Yugoslav Journal of Operations Research
xx (20xx), Number xx, xxx–xxx
DOI: https://doi.org/10.2298/YJOR240715029L

Research Article

TWO-WAREHOUSE INVENTORY MODEL FOR NON-INSTANTANEOUSLY DETERIORATING ITEMS INCORPORATING PRESERVATION TECHNOLOGY, CARBON EMISSIONS AND HYBRID PAYMENT SCHEMES

Anthony LIMI
Department of Mathematics, Saveetha School of Engineering, Saveetha Institute of Medical and Technical Sciences, Chennai 602105, Tamil Nadu, India
anthonyshyla2@gmail.com, ORCID: 0009-0005-9942-240X

Kumar RANGARAJAN*
Department of Mathematics, Saveetha School of Engineering, Saveetha Institute of Medical and Technical Sciences, Chennai 602105, Tamil Nadu, India
rangarajancheyyaru@gmail.com, ORCID: 0000-0002-7059-8373

Received: July 2024 / Accepted: July 2025

Abstract: This study proposes an advanced inventory model for non-instantaneously deteriorating items, integrating preservation technology, carbon emissions considerations, price-dependent demand, and a hybrid payment scheme within a two-warehouse framework. The hybrid payment method, combining partial upfront and deferred payments, enhances cash flow flexibility, which is critical for managing financial constraints in supply chain operations. The model aims to optimize inventory management by minimizing total costs while fostering environmental sustainability. Key features include investments in green technology to reduce carbon emissions and mitigate item deterioration, along with dynamic pricing strategies to respond to market demand fluctuations. Numerical analyses validate the model, revealing that preservation technology investments significantly lower total costs by extending product shelf life, while effective carbon management reduces

*Corresponding author

transportation expenses. The hybrid payment scheme also proves to be a strategic tool for balancing financial obligations and operational efficiency. Sensitivity analysis conducted using MATLAB R2024a highlights the impact of changes in key parameters, such as demand elasticity, deterioration rate, and carbon tax, on the total cost. The findings provide actionable insights for managers to enhance inventory efficiency and sustainability, particularly in cost-sensitive and environmentally regulated industries.

Keywords: Non-instantaneous deterioration, preservation, carbon emission, two-warehouse, time-dependent holding cost, green-technology.

MSC: 90B05, 90B06, 90C11.

1. INTRODUCTION

In an environmentally conscious marketplace, businesses are increasingly driven to integrate sustainability into their operations. This research develops a sustainable inventory model that incorporates key factors such as preservation technology, carbon emissions, and demand influenced by price-dependent variations, specifically for non-instantaneous deteriorating (NID) items. Additionally, the model introduces a hybrid payment scheme to offer a holistic framework for optimizing inventory management in a way that balances environmental goals with economic efficiency.

Preservation technology is vital for extending the shelf life of perishable goods, thereby reducing waste and minimizing the need for frequent replenishments. By enhancing product longevity, preservation methods contribute to both cost savings and a reduction in carbon emissions associated with storage and transportation. This study investigates how the integration of preservation technology within inventory models can help businesses achieve sustainability without sacrificing profitability.

Another key aspect of this research is the dual consideration of carbon emissions and demand influenced by pricing in a two-warehouse setup. Understanding how these factors interact to shape demand is crucial for developing effective inventory strategies. By accounting for both environmental impact and pricing, the proposed model helps businesses create more efficient and sustainable inventory practices.

Additionally, the study incorporates a hybrid payment scheme that combines traditional and flexible payment options to optimize financial flow and operational efficiency. This payment structure is particularly useful for businesses managing high-value, slow-moving goods, where liquidity management is critical, especially in the face of fluctuating demand.

1.1. Research Question

Based on the above discussion, our objective is to explore the following research questions:

(a) How do investments in preservation technologies and strategies to extend the NID period impact overall inventory costs and product shelf life?

(b) What role do green technologies and effective carbon emissions management play in reducing transportation costs and enhancing supply chain sustainability?

(c) How can optimizing warehouse capacities and understanding the sensitivity of key cost drivers like holding costs, market size, and purchase costs improve overall inventory efficiency and cost management?

1.2. Problem Statement

Managing inventory for NID items poses significant challenges due to product deterioration, carbon emissions, and financial constraints. Traditional models often overlook the need for sustainable practices and flexible financial strategies. Businesses require an approach that balances cost minimization, environmental sustainability, and financial efficiency. This study addresses these issues by developing a two-warehouse inventory model incorporating preservation technology, carbon emissions management, and a hybrid payment scheme. The goal is to optimize inventory management while promoting sustainability and improving cash flow flexibility.

1.3. Novelty

The novelty of this research lies in its comprehensive integration of preservation technology, carbon emissions management, and hybrid payment schemes within a two-warehouse inventory model for NID items. Unlike traditional inventory models, this study introduces preservation technology investments to extend product shelf life, thereby reducing overall inventory costs and improving product quality. Additionally, the model addresses the critical issue of carbon emissions by incorporating green technology investments and carbon tax considerations, promoting sustainability in supply chain operations. The inclusion of a hybrid payment scheme, which combines upfront and deferred payments, offers a novel financial management approach, enhancing cash flow flexibility and operational efficiency. This holistic framework not only optimizes total costs but also aligns with modern environmental and financial challenges, providing a unique and innovative solution for sustainable inventory management.

1.4. Orientation

The remainder of the paper is structured as follows: Section 2 provides a review of the relevant literature on inventory models for NID items. Section 3 outlines the notations and assumptions used in the paper. Section 4 presents the mathematical formulation of the model, detailing the integration of preservation technology, carbon emission considerations, price-dependent demand, and the hybrid payment scheme. Section 5 introduces the computational algorithm. Section 6 discusses the numerical analysis and presents the results obtained from the model. Section 7 examines the sensitivity analysis of the key parameters. In Section 8, the managerial insights derived from the study are discussed. Finally, Section 9 concludes the paper and outlines potential areas for future research.

2. LITERATURE REVIEW

The study of inventory models has evolved significantly to address the challenges posed by real-world scenarios, such as non-instantaneous deterioration, preservation technology, environmental sustainability, two-warehouse setups, hybrid payment schemes, and price-dependent demand. This literature review is organized into subsections that highlight key contributions in these domains.

2.1. Non-Instantaneous Deterioration

Non-instantaneous deterioration has been a central theme in inventory management research. Early work by Ghare and Schrader [1] introduced models for exponentially decaying inventories, forming the basis for subsequent studies. Chakraborty et al. [2] explored multi-warehouse systems with partial backlogging for NID items. Other significant contributions include optimization of inventory policies for seasonal deteriorating products by He and Huang [3], and comprehensive reviews of non-instantaneous deterioration models by Limi et al. [4].

2.2. Preservation Technology

Investment in preservation technology is crucial for managing inventory systems with deteriorating items. He and Huang [3] demonstrated its impact on seasonal products, while Dye and Dye [5] incorporated preservation technology into fluctuating demand models. Lok et al. [6] examined preservation investments under carbon emission considerations, and Mashud et al. [7] analyzed the interplay of preservation technology, trade credit, and partial backordering. Additional studies by Mishra et al. [8] highlighted the importance of preservation in sustainable inventory systems. Chiu et al. [9] studied sustainable inventory models for non-instantaneous deteriorating items with preservation and green technology. Padiyar et al. [10] proposed an imperfect production inventory model with preservation investment and inflation effects.

2.3. Green Technology Investment for Carbon Emission

Sustainability and carbon emission control have gained prominence in inventory management research. Mishra et al. [11] proposed models integrating controllable deterioration and emission rates, while Taleizadeh et al. [12] considered pricing and inventory decisions under carbon emission constraints. Wee and Daryanto [13] analysed an inventory model considering carbon emission. Recent studies by Pervin [14] and Jauhari et al. [15] emphasized green technology investment and its effects on inventory systems. Further research by Suef et al. [16] analyzed the integration of carbon emissions in multi-retailer systems. Jauhari et al. [17] and [18] integrates green investments and carbon taxes, addressing environmental and economic goals.

2.4. Two-Warehouse Setup

Two-warehouse inventory systems address storage constraints and cost optimization. Jaggi et al. [19] examined credit financing in two-storage environments, and Tiwari et al. [20] incorporated inflation effects into retailer policies. Notable advancements include multi-warehouse models with quadratic demand by Limi et al. [21], and non-instantaneously deteriorating items in two-warehouse setups by Rangarajan and Karthikeyan [22] and Rana et al. [23]. Murmu et al. [24] addressed sustainable inventory management through policies like First-In-First-Out (FIFO) and Last-In-First-Out (LIFO) to reduce waste. Rana et al. [25] proposed solutions for managing demand disruptions in a two-warehouse NID system.

2.5. Hybrid Payment Schemes

Hybrid payment schemes play a vital role in modern inventory systems, especially under inflationary conditions. Buzacott [26] analyzed economic order quantities under inflation, while Meena et al. [27] investigated partial backlog and discounting cash flow under inflationary pressures. Such schemes are also integrated into green inventory models, as explored by Datta [28] and Limi et al. [29]. Pal et al. [30] discussed credit policy and inflation effects in two-warehouse models. Choudhury and Mahata [31] analyzed inventory models for fixed lifetime deteriorating items under hybrid payment schemes.

2.6. Price-Dependent Demand

Price-dependent demand is a critical factor influencing inventory policies. Li et al. [32] optimized replenishment and preservation decisions for such items. The impact of pricing strategies on sustainable inventory systems was further analyzed by Rana et al. [33] and Jauhari et al. [34]. The interplay of pricing and environmental policies was explored by Mashud et al. [35].

This body of research demonstrates the importance of innovative inventory models for environmental sustainability, economic flexibility, and perishable characteristics. Incorporating these factors, along with green investments, hybrid payment schemes, and preservation technology, provides a comprehensive approach to sustainable NID inventory management.

2.7. Motivation

The study is motivated by the growing importance of sustainable inventory management practices in modern supply chains. With environmental concerns at the forefront and increasing regulatory pressures, integrating sustainable practices into inventory management becomes imperative. This motivation is echoed in studies such as those by Datta [32] and Mishra et al. [13], which emphasize the significance of addressing carbon emissions and green technology in warehouse operations. No researchers have studied an inventory model incorporating all the factors.

2.8. Research Gap

Existing inventory models largely focus on instantly deteriorating items, overlooking the complexities of NID products. While preservation technology and green investments have been studied separately, their combined impact with financial strategies like hybrid payment schemes remains underexplored. Current models often neglect the challenges of managing inventory across dual-warehouse systems with varying deterioration rates and holding costs. Additionally, the influence of demand fluctuations on NID inventory decisions is not thoroughly addressed. This study aims to fill these gaps by integrating sustainability, financial flexibility, and dynamic market conditions into a comprehensive inventory framework. A selection of research publications are analyzed and contrasted with the suggested model in Table 1.

This paper aims to provide a framework for managing inventories of NID items in a sustainable and economically viable manner by incorporating these diverse elements into a comprehensive model.

Table 1: Comparison of current study and previous researches.

Author(s)	Price dependent demand	NID	Two Warehouse	Payment scheme	Preservation Technology	Carbon Emission	Green Technology
Chakraborty et al. [2]	no	yes	yes	no	no	no	no
Lok et al. [6]	no	yes	no	no	yes	no	no
Mashud et al. [7]	yes	yes	no	no	yes	no	no
Mishra et al. [11]	no	yes	no	yes	no	yes	yes
Wee & Daryanto[13]	no	no	no	no	no	yes	no
Pervin et al. [14]	no	yes	no	no	no	no	no
Jauhari et al. [17]	no	no	no	no	no	yes	yes
Jaggi et al. [19]	yes	yes	yes	yes	no	no	no
Tiwari et al. [20]	yes	yes	yes	yes	no	no	no
Meena et al. [27]	yes	yes	yes	no	no	no	no
Limi et al. [29]	no	yes	yes	no	no	no	no
Pal et al. [30]	yes	yes	no	yes	no	no	no
Li et al. [32]	yes	yes	no	no	yes	yes	no
Rana et al. [33]	yes	no	no	no	yes	yes	yes
Mashud et al.[35]	yes	no	yes	yes	no	yes	yes
Proposed study	yes	yes	yes	yes	yes	yes	yes

3. NOTATIONS AND ASSUMPTIONS

3.1. Notations

The notations used in this work is presented in Table 2.

3.2. Assumptions

The following presumptions form the foundation of the inventory model.

(i) Single item is the focus of the model

(ii) Lead time is zero.

(iii) The replenishment rate is unlimited.

(iv) For a certain shape parameter (b) and market size (a), the demand is given as D= $(a-bp)$, with price having a linear influence on consumer demand as Pal et al. [6]

(v) Specific criteria $\frac{\partial m(\zeta)}{\partial \zeta} < 0$ and $\frac{\partial^2 m(\zeta)}{\partial \zeta^2} > 0$ describe the relationship between the deterioration rate and preservation technology investment. the model has considered $m(\zeta)$ as the deterioration rate with preservation technology investment; ϑ is the deterioration rate in OW and κ is the deterioration rate in RW as Mishra et al. [8].

(vi) Payment in n installments is required by the supplier. The store then obtains a loan with a certain interest rate from a financial institution. The cyclic capital cost for prepayments is determined in this work using a methodology identical to that employed in Taleizadeh et al. [12]and Mashud et al. [7].

Table 2: List of notations and their descriptions with units

Notations	Description	Units
Π	The order amount	\$
$m(\zeta)$	The deterioration rate with preservation technology investment	
κ	The deterioration rate in RW	
ϑ	The deterioration rate in OW	
U	Maximum inventory level	units
V	The OW's capacity	units
t_n	Time span during which there is no deterioration	time units
t_1	Time duration for the inventory in RW to reach zero	time units
$I_o(t), I_r(t)$	Inventory levels in the OW and RW at time t	units
c	Purchasing cost	\$/unit
$F(t)$	Holding cost per unit time in RW, $F(t) = ft,\ f > 0$	\$/unit time
O_Q	Order quantity	units
$H(t)$	Holding cost per unit time in OW, $H(t) = ht,\ h > 0$	\$/unit time
μ	Number of trips	count
q_1	The least amount of money needed for transporting an item	\$/shipment
V_t	Variable transportation cost (fuel price)	\$/L
q_2	Fuel consumption while truck is empty	L/100 km
q_3	Supplementary fuel consumption of the truck per ton of payload	L/100 km/ton
d_t	Distance traveled from OW to RW and to customer	km
w	Product weight	kg/unit
c_e	Carbon emission produced by the vehicle	\$/km
a	Constant market size	
e_x	Extra carbon emission cost for transporting one unit of an item	\$/unit/km
L_T	Total lead time for delivery of the product	time units
b	Shape parameter	
G_T	Green technology investment	\$
I_P	Interest payment	\$
n	The number of equal multiple installments at equal intervals	count
γ	Fraction of purchasing cost	\$
Decision Variables		
p	The selling price of the item	\$/unit
t_2	Maximum inventory level	units

(vii) The retailer proposes investing in green technology (G_T), including energy-efficient equipment and renewable energy sources, to promote a greener logistics system. The retailer's budget determines the maximum amount available for green technology projects. The proportion of emission reduction owing to G_T is F= $\delta(1-e^{-\chi G_T})$, where δ represents the amount of carbon emissions when green technology is invested in, and χ affects green technology's capacity to reduce emissions as Mashud et al.[35].

(viii) Both OW and RW have a limited capacity.

(ix) Once an item reaches its designated lifespan, deterioration takes place. This indicates that the product will deteriorate at a constant rate once its lifespan has passed, but it will not deteriorate during that time.

(x) The first inventory to be used up is from the rented warehouse RW and lastly from OW.

(xi) Throughout the cycle, the model's decaying components are neither replaced nor fixed.

(xii) The holding cost is a variable that follows a linear function of time.

4. DESCRIPTION OF THE MATHEMATICAL MODEL

This work establishes a two-warehouse, NID inventory model over a finite horizon having preservation, carbon emission and hybrid payment scheme. Using the notations and assumptions mentioned above, the inventory time diagram for the model's behavior over time t is shown in Figure 1. Before the items are transported, the supplier asks a merchant to deposit a percentage of the purchase price. This amount is then paid in numerous equal payments throughout the course of the lead time L_T at equal intervals. The left side of Figure 1 depicts the equal prepayments made by the retailer throughout the lead period L_T . The merchant begins selling the items at time $t = 0$. where n is the number of installments and L_T/n is the installment interval. The part of the purchase price that is paid in full before the items are delivered is shown by the blue shaded region.

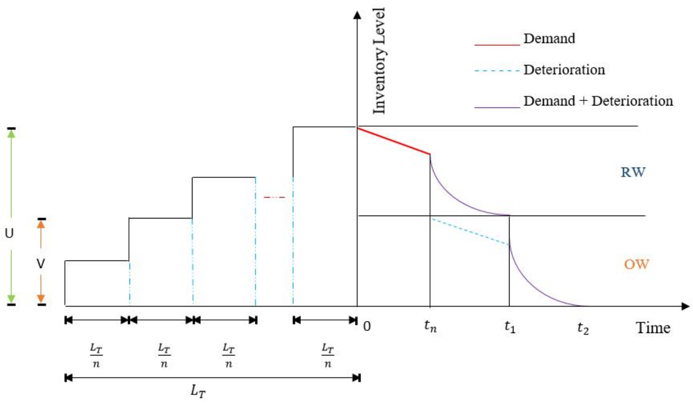

Figure 1: Two-warehouse inventory model for NID items

The differential equation during $(0,t_n)$ the demand is met and no deterioration occurs in RW is,

$$\frac{dI_{r_1}(t)}{dt} = -(a-bp); \qquad 0 < t < t_n, \tag{1}$$

The differential equation during (t_n,t_1) the system undergoes deterioration and the demand is met in RW is,

$$\frac{dI_{r_2}(t)}{dt} + \kappa m(\zeta)I_{r_2}(t) = -(a-bp); \qquad t_n < t < t_1, \tag{2}$$

The differential equation during $(0,t_n)$ in the OW neither deterioration nor demand occurs,

$$\frac{dI_{o_1}(t)}{dt} = 0; \qquad 0 < t < t_n, \tag{3}$$

During (t_n, t_1) in OW deterioration occurs and the equation is,

$$\frac{dI_{o_2}(t)}{dt} + \vartheta m(\zeta) I_{o_2}(t) = 0; \qquad t_n < t < t_1, \tag{4}$$

During (t_1, t_2) both deterioration occurs and demand is met,

$$\frac{dI_{o_3}(t)}{dt} + \vartheta m(\zeta) I_{o_3}(t) = -(a - bp); \qquad t_1 < t < t_2, \tag{5}$$

Using the following boundary conditions(b.c.):

$$I_{r_1}(0) = U - V,\ I_{r_2}(t_1) = 0,\ I_{o_1}(0) = V,\ I_{o_2}(t_n) = V,\ I_{o_3}(t_2) = 0$$

And also using the continuity at $t = t_n$, $I_{r_1}(t_n) = I_{r_2}(t_n)$ we solve the above equations to get the optimal total cost
Consider solving Eq. (1) using the b.c. at t=0, $I_{r_1}(0) = U - V$,we obtain:

$$I_{r_1}(t) = -(a - bp)t + U - V \tag{6}$$

By solving Eq. (2)using the b.c. at t=t_1, $I_{r_2}(t) = 0$, we get :

$$I_{r_2}(t) = \frac{(a - bp)}{\kappa m(\zeta)} \left[e^{\kappa m(\zeta)(t_1 - t)} - 1 \right] \tag{7}$$

By solving Eq. (3) using the b.c. at $t = 0$, $I_{o_1}(t) = V$, we get:

$$I_{o_1}(t) = V. \tag{8}$$

Solving Eq. (4) using the b.c. at $t = t_n$, $I_{o_2}(t) = V$, we get:

$$I_{o_2}(t) = \left[V e^{\vartheta m(\zeta)(t_n - t)} \right] \tag{9}$$

By solving Eq. (5) using the b.c. at $t = t_2$, $I_{o_3}(t) = 0$, we get:

$$I_{o_3}(t) = \frac{(a - bp)}{\vartheta m(\zeta)} \left[e^{\vartheta m(\zeta)(t_2 - t)} - 1 \right]. \tag{10}$$

Considering continuity at $t = t_n$, i.e., $I_{r_1}(t_n) = I_{r_2}(t_n)$, along with Eqs. (6) and (7), the maximum inventory level per cycle is:

$$U = V + (a - bp) \left[t_n + \frac{1}{\kappa m(\zeta)} \left(e^{\kappa m(\zeta)(t_1 - t_n)} - 1 \right) \right] \tag{11}$$

In this model, the total cost during the replenishment cycle is:

1. Ordering Cost= Π

2. RW Holding Cost:

$$
\begin{aligned}
HC_{RW} &= \int_0^{t_n} (ft) I_{r_1}(t) dt + \int_{t_n}^{t_1} (ft) I_{r_2}(t) dt \\
&= \frac{-f}{6} t_n^2 \left[2(a-bp)t_n + 3(U-V) \right] + \frac{f(a-bp)}{\kappa m(\zeta)} \\
&\quad \left[\frac{e^{\kappa m(\zeta)(t_1-t_n)}}{\kappa^2 m^2(\zeta)} \Big(t_n \kappa m(\zeta) + 1 \Big) - \frac{1}{2} \Big(t_1^2 - t_n^2 \Big) \right. \\
&\quad \left. + \frac{1}{\kappa m(\zeta)} \Big(t_1 \kappa m(\zeta) + 1 \Big) \right]
\end{aligned}
$$

3. OW Holding Cost:

$$
\begin{aligned}
HC_{OW} &= \int_0^{t_n} (ht) I_{o_1} dt + \int_{t_n}^{t_1} (ht) I_{o_2}(t) dt + \int_{t_1}^{t_2} (ht) I_{o_3}(t) dt \\
&= \frac{hVt_n^2}{2} + hV \left[\Big(\frac{1}{\vartheta^2 m^2(\zeta)} (t_n \vartheta m(\zeta) + 1) \Big) - \frac{e^{\vartheta m(\zeta)(t_n - t_1)}}{\vartheta^2 m^2(\zeta)} \right. \\
&\quad \left. (t_1 \vartheta m(\zeta) + 1) \right] + \frac{h(a-bp)}{\vartheta m(\zeta)} \left[\frac{e^{\vartheta m(\zeta)(t_2-t_1)}}{\vartheta^2 m^2(\zeta)} (t_1 \vartheta m(\zeta) + 1) \right. \\
&\quad \left. - \frac{1}{\vartheta^2 m^2(\zeta)} (t_2 \vartheta m(\zeta) + 1) + \frac{1}{2} (t_1^2 - t_2^2) \right]
\end{aligned}
$$

4. RW Deterioration Cost:

$$
\begin{aligned}
DC_{RW} &= c \int_{t_n}^{t_1} \kappa I_{r_2}(t) dt \\
&= \frac{c(a-bp)}{\kappa m^2(\zeta)} \left[\Big(e^{\kappa m(\zeta)(t_1 - t_n)} - 1 \Big) + \kappa m(\zeta)(t_n - t_1) \right]
\end{aligned}
$$

5. OW Deterioration Cost:

$$
\begin{aligned}
DC_{RW} &= c \left[\int_{t_n}^{t_1} \vartheta I_{o_2}(t) dt + \int_{t_1}^{t_2} \vartheta I_{o_3}(t) dt \right] \\
&= c \left[\frac{Ve^{\vartheta m(\zeta) t_n}}{m(\zeta)} \Big(e^{\vartheta m(\zeta)(t_n - t_1)} \Big) + \frac{(a-bp)}{\vartheta m^2(\zeta)} \Big(1 - e^{\vartheta m(\zeta)(t_2 - t_1)} \Big) \right. \\
&\quad \left. - \vartheta m(\zeta)(t_2 - t_1) \right]
\end{aligned}
$$

6. Cyclic Capital Cost:
   The cyclic capital cost is calculated using the same procedure as Taleizadeh et al. [12]

$$
\begin{aligned}
CCC = & \left(\frac{I_P\gamma c}{n} * O_Q * n * \frac{L_T}{n}\right) + \left(\frac{I_P\gamma c}{n} * O_Q * (n-1) * \frac{L_T}{n}\right) \\
& + \cdots + \left(\frac{I_P\gamma c}{n} * O_Q * (n-(n-2)) * \frac{L_T}{n}\right) \\
& + \left(\frac{I_P\gamma c}{n} * O_Q * (n-(n-1)) * \frac{L_T}{n}\right) \\
= & \frac{n+1}{2n} I_P\gamma c O_Q L_T
\end{aligned}
$$

7. Transportation Cost:
   The transportation cost is calculated using the same procedure as Mashud et al. [35]

$$TC = \mu\Big[q_1 + (2d_t V_t q_2 + d_t q_3 w O_Q) + (2d_t c_e + d_t e_x O_Q)\Big]$$

8. Green Technology Investment Cost(GT):

$$GTIC = G_T t_2$$

9. Reduced Transportation Cost (RTC):
   With the introduction of green technology, the new or decreased transportation costs using Mashud et al. [7] is,

$$
\begin{aligned}
RTC = & \mu\Big[q_1 + (2d_t V_t q_2 + d_t q_3 w O_Q) + (2d_t c_e + d_t e_x O_Q) \\
& * (1 - \delta(1 - e^{-\chi G_T}))\Big]
\end{aligned}
$$

Total cost: The total cost is a sum of all the above costs and is given as follows:

$$
\begin{aligned}
T^c = \frac{1}{t_2}\Bigg[& OC + HCRW + HCOW + DCRW + DCOW + CCC + \\
& GTIC + RTC\Bigg]
\end{aligned}
$$

$$T^c = \frac{1}{t_2}\Bigg[\Pi - \frac{f}{6}t_n^2\Big[2(a-bp)t_n + 3(U-V)\Big] + \frac{f(a-bp)}{\kappa m(\zeta)}\Bigg[\frac{e^{\kappa m(\zeta)(t_1-t_n)}}{\kappa^2 m^2(\zeta)}$$
$$\Big(t_n\kappa m(\zeta)+1\Big) - \frac{1}{2}\Big(t_1^2 - t_n^2\Big) + \frac{1}{\kappa m(\zeta)}\Big(t_1\kappa m(\zeta)+1\Big)\Bigg] + \frac{hVt_n^2}{2} + hV$$
$$\Bigg[\Big(\frac{1}{\vartheta^2 m^2(\zeta)}(t_n\vartheta m(\zeta)+1)\Big) - \frac{e^{\vartheta m(\zeta)(t_n-t_1)}}{\vartheta^2 m^2(\zeta)}(t_1\vartheta m(\zeta)+1)\Bigg] + \frac{h(a-bp)}{\vartheta m(\zeta)}$$
$$\Bigg[\frac{e^{\vartheta m(\zeta)(t_2-t_1)}}{\vartheta^2 m^2(\zeta)}(t_1\vartheta m(\zeta)+1) - \frac{1}{\vartheta^2 m^2(\zeta)}(t_2\vartheta m(\zeta)+1) + \frac{1}{2}(t_1^2 - t_2^2)\Bigg]$$
$$+\frac{c(a-bp)}{\kappa m^2(\zeta)}\Bigg[\Big(e^{\kappa m(\zeta)(t_1-t_n)} - 1\Big) + \kappa m(\zeta)(t_n - t_1)\Bigg] + c\Bigg[\frac{Ve^{\vartheta m(\zeta)t_n}}{m(\zeta)}$$
$$\Big(e^{\vartheta m(\zeta)(t_n-t_1)}\Big) + \frac{(a-bp)}{\vartheta m^2(\zeta)}\Bigg[\Big(1 - e^{\vartheta m(\zeta)(t_2-t_1)}\Big) - \vartheta m(\zeta)(t_2-t_1)\Bigg]\Bigg]$$
$$+\frac{n+1}{2n}I_P\gamma P_C O_Q L_T + G_T t_2 + \mu\Big[q_1 + (2d_t V_t q_2 + d_t q_3 w O_Q)$$
$$+(2d_t c_e + d_t e_x O_Q) * (1 - \delta(1 - e^{-\chi G_T}))\Big]\Bigg] \tag{12}$$

5. COMPUTATIONAL ALGORITHM

The following steps are used to obtain the optimal solution:
Step 1: Input the parameter values(using references)
Step 2: From Eq (12), find

$$\frac{\partial T^c}{\partial t_2} = 0 \quad \text{and} \quad \frac{\partial T^c}{\partial p} = 0.$$

Step 3: Find the ideal values of t_2 and p by following Steps 1 and 2.

Step 4: The following prerequisites must be met in order to minimize T^c using the Hessian matrix:

$$H^i_{ess} = \begin{bmatrix} \frac{\partial^2 T^c}{\partial t_2^2} & \frac{\partial^2 T^c}{\partial t_2 \partial p} \\ \frac{\partial^2 T^c}{\partial p \partial t_2} & \frac{\partial^2 T^c}{\partial p^2} \end{bmatrix},$$

$$\text{where} \quad \frac{\partial^2 T^c}{\partial t_2^2} > 0, \quad \frac{\partial^2 T^c}{\partial p^2} > 0, \quad \text{and} \quad \det \begin{bmatrix} \frac{\partial^2 T^c}{\partial t_2^2} & \frac{\partial^2 T^c}{\partial t_2 \partial p} \\ \frac{\partial^2 T^c}{\partial p \partial t_2} & \frac{\partial^2 T^c}{\partial p^2} \end{bmatrix} > 0.$$

Step 5: Determine the ideal values for t_2 and p. The optimal total inventory cost T^c is computed using these values.

Refer to Appendix for optimality and necessary condition.
The solution procedure for the proposed model has been shown in figure 2 as a flow chart.

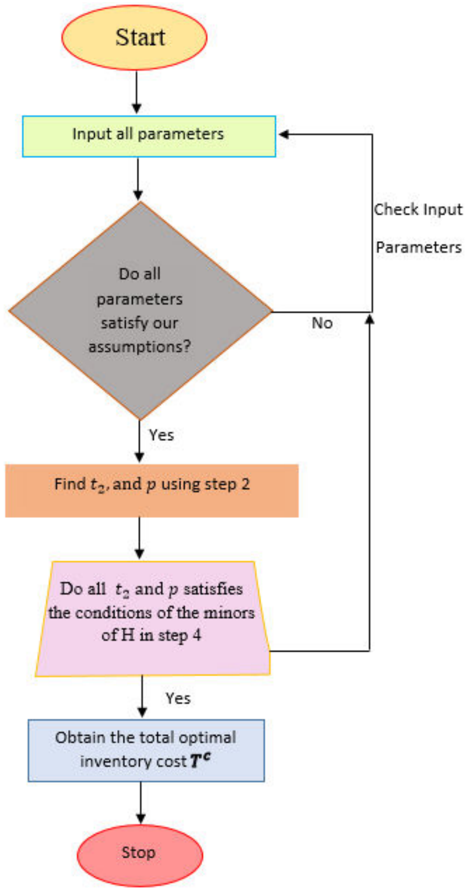

Figure 2: Flowchart of the solution procedure of the proposed model

5.1. Model Simplifications

Consider the original model we can derive several simplified versions of this model by imposing specific conditions:

- **Single Warehouse Setup**
  When the difference between the capacities of the two warehouses is zero, i.e., $U - V = 0$, the model effectively reduces to a single warehouse scenario. This condition suggests that there is no distinction between the primary and secondary storage facilities, simplifying the management and inventory calculations to a single storage unit.

- **Single Warehouse with Instantaneous Deterioration and No Green Technology** By further assuming that $U - V = 0$, $G_T = 0$ (indicating the absence of green technology), and $t_n = 0$ (indicating instantaneous deterioration), the model is further simplified. Under these conditions, we deal with a single warehouse system where items deteriorate instantaneously, and no environmental considerations (such as carbon emissions reduction) are included.

6. NUMERICAL EXAMPLE

Here, we present four numerical examples to validate the model by considering specific parameter values from Mashud et al. [35] and Pal et al. [30]. The models are as follows:

- **Model 1: Inventory model for NID items with green technology investment**

$$\begin{aligned}
&\Pi = \$250, f = \$0.4, t_n = 1.5, a = 100, t_1 = 3.3, b = 0.2, U = 350units,\\
&V = 200units, d_t = 50km, V_t = 7, \kappa = 0.3, \delta = 6, m(\zeta) = 0.8, h = \$0.6,\\
&\vartheta = 0.5, c = \$15, n = 2, I_P = \$4, \gamma = 0.5, O_Q = 9, L_T = 0.5, G_T = 6,\\
&\mu = 6, q_1 = \$0.1/shipment, \chi = 4, w = 4kg/unit, q_2 = 0.75L/100km,\\
&q_3 = 2.4L/100km/ton\ of\ payload, c_e = \$2.35/km, e_x = \$1.3/unit/km.
\end{aligned}$$

  We get the values of $t_2 = 6.3104, p = 375.523$ using MATLAB R2024a. And the total optimal cost is $T^c = 1697.7$.

- **Model 2: Inventory model for NID item without green technology investment** For the fixed values as in Model 1 but taking $G_T = 0$ we convert the model to an NID model without green technology and we get the values of $t_2 = 6.4201, p = 377.525$ and $T^c = 1725.6$ using MATLAB R2024a.

- **Model 3: Inventory model for instantaneous deteriorating items with green technology investment** For the fixed values as in Model 1 but taking $t_n = 0$ we convert the model to an instantaneous model with green technology and get the values of $t_2 = 6.3234, p = 375.729$ and $T^c = 1964.2$ using MATLAB R2024a.

- **Model 4: Inventory model for instantaneous deteriorating items without green technology investment** For the fixed values as in Model 1 but taking $t_n = 0$ & $G_T = 0$ we convert the model to an instantaneous model without green technology and get the values of $t_2 = 6.336, p = 379.354$ and $T^c = 2129.4$ using MATLAB R2024a.

6.1. Observation

When comparing the total inventory costs using ratios:

1. Model 1 serves as the baseline with a ratio of 1.

2. Model 2 has a total cost that is 1.64% higher than Model 1.
3. Model 3 results in a total cost that is 15.70% higher than Model 1.
4. Model 4 incurs the highest cost, 25.41% higher than Model 1.

Model 1, the inventory model for NID items with preservation technology, carbon emissions, price-dependent demand, and hybrid payment scheme, is the most cost-effective option. By incorporating green technology and effectively managing NID, Model 1 achieves the lowest total inventory cost. This comprehensive approach underscores the importance of integrating sustainability measures and accurate deterioration modeling to optimize inventory management costs. The analysis of the four models confirms that Model 1 is the most efficient and sustainable. Graphically it is presented in Figure 3 for NID and Figure 4 for instantaneous deteriorating items below.

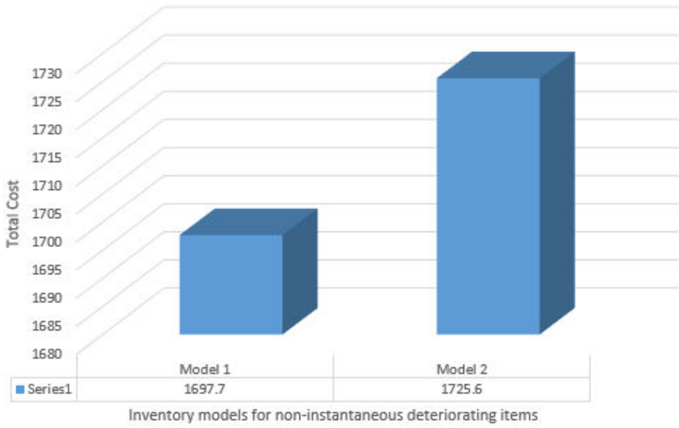

Figure 3: Comparison of inventory models for NID items vs Total cost

7. SENSITIVITY ANALYSIS

This study illustrates how a certain element affects the choice variables.This section looks at the sensitivity analysis that was done on the model's inventory parameters. The optimal values of the total cost (T^c) of the proposed model exhibit substantial variation when $-50\%, -25\%, +25\%, +50\%$ is applied to the values of the various parameters in Table 3 and Table 4.

In Table 4, the sensitivity analysis of the parameters is provided for the values $t_1 = 6.3104$ and $p = 375.523$ as computed using MATLAB R2024a.

7.1. Observations

Based on Tables 3 and 4, The following characteristics are highlighted founded on a sensitivity evaluation of parameter values.

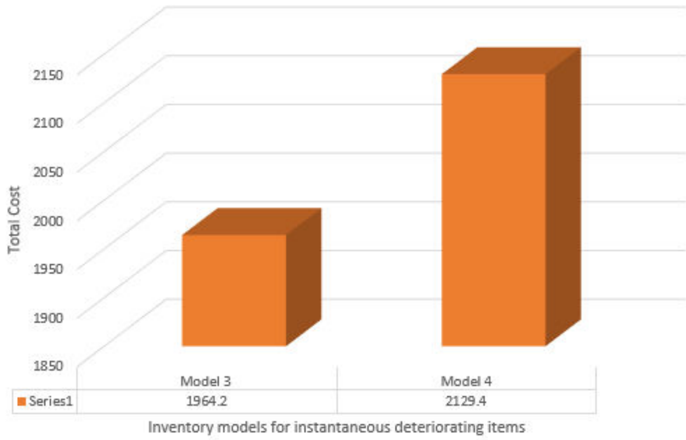

Figure 4: Comparison of inventory models for instantaneous deteriorating items vs Total cost

- Increasing the parameters U and V affects the total cost T^c differently. Expanding the capacity of the outbound warehouse (OW) and the maximum inventory level (linked to V) increases T^c, whereas increasing the capacity of the receiving warehouse (RW) (linked to U) reduces T^c. Thus, higher OW capacity and inventory levels lead to greater costs, while enhancing RW capacity lowers the total cost.
- Higher deterioration rates in the RW and OW, denoted by κ and ϑ respectively, have opposing effects on T^c. An increase in ϑ raises T^c, while an increase in κ reduces it. Both ϑ and κ are highly sensitive parameters that significantly influence the total cost.
- Extending the non-deterioration period t_n results in a reduction in T^c. This implies that prolonging the phase during which inventory does not deteriorate can effectively lower costs.
- The overall inventory management cost T^c decreases as t_n increases, indicating that this parameter is highly sensitive. Longer non-deterioration times within the inventory cycle yield greater cost savings.
- An increase in t_1, the time required for the inventory in RW to deplete completely, leads to a rise in T^c. This parameter exhibits moderate sensitivity, showing that extending this time frame increases the overall cost.
- Increasing the investment in preservation technology $m(\zeta)$ results in a decrease in T^c. This demonstrates that higher spending on preservation technology significantly reduces costs, reflecting high sensitivity to this parameter.
- Raising the parameter Π increases the total inventory cost T^c. This underscores the importance of reducing ordering costs as part of cost-reduction strategies, with Π showing moderate sensitivity.

Table 3: Sensitivity analysis concerning the parameters $f, h, t_n, t_1, \kappa, \vartheta, m(\zeta), G_T$

Parameter	Initial value	Percentage variation	Values of t_2	Values of p	Total cost (T^c)	% change in T^c
f	0.4	-50%	6.4261	328.62	1572	-7.3
		-25%	6.4334	347.34	1617	-4.7
		+25%	6.5827	436.60	1779	4.8
		+50%	6.6354	534.32	1828	7.7
h	0.6	-50%	6.7261	379.18	1329	-21.6
		-25%	6.8334	392.66	1493	-12.1
		+25%	6.9827	475.68	1884	11.1
		+50%	7.0154	512.92	1969	16.1
t_n	1.5	-50%	6.5145	340.32	1857	9.4
		-25%	6.5164	340.58	1742	2.6
		+25%	6.5190	340.94	1526	-10.2
		+50%	6.5199	341.06	1448	-14.7
t_1	3.3	-50%	6.5194	280.62	1662	-2.1
		-25%	6.5183	310.71	1671	-1.5
		+25%	6.5172	342.83	1713	0.9
		+50%	6.5166	343.87	1764	3.6
κ	0.3	-50%	7.0348	570.28	1862	9.7
		-25%	6.7568	453.95	1754	3.3
		+25%	6.5145	272.89	1599	-5.7
		+50%	6.4789	227.62	1499	-11.6
ϑ	0.5	-50%	6.7158	534.89	1380	-18.6
		-25%	6.6000	469.12	1519	-10.4
		+25%	6.4542	365.78	1839	8.4
		+50%	6.4021	316.82	1991	17.3
$m(\zeta)$	0.8	-50%	6.6434	546.21	1856	9.3
		-25%	6.4662	524.81	1762	3.8
		+25%	5.9636	512.86	1618	-4.6
		+50%	5.5077	503.83	1536	-9.7
G_T	6	-50%	6.3250	303.89	1704	0.4
		-25%	6.4201	331.33	1700	0.2
		+25%	6.5702	367.34	1693	-0.2
		+50%	6.6218	380.50	1689	-0.4

- Higher holding costs in both warehouses, represented by f and h, lead to an increase in T^c. These parameters are highly sensitive, as increased holding costs directly escalate the total inventory management cost.
- Increasing the investment in preservation technology G_T reduces T^c, indicating moderate sensitivity. Investing more in preservation technology can moderately lower total costs.
- When the market size a and the shape parameter b increase, T^c also rises. This indicates that larger market sizes and variations in demand patterns contribute to higher overall costs.
- The total cost behaves differently with variations in γ (fraction of purchase cost), I_P (interest payment), L_T (total lead time for delivery), and n (number of installments). Specifically, T^c increases with γ and I_P but decreases with L_T and n. Thus, higher

Table 4: Sensitivity analysis concerning the parameters having $t_2 = 6.3104$ and $p = 375.523$

Parameter	Initial value	Total Cost				↑/↓
		-50%	-25%	25%	50%	
Π	250	1547	1609	1745	1816	↑
U	350	1799	1724	1619	1572	↓
a	100	1528	1596	1710	1871	↑
b	0.2	1415	1522	1768	1832	↑
V	200	1489	1592	1782	1865	↑
I_P	4	1624	1663	1729	1787	↑
γ	0.5	1693	1695	1699	1701	↑
O_Q	9	1423	1567	1729	1895	↑
L_T	0.5	1795	1753	1657	1603	↓
n	2	1762	1731	1668	1631	↓
d_t	50	1129	1311	1988	2382	↑
q_1	0.1	1727	1704	1665	1612	↓
q_2	0.75	1592	1695	1701	1709	↑
q_3	2.4	1193	1348	1945	2254	↑
V_t	7	1307	1578	1821	2091	↑
w	4	1207	1478	1846	2065	↑
δ	6	1807	1721	1518	1421	↓
c_e	2.35	1791	1743	1638	1562	↓
e_x	1.3	1775	1732	1662	1616	↓
μ	6	1371	1490	1788	1927	↑
χ	4	1531	1599	1762	1821	↑
c	15	1619	1659	1735	1764	↓

Increasing(↑) and Decreasing(↓)

interest payments and purchase cost fractions raise costs, while longer lead times and more installments lower them.

- Transportation costs are influenced by various parameters. The total cost increases with μ (number of trips), q_2 (fuel consumption when the truck is empty), q_3 (additional fuel consumption per ton of payload), d_t (distance traveled to customers), V_t (transportation cost per fuel price), O_Q (order quantity), w (product weight), and χ (efficacy of green technology in reducing emissions). Conversely, costs decrease with δ (amount of carbon emissions reduced by green technology), q_1 (minimum transportation cost), c_e (carbon emission cost), and e_x (extra carbon emission cost per item).

- An increase in the purchase cost per unit time c directly raises the total cost T^c, reflecting a direct relationship between purchase costs and total inventory management costs.

The graph in Figure 5 illustrates the relationship between total cost and green technology investment. As green technology investment increases, the total cost consistently decreases, highlighting the cost-saving benefits of integrating green technology in inventory management. This supports the conclusion that Model 1, which includes green technology investment, is the most cost-effective option.

The optimality graph, presented in Figure 6, illustrates the relationship between the

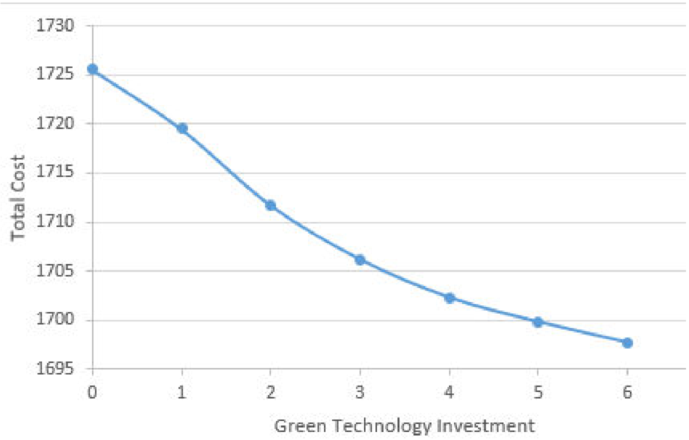

Figure 5: Total Cost with the change in green technology investment

total cost, the decision variable t_2 and p. This three-dimensional graph provides a visual representation of how variations in t_2 and p impact the overall system cost, aiding in identifying the optimal combination of these variables to minimize costs. The interplay between these factors reveals critical insights into the efficiency and cost-effectiveness of the proposed inventory model under different operational settings.

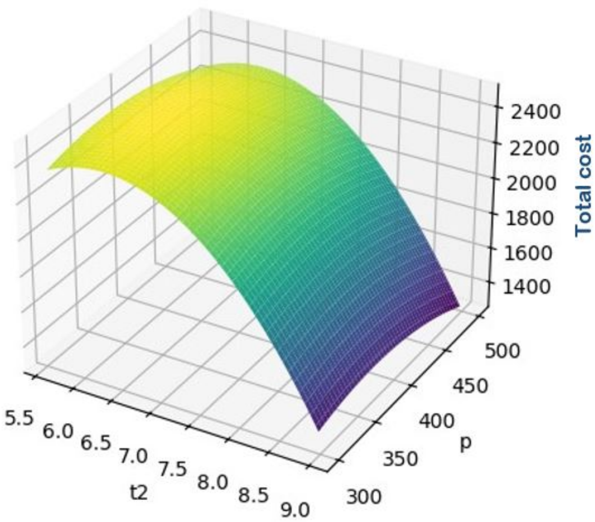

Figure 6: Total Cost vs t_2 vs p

On the other hand, Figures 7 and 8 display the convergence behavior of the decision variables t_2 and p, respectively. These graphs illustrate the iterative optimization process, showing how the values of t_2 and p evolve across iterations to reach their optimal levels. The convergence patterns highlight the stability and robustness of the optimization algorithm employed, ensuring that the derived solutions are reliable and computationally efficient.

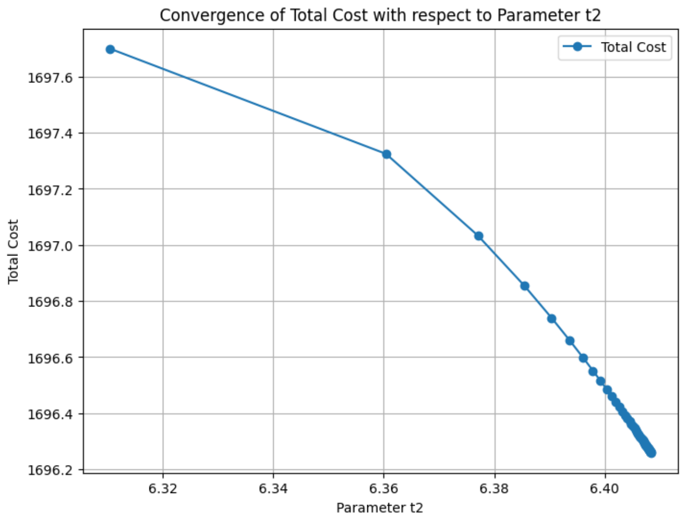

Figure 7: Convergence graph for t_2

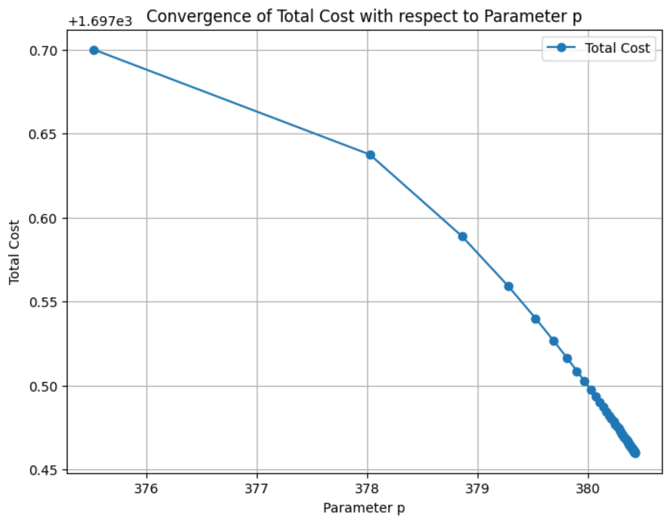

Figure 8: Convergence graph for p

7.2. Result and Discussion

The proposed inventory model integrates preservation technology, carbon emissions, and hybrid payment schemes to optimize costs for NID items. Key findings include:

- Preservation Technology: Investments significantly reduce deterioration rates and total costs, emphasizing their importance in sustainable inventory management.
- Carbon Emission Management: Incorporating green technologies reduces costs while ensuring regulatory compliance.
- Hybrid Payment Scheme: Enhances financial flexibility by balancing upfront and deferred payments, optimizing cash flow.
- Sensitivity Insights: Prolonged non-deterioration periods and increased RW capacity reduce costs, while OW capacity and higher deterioration in the OW raise costs.
- The optimality and convergence graphs confirm the model's stability, and sensitivity analysis highlights key cost drivers. These results underline the practical value of the model for sustainable and cost-efficient inventory management.

8. MANAGERIAL INSIGHT

Following are some significant findings from this study:

- Investing in preservation technologies is highly effective in reducing total inventory costs. Managers should prioritize funding for advanced preservation methods as they yield significant cost savings by extending product shelf-life and maintaining inventory quality.
- Extending the NID period, during which inventory does not degrade, can lead to substantial cost reductions. Managers should explore ways to lengthen this period through improved storage conditions, better inventory handling practices, and utilizing preservatives to delay deterioration.
- Managing carbon emissions effectively is crucial for controlling transportation costs. Implementing green technologies to reduce carbon emissions can lower overall costs associated with fuel consumption and carbon penalties. Managers should invest in eco-friendly transportation solutions and optimize logistics to minimize the environmental impact and related expenses.
- Careful management of warehouse capacities and inventory levels is essential. Increasing the capacity of receiving warehouses (RW) can help in reducing total costs, while optimizing outbound warehouse (OW) capacities and maximum inventory levels can prevent unnecessary cost escalations.
- Managers should monitor key cost drivers such as holding costs, market size, and purchase costs closely. Understanding the sensitivity of these parameters allows for better strategic planning and cost control measures. Adjusting strategies based on the sensitivity of these factors can lead to more efficient inventory and cost management.

8.1. Research Implications

- The proposed model integrates preservation technology, carbon management, and hybrid payment schemes, addressing gaps in traditional NID inventory models.
- Sensitivity analysis provides a framework for assessing the impact of key parameters, aiding the development of dynamic and adaptable inventory strategies.
- Can reduce costs and environmental impact by investing in preservation technologies and green logistics solutions.
- Financial flexibility offered by hybrid payment schemes supports liquidity management, especially in cost-sensitive industries.
- Sensitivity to storage and deterioration rates highlights the importance of designing warehouses and logistics systems tailored to product characteristics.

9. CONCLUSION

This study develops a comprehensive inventory model for NID items within a two-warehouse system, integrating preservation technology, carbon emission considerations, price-dependent demand, and a hybrid payment scheme. The model aims to optimize total costs while promoting sustainability and financial flexibility, making it particularly relevant for industries dealing with perishables and environmentally regulated products. Through numerical analysis and sensitivity evaluation, the model demonstrates its effectiveness in balancing economic and environmental objectives. The incorporation of preservation technology and green investments significantly enhances inventory efficiency by extending shelf life and reducing carbon emissions. Furthermore, the hybrid payment scheme provides businesses with improved liquidity management, allowing for strategic financial planning. Overall, the proposed model offers a valuable framework for cost-effective and sustainable inventory management.

9.1. Key Findings

This study highlights the critical role of preservation and green technology investments in minimizing inventory costs and promoting sustainability. Preservation technology effectively reduces deterioration rates, lowering holding costs and extending shelf life, while green technology adoption cuts transportation costs and carbon emissions, ensuring regulatory compliance. The hybrid payment scheme optimizes cash flow by balancing upfront and deferred payments. Sensitivity analysis underscores the impact of factors like deterioration rates, holding costs, and market size on total costs, offering valuable insights for inventory managers to enhance efficiency and sustainability.

9.2. Limitations

Several limitations should be noted. The assumption of constant deterioration rates may not reflect real-world variations due to environmental or operational factors. The model primarily focuses on warehouse inventory, overlooking broader supply chain dynamics such as logistics, supplier uncertainties, and demand fluctuations. Its complexity may also present implementation challenges for SMEs with limited resources.

9.3. Research Direction

Future work can improve the model by incorporating dynamic deterioration rates, adapting to variable storage conditions, and extending coverage to include transportation and distribution. Integrating advanced technologies such as IoT, AI, and blockchain can enhance real-time decision-making and tracking. Further exploration into multi-product scenarios and addressing inflation, demand disruptions, and regulatory uncertainties would enhance the model's practical applicability.

Acknowledgement: The authors would like to acknowledge the valuable comments of the anonymous reviewers which helped to improve the quality of the paper.

Funding: This research received no external funding.

REFERENCES

[1] P. N. Ghare and G. F. Schrader, "A model for exponentially decaying inventories," *The Journal of Industrial Engineering*, vol. 15, no. 5, pp. 238–243, 1963.

[2] D. Chakraborty, D. K. Jana, and T. K. Roy, "Multi-warehouse partial backlogging inventory system with inflation for non-instantaneous deteriorating multi-item under imprecise environment," *Soft Computing*, vol. 24, no. 19, pp. 14471–14490, 2020, doi: 10.1007/s00500-020-04920-6.

[3] Y. He and H. Huang, "Optimizing inventory and pricing policy for seasonal deteriorating products with preservation technology investment," *Journal of Industrial Engineering*, Article ID 793568, 2013.

[4] A. Limi, K. Rangarajan, and P. Rajadurai, "Recent review on non-instantaneous deteriorating items in inventory models (2006–2022)," *International Journal of Services and Operations Management*, vol. 1, no. 1, 2023 (in press), doi: 10.1504/IJSOM.2023.10061336.

[5] T. P. Dye and C. Y. Dye, "A production inventory model incorporating the effect of preservation technology investment when demand is fluctuating with time," *Journal of Computational and Applied Mathematics*, vol. 239, pp. 25–36, 2013, doi: 10.1016/j.cam.2012.09.020.

[6] Y. W. Lok, S. S. Supadi, and K. B. Wong, "Optimal investment in preservation technology for non-instantaneous deteriorating items under carbon emissions consideration," *Computers & Industrial Engineering*, vol. 183, no. 109446, 2023, doi: 10.1016/j.cie.2023.109446.

[7] A. H. M. Mashud, H.-M. Wee, and C.-V. Huang, "Preservation technology investment, trade credit, and partial backordering model for a non-instantaneous deteriorating inventory," *RAIRO - Operations Research*, vol. 55, no. S51, pp. S51–S77, 2021, doi: 10.1051/ro/2021017.

[8] U. Mishra, J. T. Aguilera, S. Tiwari, and L. E. Cárdenas-Barrón, "Retailer's joint ordering, pricing, and preservation technology investment policies for a deteriorating item under permissible delay in payments," *Mathematical Problems in Engineering*, vol. 2018, pp. 1–14, 2018, doi: 10.1155/2018/6962417.

[9] S. P. Chiu, J. J. Liao, S. L. Kang, H. M. Srivastava, and S. D. Lin, "Sustainable inventory managements for non-instantaneous deteriorating items: Preservation technology and green technology approaches with advanced purchase discounts and joint emission regulations," *Sustainability*, vol. 16, no. 6805, 2024.

[10] S. V. S. Padiyar, V. C. Kuraie, D. Makholia, S. R. Singh, V. Singh, and N. Joshi, "An imperfect production inventory model for instantaneous deteriorating items with preservation investment under inflation on time value of money," *Contemporary Mathematics*, pp. 1422–1446, 2024.

[11] U. Mishra, J. Z. Wu, Y. C. Tsao, and M. L. Tseng, "Sustainable inventory system with controllable non-instantaneous deterioration and environmental emission rates," *Journal of Cleaner Production*, vol. 244, no. 118807, 2020, doi: 10.1016/j.jclepro.2019.118807.

[12] A. A. Taleizadeh, B. Hazarkhani, and I. Moon, "Joint pricing and inventory decisions with carbon emission considerations, partial backordering and planned discounts," *Annals of Operations Research*, vol. 290, pp. 95–113, 2020, doi: 10.1007/s10479-017-2536-6.

[13] H. M. Wee and Y. Daryanto, "Imperfect quality item inventory models considering carbon emissions," in *Optimization and Inventory Management*, N. H. Shah and M. Mittal, Eds. Singapore: Springer Nature, 2020, pp. 137–159.

[14] M. Pervin, "A sustainable deteriorating inventory model with backorder and controllable carbon emission by using green technology," *Environment, Development and Sustainability*, pp. 1–37, 2024, doi: 10.1007/s10668-023-02984-9.

[15] W. A. Jauhari, I. D. Wangsa, H. Hishamuddin, and N. Rizky, "A sustainable vendor-buyer inventory model with incentives, green investment, and energy usage under stochastic demand," *Cogent Business & Management*, vol. 10, no. 1, 2023, doi: 10.1080/23311975.2022.2158609.

[16] M. Suef, W. A. Jauhari, I. Pujawan, et al., "Investigating carbon emissions in a single-manufacturer multi-retailer system with stochastic demand and hybrid production facilities," *Process Integration and Optimization for Sustainability*, vol. 7, pp. 743–764, 2023, doi: 10.1007/s41660-023-00320-3.

[17] W. A. Jauhari, D. N. Affifah, P. W. Laksono, and D. M. Utama, "A closed-loop supply chain inventory model with stochastic demand, exchange rate, green investment, and carbon tax," *Cleaner Logistics and Supply Chain*, vol. 13, no. 100168, 2024, doi: 10.1016/j.clscn.2024.100168.

[18] W. A. Jauhari, I. D. Wangsa, H. Hishamuddin, and N. Rizky, "A sustainable vendor-buyer inventory model with incentives, green investment, and energy usage under stochastic demand," *Cogent Business & Management*, vol. 10, no. 1, 2023, doi: 10.1080/23311975.2022.2158609.

[19] C. K. Jaggi, S. Tiwari, and S. K. Goel, "Credit financing in economic ordering policies for non-instantaneous deteriorating items with price dependent demand and two storage facilities," *Annals of Operations Research*, vol. 248, pp. 253–280, 2017, doi: 10.1007/s10479-016-2156-8.

[20] S. Tiwari, L. E. Cárdenas-Barrón, A. Khanna, and C. K. Jaggi, "Impact of trade credit and inflation on retailer's ordering policies for non-instantaneous deteriorating items in a two-warehouse environment," *International Journal of Production Economics*, vol. 176, pp. 154–169, 2016, doi: 10.1016/j.ijpe.2016.03.017.

[21] A. Limi, K. Rangarajan, P. Rajadurai, A. Akilbasha, and K. Parameswari, "Three warehouse inventory model for non-instantaneous deteriorating items with quadratic demand, time-varying holding costs and backlogging over finite time horizon," *Ain Shams Engineering Journal*, vol. 102826, 2024, doi: 10.1016/j.asej.2023.102826.

[22] K. Rangarajan and K. Karthikeyan, "Two warehouse optimal inventory model for non-instantaneous deteriorating item," *International Journal of Engineering and Technology (UAE)*, vol. 7, no. 4.10, pp. 946–948, 2018.

[23] R. S. Rana, D. Kumar, and K. Prasad, "Mitigating the impact of demand disruption on perishable inventory in a two-warehouse system," *Operations Management Research*, vol. 17, pp. 469–504, 2024, doi: 10.1007/s12063-023-00418-4.

[24] V. Murmu, D. Kumar, B. Sarkar, R. S. Mor, and A. K. Jha, "Sustainable inventory management based on environmental policies for perishable products under first or last in and first out policy," *Journal of Industrial and Management Optimization*, vol. 19, no. 7, pp. 4764–4803, 2023, doi: 10.3934/jimo.2022149.

[25] R. S. Rana, D. Kumar, and K. Prasad, "Two warehouse dispatching policies for perishable

items with freshness efforts, inflationary conditions, and partial backlogging," *Operations Management Research*, vol. 15, pp. 28–45, 2022, doi: 10.1007/s12063-020-00168-7.

[26] J. A. Buzacott, "Economic order quantities with inflation," *Operations Research Quarterly*, vol. 26, pp. 553–558, 1975.

[27] P. Meena, S. Meena, A. K. Sharma, P. T. Singh, and G. Kumar, "Managing inventory of non-immediately degrading items with partial backlog and discounting cash flow under inflation," *Materials Today: Proceedings*, vol. 50, pp. 155–162, 2022, doi: 10.1016/j.matpr.2021.11.348.

[28] T. K. Datta, "Effect of green technology investment on a production-inventory system with carbon tax," *Advances in Operations Research*, Article ID 4834839, 2017, doi: 10.1155/2017/4834839.

[29] A. Limi, K. Rangarajan, C. Jana, E. Ghith, T. Lamoudan, G.-W. Weber, and A. A. Abdelhamid, "Analyzing how inflation affects non-instantly decaying goods with demand linked to ads and selling price in a dual-warehouse setup," *Alexandria Engineering Journal*, vol. 106, pp. 517–529, 2024, doi: 10.1016/j.aej.2024.08.014.

[30] D. Pal, A. K. Manna, I. Ali, P. Roy, and A. A. Shaikh, "A two-warehouse inventory model with credit policy and inflation effect," *Decision Analytics Journal*, vol. 10, no. 100406, 2024, doi: 10.1016/j.dajour.2022.100406.

[31] M. Choudhury and G. C. Mahata, "Non-instantaneous deteriorating items inventory models with fixed lifetime products under hybrid partial prepayment and trade credit in supply chain," *Journal of Industrial and Management Optimization*, vol. 20, no. 1, pp. 221–259, 2024.

[32] G. Li, X. He, J. Zhou, and H. Wu, "Pricing, replenishment and preservation technology investment decisions for non-instantaneous deteriorating items," *Omega*, vol. 84, pp. 114–126, 2018, doi: 10.1016/j.omega.2018.04.010.

[33] R. S. Rana, D. Kumar, and K. Prasad, "Sustainable production-inventory system for perishables under dynamic fuel pricing and preservation technology investment," *Environmental Science and Pollution Research*, vol. 30, pp. 90121–90147, 2023, doi: 10.1007/s11356-023-28252-y.

[34] W. A. Jauhari, I. Pujawan, and M. Suef, "Sustainable inventory management with hybrid production system and investment to reduce defects," *Annals of Operations Research*, vol. 324, pp. 543–572, 2023, doi: 10.1007/s10479-022-04666-8.

[35] A. H. M. Mashud, M. Pervin, U. Mishra, Y. Daryanto, M.-L. Tseng, and M. K. Lim, "A sustainable inventory model with controllable carbon emissions in green-warehouse farms," *Journal of Cleaner Production*, vol. 298, no. 126777, 2021, doi: 10.1016/j.jclepro.2021.126777.

APPENDIX

The first order derivative of T^c with respect to t_2 is given by,

$$\frac{dT^c}{dt_2} = -\frac{1}{t_2^2}\left[\Pi - \frac{f}{6}t_n^2\left[2(a-bp)t_n + 3(U-V)\right] + \frac{f(a-bp)}{\kappa m(\zeta)}\left[\frac{e^{\kappa m(\zeta)(t_1-t_n)}}{\kappa^2 m^2(\zeta)}\right.\right.$$
$$\left.\left.\Big(t_n\kappa m(\zeta)+1\Big) - \frac{1}{2}(t_1^2-t_n^2) \quad + \frac{1}{\kappa m(\zeta)}\Big(t_1\kappa m(\zeta)+1\Big)\right] + \frac{hVt_n^2}{2}\right.$$
$$+hV\left[\frac{1}{\vartheta^2 m^2(\zeta)}\Big(t_n\vartheta m(\zeta)+1\Big) - \frac{e^{\vartheta m(\zeta)(t_n-t_1)}}{\vartheta^2 m^2(\zeta)}\Big(t_1\vartheta m(\zeta)+1\Big)\right] + G_T t_2$$
$$+\frac{h(a-bp)}{\vartheta m(\zeta)}\left[\frac{e^{\vartheta m(\zeta)(t_2-t_1)}}{\vartheta^2 m^2(\zeta)}(t_1\vartheta m(\zeta)+1) - \frac{1}{\vartheta^2 m^2(\zeta)}(t_2\vartheta m(\zeta)+1) + \frac{1}{2}(t_1^2-t_2^2)\right]$$
$$+\frac{c(a-bp)}{\kappa m^2(\zeta)}\left[\Big(e^{\kappa m(\zeta)(t_1-t_n)}-1\Big) + \kappa m(\zeta)(t_n-t_1)\right] + G_T + \frac{n+1}{2n}I_P\gamma P_C O_Q L_T$$
$$+c\left[\frac{Ve^{\vartheta m(\zeta)t_n}}{m(\zeta)}\Big(e^{\vartheta m(\zeta)(t_n-t_1)}\Big) + \frac{(a-bp)}{\vartheta m^2(\zeta)}\left[\Big(1-e^{\vartheta m(\zeta)(t_2-t_1)}\Big) - \vartheta m(\zeta)(t_2-t_1)\right]\right]$$
$$\left.+\mu\Big[q_1 + (2d_tV_tq_2 + d_tq_3wO_Q) + (2d_tc_e + d_te_xO_Q)(1-\delta(1-e^{-\chi G_T}))\Big]\right]$$

The first order derivative of T^{IC} with respect to p is given by,

$$\frac{\partial T^c}{\partial p} = \frac{1}{t_2}\left[-\frac{ft_n^2}{6}\Big(-2bt_n\Big) + \frac{f(-b)}{\kappa m(\zeta)}\left[\frac{e^{\kappa m(\zeta)(t_1-t_n)}}{\kappa^2 m^2(\zeta)}(t_n\kappa m(\zeta)+1) + \frac{1}{\kappa m(\zeta)}\right.\right.$$
$$\left.*(t_1\kappa m(\zeta)+1)\right] \quad + \frac{h(-b)}{\vartheta m(\zeta)}\left[\frac{e^{\vartheta m(\zeta)(t_2-t_1)}}{\vartheta^2 m^2(\zeta)}(t_1\vartheta m(\zeta)+1) - \frac{1}{\vartheta^2 m^2(\zeta)}\right.$$
$$\left.\left.*(t_2\vartheta m(\zeta)+1)\right] + \frac{c(-b)}{\kappa m^2(\zeta)}\right] \quad +c\left[\frac{(-b)}{\vartheta m^2(\zeta)}\left[\Big(1-e^{\vartheta m(\zeta)(t_2-t_1)}\Big) - \vartheta m(\zeta)(t_2-t_1)\right]\right]\Bigg].$$

Then compute $\frac{\partial T^c}{\partial t_2} = 0$ and $\frac{\partial T^c}{\partial p} = 0$. for getting values of t_2 and p

The second derivatives of Hessian matrix are confirmed to be positive using MATLAB R2024a, it implies that the total cost function exhibits convexity. This characteristic signifies that the cost function possesses a sole minimum at the present point. This attribute is advantageous as it suggests that the cost function demonstrates local convexity around the current solution, facilitating optimization algorithms to converge efficiently towards the global minimum.