbb{k}_1, 1), \left\{ \frac{\nabla_1}{\langle 0.2, 0.5, 0.4 \rangle}, \frac{\nabla_2}{\langle 0.7, 0.4, 0.5 \rangle} \right\} \right), \\ & \left((\mathfrak{R}_1/0.2, \mathbb{k}_2, 1), \left\{ \frac{\nabla_1}{\langle 0.4, 0.6, 0.8 \rangle}, \frac{\nabla_2}{\langle 0.8, 0.4, 0.5 \rangle} \right\} \right), \\ & \left((\mathfrak{R}_2/0.3, \mathbb{k}_2, 1), \left\{ \frac{\nabla_1}{\langle 0.4, 0.7, 0.6 \rangle}, \frac{\nabla_2}{\langle 0.6, 0.9, 0.7 \rangle} \right\} \right), \\ & \left((\mathfrak{R}_2/0.3, \mathbb{k}_1, 1), \left\{ \frac{\nabla_1}{\langle 0.1, 0.3, 0.6 \rangle}, \frac{\nabla_2}{\langle 0.9, 0.1, 0.7 \rangle} \right\} \right) \end{aligned} \right\}.$$

Proposition 33. If $(\mathfrak{J}_1, \mathbb{Q}_1), (\mathfrak{J}_2, \mathbb{Q}_2)$ and $(\mathfrak{J}_3, \mathbb{Q}_3)$ are three FPNSE-sets over $\hat{\mathfrak{K}}$, then

1. $(\mathfrak{J}_1, \mathbb{Q}_1) \cap (\mathfrak{J}_2, \mathbb{Q}_2) = (\mathfrak{J}_2, \mathbb{Q}_2) \cap (\mathfrak{J}_1, \mathbb{Q}_1)$,
2. $((\mathfrak{J}_1, \mathbb{Q}_1) \cap (\mathfrak{J}_2, \mathbb{Q}_2)) \cap (\mathfrak{J}_3, \mathbb{Q}_3) = (\mathfrak{J}_1, \mathbb{Q}_1) \cap ((\mathfrak{J}_2, \mathbb{Q}_2) \cap (\mathfrak{J}_3, \mathbb{Q}_3))$,
3. $(\mathfrak{J}, \mathbb{Q}) \cap \emptyset = \emptyset$.

Proposition 34. If $(\mathfrak{J}_1, \mathbb{Q}_1), (\mathfrak{J}_2, \mathbb{Q}_2)$ and $(\mathfrak{J}_3, \mathbb{Q}_3)$ are three FPNSE-sets over $\hat{\mathfrak{K}}$, then

1. $(\mathfrak{J}_1, \mathbb{Q}_1) \cup ((\mathfrak{J}_2, \mathbb{Q}_2) \cap (\mathfrak{J}_3, \mathbb{Q}_3)) = ((\mathfrak{J}_1, \mathbb{Q}_1) \cup ((\mathfrak{J}_2, \mathbb{Q}_2)) \cap ((\mathfrak{J}_1, \mathbb{Q}_1) \cup (\mathfrak{J}_3, \mathbb{Q}_3)))$,
2. $(\mathfrak{J}_1, \mathbb{Q}_1) \cap ((\mathfrak{J}_2, \mathbb{Q}_2) \cup (\mathfrak{J}_3, \mathbb{Q}_3)) = ((\mathfrak{J}_1, \mathbb{Q}_1) \cap ((\mathfrak{J}_2, \mathbb{Q}_2)) \cup ((\mathfrak{J}_1, \mathbb{Q}_1) \cap (\mathfrak{J}_3, \mathbb{Q}_3)))$.

Definition 35. If $(\mathfrak{J}_1, \mathbb{Q}_1)$ and $(\mathfrak{J}_2, \mathbb{Q}_2)$ are two FPNSE-sets over $\hat{\mathfrak{K}}$ then $(\mathfrak{J}_1, \mathbb{Q}_1)$ AND $(\mathfrak{J}_2, \mathbb{Q}_2)$ denoted by $(\mathfrak{J}_1, \mathbb{Q}_1) \wedge (\mathfrak{J}_2, \mathbb{Q}_2)$ is defined by $(\mathfrak{J}_1, \mathbb{Q}_1) \wedge (\mathfrak{J}_2, \mathbb{Q}_2) = (\mathfrak{J}_3, \mathbb{Q}_1 \times \mathbb{Q}_2)$, while $\mathfrak{J}_3(\varsigma, \gamma) = \mathfrak{J}_1(\varsigma) \cap \mathfrak{J}_2(\gamma), \forall (\varsigma, \gamma) \in \mathbb{Q}_1 \times \mathbb{Q}_2$.

Example 36. Retaking Example 8,

Suppose $(\mathfrak{I}_1, \mathbb{Q}_1)$ and $(\mathfrak{I}_2, \mathbb{Q}_2)$ over $\hat{\mathfrak{K}}$ are two FPNSE-sets such that

$$\begin{aligned} (\mathfrak{I}_1, \mathbb{Q}_1) &= \left\{ \left((\mathfrak{R}_1/0.2, \mathbb{k}_1, 1), \left\{ \frac{\nabla_1}{\langle 0.3, 0.5, 0.4 \rangle}, \frac{\nabla_2}{\langle 0.7, 0.4, 0.5 \rangle} \right\} \right), \left((\mathfrak{R}_1/0.2, \mathbb{k}_2, 1), \left\{ \frac{\nabla_1}{\langle 0.4, 0.6, 0.3 \rangle}, \frac{\nabla_2}{\langle 0.8, 0.4, 0.5 \rangle} \right\} \right) \right\}, \\ (\mathfrak{I}_2, \mathbb{Q}_2) &= \left\{ \left((\mathfrak{R}_1/0.2, \mathbb{k}_1, 0), \left\{ \frac{\nabla_1}{\langle 0.2, 0.3, 0.4 \rangle}, \frac{\nabla_2}{\langle 0.7, 0.3, 0.2 \rangle} \right\} \right), \left((\mathfrak{R}_1/0.2, \mathbb{k}_2, 0), \left\{ \frac{\nabla_1}{\langle 0.4, 0.3, 0.8 \rangle}, \frac{\nabla_2}{\langle 0.8, 0.3, 0.5 \rangle} \right\} \right) \right\}. \end{aligned}$$

Then $(\mathfrak{I}_1, \mathbb{Q}_1) \wedge (\mathfrak{I}_2, \mathbb{Q}_2) = (\mathfrak{I}_3, \mathbb{Q}_1 \times \mathbb{Q}_2)$

$$\left\{ \left(((\mathfrak{R}_1/0.2, \mathbb{k}_1, 1), (\mathfrak{R}_1/0.2, \mathbb{k}_1, 0)), \left\{ \frac{\nabla_1}{\langle 0.2, 0.5, 0.4 \rangle}, \frac{\nabla_2}{\langle 0.7, 0.4, 0.5 \rangle} \right\} \right), \left(((\mathfrak{R}_1/0.2, \mathbb{k}_1, 1), (\mathfrak{R}_1/0.2, \mathbb{k}_2, 0)), \left\{ \frac{\nabla_1}{\langle 0.3, 0.5, 0.8 \rangle}, \frac{\nabla_2}{\langle 0.7, 0.4, 0.5 \rangle} \right\} \right), \right. \\ \left. \left(((\mathfrak{R}_1/0.2, \mathbb{k}_2, 1), (\mathfrak{R}_1/0.2, \mathbb{k}_1, 0)), \left\{ \frac{\nabla_1}{\langle 0.2, 0.6, 0.4 \rangle}, \frac{\nabla_2}{\langle 0.7, 0.4, 0.5 \rangle} \right\} \right), \left(((\mathfrak{R}_1/0.2, \mathbb{k}_2, 1), (\mathfrak{R}_1/0.2, \mathbb{k}_2, 0)), \left\{ \frac{\nabla_1}{\langle 0.4, 0.6, 0.8 \rangle}, \frac{\nabla_2}{\langle 0.8, 0.4, 0.5 \rangle} \right\} \right) \right\}.$$

Definition 37. If $(\mathfrak{I}_1, \mathbb{Q}_1)$ and $(\mathfrak{I}_2, \mathbb{Q}_2)$ are two FPNSE-sets over $\hat{\mathfrak{K}}$, then $(\mathfrak{I}_1, \mathbb{Q}_1)$ OR $(\mathfrak{I}_2, \mathbb{Q}_2)$ denoted by $(\mathfrak{I}_1, \mathbb{Q}_1) \vee (\mathfrak{I}_2, \mathbb{Q}_2)$ is defined by $(\mathfrak{I}_1, \mathbb{Q}_1) \vee (\mathfrak{I}_2, \mathbb{Q}_2) = (\mathfrak{I}_3, \mathbb{Q}_1 \times \mathbb{Q}_2)$, while $\mathfrak{I}_3(\delta, \gamma) = \mathfrak{I}_1(\delta) \cup \mathfrak{I}_2(\gamma), \forall (\delta, \gamma) \in \mathbb{Q}_1 \times \mathbb{Q}_2$.

Example 38. Reconsidering Example 36,

Then $(\mathfrak{I}_3, \mathbb{Q}_3) \vee (\mathfrak{I}_2, \mathbb{Q}_2) = (\mathfrak{I}_3, \mathbb{Q}_1 \times \mathbb{Q}_2) =$

$$\left\{ \left(((\mathfrak{R}_1/0.2, \mathbb{k}_1, 1), (\mathfrak{R}_1/0.2, \mathbb{k}_1, 0)), \left\{ \frac{\nabla_1}{\langle 0.3, 0.3, 0.4 \rangle}, \frac{\nabla_2}{\langle 0.7, 0.3, 0.2 \rangle} \right\} \right), \left(((\mathfrak{R}_1/0.2, \mathbb{k}_1, 1), (\mathfrak{R}_1/0.2, \mathbb{k}_2, 0)), \left\{ \frac{\nabla_1}{\langle 0.4, 0.3, 0.4 \rangle}, \frac{\nabla_2}{\langle 0.8, 0.3, 0.5 \rangle} \right\} \right), \right. \\ \left. \left(((\mathfrak{R}_1/0.2, \mathbb{k}_2, 1), (\mathfrak{R}_1/0.2, \mathbb{k}_1, 0)), \left\{ \frac{\nabla_1}{\langle 0.4, 0.3, 0.3 \rangle}, \frac{\nabla_2}{\langle 0.8, 0.3, 0.2 \rangle} \right\} \right), \left(((\mathfrak{R}_1/0.2, \mathbb{k}_2, 1), (\mathfrak{R}_1/0.2, \mathbb{k}_2, 0)), \left\{ \frac{\nabla_1}{\langle 0.4, 0.3, 0.3 \rangle}, \frac{\nabla_2}{\langle 0.8, 0.3, 0.5 \rangle} \right\} \right) \right\}.$$

Proposition 39. If $(\mathfrak{I}_1, \mathbb{Q}_1), (\mathfrak{I}_2, \mathbb{Q}_2)$ and $(\mathfrak{I}_3, \mathbb{Q}_3)$ are three FPNSE-sets over $\hat{\mathfrak{K}}$, then

1. $((\mathfrak{I}_1, \mathbb{Q}_1) \wedge (\mathfrak{I}_2, \mathbb{Q}_2))^c = ((\mathfrak{I}_1, \mathbb{Q}_1))^c \vee ((\mathfrak{I}_2, \mathbb{Q}_2))^c$
2. $((\mathfrak{I}_1, \mathbb{Q}_1) \vee (\mathfrak{I}_2, \mathbb{Q}_2))^c = ((\mathfrak{I}_1, \mathbb{Q}_1))^c \wedge ((\mathfrak{I}_2, \mathbb{Q}_2))^c$.

Proof. (1) $((\mathfrak{I}_1, \mathbb{Q}_1) \wedge (\mathfrak{I}_2, \mathbb{Q}_2))^c = (\mathfrak{I}_1(\phi) \cap (\mathfrak{I}_2(\psi)))^c$

$$= (\mathfrak{I}_1^c(\phi) \cup (\mathfrak{I}_2^c(\psi)))$$

$$= c(\mathfrak{I}_1(\sim \phi) \cup c(\mathfrak{I}_2(\sim \psi)))$$

$$= ((\mathfrak{I}_1, \mathbb{Q}_1))^c \vee ((\mathfrak{I}_2, \mathbb{Q}_2))^c.$$

(2) This is similar to the first. \square

Proposition 40. If $(\mathfrak{I}_1, \mathbb{Q}_1), (\mathfrak{I}_2, \mathbb{Q}_2)$ and $(\mathfrak{I}_3, \mathbb{Q}_3)$ are three FPNSE-sets over $\hat{\mathfrak{K}}$, then

1. $((\mathfrak{I}_1, \mathbb{Q}_1) \wedge (\mathfrak{I}_2, \mathbb{Q}_2)) \wedge (\mathfrak{I}_3, \mathbb{Q}_3) = (\mathfrak{I}_1, \mathbb{Q}_1) \wedge ((\mathfrak{I}_2, \mathbb{Q}_2) \wedge (\mathfrak{I}_3, \mathbb{Q}_3))$
2. $((\mathfrak{I}_1, \mathbb{Q}_1) \vee (\mathfrak{I}_2, \mathbb{Q}_2)) \vee (\mathfrak{I}_3, \mathbb{Q}_3) = (\mathfrak{I}_1, \mathbb{Q}_1) \vee ((\mathfrak{I}_2, \mathbb{Q}_2) \vee (\mathfrak{I}_3, \mathbb{Q}_3))$
3. $(\mathfrak{I}_1, \mathbb{Q}_1) \vee ((\mathfrak{I}_2, \mathbb{Q}_2) \wedge (\mathfrak{I}_3, \mathbb{Q}_3)) = ((\mathfrak{I}_1, \mathbb{Q}_1) \vee (\mathfrak{I}_2, \mathbb{Q}_2)) \wedge ((\mathfrak{I}_1, \mathbb{Q}_1) \vee (\mathfrak{I}_3, \mathbb{Q}_3))$
4. $(\mathfrak{I}_1, \mathbb{Q}_1) \wedge ((\mathfrak{I}_2, \mathbb{Q}_2) \vee (\mathfrak{I}_3, \mathbb{Q}_3)) = ((\mathfrak{I}_1, \mathbb{Q}_1) \wedge (\mathfrak{I}_2, \mathbb{Q}_2)) \vee ((\mathfrak{I}_1, \mathbb{Q}_1) \wedge (\mathfrak{I}_3, \mathbb{Q}_3)).$

Proof. All propositions can be proved using proposition 4.16. \square

5. AN APPLICATION TO FUZZY PARAMETERIZED NEUTROSOPHIC SOFT EXPERT SETS

This section presents an algorithmically suggested use of FPNSE-set theory to a decision-making problem.

5.1. Statement of the problem

In this part, we demonstrate how fuzzy parameterized neutrosophic soft expert set theory is applied to a problem of decision-making.

Example 41. *Let's say a restaurant chain intends to invest in the construction of a new location in a city suburb. They asked some professionals to conduct an economic feasibility analysis on the place they had chosen for them. The expert's four choices are as follows: $\Lambda = \{\Upsilon_1, \Upsilon_2, \Upsilon_3, \Upsilon_4\}$ based on the criteria they utilised to arrive at their conclusions. Three parameters $\Psi_1 = \text{Market research}$, $\Psi_2 = \text{Location}$, $\Psi_3 = \text{Menu}$, $\Psi_4 = \text{Legal considerations}$, $\Psi_5 = \text{Financing}$, are used to assess the restaurant site with the important degree 0.2, 0.4, 0.5, 0.7, 0.9 respectively.*

Proposed Algorithm : Selection of area for a newly Restaurant

▷ **Start:**

▷ **Input:**

- 1. Consider set of alternatives
- 2. Consider set of parameters
- 3. Set of experts
- 4. Set opinions of experts

▷ **Construction:**

- 5. Construct FPNSE-set (ξ, K)

▷ **Computation:**

- 6. Determine agree-FPNSE-set and disagree-FPNSE-set.
- 7. Calculation of values of $|\top(0_i) - \iota(0_i) - \perp(0_i)|$ for each $0_i \in \hat{\mathfrak{K}}$.
- 8. Calculate the the highest numerical grade for agree and disagree-FPNSE-sets.
- 9. Determine the score of each element $0_i \in \hat{\mathfrak{K}}$ for agree and disagree-FPNSE-sets.
- 10. Determine the score difference for each element $\Upsilon_i \in \hat{\mathfrak{K}}$.

▷ **Output:**

- 11. Compute n, for which $M = \max j_i$ to decide the best solution of the problem.

▷ **End:**

5.2. Role of parameters

This section includes the role and explanation of some selected parameters.

1. **Market research:** It is important to understand the local market and competition to ensure that there is demand for your restaurant and to determine the best location.

2. **Location:** The location of your restaurant will have a major impact on its success. Consider factors such as foot traffic, parking availability, and proximity to other restaurants and attractions.
3. **Menu:** Determine what type of food you will serve and how it will be priced. Consider the local market and competition when developing your menu. Developing a menu for a new restaurant is an important part of the planning process. Your menu should be carefully thought out and should reflect the concept and theme of your restaurant, as well as the preferences and tastes of your target market. Here are some things to consider when developing a menu for a new restaurant: Cuisine type: Determine the type of cuisine you will serve, and consider the local market and competition when making this decision. Pricing: Determine the price point for your menu items, keeping in mind the cost of ingredients and the local market. Portion size: Decide on the portion size for each menu item, considering the price point and the desired profit margin. Specials and seasonal items: Consider offering specials or rotating menu items based on the season or local availability of ingredients. Dietary restrictions: Consider offering options for customers with dietary restrictions, such as vegetarian, vegan, or gluten-free options. Plating and presentation: The presentation of your dishes can be just as important as the taste, so consider the plating and presentation of each menu item. Menu design: The design of menu should be visually appealing and easy to read. Consider using photos of your dishes, and be sure to include the name, description, and price of each menu item.
4. **Legal considerations:** Determine how you will finance the start-up costs of your restaurant, including the cost of purchasing or leasing a location, purchasing equipment, and hiring staff.
5. **Financing:** Obtain any necessary licenses and permits, and consider any local zoning laws or regulations that may impact your restaurant.

Step-1

Let $\Theta_1, \Theta_2, \Theta_3$ be the three experts. We can determine the best place for the new restaurant to open in the network based on those findings. The committee creates the following fuzzy parameterized neutrosophic soft expert set after a thoughtful deliberation. Let $\Psi_1/0.2, \Psi_2/0.4, \Psi_3/0.5, \Psi_4/0.7, \Psi_5/0.9$ be fuzzy subset of I^\heartsuit (set of fuzzy subsets of \heartsuit). Then we can form the fuzzy parameterized neutrosophic soft expert set as

Table 1: Values $\mathfrak{K} = |\top(0) - \iota(0) - \perp(0)|$ for each $0 \in \hat{\mathfrak{K}}$ for agree-FPNSE-set

<i>Pairs</i>	ℓ_1	ℓ_2	ℓ_3	ℓ_4
$(\Psi_1/0.2, \Theta_1, 1)$	0.2	0.4	0.1	0.1
$(\Psi_1/0.2, \Theta_2, 1)$	0.0	0.1	0.2	0.3
$(\Psi_1/0.2, \Theta_3, 1)$	0.2	0.4	0.1	0.2
$(\Psi_2/0.4, \Theta_1, 1)$	0.3	0.1	0.1	0.2
$(\Psi_2/0.4, \Theta_2, 1)$	0.1	0.1	0.2	0.4
$(\Psi_2/0.4, \Theta_3, 1)$	0.2	0.2	0.4	0.1
$(\Psi_3/0.5, \Theta_1, 1)$	0.4	0.2	0.0	0.3
$(\Psi_3/0.5, \Theta_2, 1)$	0.1	0.2	0.5	0.1
$(\Psi_3/0.5, \Theta_3, 1)$	0.5	0.4	0.2	0.2
$(\Psi_4/0.7, \Theta_1, 1)$	0.1	0.0	0.3	0.1
$(\Psi_4/0.7, \Theta_2, 1)$	0.3	0.0	0.4	0.2
$(\Psi_4/0.7, \Theta_3, 1)$	0.0	0.4	0.1	0.2
$(\Psi_5/0.9, \Theta_1, 1)$	0.3	0.2	0.8	0.7
$(\Psi_5/0.9, \Theta_2, 1)$	0.6	0.1	0.5	0.4
$(\Psi_5/0.9, \Theta_3, 1)$	0.1	1.0	0.5	0.4

Table 2: Values $\mathfrak{L} = |\top(\emptyset) - \iota(\emptyset) - \perp(\emptyset)|$ for each $\emptyset \in \Omega$ for disagree-FPNSE-set

<i>Pairs</i>	ℓ_1	ℓ_2	ℓ_3	ℓ_4
$(\Psi_1/0.2, \Theta_1, 0)$	0.0	0.4	0.1	0.3
$(\Psi_1/0.2, \Theta_2, 0)$	0.4	0.2	0.0	0.8
$(\Psi_1/0.2, \Theta_3, 0)$	0.2	0.2	1.2	0.3
$(\Psi_2/0.4, \Theta_1, 0)$	0.0	0.2	0.9	0.5
$(\Psi_2/0.4, \Theta_2, 0)$	1.0	0.4	1.0	0.0
$(\Psi_2/0.4, \Theta_3, 0)$	1.0	0.1	0.9	0.1
$(\Psi_3/0.5, \Theta_1, 0)$	0.4	0.3	0.0	0.2
$(\Psi_3/0.5, \Theta_2, 0)$	0.1	0.1	0.1	1.0
$(\Psi_3/0.5, \Theta_3, 0)$	0.0	0.2	0.4	0.1
$(\Psi_4/0.7, \Theta_1, 0)$	0.5	0.1	0.2	1.2
$(\Psi_4/0.7, \Theta_2, 0)$	0.1	0.2	0.5	0.1
$(\Psi_4/0.7, \Theta_3, 0)$	0.5	0.4	0.5	0.1
$(\Psi_5/0.9, \Theta_1, 0)$	0.0	0.3	0.2	0.2
$(\Psi_5/0.9, \Theta_2, 0)$	0.5	0.6	0.5	0.1
$(\Psi_5/0.9, \Theta_3, 0)$	0.0	0.1	0.5	0.6

are FPNSE-sets.

Step-2 To calculate the values of $\top(0) - \iota(0) - \perp(0)$, Tables 1 and 2 are constructed. For the example in table 1, the value 0.2 with respect to the $\langle 0.8, 0.4, 0.6 \rangle$ is calculated by using the formula $|\top(0) - \iota(0) - \perp(0)|$, as $|0.8 - 0.4 - 0.6| = 0.2$.

Step-3

Tables 3 and 4 have been constructed for the grade values of agree and disagree-FPNSE-sets respectively.

Table 3: Numerical Grades of agree-FPNSSE-set

Pairs	ℓ_i	Grade
$(\Psi_1/0.2, \Theta_1, 1)$	ℓ_2	0.4
$(\Psi_1/0.2, \Theta_2, 1)$	ℓ_4	0.3
$(\Psi_1/0.2, \Theta_3, 1)$	ℓ_2	0.4
$(\Psi_2/0.4, \Theta_1, 1)$	ℓ_1	0.3
$(\Psi_2/0.4, \Theta_2, 1)$	ℓ_4	0.4
$(\Psi_2/0.4, \Theta_3, 1)$	ℓ_3	0.4
$(\Psi_3/0.5, \Theta_1, 1)$	ℓ_1	0.4
$(\Psi_3/0.5, \Theta_2, 1)$	ℓ_3	0.5
$(\Psi_3/0.5, \Theta_3, 1)$	ℓ_1	0.5
$(\Psi_4/0.7, \Theta_1, 1)$	ℓ_3	0.3
$(\Psi_4/0.7, \Theta_2, 1)$	ℓ_3	0.4
$(\Psi_4/0.7, \Theta_3, 1)$	ℓ_2	0.4
$(\Psi_5/0.9, \Theta_1, 1)$	ℓ_3	0.8
$(\Psi_5/0.9, \Theta_2, 1)$	ℓ_1	0.6
$(\Psi_5/0.9, \Theta_3, 1)$	ℓ_3	0.5

Table 4: Numerical Grades of disagree-FPNSSE-set

Pairs	ℓ_i	Grade
$(\Psi_1/0.2, \Theta_1, 0)$	ℓ_2	0.4
$(\Psi_1/0.2, \Theta_2, 0)$	ℓ_4	0.8
$(\Psi_1/0.2, \Theta_3, 0)$	ℓ_3	1.2
$(\Psi_2/0.4, \Theta_1, 0)$	ℓ_3	0.9
$(\Psi_2/0.4, \Theta_2, 0)$	ℓ_2	0.4
$(\Psi_2/0.4, \Theta_3, 0)$	ℓ_3	0.9
$(\Psi_3/0.5, \Theta_1, 0)$	ℓ_1	0.3
$(\Psi_3/0.5, \Theta_2, 0)$	ℓ_4	1.0
$(\Psi_3/0.5, \Theta_3, 0)$	ℓ_3	0.4
$(\Psi_4/0.7, \Theta_1, 0)$	ℓ_1	0.5
$(\Psi_4/0.7, \Theta_2, 0)$	ℓ_3	0.5
$(\Psi_4/0.7, \Theta_3, 0)$	ℓ_1	0.5
$(\Psi_4/0.9, \Theta_1, 0)$	ℓ_2	0.3
$(\Psi_4/0.9, \Theta_2, 0)$	ℓ_2	0.6
$(\Psi_4/0.9, \Theta_3, 0)$	ℓ_4	0.6

Step-(4-5)

The purpose of Table 5 is to find the difference of scores of agree and disagree-FPNSSE-sets. The scores of agree-FPNSSE-set are :

$$S(\ell_1) = 0.6, S(\ell_2) = 1.3, S(\ell_3) = 0.6 \text{ and } S(\ell_4) = 2.0$$

and scores for disagree-FPNSSE-set are:

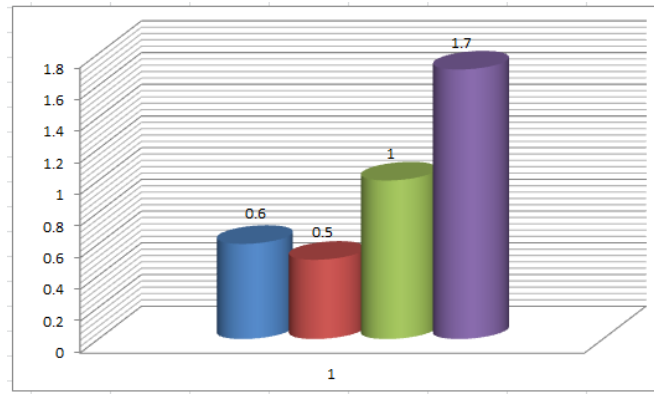
$$S(\ell_1) = 1.3, S(\ell_2) = 1.6, S(\ell_3) = 1.8 \text{ and } S(\ell_4) = 2.2.$$

Step-6; Decision

Table 5: Numerical values of $\S_i = |\dagger_i - \ddagger_i|$

\dagger_i	\ddagger_i	$\S_i = \dagger_i - \ddagger_i $
$S(\ell_1) = 1.8$	$S(\ell_1) = 1.3$	0.6
$S(\ell_2) = 1.2$	$S(\ell_2) = 1.7$	0.5
$S(\ell_3) = 2.9$	$S(\ell_3) = 3.9$	1.0
$S(\ell_4) = 0.7$	$S(\ell_4) = 2.4$	1.7

Above analysis shows that, ℓ_4 is best and it is represented in Figure 1.

**Figure 1:** Ranking of alternatives for algorithm

6. DISCUSSION AND COMPARISON

To illustrate the advantages of our proposed method using FPNSE-set as compared to fuzzy parameterised intuitionistic fuzzy soft expert set as proposed by Selvachandran and Salleh [56] which is a generalisation of fuzzy parameterised fuzzy soft expert. The FPNSE-set, as demonstrated in the example above, can provide a more detailed explanation of the universal with three membership functions, particularly when numerous parameters are involved. In contrast, fuzzy parameterised intuitionistic fuzzy soft expert set [56] can only provide a limited amount of information about the universal. The FPNSE-set is more accurate and realistic than fuzzy parameterized intuitionistic fuzzy soft expert set because it can handle problems involving imprecise, indeterminate, and inconsistent data, whereas it can only handle incomplete information when taking into account both the truth-membership and falsity-membership values [56]. The FPNSE-set gives the best resemblance, accuracy, and acceptability when compared to the already accessible soft set-like models. This will be shown by contrasting FPNSE-set to other models. This proposed model is more advantageous than others due to the inclusion of the multi-argument approximate function, which is particularly helpful in decision-making scenarios. In comparison to earlier models, including FPS-set, FPFS-set, FPIFS-set, FPSES-set, FPFSE-set, FPNS-set, and FPIFS-set, the results of the FPNSE-set study show that it is superior. In

contrast to these previous models, FPNSE-sets handle ambiguity and conflicting facts in complex decision-making more effectively by incorporating truth, falsity, and indeterminacy across different experts and attributes. By offering a more reliable framework for multi-expert evaluations, FPNSE-set improve decision accuracy in contrast to models such as FPFS-set and FPIFS-set, which do not fully address indeterminacy. By generating dependable and consistent results, skillfully handling a range of expert viewpoints, and taking into account subtle uncertainties, this study demonstrates that FPNSE-set perform better than these models in real-world situations. The comparison analysis is shown in Table 6.

Table 6: Comparison Analysis

Models	Exp. Opinions	Mship	Non-Mship	Indeterminacy
FPS-set [48]	No	No	No	No
FPSE-set [54]	Yes	No	No	No
FPFS-set [49]	Yes	Yes	No	No
FPFSE-set [55]	Yes	Yes	No	No
FPIFS-set [52]	No	Yes	Yes	No
FPIFSE-set [46]	Yes	Yes	Yes	No
FPNSE-Set	Yes	Yes	Yes	Yes

7. CONCLUSION

Together with several generalizations of the theoretic operations union, intersection, complement, AND, and OR, this paper establishes the foundations of the fuzzy parameterized neutrosophic soft expert set. Some basic ideas, such as union, and intersection, and laws, such as idempotent, absorption, domination, identity, associative, and distributive, are addressed with reference to particular cases. Ultimately, an algorithm for making decisions is developed to characterize the procedure. The some limitations of the study are (a) the process can be difficult to comprehend and apply, especially for people who are not familiar with fuzzy logic, neutrosophic sets, or decision theory (b) accurate and thorough data are necessary for the method, but they might be challenging to find. Inaccurate or partial data can have a detrimental effect on the decision's quality. Though, it has certain advantages over different structures such as (a) with its strong foundation for integrating fuzzy and neutrosophic factors, it successfully addresses the innate ambiguity and uncertainty in decision-making processes (b) the decision-making process is more accurate and reliable when expert information and opinions are integrated using this strategy (c) it gives a more thorough examination of possible building places by taking into account a variety of criteria and features, resulting in more informed and balanced conclusions. This new study inspires further advancements of related research and practical applications while providing an excellent expansion to current theories for handling indeterminacy, falsehood, and truthiness.

Funding: This research received no external funding.

REFERENCES

- [1] L. A. Zadeh, "Fuzzy sets," *Information and Control*, vol. 8, no. 3, pp. 338-353, 1965.
- [2] Z. Pawlak, "Rough sets," *International Journal of Computer Information Science*, vol. 11, no. 5, pp. 341-356, 1982.
- [3] H. Zare Ahmadabadi, A. Saffari Darberazi, F. Zamzam, M. S. Babakhanifard, M. Kiani, & E. Mofatehzadeh, "A model of the factors affecting supply chain resilience: an integrative approach incorporating hesitant fuzzy TOPSIS and meta-synthesis," *Journal of Applied Research on Industrial Engineering*, vol. 11, no. 2, pp. 195-211, 2024.
- [4] D. Rahadian, A. Firli, H. Dinçer, S. Yüksel, Ü. Hacıoğlu, Ş. C. Gherghina, & T. Aksoy, "An evaluation of e7 countries' sustainable energy investments: a decision-making approach with spherical fuzzy sets," *Economics*, vol. 17, no. 1, pp. 20220051, 2023.
- [5] F. Smarandache, "Neutrosophy, neutrosophic probability, set, and logic, analytic synthesis and synthetic analysis," *American Research Press*, 1998.
- [6] H. Wang, F. Smarandache, Y. Zhang, & R. Sunderraman, "Single valued neutrosophic sets," *review of the air force academy*, vol. 17, no. 1, pp. 10, 2010.
- [7] M. Ghaforiyan, A. Sorourkhah, & S. A. Edalatpanah, "Identifying and prioritizing antifragile tourism strategies in a Neutrosophic environment," *Journal of Fuzzy Extension and Applications*, vol. 5, no.3, pp. 374-394, 2024.
- [8] A. Nafei, S. P. Azizi, S. A. Edalatpanah, & C. Y. Huang, "Smart topsis: a neural network-driven topsis with neutrosophic triplets for green supplier selection in sustainable manufacturing," *Expert Systems with Applications*, vol. 255, pp. 124744, 2024.
- [9] A. Dubey, & R. Kumar, "Neutrosophic Inventory Management: A Cost-Effective Approach," *Economics*, vol. 18, no. 1, pp. 20220101, 2024.
- [10] T. N. Chusi, S. Qian, S. A. Edalatpanah, Y. Qiu, M. Bayane Bouraima, & A. B. Ajayi, "Interval-valued spherical fuzzy extension of SWARA for prioritizing strategies to unlock Africa's potential in the carbon credit market," *Computational Algorithms and Numerical Dimensions*, vol. 3, no. 3, pp. 217-227, 2024.
- [11] M. Ihsan, A. U. Rahman, & M. Saeed, "Single valued neutrosophic hypersoft expert set with application in decision making," *Neutrosophic Sets and Systems*, vol. 47, pp. 451-471, 2021.
- [12] M. Ihsan, M. Saeed, & A. U. Rahman, "Neutrosophic hypersoft expert set: theory and applications," *Neutrosophic Sets Syst*, vol. 50, pp. 431-458, 2022.
- [13] D. Molodtsov, "Soft set theory—first results," *Computers and Mathematics with Applications*, vol. 37, no. (4-5), pp. 19-31, 1999.
- [14] R. Vellapandi, & S. Gunasekaran, "A new decision making approach for winning strategy based on mutu soft set logic," *Journal of fuzzy extension and applications*, vol. 1, no. 2, pp. 112-121, 2020.
- [15] E. Karatas, A. Yolcu, & T. Y. Ozturk, (2023). Effective neutrosophic soft set theory and its application to decision-making. *Afrika Matematika*, vol. 34, no. 4, pp. 62, 2023.
- [16] J. Zhao, B. Li, A. U. Rahman, & M. Saeed, "An intelligent multiple-criteria decision-making approach based on sv-neutrosophic hypersoft set with possibility degree setting for investment selection," *Management Decision*, vol. 61, no. 2, pp. 472-485, 2023.
- [17] M. Pethaperumal, V. Jayakumar, S. A. Edalatpanah, A. B. K. Mohideen, & S. Annamalai, "An enhanced madm with l q q-rung orthopair multi-fuzzy soft set in healthcare supplier selection," *Journal of Intelligent & Fuzzy Systems*, (Preprint), pp. 1-12, 2024.
- [18] S. Debnath, "Fuzzy hypersoft sets and its weightage operator for decision making," *Journal of fuzzy extension and applications*, vol. 2, no. 2, pp. 163-170, 2021.
- [19] F. Smarandache, "Soft set product extended to hypersoft set and indetermsoft set cartesian

- product extended to indeterministic soft set," *Journal of fuzzy extension and applications*, vol. 3, no. 4, pp. 313-316, 2022.
- [20] A. Mehmood, A. Ahmad, M. Nawaz, M. M. Saeed, & G. Nardo, "Discussion on entropy and similarity measures and their few applications because of vague soft sets," *Systemic Analytics*, vol. 2, no. 1, pp. 157-173, 2024.
- [21] A. Sezgin, & A. İlgin, "Soft intersection almost bi-quasi ideals of semigroups," *Soft Computing Fusion with Applications*, vol. 1, no. 1, pp. 27-42, 2024.
- [22] P. K. Maji, R. Biswas, and A. R. Roy, "Soft set theory," *Computers and Mathematics with Applications*, vol. 45, no. (4-5), pp. 555-562, 2003.
- [23] M. I. Ali, F. Feng, X. Liu, W. K. Min, M. Shabir, "On some new operations in soft set theory," *Comput Math Appl*, vol. 57, no. 9, pp. 1547-1553, 2009.
- [24] P. K. Maji, A. R. Roy, and R. Biswas, "Fuzzy soft sets," *Journal of Fuzzy Mathematics*, vol. 9, pp. 589-602, 2001.
- [25] P. K. Maji, A. R. Roy, and R. Biswas, "An application of soft sets in a decision making problem," *Computers and Mathematics with Applications*, vol. 44, no. (8-9), pp. 1077-1083, 2002.
- [26] N. Çağman, S. Enginoğlu, "Soft matrix theory and its decision making," *Comput Math Appl*, vol. 59, no. 10, pp. 308-3314, 2010.
- [27] N. Çağman, S. Enginoğlu, "Soft set theory and uni-int decision making," *Eur J Oper Res*, vol. 207, no. 2, pp. 848-855, 2010.
- [28] D. Chen, E. C. C. D. Tsang, S. Yeung, and X. Wang, "The parameterization reduction of soft sets and its applications," *Computers and Mathematics with Applications*, vol. 49, no. (5-6), pp. 757-763, 2005.
- [29] F. Feng, Y. Li, V. Leoreanu-Fotea, "Application of level soft sets in decision making based on interval-valued fuzzy soft sets," *J Comput Math Appl*, vol. 60, no. 6, pp. 1756-1767, 2010.
- [30] Y. Jiang, Y. Tang, Q. Chen, "An adjustable approach to intuitionistic fuzzy soft sets based decision making," *Appl Math Model*, vol. 35, no. 2, pp. 824-836, 2011.
- [31] F. Feng, X. Y. Liu, V. Leoreanu-Fotea, Y. B. Jun, "Soft sets and soft rough sets," *Inform Sci*, vol. 181, no. 6, pp. 1125-1137, 2011.
- [32] P. K. Maji, A. R. Roy, R. Biswas, "An applications of soft sets in a decision making problem," *Comput Math Appl*, vol. 44, no. (8-9), PP. 1077-1083, 2002.
- [33] F. Feng, Y. B. Jun, X. Liu, L. Li, "An adjustable approach to fuzzy soft set based decision making," *J Comput Appl Math.*, vol. 234, no. 1, pp. 10-20, 2010.
- [34] P. K. Maji, R. Biswas, A. R. Roy, "Fuzzy soft sets," *J.Fuzzy Math*, vol. 9, no. 3, pp. 589-602, 2001.
- [35] J. Zhan, Q. Liu, B. Davvaz, "A new rough set theory: rough soft hemirings," *J. Intell. Fuzzy System*, vol. 28, no. 4, pp. 1687-1697, 2015.
- [36] K. Muniba, M. N. Jafar, A. Riffat, J. Mukhtar, & A. Saleem, "Climate change prediction model using mcdm technique based on neutrosophic soft functions with aggregate operators," *Neutrosophic Systems with Applications*, vol. 21, pp. 25-35, 2024.
- [37] F. Smarandache, "Foundation of the superhypersoft set and the fuzzy extension superhypersoft set: a new vision," *Neutrosophic Systems with Applications*, vol. 11, pp. 48-51, 2023.
- [38] T. M. Basu, N. K. Mahapatra, & S. K. Mondal, "A balanced solution of a fuzzy soft set based decision making problem in medical science," *Applied soft computing*, vol. 12, no. 10, pp. 3260-3275, 2012.
- [39] Y. Jiang, Y. Tang, & Q. Chen, "An adjustable approach to intuitionistic fuzzy soft sets based decision making," *Applied Mathematical Modelling*, vol. 35, no. 2, pp. 824-836, 2011.
- [40] A. U. Rahman, M. Saeed, M. Arshad, M. Ihsan, M. R. Ahmad, "(m; n)-Convexity-cum-concavity on fuzzy soft set with applications in first and second Sense," *Punjab University*

- Journal of Mathematics*, vol. 53, no. 1, pp. 19-33, 2021.
- [41] A. U. Rahman, M. Saeed, M. Arshad, M. Ihsan, S. Ayaz, "A Conceptual framework of m-convex and m-concave sets under soft set environment with properties," *Transactions in Mathematical and Computational Sciences*, vol. 1, no. 1, pp. 40-60, 2021.
 - [42] S. Alkhazaleh, A. R. Salleh, "Soft expert sets," *Adv. Decis. Sci.*, no. 757868-1, 2011.
 - [43] M. Ihsan, M. Saeed, & A. U. Rahman, "A rudimentary approach to develop context for convexity cum concavity on soft expert set with some generalized results," *Punjab University Journal of Mathematics*, vol. 53, no. 9, pp. 621-629, 2021.
 - [44] S. Alkhazaleh, and A. R. Salleh, "Fuzzy soft expert set and its application," *Applied Mathematics*, vol. 5, pp. 1349-1368, 2014.
 - [45] M. Ihsan, A. U. Rahman, M. Saeed, & H. A. E. W. Khalifa, "Convexity-cum-concavity on fuzzy soft expert set with certain properties," *International Journal of Fuzzy Logic and Intelligent Systems*, vol. 21, no. 3, pp. 233-242, 2021.
 - [46] S. Broumi, and F. Smarandache, "Intuitionistic fuzzy soft expert sets and its application in decision making," *Journal of new theory*, vol. 1, pp. 89-105, 2015.
 - [47] V. Uluçay, & M. Şahin, "Intuitionistic fuzzy soft expert graphs with application," *Uncertainty discourse and applications*, vol. 1, no. 1, pp. 1-10, 2024.
 - [48] Çağman, N., Çitak, F., & Enginoğlu, S. "FP-soft set theory and its applications," *Annals of Fuzzy Mathematics and Informatics*, vol. 2, no. 2, pp. 219-226, 2011.
 - [49] N. Çağman, F. Çitak, & S. Enginoğlu, "Fuzzy parameterized fuzzy soft set theory and its applications," *Turkish Journal of Fuzzy Systems*, vol. 1, no. 1, pp. 21-35, 2010.
 - [50] K. Zhu, & J. Zhan, "Fuzzy parameterized fuzzy soft sets and decision making," *International Journal of Machine Learning and Cybernetics*, vol. 7, no. 6, pp. 1207-1212, 2016.
 - [51] M. Riaz, & M. R. Hashmi, "Certain applications of fuzzy parameterized fuzzy soft sets in decision-making problems," *International Journal of Algebra and Statistics*, vol. 5, no. 2, pp. 135-146, 2016.
 - [52] E. Sulukan, N. Çağman, & T. Aydin, "Fuzzy parameterized intuitionistic fuzzy soft sets and their application to a performance-based value assignment problem," *Journal of New Theory*, vol. 29, pp. 79-88, 2019.
 - [53] M. Ihsan, M. Saeed, and A. U. Rahman, "Optimizing hard disk selection via a fuzzy parameterized single-valued neutrosophic soft set approach," *J. Oper. Strateg Anal*, vol. 1, no. 2, pp. 62-69, 2023.
 - [54] M. Bashir, & A. R. Salleh, "Fuzzy parameterized soft expert set," *Abstract and Applied Analysis*, pp. 1-15, 2012.
 - [55] A. Hazaymeh, I. B. Abdullah, Z. Balkhi, & R. Ibrahim, "Fuzzy parameterized fuzzy soft expert set," *Applied Mathematical Sciences*, vol. 6, no. 112, pp. 5547-5564, 2012.
 - [56] G. Selvachandran, & A. R. Salleh, "Fuzzy parameterized intuitionistic fuzzy soft expert set theory and its application in decision making," *Int. J. Soft Comput*, vol. 11, no. 2, pp. 52-63, 2016.
 - [57] K. H. Gazi, , S. P. Mondal, B. Chatterjee, N. Ghorui, A. Ghosh, & D. De, "A new synergistic strategy for ranking restaurant locations: A decision-making approach based on the hexagonal fuzzy numbers," *RAIRO-Operations Research*, vol. 57, no. 2, pp. 571-608, 2023.