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Research article

A MIXED INTEGER LINEAR PROGRAMMING APPROACH TO PRODUCTION OPTIMIZATION IN A SURFACE TREATMENT AREA OF THE AEROSPACE INDUSTRY

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Abstract: Production scheduling, especially in intricate sectors like aerospace, is essential. This study presents a Mixed Integer Linear Programming (MILP) model to optimize production in the surface treatment area of an aerospace company. The complex task involves several constraints, including raw material alloy type, tank capacity, and fixture design for parts. The goal is to find the optimal combination to maximize production, efficiency, and safety. Unique to this problem is the integration of factors such as different tanks, fixtures, scheduling, and geometric constraints. Technical requirements have been integrated as constraints to satisfy aircraft manufacturing certifications. Experiments with

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thirty real and thirty simulated scenarios showed that the proposed model significantly outperformed the company's existing methods. In the best case, it demonstrated the potential to more than double the production with the same resources. The results underscore the existing capacity to take on new products and clients, highlighting the potential for increased efficiency. Spatial optimization allows for processing more parts per cycle, thus reducing time, costs, and environmental impact. This study adds value to optimization literature, offering a novel approach to a real and multifaceted problem in the surface treatment industry, with applications extending to other industrial contexts where spatial optimization and efficient scheduling are key.

Keywords: Optimization, lot sizing, surface treatment, mixed integer linear programming, aerospace industry.

MSC: 90B50

1. INTRODUCTION

The aerospace industry is a prominent sector in the global marketplace, distinguished by advanced technological innovation and highly specialized products with significant value-added components. It operates predominantly within the broader capital-goods segment, characterized by Engineer-to-Order (ETO) and Make-to-Order (MTO) strategies that entail low production volumes, high levels of customization, long lead times, and tight integration between engineering design and shop-floor control [1,2]. Recent studies reinforce how these inherent conditions amplify the complexity of planning and scheduling. Neumann et al. [3], for instance, highlight ongoing challenges optimization techniques face when addressing product variability and resource constraints inherent to ETO manufacturing. Additionally, Love et al. [4] emphasize the operational risks arising from rigorous certification requirements and customer-specific demands prevalent in aerospace-related ETO production environments. These complexities reduce the feasibility of maintaining inventory buffers, escalate the potential costs associated with rework, and consequently necessitate the development of advanced scheduling tools, such as the one proposed in this study.

Organizationally, the aerospace supply chain adopts a multi-tier hierarchy, culminating in Original Equipment Manufacturers (OEMs), such as Boeing, Airbus, and Embraer, that execute the final assembly of aircraft. Within this structure, suppliers are classified according to the complexity and criticality of their supplied components: Tier-1 firms produce primary systems and modules directly for OEMs; Tier-2 and Tier-3 companies supply subsystems, individual components, and specialized services; and Tier-4 suppliers primarily handle raw materials [5].

This research specifically addresses a complex production-planning challenge in the surface-treatment division of a Tier-1 aerospace supplier based in Guaratinguetá, São Paulo, Brazil. This facility manufactures sophisticated aeronautical systems, including landing gears, pressurization systems, flight controls, and avionics components. To guarantee that only confirmed customer orders proceed to galvanoplasty operations, the firm employs a robust backlog management system. Effective production planning must therefore consider the alloys of raw materials, specialized surface-treatment protocols, and the capacity limitations of two chromic anodizing tanks, each capable of processing up to 40 lots daily.

Surface treatment operations within aerospace manufacturing represent substantial financial commitments, primarily driven by high electricity consumption and strict process control standards. Nevertheless, improvements in operational efficiency promise significant economic returns, enabling the treatment of more components per production cycle while simultaneously reducing environmental impacts.

Considering this context, the central research question guiding our study is: "What configuration of products and quantities per lot and per tank, respecting all technical and operational constraints, maximizes daily productivity?" To answer this, we develop and implement a customized Mixed-Integer Linear Programming (MILP) model explicitly designed to optimize the scheduling of chromic-anodizing tanks in the analyzed Tier-1 aerospace plant.

The proposed MILP model was implemented within the GAMS IDE and solved using the CPLEX optimization solver. Real-world production scenarios from the company were utilized for validation purposes, allowing direct comparisons between current scheduling practices and solutions derived from our optimization approach. Additionally, supplementary experiments involving simulated demand conditions were carried out to assess the robustness and adaptability of the proposed scheduling method.

The decision problems tackled by this study align closely with classical optimization frameworks such as Lot Sizing Problems, which are recognized in the literature as NP-hard [6,7]. Additional theoretical insights were drawn from related Knapsack-type problem formulations [8,9].

Although the broader context and general motivation for this research are presented here, the specific gaps in existing knowledge that justify our study's contributions are detailed comprehensively in Section 2.4.

The remainder of the article is structured as follows: Section 2 provides an in-depth review of foundational concepts, existing optimization methods, and analytical tools pertinent to our approach. Section 3 introduces the industrial context, describing the production steps and operational constraints of the surface-treatment process, followed by a detailed presentation of our MILP formulation. Section 4 presents the computational results obtained from analyzing thirty actual production days and an additional thirty simulated scenarios, contrasting the effectiveness of the firm's current scheduling methods with the proposed MILP-generated schedules. Finally, Section 5 summarizes the study's key findings, discusses managerial implications, and outlines potential avenues for future research.

2. LITERATURE REVIEW

2.1. Production Scheduling and PPC in the Capital-Goods Industry

Production scheduling is a critical function within Production Planning and Control (PPC), particularly in capital-goods industries such as aerospace, heavy machinery, and power-generation equipment. These industries typically employ production strategies such as Make-to-Order (MTO) or Engineer-to-Order (ETO), characterized by low production volumes, high customization, long lead times, and close integration between engineering activities and manufacturing operations [10,11].

Unlike the standardized environments typical of consumer-goods manufacturing, capital-goods production commonly occurs within complex job-shop or project-based settings, involving deep bills-of-materials and multi-level assemblies. These environments require precise coordination across various manufacturing and assembly processes, frequently complicated by stringent precedence constraints. Such constraints include concurrent production modes within the same facility, extended processing sequences, and finite-capacity limitations [12,13].

ETO environments further intensify these complexities. Each order represents a unique, customized project with minimal repetition, requiring specialized planning and scheduling approaches. Standard enterprise systems such as Enterprise Resource Planning (ERP) and Material Requirements Planning (MRP) often prove insufficient in addressing dynamic and non-repetitive demands inherent in ETO production. Additionally, logistical constraints, such as spatial packing restrictions for oversized components and alloy compatibility requirements in surface-treatment operations, frequently demand dynamic adjustments to schedules due to evolving design specifications or unforeseen supply disruptions [14,15].

Historically, production scheduling in capital-goods manufacturing has predominantly relied on human expertise combined with simple heuristic rules. Formal scheduling methods have seen limited practical application, contributing to suboptimal performance outcomes, including delays in delivery and excess inventory [16]. Even today, production planners face significant workloads, frequently required to adapt and re-plan schedules in response to unexpected disruptions. Contemporary research thus emphasizes the growing necessity of robust decision-support systems capable of enabling rapid and effective rescheduling within these highly dynamic production environments [11].

In summary, Production Planning and Control (PPC) in capital-goods industries involves the complex task of coordinating diverse, customized production processes under stringent technical and operational constraints. Effective PPC strategies must therefore remain flexible, responsive, and capable of integrating detailed scheduling tools adapted to the unique realities of MTO and ETO production contexts.

2.2. Historical and Contemporary Optimization Approaches

Historically, production scheduling in complex industrial environments has relied predominantly on heuristic and dispatching rules due to their simplicity, ease of implementation, and rapid decision-making capabilities. Common examples include the Earliest Due Date (EDD) and Critical Ratio (CR) rules. However, despite their practical advantages, these approaches generally lack robustness in dynamic environments, particularly in scenarios subject to stochastic disruptions or uncertainties [16].

To overcome these limitations, researchers in the late twentieth and early twenty-first centuries began exploring advanced heuristic approaches known as metaheuristics. Techniques such as Genetic Algorithms (GA), Simulated Annealing (SA), and Tabu Search were introduced and gained prominence due to their powerful search capabilities, which significantly improved scheduling outcomes compared to traditional heuristic methods [17]. Subsequent developments in metaheuristics further incorporated algorithms such as Particle Swarm Optimization (PSO), Artificial Bee Colony (ABC), and Grey Wolf Optimizer (GWO). These methods were particularly suitable for capital-goods manufacturing scenarios characterized by sequence-dependent setups, multiple resource constraints, and highly customized production requirements [18].

Parallel to these developments, MILP emerged as a rigorous mathematical approach for optimizing production scheduling. Historically, MILP models were seldom applied at industrial scale due to computational limitations. However, recent advancements in solver technologies and computational power have significantly improved their viability, enabling the formulation and practical resolution of more realistic and detailed scheduling problems. Modern MILP formulations now routinely integrate complex constraints, including preventive maintenance schedules, sequence-dependent setups, and resource limitations, effectively addressing the detailed scheduling needs of capital-goods manufacturers [19].

This modeling approach aligns with previous applications of MILP and structured decision-support models in various domains, including production [20], urban mobility [21], innovation management [22], and logistics and healthcare systems [23,24]. These applications demonstrate the versatility and robustness of MILP frameworks for solving complex problems involving capacity constraints, resource compatibility, and operational efficiency.

Despite these advancements, purely mathematical (exact) approaches can still face challenges related to computational scalability in real-world industrial contexts. Consequently, contemporary research increasingly advocates hybrid optimization methods that combine strengths from both heuristic/metaheuristic and exact MILP techniques. Hybrid approaches often utilize heuristic or metaheuristic algorithms to generate initial solutions, which subsequently inform MILP models, reducing the computational burden and improving convergence times. For example, hybrid methods that integrate metaheuristic algorithms such as Genetic Algorithms or Grey Wolf Optimization with MILP models have demonstrated superior performance, particularly in minimizing makespan and tardiness in complex, customized scheduling scenarios [19].

Additionally, hybrid approaches that integrate simulation techniques alongside optimization models have proven effective in evaluating the robustness of production schedules under realistic disruptive conditions. Such simulation-based hybrid methods enable schedulers to better understand how schedule performance degrades under operational disruptions, providing critical insights into the resilience and adaptability of scheduling solutions typical in capital-goods production environments [25].

2.3. Classical Exact Formulations

Classical exact formulations for production planning and scheduling have their roots in early inventory management theory, beginning with the Economic Order Quantity (EOQ) model, originally proposed by Harris [26] and subsequently applied in practice by Wilson [27]. EOQ provided a foundational framework but was limited by its restrictive assumptions, particularly the absence of capacity constraints and the simplification to single-level production scenarios.

To address some of these limitations, Rogers [28] introduced the Economic Lot Scheduling Problem (ELSP), extending inventory optimization to multi-product contexts with explicit capacity constraints [29]. Further advancements in this area culminated in the Dynamic Lot Sizing Problem (DLSP) by Wagner and Whitin [30], which incorporated dynamic, time-variable demand into inventory optimization models, significantly broadening the practical applicability of lot-sizing formulations [31].

Building upon these foundations, contemporary lot-sizing models often adopt MILP formulations. Among them, the Capacitated Lot-Sizing Problem (CLSP) stands out as a widely applied approach. The CLSP considers multiple items, discrete time periods, capacity limits, setup requirements, and inventory holding costs. It is particularly suitable for MTO and ETO environments where resource allocation and production eligibility, rather than sequencing, are the primary concerns.

The CLSP formulation explicitly incorporates inventory costs, setup costs, and capacity constraints without explicitly modeling product sequencing within each period. A typical CLSP formulation [32,33,34] can be summarized as follows:

Decision Variables:

x_{jt} Quantity of product j produced in period t.
y_{jt} Binary variable equal to 1 if product j is produced in period t, 0 otherwise.
I_{jt} Inventory level of product j at the end of period t.

Parameters:

D_{jt} Demand for product j in period t.
s_j Setup cost for product j.
h_j Unit inventory holding cost for product j.
C_t Available production capacity in period t.

CLSP Formulation:

$$Minimize\ Z = \sum_{t=1}^{T}\sum_{j=1}^{J}(s_j y_{jt} + h_j I_{jt}) \tag{1}$$

Subject to:

$$I_{j(t-1)} + x_{jt} = D_{jt} + I_{jt},\ \forall\ j,\ t \tag{2}$$

$$\sum_{j=1}^{J} p_j x_{jt} \leq C_t,\ \forall\ t \tag{3}$$

$$x_{jt} \leq M y_{jt},\ \forall\ j,\ t \tag{4}$$

$$I_{jt}, x_{jt}\ \geq 0\ ,\ \forall\ j,\ t \tag{5}$$

$$y_{jt} \in \{0,1\},\ \forall\ j,\ t \tag{6}$$

The objective function (1) minimizes the total cost, which consists of setup costs and inventory holding costs incurred over the planning horizon. Constraint (2) ensures inventory balance by requiring that the sum of the previous period's ending inventory and the current period's production equals the current period's demand plus ending inventory. Constraint (3) imposes capacity restrictions: the total production workload in each period must not exceed the available capacity. Constraint (4) is a big-M constraint that enforces setup logic, allowing production of a given product only when the corresponding setup variable is activated. Constraints (5) define non-negativity for the production and inventory variables, while constraint (6) ensures the binary nature of the setup variables.

This classical CLSP structure is particularly appropriate for industrial environments where capacity constraints and production eligibility are the dominant factors, but sequencing decisions can be abstracted or predetermined. In the context of this study, the CLSP formulation provides a solid foundation for modeling lot-sizing and capacity allocation in a constrained surface-treatment environment, enabling the representation of real operational requirements without the additional complexity of sequencing logic.

Despite their modeling rigor, exact MILP formulations often face computational scalability challenges when applied to large-scale, highly customized industrial settings such as those found in ETO and MTO contexts. Solving real-world scheduling problems with pure MILP models may demand advanced solution strategies, such as decomposition methods, heuristics, or solver-specific enhancements, to remain tractable within acceptable computational timeframes [11]. In the context of this study, however, the problem structure and scale are such that a classical CLSP-based MILP formulation remains both feasible and effective. This justifies the choice of a pure MILP model in this study, as it offers sufficient modeling power to address the operational complexity of the industrial problem without incurring prohibitive computational costs.

2.4. Research Gaps and Contributions of the Present Study

Although classical MILP models such as CLSP provide robust formulations, they often fall short in representing the full operational reality of capital-goods production, particularly in aerospace settings. The literature review highlights two key gaps that this study addresses:

First, comprehensive optimization models that fully capture the intricate realities of ETO and Make-to-Order (MTO) scheduling environments remain relatively scarce. The majority of existing formulations in the literature simplify the complexity of real-world industrial operations, neglecting detailed multi-stage manufacturing processes, spatial constraints inherent to large and customized components, and operational specifics such as alloy compatibility in surface-treatment processes. Such simplifications severely limit the practical applicability and accuracy of these scheduling solutions in actual industrial scenarios [15,35].

Second, despite the growing body of theoretical literature on advanced scheduling optimization, empirical validation and real-world applicability remain significantly underrepresented. Few studies provide comprehensive empirical evidence demonstrating the effectiveness and practical feasibility of proposed methods within actual industrial environments, particularly in capital-intensive industries such as aerospace. The scarcity of detailed industrial case studies limits the potential for generalizing theoretical results, thereby creating uncertainty regarding the robustness, adaptability, and practical impact of these advanced scheduling methods [36].

Addressing precisely these identified gaps, the present study makes several important contributions. First, it develops a tailored MILP model explicitly designed for the scheduling challenges faced by a Tier-1 aerospace manufacturer. This MILP model integrates critical real-world operational constraints, including finite capacities, spatial packing limitations, alloy compatibility constraints associated with chromic anodizing surface treatments.

Second, the study provides extensive empirical validation, leveraging authentic production data obtained directly from the aerospace company's surface-treatment area. By systematically comparing current scheduling practices employed by the company with optimized schedules produced by the proposed MILP approach, the research delivers concrete evidence of substantial efficiency gains, improved resource utilization, and enhanced responsiveness to demand fluctuations and operational disruptions.

Furthermore, the methodological rigor of this empirical validation extends through computational experiments involving both actual historical production scenarios and realistic simulated demand conditions. These additional tests validate the robustness and adaptability of the proposed model, demonstrating its potential for broader application across similar ETO and MTO manufacturing contexts.

Finally, from a theoretical and practical standpoint, this research offers significant insights into the challenges associated with implementing advanced optimization methods in industrial settings, particularly regarding solver performance, computational feasibility, and operational integration within the context of highly customized aerospace manufacturing.

3. DESCRIPTION AND MATHEMATICAL FORMULATION OF THE PROBLEM

This section outlines the problem identified in the surface treatment area of the company under study. The goal is to maximize the utilization of available resources, avoiding bottlenecks and underutilization throughout the day.

3.1. Preparatory Stages

Before chromic anodizing, there are eight preparatory stages focused on cleaning and activating the product's surface. Figure 1 is a schematic representation of the entire process in the surface treatment area.

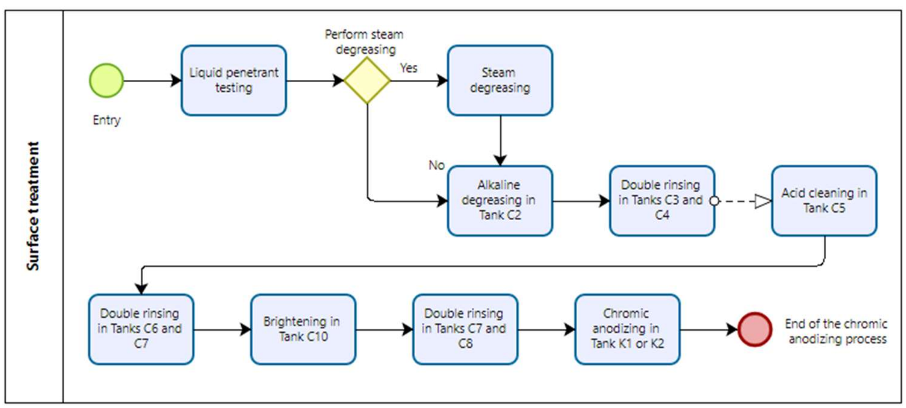

Figure 1: Process steps in the surface treatment area

The first phase involves receiving items released by the Machining process, executed by a material movement route at predefined times. The second stage, optional and dependent on the process defined by the customer, may involve a liquid penetrant test to detect mechanical or material defects.

The degreasing (tank C2), double rinse (tanks C3 and C4) until (tanks C7 and C8) stages refer to the cleaning and activation process of the surface to be anodized. These phases are crucial, as inefficient cleaning can lead to quality issues. However, these stages are not considered in the model's elaboration.

3.2. Production Process

Scheduling in the electroplating sector is complex and burdensome, conducted based on items delivered by the machining area and ready for treatment. The scheduling aims to optimize the use of chromic anodizing tanks, the sector's bottleneck.

Assembly of Parts: Parts are mounted on hooks or specific devices, with dimensions determining the quantity and manner of attachment (Figure 2).

(a) (b)

Figure 2: Fixing devices with assembled products, 8 items (a) and 3 items (b)

Treatment in the Tank: Fixing devices are fitted into the busbar of the chromic anodizing tank, adjusted according to the geometry of each piece to occupy the maximum available space (Figure. 3).

Busbar Constraints: The busbar is fixed, and its position cannot be altered. Optimization must explore the tank's depth, width, and thickness dimensions.

Figure 3: Fixing devices fitted into the busbar of the chromic anodizing tank

Spatial optimization is vital for the efficiency of the treatment, allowing more pieces to be processed per cycle and minimizing environmental impact. Overloading or improper placement can compromise quality and safety, requiring experienced professionals.

3.3. Production Details

Each scenario represents a production day, with up to 40 subperiods (starts), and 56 different products recorded over two months. The company has two chromic anodizing tanks (K1 and K2), with different capacities and limitations on fixing devices (Tables 1 and 2).

Table 1: Maximum quantity of fixation devices with the item in Tank 'K1'

Quantity of fixation devices for the item in the tank	ITEMS				
4	1964A2200				
	830D0100	829D0100	1320_0101	1312_0101	4774_0002
	4775_0002	810A0400	786A0700	926_0102	3957A0600
	4778_0102	3958A0600	1386A2200	6896A0100	1556A0200
5	1559A0200	1555A0200	1553A0200	1554A0200	4781A0100
	S4_9504612	S4_9504622	918_0102	920A0100	1394_0101
	1568_0101	6884_2011	6885_2011	6886_2011	6887_2011
	6898A0100	6882_2011	6883_2011	6888_2011	962A0100
	70069_23	2434A2200	798_0184		
6	1806_041	1806_5051	1806A0200	6864_0280	1642A0400
	787A0100	1806A0800	786A0600	341_0250	60091_011
7	6774_01030	6754_0481	3470_085	6753_02011	5450A0000
	6724_0941	3470_028			
8	1567A0100	3445A1800	60078_0900	S4_9704300	6724_0271
9	831A0600	786A0900	342_0460	S4_9704411	
10	1806D0900				

Table 2: Maximum quantity of fixation devices with the item in Tank 'K2'

Quantity of fixation devices for the item in the tank	ITEMS				
1	1964A2200				
2	830D0100	829D0100	1320_0101	1312_0101	4774_0002
	4775_0002	810A0400	786A0700	926_0102	3957A0600
	4778_0102	3958A0600	1386A2200	6896A0100	1556A0200
	1559A0200	1555A0200	1553A0200	1554A0200	4781A0100
	S4_9504612	S4_9504622	918_0102	920A0100	1394_0101
	1568_0101	6884_2011	6885_2011	6886_2011	6887_2011
	6898A0100	6882_2011	6883_2011	6888_2011	962A0100
	70069_23	2434A2200	1806_041	786A0600	6864_0280
3	787A0100	1806A0800	1806_5051	1806A0200	1567A0100
	341_0250	60091_011	6774_01030	6754_0481	3470_085
	6753_02011	S4_9704411	5450A0000	6724_0941	798_0184
	S4_9704300	1642A0400	6724_0271	3470_028	
4	831A0600	1806D0900	786A0900	3445A1800	60078_0900
	342_0460				

Fr each fixing device (all identical), there is a maximum quantity of parts defined by the dimensions (Table 3).

Table 3: Maximum quantity of the item in the fixation device

Quantity of items in the fixation device	**ITEMS**				
2	1806_041	1806A0200			
3	787A0100	786A0600			
4	6864_0280				
6	1567A0100	786A0900	3445A1800	3470_085	S4_9704411
	2434A2200	3470_028	962A0100	S4_9704300	1642A0400
5	798_0184				
	786A0700	4778_0102	1554A0200	4781A0100	920A0100
8	6884_2011	6885_2011	6886_2011	6887_2011	1964A2200
	6882_2011	6883_2011	6888_2011		
10	831A0600	1806_5051			
	830D0100	829D0100	1806A0800	4774_0002	4775_0002
	810A0400	926_0102	1386A2200	6896A0100	1556A0200
12	1559A0200	1555A0200	60078_0900	1553A0200	341_0250
	60091_011	342_0460	6754_0481	6753_02011	6774_01030
	5450A0000	6898A0100	6724_0941	70069_23	6724_0271
15	918_0102				
21	S4_9504612	S4_9504622			
24	1320_0101	1312_0101	3957A0600	3958A0600	1394_0101
	1568_0101				
60	1806D0900				

Note the requirement that all items in the tank must belong to the same raw material group. As an example, in Table 4, the items 'S4_9704300' and '1642A0400' belong to the same group (B6). Therefore, in the tank, only fixation devices assembled with these specific products can exist.

3.4. Model Assumptions

To support the formulation of the proposed MILP model and ensure alignment with the real production environment, the following assumptions were established:

1. The model considers a single production day, divided into 40 subperiods (starts). All demand data are known in advance.
2. Each chromic anodizing tank (K1 and K2) has a maximum number of fixation devices, depending on geometric restrictions.
3. All devices allocated to a tank in the same period are processed simultaneously. Therefore, no sequencing is required within the tank.

4. Each fixing device contains only one type of item with standardized surface-treatment parameters.
5. Each tank can only process items from one raw material group (e.g., B1, B2, B6, LTS30) in each period. Groups are mutually exclusive.
6. For each item, there are known limits on the quantity of items per fixation device.
7. All items are considered available at the start of the planning horizon.

These assumptions were validated with the engineering and production teams of the case company and reflect operational practices within the surface-treatment division.

Table 4: Raw material group of the item

Raw material	ITEMS				
B1	1806A0200	1567A0100	786A0600	786A0700	926_0102
	S4_9504612	S4_9504622	918_0102		
B2	830D0100	810A0400	1559A0200	920A0100	6898A0100
	829D0100	787A0100	831A0600	4774_0002	4775_0002
	786A0900	4778_0102	1386A2200	6896A0100	1556A0200
	1555A0200	1553A0200	1554A0200	4781A0100	S4_9704411
	5450A0000	6884_2011	6885_2011	6886_2011	6887_2011
	1964A2200	6882_2011	6883_2011	6888_2011	2434A2200
B6	S4_9704300	1642A0400			
LTS30	1320_0101	3445A1800	342_0460	6864_0280	6724_0271
	1806_041	1806D0900	1806A0800	1312_0101	1806_5051
	3957A0600	3958A0600	60078_0900	341_0250	60091_011
	6774_01030	6754_0481	3470_085	6753_02011	1394_0101
	1568_0101	6724_0941	798_0184	962A0100	70069_23
	3470_028				

3.5. Mathematical formulation of the MILP problem

In this context, a MILP model is presented to optimize daily scheduling, considering specific constraints such as the need for products of the same alloy and anodizing process.

For the proposed model, the indices, parameters, and decision variables are:

Indices

J	Item: *1..56*
T	Production Period: *1..40* (a production day)
K	Tank: ‘*K1*’, ‘*K2*’
G	Group (type of process) by raw material: ‘*B1*’, ‘*B2*’, ‘*B6*’, ‘*LTS30*’
F	Index of the fixing device in the tank: *1..10*

Parameters

D_j	Available quantity of the item *j*
JF_j	Maximum quantity of item *j* in a device
FK_{jk}	Maximum quantity of devices with item *j* per tank *k*
GM_{jg}	Group of raw material *g* to which item *j* belongs

Decision Variables

x_{tkfj}	Quantity of product *j* in device *f* allocated in tank *k* during period *t* (positive integer variables).
y_{tkfj}	$y_{tkfj} = 1$ if product *j* is in device *f* in tank *k* during period *t*, or $y_{tkf\mathrm{j}} = 0$ otherwise. These variables play a role similar to that of setup (whether the device is prepared or not to produce that item). They are auxiliary binary variables that facilitate modeling.
w_{tkg}	$w_{tkg} = 1$ if the raw material *g* is used in tank *k* during period *t*, or $w_{tkg} =$ 0 otherwise.
L_j	Available unproduced quantity of item *j* (positive integer variables).

$$Minimize\ z = \sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{f=1}^{F}\sum_{j=1}^{J} t \cdot y_{tkfj} + M \cdot \sum_{j=1}^{I} L_j \quad (7)$$

Subject to:

$$\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{f=1}^{F} X_{tkfj} + L_j = D_j, \forall\ j \quad (8)$$

$$\sum_{f=1}^{F}\sum_{j=1}^{J} Y_{tkfj}\left(\frac{1}{FK_{jk}}\right) \leq 1, \forall\ t,k (9)$$

$$X_{tkfj} \leq IF_j * Y_{tkfj}, \forall\ t,k,f,j \quad (10)$$

$$\sum_{j=1}^{J} Y_{tkfj} \leq 1, \forall\ t,k,f \quad (11)$$

$$\sum_{g=1}^{G} W_{tkg} \leq 1, \forall\ t,k \quad (12)$$

$$Y_{tkfj} \leq \sum_{g=1}^{G} W_{tkg} GM_{jg}, \forall\ t,k,f,j \quad (13)$$

$$\sum_{j=1}^{J} Y_{tkfj} \geq \sum_{j=1}^{J} Y_{tkf+1j}, \forall\ t,k,f \quad (14)$$

$$X_{tkfj} \geq Y_{tkfj}, \forall\ t,k,f,j \quad (15)$$

The optimization of the first term in the objective function (7) aims to ensure that production occurs as early as possible within the planning horizon, thereby maximizing the utilization of tank capacity. Without this term, and assuming the tanks are capable of completing all production within a day, the solution might result in underutilized capacity, with more starts (periods) being used than necessary. The parameter 'M' is a sufficiently large number that ensures the second term, related to the unproduced quantity of each item, takes precedence over the first.

Constraints (8) governs daily production based on the availability of each item. The variable L_j represents the quantity of item *j* that remains unproduced at the end of the planning horizon.

The number of fixing devices per tank is determined by the geometries of the items, provided that all items assigned to the same tank share the same raw material alloy and anodizing process.

Constraint (9) addresses tank occupancy. Each item has a maximum allowable number of devices that can be installed in a tank. Since different items can share the same tank, the occupancy ratio for each device must be computed, and the total occupancy cannot exceed the tank's capacity.

Constraint (10) ensures that the quantity of items assigned to each device does not exceed the maximum allowed for that item and device type within the tank.

Constraint (11) stipulates that each fixing device can hold only one type of item, regardless of the quantity.

Constraints (12) ensures that only one alloy group is assigned per tank in each period.

Constraints (13) guarantees that each item assigned to a fixing device belongs to the raw material group specified for the tank in that period.

Constraint (14) organizes the utilization of fixing devices within each tank, enforcing a sequential order starting from device index 1. While not strictly required for feasibility, this constraint improves the interpretability of the model's output.

Finally, constraint (15) ensures that production occurs for all configurations where $Y_{tkfj} = 1$, meaning whenever a device is activated for a given item.

4. COMPUTATIONAL EXPERIMENTS

This section presents and analyzes the results obtained from computational experiments using the proposed MILP model, compared with the company's current scheduling practices. The optimization model was implemented in GAMS IDE version 23.6 and solved using IBM ILOG CPLEX version 12, ensuring numerical consistency across all test instances.

The use of the parallelized CPLEX solver enhances computational efficiency and reduces solution times. By leveraging multicore processing, multiple threads run simultaneously, accelerating the resolution of complex optimization problems [37].

A total ofthirty scenarios were solved, each representing one day of real operations during the months of July and August/2022. All experiments were performed on a computer equipped with an Intel i7 3.1 GHz processor, 8GB of RAM, and an solid-state drive (SSD), running Windows 10.

4.1 Results with the company’s real demand

The results (Table 5) reveal significant improvements compared to the company’s existing scheduling approach. In all scenarios, the proposed MILP model generated considerably more efficient production plans, with the total number of scheduled items nearly four times higher than those achieved by the company's manual scheduling.

It is noteworthy that no production incidents or equipment downtime occurred during the observed period that could have influenced these results.

The maximum computation time per scenario was limited to 600 seconds, within which the generated feasible solutions that scheduled the production of all available items in every instance.

As shown in Table 5, the optimality gap, defined as the difference between the best integer solution and the best relaxed solution, is primarily attributed to the first term in the objective function (7), which encourages early utilization of production periods, rather than the total number of items produced. It is important to note that computational time tends to increase with the number of items to be scheduled, as larger problem sizes lead to more complex tank-loading combinations and batching configurations. Table 5 Comparison of the MILP model performance and the company's actual scheduling across production days.

Table 5: MILP Model vs. Actual Scheduling Performance.

Day (DD/MM)	Available items	Actual Scheduling	MILP Model	GAP (%) 600seconds	Δ (%) Improvement
7/10	1,737	321	1,737	1.37	541
7/11	1,759	278	1,759	1.09	633
7/12	1,814	235	1,814	1.77	772
7/15	1,582	247	1,582	1.84	640
7/16	1,649	357	1,649	2.68	462
7/17	1,446	298	1,446	4.30	485
7/18	961	264	961	2.56	364
7/19	1,329	272	1,329	0.79	489
7/22	1,789	237	1,789	1.09	755
7/23	1,940	245	1,940	1.64	792
7/24	1,039	261	1,039	0.41	398
7/25	933	328	933	2.70	284
7/26	1,397	246	1,397	2.81	568
7/29	1,175	224	1,175	2.98	525
7/30	945	249	945	1.52	380
8/1	697	284	697	2.78	245
8/2	803	279	803	2.26	288
8/5	772	311	772	2.56	248
8/6	754	299	754	2.35	252
8/7	852	267	852	1.08	319
8/8	419	242	419	0 (172s)	173
8/9	566	229	566	2.22	247
8/12	1,065	271	1,065	1.59	393
8/13	937	290	937	2.23	323
8/14	495	310	495	0 (33s)	160
8/15	324	273	324	0 (133s)	119
8/16	323	198	323	0.75	163
8/19	480	226	480	0 (59s)	212
8/20	918	224	918	1.69	410
TOTALS	30,900	7,765	30,900	1.66	398

Analysis of the 30 daily scenarios shows that four instances with fewer than 500 items were solved in 33s – 172s, whereas all remaining runs reached the 600s cut-off. Fitting a simple linear regression between problem size (items) and elapsed solver time (n = 30, assigning 600s when the limit was hit) yields a slope of 0.29s per item and an adjusted R^2=0.88. This confirms an almost linear growth of solution time with problem size within the 10-minute planning window.

To facilitate visual interpretation, the line graph presented in Figure 4 compares the number of items scheduled by the company's current method and by the proposed MILP model across different days. This graphical representation highlights the contrast between the company's actual scheduling and the optimized schedules generated by the model. The

X-axis represents the production days (from July 10 to August 20), while the Y-axis indicates the total number of scheduled items. The blue line corresponds to the company's actual scheduling, whereas the orange line represents the scheduling results obtained with the proposed MILP model.

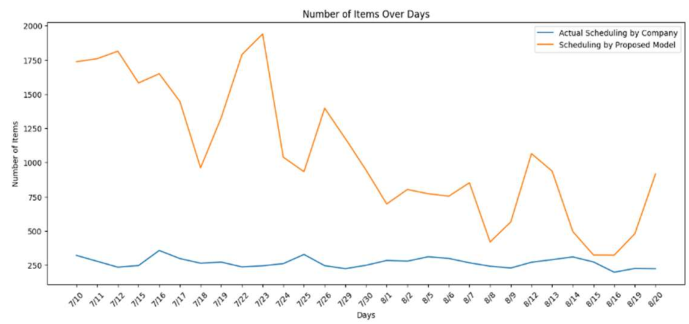

Figure 4: Number of scheduling items per day: Comparison between the MILP model and the company's current scheduling

The bar graph presented in Figure 5 provides a clear visual comparison of both the optimality gap (%) and the improvement percentage (Δ) for each production day within the analyzed period. The X-axis represents the production days (from July10 to August 20), while the Y-axis indicates the percentage values. In the graph, blue bars represent optimality gap, whereas green bars indicate the corresponding improvement percentage (Δ) achieved by the proposed MILP model compared to the company's current scheduling. This visualization facilitates an intuitive understanding of the model's performance relative to the company's existing approach illustrates how closely the model approaches the optimal solution across different days.

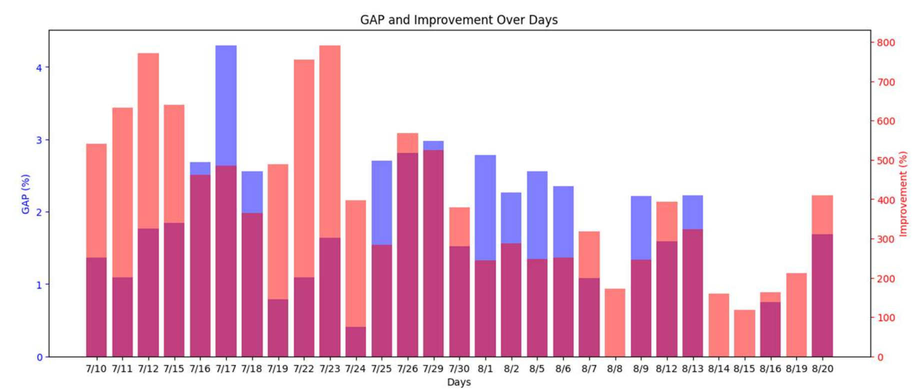

Figure 5: GAP and improvement over days

4.2. Results with accumulated demand simulation

To further evaluate the robustness and scalability of the proposed model, and to provide strategic insights for the company's management, an additional set of scenarios with accumulated demand was developed. These simulations were designed to assess how the model performs when faced with higher workloads resulting from the aggregation of demand over two consecutive months. This analysis is particularly relevant given the significant improvements observed in the baseline experiments, which could support the company's commercial decisions to onboard new clients or expand its product portfolio.

A total of thirty new scenarios were created by combining the demand of two consecutive days from the company's historical production data used in the previous experiments. Each scenario was executed twice, with maximum computation times set at 600 seconds and 3,600 seconds, respectively.

The results of each scenario, including the demand volume, number of production periods utilized, scheduled items, optimality gaps, and the impact of extended solution times, are summarized in Table 6.

Table 6: Results for simulated scenarios with accumulated demand

Scenario	Available items	Items 600s	GAP (%) 600s	Items 3,600s	GAP (%) 3,600s
01	3,496	3,136	0.28	3,136	0.27
02	3,573	3,262	4.41	3,272	1.32
03	3,396	3,180	8.04	3,194	1.90
04	3,231	3,043	11.16	3,062	1.58
05	3,095	3,005	23.54	3,023	0.60
06	2,407	2,399	64.61	2,407*	0.83
07	2,290	2,283	53.35	2,290*	0.72
08	3,118	2,938	5.11	2,943	2.31
09	3,729	3,445	13.13	3,479	1.63
10	2,979	2,950	53.63	2,963	26.99
11	1,972	1,972	0.69	1,972	0.64
12	2,330	2,314	81.89	2,330*	1.07
13	2,572	2,555	77.12	2,572*	0.96
14	2,120	2,120	1.25	2,120	1.00
15	1,493	1,493	1.73	1,493	1.60
16	1,245	1,245	2.02	1,245	1.34
17	1,500	1,500	2.70	1,500	1.49
18	1,575	1,575	1.49	1,575	1.36
19	1,526	1,526	1.98	1,526	0.13
20	1,606	1,606	3.28	1,606	0.99
21	1,271	1,271	1.10	1,271	0.94
22	985	985	2.35	985	2.13
23	1,631	1,609	91.95	1,631*	2.04
24	2,002	2,000	41.33	2,002*	0.97
25	1,432	1,432	1.51	1,432	1.05
26	819	819	3.50	819	1.75
27	647	647	1.20	647	0.38
28	803	803	2.79	803	1.79
29	1,398	1,398	1.00	1,398	0.62
30	918	918	3.54	918	0.87
TOTALS	61,159	59,429	18.72	59,614	2.04

A critical observation from the thirty tested scenarios is that when the input demand is particularly high, it becomes infeasible to schedule the production of all items due to spatial constraints in the surface-treatment area. Notably, the average optimality gap when the solver was limited to 600 seconds was 18.72%. When the maximum computational time was extended to 3,600 seconds, this gap dropped significantly to 2.04%, resulting in increased production in 13 out of the 14 scenarios where full production had previously been infeasible.

No improvement was observed in Scenario 1, indicating that the optimal solution was likely reached within the initial 600 second time limit. All other scenarios, except for Scenario 10, exhibited low optimality gaps after the extended execution, suggesting that the solver was close to or achieved optimal solutions, requiring only additional time for confirmation.

In Table 6, bold values in the fifth column indicate improvement in the number of items produced compared to the results in the third column. Asterisks (*) mark scenarios in which the model succeeded in producing the total quantity of available items when additional computational time was allocated.

In the 30 daily scenarios (Table 5) the model reached the 600s time limit; in four of those cases the solver proved optimality earlier (33s–172s), indicating that smaller instances close quickly. For the 30 accumulated-demand scenarios (Table 6), extending the limit to 3600s reduced the average optimality gap from 18.7 % to 2.0 %. Consequently, for routine daily loads the MILP provides results within ≤ 10 min, and for demand peaks a 60-min window is sufficient to obtain near-optimal solutions.

The line graph presented in Figure 6 illustrates the optimality gap (%) at two distinct computational time limits: 600 seconds (blue line) and 3,600 seconds (orange line) across all tested scenarios. This visual comparison provides a clear understanding of the model’s behavior under varying time constraints. It is evident that the optimality gap decreases substantially in most scenarios when the solver is allowed more time, highlighting the model's responsiveness to increased computational effort and its ability to approach near-optimal solutions in larger, more complex instances.

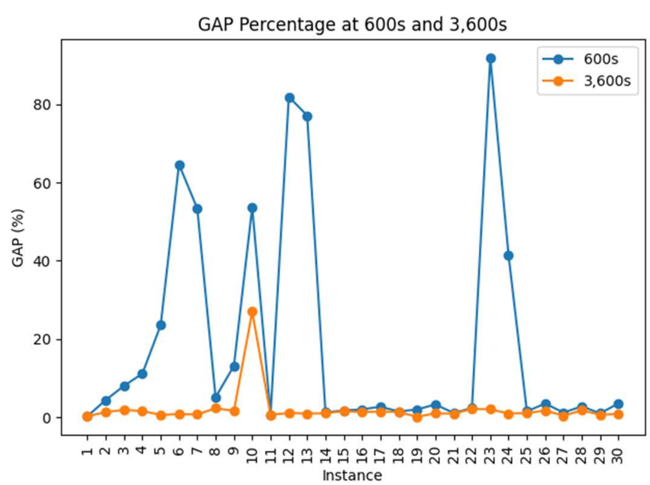

Figure 6: Optimality gap (%) at 600 and 3,600 seconds for each simulated scenario

The bar chart presented in Figure 7 compares the number of scheduled items at two distinct computational time limits, 600 seconds and 3,600 seconds, for each accumulated demand scenario. In each pair of bars representing a scenario, the blue bar indicates the number of items scheduled within 600 seconds, while the orange bar represents the number achieved with the extended time of 3,600 seconds. A visual inspection reveals that the total number of scheduled items changes only marginally between these two-time limits in most scenarios, highlighting the model's stability and the consistency of its performance across the analyzed cases.

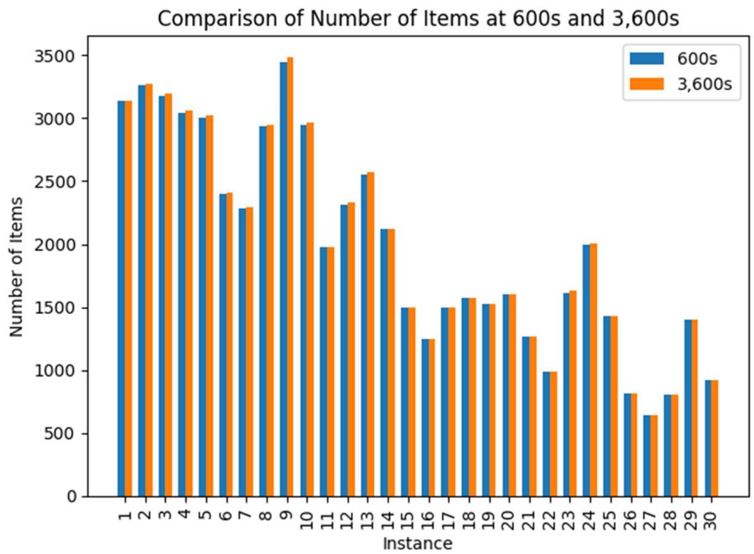

Figure 7: Comparison of the number of scheduled items at 600 and 3,600 seconds for each accumulated demand scenario

Table 7 offers a decision guideline for production schedulers: daily loads under 2,000 items are solved within the regular 10-minute window, whereas exceptionally high loads (≥ 2,000 items) should be submitted to the MILP with an overnight limit of 3,600 s to ensure gaps below 5 %.

Table 7: Solver runtime and recommended MILP configuration by problem size

Problem size (items per day or scenario)	Observed runtime (Intel i7 / 8 GB)	Typical gap	Recommended approach
≤ 500 items	≤ 3 min	0 %	MILP, 180s limit (near-instant run)
500 – 2000 items	3 – 10 min	0 – 3 %	MILP, 600s limit (daily routine)
2000 – 3 00 items	10 – 60 min	2 – 5 %	MILP, 3,600s limit (overnight run)

Although sensitivity analysis is a valuable technique for assessing model robustness under parameter uncertainty, its application was deemed unnecessary in this study due to the deterministic nature of the production environment under investigation. The entire production process operates on a strict Engineer-to-Order (ETO) basis, where all demand is confirmed in advance through customer orders, and no production occurs without a validated request.

The MILP model formulation incorporates all relevant operational constraints, including tank capacity, fixture-device limitations, geometric fit, and alloy compatibility. These are not subject to daily variation and reflect standardized practices validated with the company's engineering team. As such, uncertainty in parameter estimates is minimal to nonexistent within the model's operational scope.

5. CONCLUSION

This research addressed a critical operational challenge faced by companies in the aerospace supply chain, particularly within the electroplating sector, which was identified as a major bottleneck in the company under study. To overcome this challenge, a MILP model was developed to optimize production scheduling in the chromic anodizing process.

The results fully achieved both the general and specific research objectives by determining the optimal combination of products and quantities per tank, while respecting all technical and operational constraints, with the overarching goal of maximizing productivity.

When applied to real operational data, the proposed MILP model consistently outperformed the company's existing scheduling practices. In several scenarios, it enabled the complete scheduling of all available items for production, demonstrating substantial improvements in efficiency.

Beyond improving operational outcomes, the model also offers the potential to automate the scheduling process, reducing the need for manual intervention by production managers. This not only enhances the reliability and consistency of the scheduling process but also allows managers to focus on higher-value tasks.

Additionally, the simulation results revealed that the current anodizing capacity is sufficient to accommodate additional business demands, such as onboarding new clients or expanding the product portfolio. This available capacity has the potential to support cost reduction strategies and contribute positively to the company's financial performance.

In summary, the findings of this study provide both practical and managerial insights that can support decision-making in the surface treatment area. Furthermore, the modeling approach presented here can be extended to other operational areas within the company or adapted to similar industries facing complex production scheduling challenges. The demonstrated success of the model suggests that optimization-based solutions hold significant promise for improving operational efficiency across various organizational contexts.

Building on the present findings, three avenues merit further investigation: (i) robust optimization to explicitly hedge against demand volatility, tank-capacity deviations, and fixture-geometry tolerances; (ii) hybrid meta-heuristics, combining GRASP or genetic algorithms with MILP-based large-neighborhood search, to handle instances exceeding 5,000 items or multi-day horizons with real-time responsiveness; and (iii) integration with cyber-physical production systems, coupling the MILP scheduler with digital twins and IoT sensors to enable 15-minute rescheduling cycles and to incorporate additional objectives such as energy consumption and chemical waste reduction. These extensions would enhance the model's applicability in highly dynamic or sustainability-driven manufacturing environments.

5.1. Data Availability

The datasets generated and analyzed during the current study are publicly available in the Mendeley Data repository under the title "Dataset for Scheduling and Capacity Planning in Aerospace Surface Treatment: Real and Simulated Scenarios", DOI: https://doi.org/10.17632/p7g96yvvgb.1 . The repository includes both real production data (30 daily scenarios) and simulated scenarios generated by aggregating the demand from two consecutive production days (30 accumulated-demand scenarios). Additionally, the repository contains text files for each scenario and a metadata file detailing the constraints and characteristics of each item.

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