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Research Article

A NOVEL PYTHAGOREAN FUZZY DISTANCE MEASURE FOR MULTICRITERIA DECISION MAKING IN RENEWABLE ENERGY RESOURCE SELECTION

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Abstract: Selecting the most suitable renewable energy resource is vital for ensuring long-term sustainability, reducing environmental impact, and decreasing dependence on conventional fossil fuels. However, this selection process involves managing complex, uncertain, and imprecise information arising from diverse evaluation criteria. Pythagorean fuzzy sets (PFSs), combined with distance measures, offer a more expressive and flexible framework than traditional fuzzy or intuitionistic fuzzy sets for modeling such uncertainty, particularly in multicriteria decision-making (MCDM) problems. However, existing distance measures for PFSs often suffer from limitations such as zero-divisor problems, counterintuitive outcomes, and violations of axiomatic conditions. To address these issues, this study introduces a novel distance measure for PFSs, whose mathematical properties are thoroughly analyzed to ensure consistency, boundedness, and interpretability. A hybrid Pythagorean fuzzy- stepwise weight assessment ratio analysis–technique for order preference by similarity to ideal solution (PF-SWARA–TOPSIS) framework is proposed, where the SWARA method determines the importance weights of the criteria, and the TOPSIS method ranks the alternatives using the proposed distance measure. The

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approach is applied to evaluate five renewable energy resources based on multiple criteria. The results indicate that solar energy (alternative D_1) is the most preferred option, followed by the ranking order: $D_1 > D_3 > D_4 > D_2 > D_5$. Comparative analysis with existing methods validates the robustness and effectiveness of the proposed model, offering a reliable tool for sustainable energy planning under uncertainty. Finally, the study highlights the future potential of the proposed distance measure and methodology, providing insights for further advancements in renewable energy resource selection.

Keywords: Pythagorean fuzzy sets, distance measure, renewable energy resource, MCDM, hybrid PF-SWARA-TOPSIS approaches..

MSC: 03E72, 90C70, 90B50.

1. INTRODUCTION

Non-renewable energy sources such as nuclear power, natural gas, oil, and coal are derived from finite natural reserves. These resources are gradually depleting and are expected to become inadequate to meet future global energy demands. In addition, the use of nonrenewable energy significantly contributes to various environmental issues, including pollution, greenhouse gas emissions, and ecological degradation. To address these challenges, researchers are increasingly focusing on the development of alternative energy solutions. In this context, renewable energy sources such as solar and wind power are gaining considerable attention due to their sustainability and availability. Unlike non-renewable sources, renewable energy systems have minimal environmental impact and can be naturally replenished, offering a long term, environmentally friendly solution to the world's growing energy demands [1]. Various forms of energy are derived from natural sources such as geothermal heat, wind, solar radiation, biomass, and hydropower. Although these energy sources offer numerous advantages, they also present certain limitations. One major challenge is the substantial initial investment required for their installation, which can result in significant financial burdens. Additionally, the availability of these resources is often subject to seasonal variations, making consistent energy supply more difficult. Therefore, it is essential to carefully evaluate these factors when selecting the most appropriate renewable energy source. The issue of energy resources holds critical importance for many nations and for humanity as a whole. In recent years, a growing number of countries have shifted toward renewable energy as a primary alternative to overcome the limitations of finite fossil fuel reserves and address escalating environmental concerns, particularly those related to the reduction of carbon dioxide and other greenhouse gas emissions, as outlined in the Kyoto Protocol of 1997 under the united nations framework convention on climate change (UNFCCC). Renewable energy resources (RERs) are widely recognized as environmentally friendly options that help mitigate greenhouse gas emissions, reduce secondary waste, and promote sustainability in line with current and future social and economic needs [2]. These resources, also referred to as alternative energy sources, harness natural phenomena such as solar radiation, biomass, geothermal activity, wind, hydropower, ocean energy, and fuel cells. However, the renewable energy sector continues to evolve, influenced by advancements in technology, policy reforms, and market dynamics. This ongoing uncertainty regarding future developments can make decision makers hesitant to commit to a specific renewable energy resource. To address the challenges arising from uncertainty and imprecision in real-world problems, Zadeh [3] introduced fuzzy set theory, which provides a mathematical framework for modeling vagueness by assigning to each element a membership degree (MD) in the interval $[0,1]$. As an extension of this concept, Atanassov [4] proposed intuitionistic fuzzy sets (IFSs), wherein each element is associated with a MD and a nonmembership degree (NMD), both taking values in $[0,1]$, with the constraint that $\mathrm{MD}+\mathrm{NMD} \leq 1$. Although IFSs offer an improved structure over classical fuzzy sets, they may be inadequate in decision making scenarios where expert evaluations assign, for instance, 0.8 as the MD and 0.5 as the NMD, resulting in a sum greater than one, thereby violating the fundamental constraint of IFSs. To address this limitation, Yager [5] introduced Pythagorean fuzzy sets (PFSs), which generalize IFSs by replacing the linear constraint with a quadratic one, requiring that $\sigma^2+\delta^2 \leq 1$, where σ and δ denote the MD and NMD, respectively. This generalization provides greater flexibility in representing imprecise and conflicting information. Consequently, PFSs have been employed in various domains, particularly in multi-criteria decision making, as a more robust mathematical framework for capturing real-world uncertainty, especially when integrated with distance and similarity measures.

The remainder of the paper is organized as follows: Section 2 explores the fundamentals of PFSs and provides a comprehensive review of existing distance measures and MCDM techniques developed within the PFSs framework. The limitations and drawbacks of these existing measures and techniques are critically discussed in Section 3. Section 4 outlines the motivation and clearly defines the objectives of the current study. In Section 5, essential mathematical definitions and properties related to PFSs are discussed. Section 6 extends the existing distance measures by presenting general mathematical formulations, while Section 7 introduces a novel distance measure specifically designed for PFSs. A comparative analysis between the proposed and existing distance measures using various numerical examples is carried out in Section 8. Section 9 presents a hybrid decision-making methodology based on PFSs. In Section 10, the proposed methodology is applied to a real-life decision-making problem, and Section 11 compares the results obtained with those from existing approaches. Finally, Section 12 summarizes the key findings of the study and discusses future research directions and potential challenges.

2. LITERATURE REVIEW

2.1. Existing distance measure for PFSs

Distance measures (DMs) served as invaluable tools for quantifying the dissimilarities between two objects. Zhang and Xu [6] introduced a DM specifically tailored for PFS, utilizing three parameters: MD, NMD, and hesitation degree (HD). Subsequently, Peng et al. [7] provided an axiomatic definition of DM, comprising five axioms, and standardized the earlier Zhang DM. Despite their utility, existing DMs only accounted for three parameters, neglecting the directional influence of Pythagorean fuzzy numbers. To address this limitation, Li and Zeng [8] expanded the DM formulation to include four parameters, aiming to mitigate such oversights. Building upon this work, Zeng et al. [9] further extended the DM by incorporating five parameters, establishing it as a more robust analytical tool compared to its predecessors. However, Peng [10] identified deficiencies in existing Pythagorean fuzzy distance measures (PFDMs), particularly their failure to adhere to the third and fourth axioms, inability to discern between positive and negative differences, and susceptibility to division by zero issues. In response, the author proposed a novel DM for PFS, addressing these shortcomings and yielding dependable outcomes. Furthermore, Hussein and Yang [11] delineated a PFS DM based on the Hausdorff metric, leveraging it within the TOPSIS framework for resolving decision-making challenges. Ejegwa and Awolola [12] observed that current DMs needed to yield dependable and rational outcomes. Consequently, the authors introduced novel DMs tailored for PFS, integrated traditional parameters, and applied them to pattern recognition and decision-making in real-world scenarios characterized by uncertainty. Additionally, Ejegwa [13] scrutinized the existing DM proposed by Zhang and Xu [6], noting its failure to meet metric properties. To address this deficiency, the author devised a new DM that complied with all metric conditions. Further, He and Xiao [14] addressed that some existing methods did not accurately reveal the difference between PFSs and did not satisfy the metric property. To address this issue, the authors defined a new DM for PFSs based on a matrix that met all the circumstances of metric conditions and applied it to medical diagnosis. Mahanta and Panda [15] analyzed that most of the existing distances were generalizations of IFSs DM; few distances were not normalized, and most of the DMs were unable to distinguish high uncertainty for PFSs. To address these aspects, they introduced a novel DM for PFSs in a simple mathematical form. Then, for the applicability of DM, they applied it in pattern recognition and decision-making sectors for selecting face masks. Additionally, He and Xiao [14] pointed out that certain current methods failed to accurately differentiate between PFSs and lacked fulfillment of the metric property. To address this issue, the authors proposed a new distance measure for PFSs, grounded on a matrix that adhered to all metric conditions, and applied it to medical diagnosis.

2.2. Existing MCDM techniques for RERs

Recent methodologies have emerged to utilize RERs effectively, with researchers advocating MCDM processes to optimize their selection and deployment. Decision support tools play a vital role in addressing complex problems involving uncertainty and conflicting criteria. As countries shift to renewables to reduce dependence on fossil fuels, the selection of appropriate energy alternatives becomes increasingly important. In the Indian context, Rani et al. [16] applied a fuzzy TOPSIS approach to evaluate RERs, while Sen et al. [17] highlighted the role of renewables in energy security and emission reduction, emphasizing collaboration among stakeholders. Dwivedi et al. [18] employed the Entropy-WASPAS (Weighted Aggregated Sum Product Assessment) method to identify suitable renewable sources, and Mukherjee et al. [19] examined strategic implementation from technical and social perspectives. Li et al. [20] and Shah et al. [21] analyzed factors influencing RER selection globally. Suvitha et al. [22] developed a hybrid MCDM framework using triangular fuzzy neutrosophic numbers and the MABAC (multi-attributive border approximation area comparison) method to assess hydropower sustainability, identifying pumped storage as the most stable option. Manirathinam et al. [23] used a Fermatean neutrosophic fuzzy approach to evaluate smart home energy systems, identifying hybrid electric systems as the most viable. Geetha et al. [24] proposed

HPF-ELECTRE III (ELimination and Choice Expressing REality), extending ELECTRE III with hesitant Pythagorean fuzzy sets for plastic recycling decisions. Later, Geetha et al. [25] developed an integrated hybrid model to evaluate RER alternatives based on intercriteria relationships and economic, environmental, and technological factors. Suvitha et al. [26] introduced a hybrid AHP (Analytic Hierarchy Process)-CODAS (Combinative Distance-Based Assessment) method under the Fermatean probabilistic hesitant fuzzy set (FPHFS) framework to assess plastic waste collection systems, highlighting the effectiveness of deposit-refund mechanisms. Sandra et al. [27] used T-Spherical Hesitant Fuzzy Rough sets to identify optimal geothermal drilling techniques, revealing directional drilling as the most sustainable. Suvitha et al. [28] combined Pythagorean probabilistic hesitancy fuzzy sets with SPA (Set-Pair Analysis) to assess biomass energy sources. Their PRSRV (Projection Ranking by Similarity to Referencing Vector) - SECA (Simultaneous Evaluation of Criteria and Alternatives) method ranked animal residues as the most sustainable for bioenergy production. Kang et al. [29] proposed a stratified fuzzy MCDM framework combining improved intuitionistic AHP with WASPAS to evaluate waste-to-energy technologies, identifying plasma technology as the most feasible option. Clearly defined criteria are essential for constructing effective decision models.

However, assigning appropriate weights to these criteria remains a significant challenge, particularly when the weights are uncertain or subjective. To address this, Kersuliene et al. [30] demonstrated the effectiveness of the SWARA method for determining subjective weights, supported by Chen [31] and Stevic et al. [32]. Integrated approaches such as SWARA-ARAS [33], SWARA-COPRAS (Complex Proportional Assessment) [34], and hesitant fuzzy SWARA-MULTIMOORA ((Multi-Objective Optimization on the basis of Ratio Analysis) [35] have been proposed for healthcare waste, supplier evaluation, and online education, respectively. Peng et al. [36] applied CRITIC (CRiteria Importance Through Intercriteria Correlation) - COCOSO (Combined Compromise Solution) for evaluating the 5G industry. Other weight-determination techniques are also discussed by Hezam et al. [37] and Rani et al. [38]. Among various ranking methods, the TOPSIS, introduced by Hwang and Yoon [39], remains a widely used tool due to its simplicity and effectiveness in evaluating alternatives based on proximity to the ideal solution. TOPSIS has been applied in diverse contexts such as domestic airline service evaluation [6], energy project selection [40], smartphone procurement [41], and property acquisition [42], confirming its broad applicability in MCDM problems.

3. LIMITATIONS AND DRAWBACKS OF EXISTING DISTANCE MEASURES AND METHODOLOGIES

During the literature review, it was observed that the primary objective of many existing DMs is to distinguish between two PFSs. However, several errors and shortcomings were identified in these definitions. The common limitations of current methods include:

- Some existing distances fail to satisfy the axioms of a DM.
- Many existing DMs produce counterintuitive or illogical results.
- Several DMs are merely generalizations of IFS-based distances.
- Existing DMs often struggle to effectively differentiate between PFSs with high uncertainty.

In addition, while analyzing methodologies for MCDM, it became apparent that certain approaches have been applied without incorporating expert opinions or assigning weights to criteria. The lack of expert involvement and proper weighting may lead to biased or incomplete decision outcomes, thereby compromising the reliability and precision of the MCDM process. Dependence on fixed methodologies without expert consultation can also exclude critical domain insights necessary for sound decision-making. Furthermore, some ranking techniques used in MCDM display significant limitations when dealing with high levels of uncertainty. These methods may be unable to clearly distinguish between alternatives with closely similar or uncertain evaluations, potentially leading to inaccurate or misleading rankings. As a result, decision-makers may face difficulties in correctly prioritizing options, which can adversely affect the quality and robustness of the final decisions.

4. MOTIVATION AND OBJECTIVES

The transition to renewable energy has become imperative due to escalating environmental concerns, the depletion of fossil fuel reserves, and the growing global demand for clean and sustainable energy. Identifying the most suitable renewable energy source is critical for national energy planning, minimizing carbon emissions, and ensuring long-term energy security. However, this selection process involves evaluating multiple, often conflicting, criteria such as cost, efficiency, environmental impact, and availability, which are typically uncertain or imprecise. Therefore, a robust MCDM framework capable of effectively handling such uncertainty is essential for supporting informed and sustainable decision-making. During the literature review, it was observed that several existing distance functions are extensions of previously established ones. However, many of these suffer from issues such as failing to satisfy the boundedness condition of a valid distance measure, leading to unreasonable or counterintuitive results when applied to different PFSs. This highlighted the need for improvements in the design of distance functions (see Table 2 and Figure 1).

To address this issue, the present study proposes a novel distance measure for PFSs. The key contributions of the paper are summarized as follows:

- Introduction of a novel distance measure tailored for Pythagorean fuzzy sets.
- Discussion of the mathematical properties inherent in the proposed distance function.
- Conducting a comparative study through numerical examples to demonstrate the superiority and validity of the proposed measure.
- Development of hybrid methodologies integrating PF-SWARA-TOPSIS using the proposed distance measure for PFSs.
- Determination of criteria weights using the SWARA technique and ranking of alternatives through the TOPSIS technique.
- Application of the proposed methodology to a real-life scenario to identify the best renewable energy resource.
- Comparative analysis of existing methodologies versus the proposed methodology to validate its effectiveness.

5. PRELIMINARIES

We reaffirm the concept of PFSs. We consider S to be the universe of discourse in the work.

Definition 1. *Consider the structure*

$$\mathbb{N} = \{\langle s_i, \sigma_{\mathbb{N}}(s_i), \delta_{\mathbb{N}}(s_i)\rangle | s_i \in S\},$$

under the circumstances, $\sigma_{\mathbb{N}}, \delta_{\mathbb{N}} : S \to [0,1]$ *signify* MD *and* NMD *of* $s_i \in S$.

1. $\mathbb{N}$ *is called an IFS in S if* $\sigma_{\mathbb{N}}(s_i) + \delta_{\mathbb{N}}(s_i) \in [0,1]$, *and* $\eta_{\mathbb{N}}(s_i) = 1 - \sigma_{\mathbb{N}}(s_i) - \delta_{\mathbb{N}}(s_i)$ *is the hesitation margin of* $\mathbb{N}$ *[43].*

2. $\mathbb{N}$ *is called an PFS in S if* $\sigma^2_{\mathbb{N}}(s_i) + \delta^2_{\mathbb{N}}(s_i) \in [0,1]$, *and* $\eta_{\mathbb{N}}(s_i) = \sqrt{1 - \sigma^2_{\mathbb{N}}(s_i) - \delta^2_{\mathbb{N}}(s_i)}$ *is the hesitation margin of* $\mathbb{N}$ *[5].*

PFS $\mathbb{N}$ can also be represented by $\mathbb{N} = \langle \sigma_{\mathbb{N}}(s_i), \delta_{\mathbb{N}}(s_i)\rangle$, called the Pythagorean fuzzy number (PFN).

Definition 2. *[6] Suppose* $\eta = \mathbb{Y}\langle \sigma_\eta, \delta_\eta\rangle$ *to be a PFN. The score function of* η *are described as*

$$\mathbb{S}(\eta) = \sigma^2_\eta - \delta^2_\eta \quad \textit{where} \quad \mathbb{S}(\eta) \in [-1,1].$$

Since $\mathbb{S}(\eta) \in [-1,1]$, *Thus, PFN score function has been modified as*

$$\mathbb{S}^\star(\eta) = \frac{1}{2}\big((\sigma^2_\eta - \delta^2_\eta) + 1\big), \quad \textit{where} \quad \mathbb{S}^\star(\eta) \in [0,1]. \tag{1}$$

Definition 3. *[5] Assume that* $\mathbb{N}$, $\mathbb{N}_1$ *and* $\mathbb{N}_2$ *are PFSs in* S *. Then,*

1. *Equality:* $\mathbb{N}_1 = \mathbb{N}_2$ *iff* $\sigma_{\mathbb{N}_1}(s_i) = \sigma_{\mathbb{N}_2}(s_i)$ *and* $\delta_{\mathbb{N}_1}(s_i) = \delta_{\mathbb{N}_2}(s_i)$, $\forall s_i \in S$.
2. *Inclusion:* $\mathbb{N}_1 \subseteq \mathbb{N}_2$ *iff* $\sigma_{\mathbb{N}_1}(s_i) \le \sigma_{\mathbb{N}_2}(s_i)$ *and* $\delta_{\mathbb{N}_1}(s_i) \ge \delta_{\mathbb{N}_2}(s_i)$, $\forall s_i \in S$.
3. *Complement:* $\overline{\mathbb{N}} = \{s_i, \delta_{\mathbb{N}}(s_i), \sigma_{\mathbb{N}}(s_i) | s_i \in S\}$.
4. *Union:* $\mathbb{N}_1 \cup \mathbb{N}_2 = \{\langle s_i, \max\{\sigma_{\mathbb{N}_1}(s_i), \sigma_{\mathbb{N}_2}(s_i)\}, \min\{\delta_{\mathbb{N}_1}(s_i), \delta_{\mathbb{N}_2}(s_i)\rangle | s_i \in S\}$.
5. *Intersection:* $\mathbb{N}_1 \cap \mathbb{N}_2 = \{\langle s_i, \min\{\sigma_{\mathbb{N}_1}(s_i), \sigma_{\mathbb{N}_2}(s_i)\}, \max\{\delta_{\mathbb{N}_1}(s_i), \delta_{\mathbb{N}_2}(s_i)\rangle | s_i \in S\}$.

Definition 4. *[12] If* $\mathbb{N}$, $\mathbb{N}_1$, *and* $\mathbb{N}_2$ *are PFSs in S, then the distance measure between PFSs is denoted by* $\Gamma(\mathbb{N}_1, \mathbb{N}_2)$, *where* $\Gamma : PFS \times PFS \to [0,1]$, *satisfying:*

1. $0 \le \Gamma(\mathbb{N}_1, \mathbb{N}_2) \le 1$.
2. $\Gamma(\mathbb{N}_1, \mathbb{N}_2) = 0 \Leftrightarrow \mathbb{N}_1 = \mathbb{N}_2$.
3. $\Gamma(\mathbb{N}_1, \mathbb{N}_2) = \Gamma(\mathbb{N}_2, \mathbb{N}_1)$.
4. $\Gamma(\mathbb{N}_1, \mathbb{N}) \le \Gamma(\mathbb{N}_1, \mathbb{N}_2) + \Gamma(\mathbb{N}_2, \mathbb{N})$.

Table 1 elucidates the behavior of the distance function.

Table 1: Behavior of $\Gamma(\mathbb{N}_1,\mathbb{N}_2)$

Behavior	Explanations
$\Gamma(\mathbb{N}_1,\mathbb{N}_2)=0$	$\mathbb{N}_1$ and $\mathbb{N}_1$ are perfectly identical.
$\Gamma(\mathbb{N}_1,\mathbb{N}_2)=1$	$\mathbb{N}_1$ and $\mathbb{N}_2$ have a perfect dissimilarity.
$\Gamma(\mathbb{N}_1,\mathbb{N}_2)\approx 0$	$\mathbb{N}_1$ and $\mathbb{N}_2$ are nearly identical.
$\Gamma(\mathbb{N}_1,\mathbb{N}_2)\approx 1$	$\mathbb{N}_1$ and $\mathbb{N}_2$ are nearly maximally dissimilar.

6. SOME EXTANT DISTANCE MEASURES FOR PYTHAGOREAN FUZZY SETS

In real-world scenarios, distinguishing between elements, objects, and concepts can be challenging. Distance measures play a crucial role in delineating such entities. This literature review explores some of the prominent distance functions, focusing on those dependent on a parameter λ, along with other relevant distance metrics. Let's consider that we have two PFSs

$$\mathbb{N}_1=\{\langle s_i,\sigma_{\mathbb{N}_1}(s_i),\delta_{\mathbb{N}_1}(s_i),\pi_{\mathbb{N}_1}(s_i)\rangle|s_i\in S\},$$
$$\mathbb{N}_2=\{\langle s_i,\sigma_{\mathbb{N}_2}(s_i),\delta_{\mathbb{N}_2}(s_i),\pi_{\mathbb{N}_2}(s_i)\rangle|s_i\in S\}.$$

for $S=\{s_1,s_2,....,s_k\}$.

1. Qin et al. [44] underscore the significance of PFSs and propose novel distance measures within this framework. These measures find applications in service quality among domestic airlines in MCDM contexts.

$$\Gamma_Q(\mathbb{N}_1,\mathbb{N}_2)=\frac{1}{2}\sum_{i=1}^{k}\Big(\big|\sigma^2_{\mathbb{N}_1}(s_i)-\sigma^2_{\mathbb{N}_2}(s_i)\big|^\lambda+\big|\delta^2_{\mathbb{N}_1}(s_i)-\delta^2_{\mathbb{N}_2}(s_i)\big|^\lambda + \big|\pi^2_{\mathbb{N}_1}(s_i)-\pi^2_{\mathbb{N}_2}(s_i)\big|^\lambda\Big) \quad \text{for } \lambda\geq 1.$$

2. Chen [45] introduces innovative remoteness index-based Pythagorean fuzzy VIKOR methods, departing from conventional techniques. By leveraging Pythagorean fuzzy values, a generalized distance measure and remoteness indices are devised, enhancing the handling of uncertain information.

$$\Gamma_C(\mathbb{N}_1,\mathbb{N}_2)=\Big[\frac{1}{2}\Big(\mid\sigma^2_{\mathbb{N}_1}(s_i)-\sigma^2_{\mathbb{N}_2}(s_i)\mid^\lambda+\mid\delta^2_{\mathbb{N}_1}(s_i)-\delta^2_{\mathbb{N}_2}(s_i)\mid^\lambda + \mid\pi^2_{\mathbb{N}_1}(s_i)-\pi^2_{\mathbb{N}_2}(s_i)\mid^\lambda\Big)\Big]^{\frac{1}{\lambda}},\lambda\geq 1.$$

3. Hussian and Yang [11] propose novel distance and similarity measures for PFSs based on the Hausdorff metric. These measures are validated in pattern recognition and linguistic variables and prove effective in MCDM scenarios.

$$\Gamma_{HY}(\mathbb{N}_1,\mathbb{N}_2)=\frac{1}{k}\sum_{i=1}^{k}\max\Big[\mid\sigma^2_{\mathbb{N}_1}(s_i)-\sigma^2_{\mathbb{N}_2}(s_i)\mid,\mid\delta^2_{\mathbb{N}_1}(s_i)-\delta^2_{\mathbb{N}_2}(s_i)\mid\Big].$$

4. Biswas and Sarkar [46] introduce a novel approach for MCDM utilizing a new distance measure based on PFSs. Their method modifies the AHP to calculate both subjective and objective weights of criteria simultaneously, as demonstrated in a case study on selecting a transportation company.

$$\Gamma_{SB}(\mathbb{N}_1,\mathbb{N}_2)=\Bigg[\frac{1}{4k}\sum_{i=1}^{k}\Big(\big|\sigma^2_{\mathbb{N}_1}(s_i)-\sigma^2_{\mathbb{N}_2}(s_i)\big|^{\lambda}+\big|\delta^2_{\mathbb{N}_1}(s_i)-\delta^2_{\mathbb{N}_2}(s_i)\big|^{\lambda} + \big|\pi^2_{\mathbb{N}_1}(s_i)-\pi^2_{\mathbb{N}_2}(s_i)\big|^{\lambda}+\big|\max\big(\sigma^2_{\mathbb{N}_1}(s_i),\delta^2_{\mathbb{N}_1}(s_i)\big) - \max\big(\sigma^2_{\mathbb{N}_2}(s_i),\delta^2_{\mathbb{N}_2}(s_i)\big)^{\lambda}\Big)\Bigg]^{\frac{1}{\lambda}},\quad \lambda\geq 1.$$

5. Anshu [47] presents a novel exponential function-based distance measure for PFSs, extending it to weighted measures and comparing it with existing methods. A decision-making approach utilizing this measure is also introduced and validated through numerical examples.

$$\Gamma_{A}(\mathbb{N}_1,\mathbb{N}_2)=\sum_{i=1}^{k}\Bigg[2-\left(1-\frac{(\sigma^2_{\mathbb{N}_1}(s_i)-\sigma^2_{\mathbb{N}_2}(s_i))-(\delta^2_{\mathbb{N}_1}(s_i)-\delta^2_{\mathbb{N}_2}(s_i))}{2}\right)e^{\frac{(\sigma^2_{\mathbb{N}_1}(s_i)-\sigma^2_{\mathbb{N}_2}(s_i))-(\delta^2_{\mathbb{N}_1}(s_i)-\delta^2_{\mathbb{N}_2}(s_i))}{2}} - \left(1+\frac{(\sigma^2_{\mathbb{N}_1}(s_i)-\sigma^2_{\mathbb{N}_2}(s_i))-(\delta^2_{\mathbb{N}_1}(s_i)-\delta^2_{\mathbb{N}_2}(s_i))}{2}\right)e^{\frac{(\delta^2_{\mathbb{N}_1}(s_i)-\delta^2_{\mathbb{N}_2}(s_i))-(\sigma^2_{\mathbb{N}_1}(s_i)-\sigma^2_{\mathbb{N}_2}(s_i))}{2}}\Bigg].$$

6. Khan et al. [48] address the limitations of the VIKOR method for PFSs in MCDM by introducing a new dissimilarity measure and refining the VIKOR method accordingly, aiming to achieve closer approximation to the positive ideal solution. $\Gamma_{K}(\mathbb{N}_1,\mathbb{N}_2)=\Big(\frac{1}{2}\Big(\mid\sigma^2_{\mathbb{N}_1}(s_i)-\sigma^2_{\mathbb{N}_2}(s_i)\mid^{\lambda}+\mid\delta^2_{\mathbb{N}_1}(s_i)-\delta^2_{\mathbb{N}_2}(s_i)\mid^{\lambda}\Big)\times(1-\frac{1}{2})\mid\pi^2_{\mathbb{N}_1}(s_i)-\pi^2_{\mathbb{N}_2}(s_i)\mid^{\lambda}\Big)^{\frac{1}{\lambda}}$, $\lambda\geq 1$.

7. Dutta et al. [49] introduce innovative nonlinear distance measures for PFSs, addressing limitations of linear measures and demonstrating their effectiveness through comprehensive analysis and practical applications, including medicine selection for COVID-19 transmission rate reduction.

$$\Gamma_{D}(\mathbb{N}_1,\mathbb{N}_2)=\Big[\frac{1}{2^{(1-\frac{\lambda}{2})}.k}\sum_{i=1}^{k}\Big(\frac{\mid\mid\sigma_{\mathbb{N}_1}(s_i)-\sigma_{\mathbb{N}_2}(s_i)\mid\mid}{\sqrt{1+\mid\mid\sigma_{\mathbb{N}_1}(s_i)\mid\mid^2}.\sqrt{1+\mid\mid\sigma_{\mathbb{N}_2}(s_i)\mid\mid^2}}\Big)^{\lambda} + \Big(\frac{\mid\mid\delta_{\mathbb{N}_1}(s_i)-\delta_{\mathbb{N}_2}(s_i)\mid\mid}{\sqrt{1+\mid\mid\delta_{\mathbb{N}_1}(s_i)\mid\mid^2}.\sqrt{1+\mid\mid\delta_{\mathbb{N}_2}(s_i)\mid\mid^2}}\Big)^{\lambda}\Big]^{\frac{1}{\lambda}},\quad \lambda\geq 1.$$

7. A NOVEL DISTANCE MEASURE FOR PFSs

This section introduces a novel distance measure tailored for comparing PFS. Traditional distance measures often need to adequately capture the nuances of PFS, especially when considering membership and non-membership values. Our proposed approach addresses this limitation by incorporating the modulus difference of membership and non-membership values while also considering the maximum disparity between the two sets.

Definition 5. *Given we have two PFSs*

$$\mathbb{N}_1 = \{\langle s_i, \sigma_{\mathbb{N}_1}(s_i), \delta_{\mathbb{N}_1}(s_i)\rangle | s_i \in S\},$$
$$\mathbb{N}_2 = \{\langle s_i, \sigma_{\mathbb{N}_2}(s_i), \delta_{\mathbb{N}_2}(s_i)\rangle | s_i \in S\}$$

for feature space $S = \{s_1, s_2, ..., s_k\}$*, we define the new distance function for* $\mathbb{N}_1$ *and* $\mathbb{N}_2$ *as follows:*

$$\Gamma(\mathbb{N}_1, \mathbb{N}_2) = \frac{1}{2k} \sum_{i=1}^{k} \frac{|\sigma^2_{\mathbb{N}_1}(s_i) - \sigma^2_{\mathbb{N}_2}(s_i)| + |\delta^2_{\mathbb{N}_1}(s_i) - \delta^2_{\mathbb{N}_2}(s_i)|}{\max\left\{|\sigma^2_{\mathbb{N}_1}(s_i) - \sigma^2_{\mathbb{N}_2}(s_i)|, |\delta^2_{\mathbb{N}_1}(s_i) - \delta^2_{\mathbb{N}_2}(s_i)|\right\}} \tag{2}$$

Theorem 6. *The distance measure* $\Gamma(\mathbb{N}_1, \mathbb{N}_2)$ *is satisfies the axioms of the distance property outlined in Definition* 4*.*

Proof. 1. Let we know that $0 \le |\sigma^2_{\mathbb{N}_1}(s_i) - \sigma^2_{\mathbb{N}_2}(s_i)| \le 1$, $0 \le |\delta^2_{\mathbb{N}_1}(s_i) - \delta^2_{\mathbb{N}_2}(s_i)| \le 1$.

$$0 \le \max\left\{|\sigma^2_{\mathbb{N}_1}(s_i) - \sigma^2_{\mathbb{N}_2}(s_i)|, |\delta^2_{\mathbb{N}_1}(s_i) - \delta^2_{\mathbb{N}_2}(s_i)|\right\} \le 1 \tag{3}$$

$$0 \le |\sigma^2_{\mathbb{N}_1}(s_i) - \sigma^2_{\mathbb{N}_2}(s_i)| + |\delta^2_{\mathbb{N}_1}(s_i) - \delta^2_{\mathbb{N}_2}(s_i)| \le 2 \tag{4}$$

Equation (4) divide by $\max\left\{|\sigma^2_{\mathbb{N}_1}(s_i) - \sigma^2_{\mathbb{N}_2}(s_i)|, |\delta^2_{\mathbb{N}_1}(s_i) - \delta^2_{\mathbb{N}_2}(s_i)|\right\}$, so we get

$$\begin{aligned} 0 &\le \frac{|\sigma^2_{\mathbb{N}_1}(s_i) - \sigma^2_{\mathbb{N}_2}(s_i)| + |\delta^2_{\mathbb{N}_1}(s_i) - \delta^2_{\mathbb{N}_2}(s_i)|}{\max\left\{|\sigma^2_{\mathbb{N}_1}(s_i) - \sigma^2_{\mathbb{N}_2}(s_i)|, |\delta^2_{\mathbb{N}_1}(s_i) - \delta^2_{\mathbb{N}_2}(s_i)|\right\}} \\ &\le \frac{2}{\max\left\{|\sigma^2_{\mathbb{N}_1}(s_i) - \sigma^2_{\mathbb{N}_2}(s_i)|, |\delta^2_{\mathbb{N}_1}(s_i) - \delta^2_{\mathbb{N}_2}(s_i)|\right\}} \end{aligned}$$

$$0 \le \frac{|\sigma^2_{\mathbb{N}_1}(s_i) - \sigma^2_{\mathbb{N}_2}(s_i)| + |\delta^2_{\mathbb{N}_1}(s_i) - \delta^2_{\mathbb{N}_2}(s_i)|}{\max\left\{|\sigma^2_{\mathbb{N}_1}(s_i) - \sigma^2_{\mathbb{N}_2}(s_i)|, |\delta^2_{\mathbb{N}_1}(s_i) - \delta^2_{\mathbb{N}_2}(s_i)|\right\}} \le 2 \quad \text{(using equation (3))}$$

$$0 \le \sum_{i=1}^{k} \frac{|\sigma^2_{\mathbb{N}_1}(s_i) - \sigma^2_{\mathbb{N}_2}(s_i)| + |\delta^2_{\mathbb{N}_1}(s_i) - \delta^2_{\mathbb{N}_2}(s_i)|}{\max\left\{|\sigma^2_{\mathbb{N}_1}(s_i) - \sigma^2_{\mathbb{N}_2}(s_i)|, |\delta^2_{\mathbb{N}_1}(s_i) - \delta^2_{\mathbb{N}_2}(s_i)|\right\}} \le \sum_{i=1}^{k} 2$$

$$0 \le \frac{1}{2k} \sum_{i=1}^{k} \frac{|\sigma^2_{\mathbb{N}_1}(s_i) - \sigma^2_{\mathbb{N}_2}(s_i)| + |\delta^2_{\mathbb{N}_1}(s_i) - \delta^2_{\mathbb{N}_2}(s_i)|}{\max\left\{|\sigma^2_{\mathbb{N}_1}(s_i) - \sigma^2_{\mathbb{N}_2}(s_i)|, |\delta^2_{\mathbb{N}_1}(s_i) - \delta^2_{\mathbb{N}_2}(s_i)|\right\}} \le 1$$

$\Rightarrow 0 \le \Gamma(\mathbb{N}_1, \mathbb{N}_2) \le 1.$

2. Let $\Gamma(\mathbb{N}_1,\mathbb{N}_2)=0 \Leftrightarrow \mathbb{N}_1=\mathbb{N}_2$
$\Leftrightarrow \frac{1}{2k}\sum_{i=1}^{k}\frac{|\sigma^2_{\mathbb{N}_1}(s_i)-\sigma^2_{\mathbb{N}_2}(s_i)|+|\delta^2_{\mathbb{N}_1}(s_i)-\delta^2_{\mathbb{N}_2}(s_i)|}{\max\left\{|\sigma^2_{\mathbb{N}_1}(s_i)-\sigma^2_{\mathbb{N}_2}(s_i)|,|\delta^2_{\mathbb{N}_1}(s_i)-\delta^2_{\mathbb{N}_2}(s_i)|\right\}}=0$
$\Leftrightarrow |\sigma^2_{\mathbb{N}_1}(s_i)-\sigma^2_{\mathbb{N}_2}(s_i)|+|\delta^2_{\mathbb{N}_1}(s_i)-\delta^2_{\mathbb{N}_2}(s_i)|=0$
$\Leftrightarrow |\sigma^2_{\mathbb{N}_1}(s_i)-\sigma^2_{\mathbb{N}_2}(s_i)|=0; \quad |\delta^2_{\mathbb{N}_1}(s_i)-\delta^2_{\mathbb{N}_2}(s_i)|=0$
$\Leftrightarrow \sigma^2_{\mathbb{N}_1}(s_i)=\sigma^2_{\mathbb{N}_2}(s_i); \quad \delta^2_{\mathbb{N}_1}(s_i)=\delta^2_{\mathbb{N}_2}(s_i) \Leftrightarrow \sigma_{\mathbb{N}_1}(s_i)=\sigma_{\mathbb{N}_2}(s_i); \quad \delta_{\mathbb{N}_1}(s_i)=\delta_{\mathbb{N}_2}(s_i)$
$\Leftrightarrow \mathbb{N}_1=\mathbb{N}_2$

3. To verify the symmetric behavior of $\Gamma(\mathbb{N}_1,\mathbb{N}_2)$, we show that $\Gamma(\mathbb{N}_1,\mathbb{N}_2)=\Gamma(\mathbb{N}_2,\mathbb{N}_1)$. Thus,

$$\begin{aligned}\Gamma(\mathbb{N}_1,\mathbb{N}_2)&=\frac{1}{2k}\sum_{i=1}^{k}\frac{|\sigma^2_{\mathbb{N}_1}(s_i)-\sigma^2_{\mathbb{N}_2}(s_i)|+|\delta^2_{\mathbb{N}_1}(s_i)-\delta^2_{\mathbb{N}_2}(s_i)|}{\max\left\{|\sigma^2_{\mathbb{N}_1}(s_i)-\sigma^2_{\mathbb{N}_2}(s_i)|,|\delta^2_{\mathbb{N}_1}(s_i)-\delta^2_{\mathbb{N}_2}(s_i)|\right\}}\\&=\frac{1}{2k}\sum_{i=1}^{k}\frac{|\sigma^2_{\mathbb{N}_2}(s_i)-\sigma^2_{\mathbb{N}_1}(s_i)|+|\delta^2_{\mathbb{N}_2}(s_i)-\delta^2_{\mathbb{N}_1}(s_i)|}{\max\left\{|\sigma^2_{\mathbb{N}_2}(s_i)-\sigma^2_{\mathbb{N}_1}(s_i)|,|\delta^2_{\mathbb{N}_2}(s_i)-\delta^2_{\mathbb{N}_1}(s_i)|\right\}}\end{aligned}$$

i.e., $\Gamma(\mathbb{N}_1,\mathbb{N}_2)=\Gamma(\mathbb{N}_2,\mathbb{N}_1)$

4. To verify the triangular property, let's consider $\mathbb{N}_1$, $\mathbb{N}_2$ and $\mathbb{N}$ are three PFSs. Then

$$\Gamma(\mathbb{N}_1,\mathbb{N})=\frac{1}{2k}\sum_{i=1}^{k}\frac{|\sigma^2_{\mathbb{N}_1}(s_i)-\sigma^2_{\mathbb{N}}(s_i)|+|\delta^2_{\mathbb{N}_1}(s_i)-\delta^2_{\mathbb{N}}(s_i)|}{\max\left\{|\sigma^2_{\mathbb{N}_1}(s_i)-\sigma^2_{\mathbb{N}}(s_i)|,|\delta^2_{\mathbb{N}_1}(s_i)-\delta^2_{\mathbb{N}}(s_i)|\right\}}$$

$$=\frac{1}{2k}\sum_{i=1}^{k}\frac{|\sigma^2_{\mathbb{N}_1}(s_i)-\sigma^2_{\mathbb{N}_2}(s_i)+\sigma^2_{\mathbb{N}_2}(s_i)-\sigma^2_{\mathbb{N}}(s_i)|+|\delta^2_{\mathbb{N}_1}(s_i)-\delta^2_{\mathbb{N}_2}(s_i)+\delta^2_{\mathbb{N}_2}(s_i)-\delta^2_{\mathbb{N}}(s_i)|}{\max\left\{|\sigma^2_{\mathbb{N}_1}(s_i)-\sigma^2_{\mathbb{N}_2}(s_i)+\sigma^2_{\mathbb{N}_2}(s_i)-\sigma^2_{\mathbb{N}}(s_i)|,|\delta^2_{\mathbb{N}_1}(s_i)-\delta^2_{\mathbb{N}_2}(s_i)+\delta^2_{\mathbb{N}_2}(s_i)-\delta^2_{\mathbb{N}}(s_i)|\right\}}$$

$$\leq \frac{1}{2k}\sum_{i=1}^{k}\frac{|\sigma^2_{\mathbb{N}_1}(s_i)-\sigma^2_{\mathbb{N}_2}(s_i)|+|\delta^2_{\mathbb{N}_1}(s_i)-\delta^2_{\mathbb{N}_2}(s_i)|}{\max\left\{|\sigma^2_{\mathbb{N}_1}(s_i)-\sigma^2_{\mathbb{N}_2}(s_i)|,|\delta^2_{\mathbb{N}_1}(s_i)-\delta^2_{\mathbb{N}_2}(s_i)|\right\}}+\frac{1}{2k}\sum_{i=1}^{k}\frac{|\sigma^2_{\mathbb{N}_2}(s_i)-\sigma^2_{\mathbb{N}}(s_i)|+|\delta^2_{\mathbb{N}_2}(s_i)-\delta^2_{\mathbb{N}}(s_i)|}{\max\left\{|\sigma^2_{\mathbb{N}_2}(s_i)-\sigma^2_{\mathbb{N}}(s_i)|,|\delta^2_{\mathbb{N}_2}(s_i)-\delta^2_{\mathbb{N}}(s_i)|\right\}}$$

$$\leq \Gamma(\mathbb{N}_1,\mathbb{N}_2)+\Gamma(\mathbb{N}_2,\mathbb{N})$$

Given that the novel distance function adheres to all the axioms of a distance measure, it follows that the resulting distance measures are valid. □

Theorem 7 (Avoidance of zero divisor). *The proposed distance measure $\Gamma(\mathbb{N}_1,\mathbb{N}_2)$ is mathematically well-defined for all possible input values, including cases where the denominator might become zero. Specifically, the measure avoids division by zero through a conditional formulation, and is defined as:*

$$\Gamma(\mathbb{N}_1,\mathbb{N}_2)=\frac{1}{2k}\sum_{i=1}^{k}\begin{cases}0, & \text{if } \max\left\{\begin{array}{l}|\sigma^2_{\mathbb{N}_1}(s_i)-\sigma^2_{\mathbb{N}_2}(s_i)|,\\|\delta^2_{\mathbb{N}_1}(s_i)-\delta^2_{\mathbb{N}_2}(s_i)|\end{array}\right\}=0\\ \dfrac{|\sigma^2_{\mathbb{N}_1}(s_i)-\sigma^2_{\mathbb{N}_2}(s_i)|+|\delta^2_{\mathbb{N}_1}(s_i)-\delta^2_{\mathbb{N}_2}(s_i)|}{\max\left\{\begin{array}{l}|\sigma^2_{\mathbb{N}_1}(s_i)-\sigma^2_{\mathbb{N}_2}(s_i)|,\\|\delta^2_{\mathbb{N}_1}(s_i)-\delta^2_{\mathbb{N}_2}(s_i)|\end{array}\right\}}, & \text{otherwise}\end{cases}$$

Theorem 8. *The distance function $\Gamma(\mathbb{N}_1,\mathbb{N}_2)$ between two PFSs $\mathbb{N}_1$ and $\mathbb{N}_2$ satisfies*

1. $\Gamma(\mathbb{N}^c_1,\mathbb{N}^c_2)$=$\Gamma(\mathbb{N}_1,\mathbb{N}_2)$
2. $\Gamma(\mathbb{N}_1,\mathbb{N}^c_2)$=$\Gamma(\mathbb{N}^c_1,\mathbb{N}_2)$

Proof. *1. Let* $\mathbb{N}_1=\{\langle s_i,\sigma_{\mathbb{N}_1}(s_i),\delta_{\mathbb{N}_1}(s_i)\rangle|s_i\in S\}$ *and* $\mathbb{N}_2=\{\langle s_i,\sigma_{\mathbb{N}_2}(s_i),\delta_{\mathbb{N}_2}(s_i)\rangle|s_i\in S\}$,
$\Rightarrow \mathbb{N}_1^c=\{\langle s_i,\delta_{\mathbb{N}_1}(s_i),\sigma_{\mathbb{N}_1}(s_i)\rangle|s_i\in S\}$ *and* $\mathbb{N}_2^c=\{\langle s_i,\delta_{\mathbb{N}_2}(s_i),\sigma_{\mathbb{N}_2}(s_i)\rangle|s_i\in S\}$ *(from Definition 3)*
Using equation (2) the distance between $\mathbb{N}_1^c$ *and* $\mathbb{N}_2^c$ *are*

$$\Gamma(\mathbb{N}_1^c,\mathbb{N}_2^c)=\frac{1}{2k}\sum_{i=1}^{k}\frac{|\delta^2_{\mathbb{N}_1}(s_i)-\delta^2_{\mathbb{N}_2}(s_i)|+|\sigma^2_{\mathbb{N}_1}(s_i)-\sigma^2_{\mathbb{N}_2}(s_i)|}{\max\{|\delta^2_{\mathbb{N}_1}(s_i)-\delta^2_{\mathbb{N}_2}(s_i)|,|\sigma^2_{\mathbb{N}_1}(s_i)-\sigma^2_{\mathbb{N}_2}(s_i)|\}}$$

$$=\frac{1}{2k}\sum_{i=1}^{k}\frac{|\sigma^2_{\mathbb{N}_1}(s_i)-\sigma^2_{\mathbb{N}_2}(s_i)|+|\delta^2_{\mathbb{N}_1}(s_i)-\delta^2_{\mathbb{N}_2}(s_i)|}{\max\{|\sigma^2_{\mathbb{N}_1}(s_i)-\sigma^2_{\mathbb{N}_2}(s_i)|,|\delta^2_{\mathbb{N}_1}(s_i)-\delta^2_{\mathbb{N}_2}(s_i)|\}}=\Gamma(\mathbb{N}_1,\mathbb{N}_2).$$

2. *The distance between* $\mathbb{N}_1$ *and* $\mathbb{N}_2^c$ *are*

$$\Gamma(\mathbb{N}_1,\mathbb{N}_2^c)=\frac{1}{2k}\sum_{i=1}^{k}\frac{|\sigma^2_{\mathbb{N}_1}(s_i)-\delta^2_{\mathbb{N}_2}(s_i)|+|\delta^2_{\mathbb{N}_1}(s_i)-\sigma^2_{\mathbb{N}_2}(s_i)|}{\max\{|\sigma^2_{\mathbb{N}_1}(s_i)-\delta^2_{\mathbb{N}_2}(s_i)|,|\delta^2_{\mathbb{N}_1}(s_i)-\sigma^2_{\mathbb{N}_2}(s_i)|\}}$$

after rearranging the term, we get

$$=\frac{1}{2k}\sum_{i=1}^{k}\frac{|\delta^2_{\mathbb{N}_1}(s_i)-\sigma^2_{\mathbb{N}_2}(s_i)|+|\sigma^2_{\mathbb{N}_1}(s_i)-\delta^2_{\mathbb{N}_2}(s_i)|}{\max\{|\delta^2_{\mathbb{N}_1}(s_i)-\sigma^2_{\mathbb{N}_2}(s_i)|,|\sigma^2_{\mathbb{N}_1}(s_i)-\delta^2_{\mathbb{N}_2}(s_i)|\}}=\Gamma(\mathbb{N}_1^c,\mathbb{N}_2).$$

□

Theorem 9. *Let* $\mathbb{N}_1$, $\mathbb{N}_2$ *and* $\mathbb{N}_3$ *be three PFSs. The distance functions are satisfies following properties*

1. $\Gamma(\mathbb{N}_1,\mathbb{N}_2)$=$\Gamma(\mathbb{N}_1\cap\mathbb{N}_2,\mathbb{N}_1\cup\mathbb{N}_2)$
2. $\Gamma(\mathbb{N}_1,\mathbb{N}_1\cap\mathbb{N}_2)$=$\Gamma(\mathbb{N}_2,\mathbb{N}_1\cup\mathbb{N}_2)$
3. $\Gamma(\mathbb{N}_1,\mathbb{N}_1\cup\mathbb{N}_2)$=$\Gamma(\mathbb{N}_2,\mathbb{N}_1\cap\mathbb{N}_2)$
4. $\Gamma(\mathbb{N}_1\cup\mathbb{N}_2,\mathbb{N}_3)+\Gamma(\mathbb{N}_1\cap\mathbb{N}_2,\mathbb{N}_3)$=$\Gamma(\mathbb{N}_1,\mathbb{N}_3)+\Gamma(\mathbb{N}_2,\mathbb{N}_3)$

Proof. *1. Let* $\mathbb{N}_1=\{\langle s_i,\sigma_{\mathbb{N}_1}(s_i),\delta_{\mathbb{N}_1}(s_i)\rangle|s_i\in S\}$ *and* $\mathbb{N}_2=\{\langle s_i,\sigma_{\mathbb{N}_2}(s_i),\delta_{\mathbb{N}_2}(s_i)\rangle|s_i\in S\}$
We know that $\mathbb{N}_1\cup\mathbb{N}_2=\{\langle s_i,\max\{\sigma_{\mathbb{N}_1}(s_i),\sigma_{\mathbb{N}_2}(s_i)\},\ \min\{\delta_{\mathbb{N}_1}(s_i),\delta_{\mathbb{N}_2}(s_i)\rangle|s_i\in S\}$
$\mathbb{N}_1\cap\mathbb{N}_2=\{\langle s_i,\min\{\sigma_{\mathbb{N}_1}(s_i),\sigma_{\mathbb{N}_2}(s_i)\},\ \max\{\delta_{\mathbb{N}_1}(s_i),\delta_{\mathbb{N}_2}(s_i)\rangle|s_i\in S\}$ *(from Definition 3)*
Case-I *If* $\mathbb{N}_1\subseteq\mathbb{N}_2$, *then* $\sigma_{\mathbb{N}_1}(s_i)\le\sigma_{\mathbb{N}_2}(s_i)$ *and* $\delta_{\mathbb{N}_1}(s_i)\ge\delta_{\mathbb{N}_2}(s_i)$ *(using inclusion property from Definition 3)*
$\Rightarrow\mathbb{N}_1\cap\mathbb{N}_2=\{\langle s_i,\sigma_{\mathbb{N}_1}(s_i),\delta_{\mathbb{N}_1}(s_i)\rangle|s_i\in S\}$, *and* $\mathbb{N}_1\cup\mathbb{N}_2=\{\langle s_i,\sigma_{\mathbb{N}_2}(s_i),\delta_{\mathbb{N}_2}(s_i)\rangle|s_i\in S\}$
Now, utilizing equation (2), the distance measures are

$$\Gamma(\mathbb{N}_1\cap\mathbb{N}_2,\mathbb{N}_1\cup\mathbb{N}_2)=\frac{1}{2k}\sum_{i=1}^{k}\frac{|\sigma^2_{\mathbb{N}_1}(s_i)-\sigma^2_{\mathbb{N}_2}(s_i)|+|\delta^2_{\mathbb{N}_1}(s_i)-\delta^2_{\mathbb{N}_2}(s_i)|}{\max\{|\sigma^2_{\mathbb{N}_1}(s_i)-\sigma^2_{\mathbb{N}_2}(s_i)|,|\delta^2_{\mathbb{N}_1}(s_i)-\delta^2_{\mathbb{N}_2}(s_i)|\}}=\Gamma(\mathbb{N}_1,\mathbb{N}_2)$$

Case-II *If* $\mathbb{N}_2 \subseteq \mathbb{N}_1$, *then* $\sigma_{\mathbb{N}_2}(s_i) \leq \sigma_{\mathbb{N}_1}(s_i)$ *and* $\delta_{\mathbb{N}_2}(s_i) \geq \delta_{\mathbb{N}_1}(s_i)$ *(using inclusion property from Definition 3)*
$\Rightarrow \mathbb{N}_1 \cap \mathbb{N}_2 = \{\langle s_i, \sigma_{\mathbb{N}_2}(s_i), \delta_{\mathbb{N}_2}(s_i)\rangle | s_i \in S\}$, *and* $\mathbb{N}_1 \cup \mathbb{N}_2 = \{\langle s_i, \sigma_{\mathbb{N}_1}(s_i), \delta_{\mathbb{N}_1}(s_i)\rangle | s_i \in S\}$
Now, utilizing equation (2), the distance measures are

$$\Gamma(\mathbb{N}_1 \cap \mathbb{N}_2, \mathbb{N}_1 \cup \mathbb{N}_2) = \frac{1}{2k}\sum_{i=1}^{k} \frac{|\sigma^2_{\mathbb{N}_2}(s_i) - \sigma^2_{\mathbb{N}_1}(s_i)| + |\delta^2_{\mathbb{N}_2}(s_i) - \delta^2_{\mathbb{N}_1}(s_i)|}{\max\left\{|\sigma^2_{\mathbb{N}_2}(s_i) - \sigma^2_{\mathbb{N}_1}(s_i)|, |\delta^2_{\mathbb{N}_2}(s_i) - \delta^2_{\mathbb{N}_1}(s_i)|\right\}}$$

After arranging the terms in the equation, we can express it in a simplified form

$$\Gamma(\mathbb{N}_1 \cap \mathbb{N}_2, \mathbb{N}_1 \cup \mathbb{N}_2) = \frac{1}{2k}\sum_{i=1}^{k} \frac{|\sigma^2_{\mathbb{N}_1}(s_i) - \sigma^2_{\mathbb{N}_2}(s_i)| + |\delta^2_{\mathbb{N}_1}(s_i) - \delta^2_{\mathbb{N}_2}(s_i)|}{\max\left\{|\sigma^2_{\mathbb{N}_1}(s_i) - \sigma^2_{\mathbb{N}_2}(s_i)|, |\delta^2_{\mathbb{N}_1}(s_i) - \delta^2_{\mathbb{N}_2}(s_i)|\right\}}$$

$\Rightarrow \Gamma(\mathbb{N}_1 \cap \mathbb{N}_2, \mathbb{N}_1 \cup \mathbb{N}_2) = \Gamma(\mathbb{N}_1, \mathbb{N}_2)$ *(from equation (2))*
Similarly, with the assistance of the aforementioned conditions, we can acquire 2, 3, and 4.
□

8. COMPARISON ON EXISTING AND NOVEL DISTANCE MEASURES USING DIFFERENT PFSs

To assess the superiority and validity of the novel distance function in this section, we discuss a comparison between various existing distance functions and the proposed one through numerical examples. To facilitate this comparison, we consider two sets of PFSs, denoted as $\mathbb{N}_i$ and $\mathbb{M}_i$ for $i = 1, 2$, across four distinct cases: cases 1-4. In cases 1 and 2, the sets $\mathbb{N}_i$ are identical for $i = 1, 2$, while in cases 1 and 3, the sets $\mathbb{M}_i$ are identical for $i = 1, 2$. Case 4 represents a boundary set of PFSs. These cases are applied to both the existing extant distance function and the proposed one to compute the distance value between $\mathbb{N}_i$ and $\mathbb{M}_i$ for $i = 1, 2$. The results produced by both the extant and proposed distance functions are presented in Table 2.

Table 2: Comparison of existing and proposed distance measure via examples

	Case 1	Case 2	Case 3	Case 4
$\mathbb{N}_i$	$\{\langle 0.9, 0.2\rangle, \langle 0.8, 0.3\rangle\}$	$\{\langle 0.9, 0.2\rangle, \langle 0.8, 0.3\rangle\}$	$\{\langle 0.9, 0.2\rangle, \langle 0.8, 0.4\rangle\}$	$\{\langle 1, 0\rangle, \langle 0, 1\rangle\}$
$\mathbb{M}_i$	$\{\langle 0.7, 0.4\rangle, \langle 0.6, 0.5\rangle\}$	$\{\langle 0.4, 0.9\rangle, \langle 0.3, 0.9\rangle\}$	$\{\langle 0.7, 0.4\rangle, \langle 0.6, 0.5\rangle\}$	$\{\langle 0, 1\rangle, \langle 1, 0\rangle\}$
Γ_Q [44]	**0.5999**	**1.4900**	**0.5999**	**2**
Γ_C [45]	**0.5999**	**1.4900**	**0.5999**	**2**
Γ_{HY} [11]	**0.3000**	0.7450	**0.3000**	1
Γ_{SB} [46]	**0.3000**	0.7250	**0.3000**	1
Γ_A [47]	0.0980	**1.0142**	0.0835	**2.5285**
Γ_K [48]	0.4048	1.2484	0.3654	**2**
Γ_D [49]	0.8614	**2.3425**	0.7366	**4**
Proposed	0.7366	0.9020	0.6741	1

Bold numerals indicate the unreasonable results

*

Discussion: In our literature review, we identified a significant drawback in the extant distance function: it fails to meet the separability condition required of a distance measure. Table 2 presents an analysis of several extant distance functions, including Γ_Q [44],

Γ_C [45], Γ_A [47], and Γ_D [49], all of which fail to satisfy the bounded condition outlined in Definition 4 for cases 2 and 4. Additionally, Γ_K [48] fails to meet the bounded condition for case 4, as its distance values fall within the interval $[0,1]$. Some extant distance functions, such as Γ_Q [44], Γ_C [45], Γ_{HY} [11], and Γ_{SB} [46], yield unreasonable results for cases 1 and 3 by producing identical outcomes for different sets of PFSs.

Additionally, we identified a common drawback in some extant distance functions, which depends on a parameter λ . Specifically, increasing the value of λ causes the extended distance function [44, 45, 46, 48] to tend towards zero. This trend violates the separability condition, expressed as $\Gamma(\mathbb{N}_1,\mathbb{N}_2) = 0 \Leftrightarrow \mathbb{N}_1 = \mathbb{N}_2$, where $\mathbb{N}_1 \neq \mathbb{N}_2$. We illustrate this drawback through numerical examples.

Example 10. *Let* $\mathbb{N}_1 = \{\langle 0.5,0.8\rangle,\langle 0.6,0.7\rangle\}$ *and* $\mathbb{N}_2 = \{\langle 0.4,0.7\rangle,\langle 0.7,0.7\rangle\}$ *be two PFSs. Then the distance measure for some existing distance function* $\Gamma(\mathbb{N}_1,\mathbb{N}_2)$ *[44, 45, 46, 48] values for different parameter* λ *are visually represented in Figure 1.*

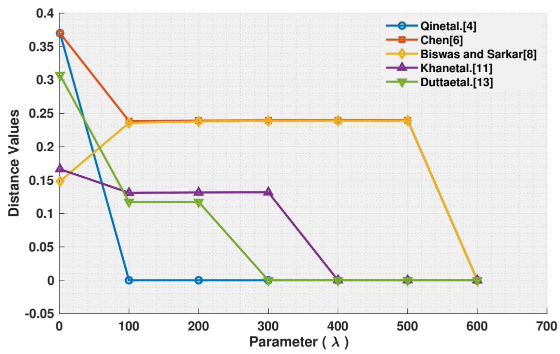

Figure 1: Drawbacks of some extant distance measure based on Parameter λ

In Figure 1, it is evident that as we increase the values of parameter λ, the existing distance values tend toward zero. This presents a significant drawback, as the sets $\mathbb{N}_1$ and $\mathbb{N}_2$ are distinct, yet we obtain the same values, resulting in unreasonable outcomes. In contrast, our proposed distance function consistently provides reliable and valid results without failing any condition outlined in Definition 4. This demonstrates the superiority of our proposed distance function over extant distance functions.

9. A NOVEL HYBRID MCDM METHODOLOGY

In this section, we present the proposed methodology. First, the criteria weights are determined using the SWARA method, with corresponding mathematical expressions provided. Next, the ranking of alternatives is carried out using the TOPSIS method, along with the relevant formulations. The integration of PFSs with the SWARA–TOPSIS framework is adopted due to its effectiveness in managing uncertainty, vagueness, and subjective expert judgments. Compared to classical and IFSs, Pythagorean fuzzy sets offer a more flexible representation of hesitation by allowing the squared sum of membership and non-membership degrees to be at most one. This enhances the modeling of imprecision often encountered in real-world decision-making. The SWARA technique is selected for its simplicity and ability to capture expert opinions without relying on complex pairwise comparisons, unlike methods such as AHP. It efficiently reflects the relative importance of criteria in a sequential, expert-driven manner. When combined with TOPSIS, the approach ensures a robust and objective ranking of alternatives based on their proximity to ideal and anti-ideal solutions in the fuzzy context. Compared to other hybrid decision-making frameworks, such as AHP–VIKOR or DEMATEL–TOPSIS under a fuzzy environment, the proposed method offers a practical balance between computational simplicity and decision accuracy. It is especially suitable for scenarios involving both qualitative and quantitative factors, making it a reliable and interpretable tool for complex decision problems under uncertainty. The algorithm includes the following steps.

Step 1: Creating a decision matrix

In the process of MCDM, we analyze a set of alternatives (D) and criteria (C), represented as $\mathrm{D} = \{\mathrm{D}_1, \mathrm{D}_2, ..., \mathrm{D}_m\}$ and $\mathrm{C} = \{\mathrm{C}_1, \mathrm{C}_2, ..., \mathrm{C}_n\}$ respectively. To determine the most suitable renewable energy source, we assemble a group of decision experts (DEs), denoted as $\mathrm{E} = \{\mathrm{E}_1, \mathrm{E}_2, ..., \mathrm{E}_{\mathscr{L}}\}$. Let $\mathrm{M} = \left(g_{ij}^{k}\right)$, where $i = 1, 2..., m$ and $j = 1, 2..., n$, represents a decision matrix provided by DEs. This matrix evaluates the alternatives D_m with respect to the criteria C_n as perceived by the k^{th} DEs.

Step 2: Determine the weights assigned to the DEs

To calculate the weights of DEs [50] in PFN, $\mathrm{D}_k = \mathbb{Y}\langle \sigma_k, \delta_k, \pi_k \rangle$ for the k^{th} evaluation, we employ equation (5).

$$\mathrm{W}_k = \frac{\left(\sigma_k^2 + \pi_k^2 \times \left(\frac{\sigma_k^2}{\sigma_k^2 + \delta_k^2}\right)\right)}{\sum_{k=1}^{\mathscr{L}} \left(\sigma_k^2 + \pi_k^2 \times \left(\frac{\sigma_k^2}{\sigma_k^2 + \delta_k^2}\right)\right)}, \quad k = 1, 2, ...\mathscr{L}; \mathrm{W}_k \geq 0, \sum_{k=1}^{\mathscr{L}} \mathrm{W}_k = 1 \tag{5}$$

Step 3: Generation of the Aggregated Pythagorean fuzzy decision (APFD) **matrix**

To create the APFD, the individual decision matrices are combined into a single decision matrix using the opinions of DEs. This is achieved by employing the Pythagorean fuzzy weighted averaging (PFWA) operator [5]. The resulting APFD matrix, denoted as $\mathrm{Z} = \left(z_{ij}\right)_{m \times n}$, represents the consolidated decision matrix.

$$z_{ij} = \mathbb{Y}\langle \sigma_{ij}, \delta_{ij} \rangle = \mathrm{PFWA}\left(g_{ij}^{(1)}, g_{ij}^{(2)}, ..., g_{ij}^{(\mathscr{L})}\right) = \mathbb{Y}\left\langle \sqrt{1 - \prod_{k=1}^{\mathscr{L}} (1 - \sigma_k^2)^{\mathrm{W}_k}}, \prod_{k=1}^{\mathscr{L}} (\delta_k)^{\mathrm{W}_k} \right\rangle \tag{6}$$

where σ_k, δ_k are membership and non- membership value, W_k is the weights of experts w. r. t k^{th} evaluations.

Step 4: The SWARA method for subjective weights:

We adhere to the following steps in order to gauge the subjective weight of criteria.

Step 4.a: The precise values of PFNs are established by assessing equation (1) as outlined in Definition (2) to derive the score values $\mathbb{S}^{\star}(\widetilde{z}_{kj})$.

Step 4.b: The criteria are prioritized based on the preferences of the DE, arranged from the most to the least significant.
Step 4.c: Assess the importance of score values by analyzing the criteria listed second. Relative significance is determined by comparing criteria j with criteria $j-1$.
Step 4.d: Determine the relative coefficient by calculating the coefficient k_j using the equation provided below.

$$k_j = \begin{cases} 1 & j = 1, \\ s_j + 1, & j > 1, \end{cases} \tag{7}$$

The notation s_j signifies the relative importance of the score value [51].
Step 4.e: Establish the weight. The recalculated weight p_j is denoted by

$$p_j = \begin{cases} 1 & j = 1, \\ \frac{k_{j-1}}{k_j}, & j > 1, \end{cases} \tag{8}$$

Step 4.f: Review the allocated weight of the criteria.
The allocated criteria weights are determined by

$$\mathrm{SW}_j = \frac{p_j}{\sum\limits_{j=1}^{n} p_j}. \tag{9}$$

Step 5: The TOPSIS method for alternatives ranking
To gauge the ranking of alternatives, we adhere to the subsequent procedures.
Step 5.a: Normalize the APFD matrix utilizing equation (10).

$$\widetilde{z_{ij}} = \mathbb{Y}\left\langle \widetilde{\sigma_{ij}}, \widetilde{\delta_{ij}} \right\rangle = \begin{cases} z_{ij} = \mathbb{Y}\left\langle \sigma_{ij}, \delta_{ij} \right\rangle & j \in \mathbb{T}_b \\ (z_{ij})^c = \mathbb{Y}\left\langle \delta_{ij}, \sigma_{ij} \right\rangle & j \in \mathbb{T}_n \end{cases} \quad ; i = 1, 2, .., m \tag{10}$$

where $\mathbb{T}_b$= beneficial sets and $\mathbb{T}_n$= Non beneficial sets, respectively.
Step 5.b: Compute the PF positive and negative ideal solutions
To calculate the positive and negative ideal solutions for PF, apply equation (11) for the positive case and equation (12) for the negative case.

$$\mathbb{N}^{p+} = \{z_1^{p+}, z_2^{p+}, \ldots.., z_n^{p+}\} = \left\{ \left(\max_i(z_{ij}^p), j \in \mathbb{T}_b \right), \left(\min_i(z_{ij}^p), j \in \mathbb{T}_n \right) \right\} \tag{11}$$

$$\mathbb{N}^{p-} = \left\{z_1^{p-}, z_2^{p-}, \ldots.., z_n^{p-}\right\} = \left\{ \left(\min_i(z_{ij}^p), j \in \mathbb{T}_b \right), \left(\max_i(z_{ij}^p), j \in \mathbb{T}_n \right) \right\} \tag{12}$$

Step 5.c: Calculate the PF distance of each alternative from the positive and negative ideal solution
To compute the PF distance of every option from both the positive and negative ideal solutions, utilize equations (13) and (14) correspondingly.

$$\Gamma(z_{ij}, \mathbb{N}^{p+}) = \frac{1}{2k} \sum_{i=1}^{k} \mathrm{SW}_i \left[\frac{|\sigma^2(s_i) - \sigma^2_{\mathbb{N}^{p+}}(s_i)| + |\delta^2(s_i) - \delta^2_{\mathbb{N}_{p+}}(s_i)|}{\max\left\{ |\sigma^2(s_i) - \sigma^2_{\mathbb{N}^{p+}}(s_i)|, |\delta^2(s_i) - \delta^2_{\mathbb{N}^{p+}}(s_i)| \right\}} \right] \tag{13}$$

$$\Gamma(z_{ij}, \mathbb{N}^{p-}) = \frac{1}{2k}\sum_{i=1}^{k} \mathrm{SW}_i \left[\frac{|\sigma^2(s_i) - \sigma^2_{\mathbb{N}^{p-}}(s_i)| + |\delta^2(s_i) - \delta^2_{\mathbb{N}p-}(s_i)|}{\max\left\{|\sigma^2(s_i) - \sigma^2_{\mathbb{N}^{p-}}(s_i)|, |\delta^2(s_i) - \delta^2_{\mathbb{N}^{p-}}(s_i)|\right\}} \right] \tag{14}$$

Step 5.d: Calculate the PF closeness coefficient
Determine the proximity coefficient utilizing equation (15).

$$\mathbb{R}_i = \frac{\Gamma(\mathbb{N}_i, \mathbb{N}_i^-)}{\Gamma(\mathbb{N}_i, \mathbb{N}_i^-) + \Gamma(\mathbb{N}_i, \mathbb{N}_i^+)}, \tag{15}$$

where $0 \leq \mathbb{R}_i \leq 1$, $i = 1, 2, ...k$.
Step 5.e: Ranking the alternatives
When assessing the alternatives, the order of preference ranges from high to low.
6: End.

The developed PF-SWARA–TOPSIS methodology offers a comprehensive and mathematically sound approach for handling decision-making problems characterized by uncertainty and subjective assessments. To demonstrate its practical utility, the next section applies this hybrid framework to the selection of renewable energy resources, a domain where conflicting criteria and fuzzy evaluations are prevalent.

10. IMPLEMENTING MCDM FOR SELECTING RENEWABLE ENERGY RESOURCES

In this section, the proposed PF-SWARA–TOPSIS approach is implemented to evaluate and rank renewable energy alternatives. The methodology described in Section 9 is followed step-by-step, using real-world data and expert inputs to illustrate the model's effectiveness in supporting complex, uncertainty-driven decision scenarios. Renewable energy resources play a dynamic role in attaining sustainable energy with minimal emissions. It is widely acknowledged that these resources have the capacity to substantially fulfill electricity needs while reducing emissions. In recent times, the nation has embraced a sustainable pathway for its energy provision. To validate the effectiveness of this approach, we present a comprehensive numerical analysis of optimal renewable energy selection using data sourced from [52].

The potential renewable energy sources under consideration include Solar power (D_1), Wind power(D_2), Hydroelectric power(D_3), Biomass(D_4), Geothermal energy(D_5). In addition, 12 criteria were evaluated: Pollution Output (C_1), Waste Management Containment Requirements (C_2), Air, Water, and Land Pollution (C_3), Resource Availability (C_4), Investment Cost (C_5), Operation and Maintenance Expenses (C_6), Energy Supply Security (C_7), Development Centralization (C_8), Climate Impact (C_9), Alignment with National Energy Policy Objectives (C_{10}), Technological Risk (C_{11}), and Job Creation Potential (C_{12}). These criteria are shown visually in the Figure 2.

In Figure 2, the criteria can be categorized into two distinct groups: beneficial and non-beneficial. In the beneficial category, we find Pollution Output (C_1), Waste Management Containment Requirements (C_2), Air, Water, and Land Pollution (C_3), Resource Availability (C_4), Energy Supply Security (C_7), Development Centralization (C_8), Alignment with National Energy Policy Objectives (C_{10}), and Job Creation Potential (C_{12}).

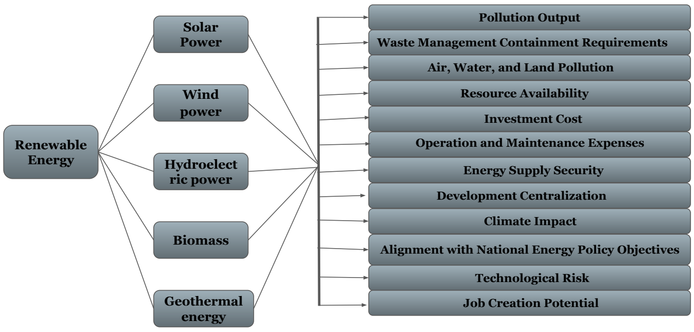

Figure 2: The systematic framework for choosing renewable energy resources

Conversely, in the non-beneficial category, we have Investment Cost (C_5), Operation and Maintenance Expenses (C_6), Climate Impact (C_9), and Technological Risk (C_{11}).

Table 3 presents the comparative weights assigned to linguistic terms for PFNs. Table 4 exhibits the weight assigned to each decision expert, as determined using equation (5). Decision experts have converted linguistic terms into PFNs to estimate the alternatives, as illustrated in Table 5.

Table 3: Linguistic concepts expressed through PFNs

linguistic terms	Mark	PFNs
Excellent	E	$\langle 0.9500, 0.1000 \rangle$
Unmatched	UM	$\langle 0.8000, 0.2500 \rangle$
Acceptable	AC	$\langle 0.5000, 0.5500 \rangle$
Noteworthy	NW	$\langle 0.3000, 0.6000 \rangle$
Insignificant	IS	$\langle 0.1500, 0.9000 \rangle$

Table 4: Decision expert DEs weights

DEs	linguistic terms	PFNs	Weights
E_1	UM	$\langle 0.8000, 0.2500 \rangle$	0.3871
E_2	AC	$\langle 0.5000, 0.5500 \rangle$	0.1923
E_3	E	$\langle 0.9500, 0.1000 \rangle$	0.4203

Table 5: Utilizing linguistic terminology to assess alternatives

linguistic terms	Abbreviation	PFNs
Outstanding	OS	$\langle 0.9800, 0.2000 \rangle$
Superb	SB	$\langle 0.8700, 0.3500 \rangle$
Highly commendable	HC	$\langle 0.7000, 0.4000 \rangle$
Commendable	CM	$\langle 0.6500, 0.4500 \rangle$
Satisfying	SF	$\langle 0.5000, 0.5500 \rangle$
Acceptable	AC	$\langle 0.4000, 0.7000 \rangle$
Partly Acceptable	PA	$\langle 0.3600, 0.8000 \rangle$
Inadequate	IN	$\langle 0.2500, 0.8700 \rangle$
Highly Inadequate	HI	$\langle 0.2000, 0.9800 \rangle$

In Table 6, we illustrate the considerations of decision makers (DEs) with respect to each alternative (D_m) according to various criteria (C_n), using linguistic terms defined in Table 5. In Table 7, the linguistic expressions provided by DEs in Table 6 undergo conversion to AFDM using equation (6).

Table 6: Linguistic assessments provided by different experts for the identification of RERs

Alternatives	Experts	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}
	E_1	HI	HI	HI	IN	IN	CM	IN	IN	SF	SF	CM	SF
D_1	E_2	HI	HI	IN	IN	PA	HC	PA	PA	SF	AC	SF	CM
	E_3	HI	IN	IN	IN	HI	CM	HI	HI	SF	SF	CM	AC
	E_1	HI	PA	IN	IN	HI	OS	HI	IN	HC	CM	CM	HC
D_2	E_2	HI	IN	PA	PA	HI	SB	HI	PA	HC	SB	HC	CM
	E_3	IN	HI	PA	HI	IN	SB	IN	PA	SB	SB	SB	CM
	E_1	IN	HI	IN	IN	IN	HC	HI	HI	CM	HC	CM	CM
D_3	E_2	IN	HI	HI	AC	HI	SB	HI	IN	CM	CM	CM	SB
	E_3	IN	IN	IN	PA	HI	SB	IN	AC	HC	HC	SB	SF
	E_1	PA	PA	IN	HI	IN	HC	IN	PA	CM	AC	HC	CM
D_4	E_2	IN	PA	IN	HI	IN	CM	PA	IN	HC	SF	HC	CM
	E_3	IN	IN	IN	IN	PA	HC	PA	IN	AC	AC	SF	SF
	E_1	IN	IN	HI	IN	IN	CM	IN	AC	AC	SF	CM	SF
D_5	E_2	IN	IN	HI	IN	PA	CM	IN	PA	SF	AC	CM	CM
	E_3	IN	HI	IN	IN	IN	HC	PA	IN	SF	SF	SF	SF

Table 7: Evaluating RERs: Aggregated Pythagorean Fuzzy Decision Matrix

$\mathbf{z_{ij}}$	D_1	D_2	D_3	D_4	D_5
C_1	$\mathbb{Y}\langle 0.1999, 0.9700\rangle$	$\mathbb{Y}\langle 0.2224, 0.9321\rangle$	$\mathbb{Y}\langle 0.2501, 0.8701\rangle$	$\mathbb{Y}\langle 0.2707, 0.8421\rangle$	$\mathbb{Y}\langle 0.2501, 0.8701\rangle$
C_2	$\mathbb{Y}\langle 0.2224, 0.9321\rangle$	$\mathbb{Y}\langle 0.2531, 0.8853\rangle$	$\mathbb{Y}\langle 0.2224, 0.9321\rangle$	$\mathbb{Y}\langle 0.2801, 0.8286\rangle$	$\mathbb{Y}\langle 0.2303, 0.8398\rangle$
C_3	$\mathbb{Y}\langle 0.2321, 0.9111\rangle$	$\mathbb{Y}\langle 0.2817, 0.8263\rangle$	$\mathbb{Y}\langle 0.2411, 0.8901\rangle$	$\mathbb{Y}\langle 0.2501, 0.8701\rangle$	$\mathbb{Y}\langle 0.2224, 0.9321\rangle$
C_4	$\mathbb{Y}\langle 0.2501, 0.8701\rangle$	$\mathbb{Y}\langle 0.2420, 0.9001\rangle$	$\mathbb{Y}\langle 0.3057, 0.8055\rangle$	$\mathbb{Y}\langle 0.2224, 0.9321\rangle$	$\mathbb{Y}\langle 0.2501, 0.8701\rangle$
C_5	$\mathbb{Y}\langle 0.2581, 0.9001\rangle$	$\mathbb{Y}\langle 0.2224, 0.9321\rangle$	$\mathbb{Y}\langle 0.2210, 0.9359\rangle$	$\mathbb{Y}\langle 0.3021, 0.8400\rangle$	$\mathbb{Y}\langle 0.2754, 0.8562\rangle$
C_6	$\mathbb{Y}\langle 0.6603, 0.4400\rangle$	$\mathbb{Y}\langle 0.9378, 0.2817\rangle$	$\mathbb{Y}\langle 0.8222, 0.3687\rangle$	$\mathbb{Y}\langle 0.6911, 0.4282\rangle$	$\mathbb{Y}\langle 0.6721, 0.4282\rangle$
C_7	$\mathbb{Y}\langle 0.2420, 0.9001\rangle$	$\mathbb{Y}\langle 0.2224, 0.9321\rangle$	$\mathbb{Y}\langle 0.3107, 0.8316\rangle$	$\mathbb{Y}\langle 0.2707, 0.8423\rangle$	$\mathbb{Y}\langle 0.3267, 0.7871\rangle$
C_8	$\mathbb{Y}\langle 0.2420, 0.9001\rangle$	$\mathbb{Y}\langle 0.2817, 0.8263\rangle$	$\mathbb{Y}\langle 0.3107, 0.8314\rangle$	$\mathbb{Y}\langle 0.2705, 0.8421\rangle$	$\mathbb{Y}\langle 0.3267, 0.7871\rangle$
C_9	$\mathbb{Y}\langle 0.5001, 0.5501\rangle$	$\mathbb{Y}\langle 0.7916, 0.3783\rangle$	$\mathbb{Y}\langle 0.6723, 0.4284\rangle$	$\mathbb{Y}\langle 0.5832, 0.5298\rangle$	$\mathbb{Y}\langle 0.4653, 0.6039\rangle$
C_{10}	$\mathbb{Y}\langle 0.4833, 0.5762\rangle$	$\mathbb{Y}\langle 0.8126, 0.3859\rangle$	$\mathbb{Y}\langle 0.6913, 0.4093\rangle$	$\mathbb{Y}\langle 0.4221, 0.6684\rangle$	$\mathbb{Y}\langle 0.4833, 0.5762\rangle$
C_{11}	$\mathbb{Y}\langle 0.6266, 0.4676\rangle$	$\mathbb{Y}\langle 0.7797, 0.3957\rangle$	$\mathbb{Y}\langle 0.7736, 0.4048\rangle$	$\mathbb{Y}\langle 0.6325, 0.4572\rangle$	$\mathbb{Y}\langle 0.5961, 0.4895\rangle$
C_{12}	$\mathbb{Y}\langle 0.5021, 0.5857\rangle$	$\mathbb{Y}\langle 0.6706, 0.4300\rangle$	$\mathbb{Y}\langle 0.6741, 0.4666\rangle$	$\mathbb{Y}\langle 0.5963, 0.4897\rangle$	$\mathbb{Y}\langle 0.5355, 0.5292\rangle$

In the Pythagorean fuzzy SWARA approach, interdependencies are captured through a sequential evaluation process. Experts initially rank the criteria and then assess the relative importance of each criterion with respect to the one ranked immediately higher. These evaluations are represented using Pythagorean fuzzy numbers, offering greater flexibility in modeling uncertainty and hesitation. This stepwise structure inherently incorporates the influence of previously ranked criteria on the subsequent ones, allowing the derived weights to reflect interdependencies more accurately and reliably.

Criteria weight via the SWARA method:

$SW_j=\{0.0837, 0.0829, 0.0794, 0.0872, 0.0835, 0.0833, 0.0759, 0.0942, 0.0779, 0.0889, 0.0841, 0.0790\}$

Table 8: Criteria weighting by DE*s* using linguistic variables for RERs

Criteria	E_1	E_2	E_3	Aggregated PFNs	Score Values
C_1	SB	HC	SF	$\mathbb{Y}\langle 0.7414, 0.4344\rangle$	0.6805
C_2	HC	CM	SF	$\mathbb{Y}\langle 0.6210, 0.4679\rangle$	0.5834
C_3	CM	HC	AC	$\mathbb{Y}\langle 0.5830, 0.5298\rangle$	0.5296
C_4	HC	SF	SF	$\mathbb{Y}\langle 0.5949, 0.4863\rangle$	0.5587
C_5	SB	CM	SF	$\mathbb{Y}\langle 0.7340, 0.4443\rangle$	0.6707
C_6	AC	PA	SF	$\mathbb{Y}\langle 0.4398, 0.6490\rangle$	0.3861
C_7	HC	SB	AC	$\mathbb{Y}\langle 0.6741, 0.4934\rangle$	0.6055
C_8	SF	AC	AC	$\mathbb{Y}\langle 0.4427, 0.6377\rangle$	0.3947
C_9	AC	PA	PA	$\mathbb{Y}\langle 0.3761, 0.7598\rangle$	0.2821
C_{10}	PA	AC	AC	$\mathbb{Y}\langle 0.3851, 0.7372\rangle$	0.3024
C_{11}	AC	IN	PA	$\mathbb{Y}\langle 0.3593, 0.7721\rangle$	0.2665
C_{12}	PA	IN	IN	$\mathbb{Y}\langle 0.2984, 0.8423\rangle$	0.1899

Table 9: Results of RER analysis employing the SWARA technique

Criteria	**Score PFN Values**	$\mathbf{s_j}$	$\mathbf{k_j}$	$\mathbf{p_j}$	SW_j
C_1	0.6805	–	1.0000	1.0000	0.0837
C_2	0.6707	0.0098	1.0098	0.9903	0.0829
C_3	0.6055	0.0652	1.0652	0.9480	0.0794
C_4	0.5834	0.0221	1.0221	1.0422	0.0872
C_5	0.5587	0.0247	1.0247	0.9975	0.0835
C_6	0.5296	0.0291	1.0291	0.9957	0.0833
C_7	0.3947	0.1349	1.1349	0.9068	0.0759
C_8	0.3861	0.0086	1.0086	1.1252	0.0942
C_9	0.3024	0.0837	1.0837	0.9307	0.0779
C_{10}	0.2821	0.0203	1.0203	1.0621	0.0889
C_{11}	0.2665	0.0156	1.0156	1.0046	0.0841
C_{12}	0.1899	0.0766	1.0766	0.9433	0.0790

By employing equations (11) and (12), we compute the optimal and sub optimal values for the renewable energy resource options.

$$
\begin{aligned}
\mathbb{N}^{p+} = \big\{ & \mathbb{Y}\langle 0.2707, 0.8421), \mathbb{Y}\langle 0.2801, 0.8286), \mathbb{Y}\langle 0.2817, 0.8263), \mathbb{Y}\langle 0.3057, 0.8055), \\
& \mathbb{Y}\langle 0.8400, 0.3021), \mathbb{Y}\langle 0.2817, 0.9378), \mathbb{Y}\langle 0.3267, 0.7871), \mathbb{Y}\langle 0.3267, 0.7871) \\
& \mathbb{Y}\langle 0.3783, 0.7916), \mathbb{Y}\langle 0.8126, 0.3859), \mathbb{Y}\langle 0.3957, 0.7797), \mathbb{Y}\langle 0.6741, 0.4300) \big\} \\
\mathbb{N}^{p-} = \big\{ & \mathbb{Y}\langle 0.1999, 0.9700\rangle, \mathbb{Y}\langle 0.2224, 0.9321\rangle, \mathbb{Y}\langle 0.2224, 0.9321\rangle, \mathbb{Y}\langle 0.2224, 0.9321\rangle, \\
& \mathbb{Y}\langle 0.9359, 0.2210\rangle, \mathbb{Y}\langle 0.4400, 0.6603\rangle, \mathbb{Y}\langle 0.2224, 0.9321\rangle, \mathbb{Y}\langle 0.2420, 0.9001\rangle \\
& \mathbb{Y}\langle 0.6039, 0.4653\rangle, \mathbb{Y}\langle 0.4221, 0.6684\rangle, \mathbb{Y}\langle 0.4895, 0.5961\rangle, \mathbb{Y}\langle 0.5021, 0.5857\rangle \big\}
\end{aligned}
$$

10.1. Results and Discussion

The distances between each alternative and the positive ideal solution were calculated using equation (13), and the distances from the negative ideal solution using equation (14). The computed results are presented in Table 11. Subsequently, the closeness coefficients were determined using equation (15), enabling the ranking of the alternatives based on

Table 10: Evaluating RERs: Normalized Aggregated Pythagorean Fuzzy Decision Matrix

$\mathbf{z_{ij}}$	D_1	D_2	D_3	D_4	D_5
C_1	$\mathbb{Y}\langle 0.1999, 0.9700\rangle$	$\mathbb{Y}\langle 0.2224, 0.9321\rangle$	$\mathbb{Y}\langle 0.2501, 0.8701\rangle$	$\mathbb{Y}\langle 0.2707, 0.8421\rangle$	$\mathbb{Y}\langle 0.2501, 0.8701\rangle$
C_2	$\mathbb{Y}\langle 0.2224, 0.9321\rangle$	$\mathbb{Y}\langle 0.2531, 0.8853\rangle$	$\mathbb{Y}\langle 0.2224, 0.9321\rangle$	$\mathbb{Y}\langle 0.2801, 0.8286\rangle$	$\mathbb{Y}\langle 0.2303, 0.8398\rangle$
C_3	$\mathbb{Y}\langle 0.2321, 0.9111\rangle$	$\mathbb{Y}\langle 0.2817, 0.8263\rangle$	$\mathbb{Y}\langle 0.2411, 0.8901\rangle$	$\mathbb{Y}\langle 0.2501, 0.8701\rangle$	$\mathbb{Y}\langle 0.2224, 0.9321\rangle$
C_4	$\mathbb{Y}\langle 0.2501, 0.8701\rangle$	$\mathbb{Y}\langle 0.2420, 0.9001\rangle$	$\mathbb{Y}\langle 0.3057, 0.8055\rangle$	$\mathbb{Y}\langle 0.2224, 0.9321\rangle$	$\mathbb{Y}\langle 0.2501, 0.8701\rangle$
C_5	$\mathbb{Y}\langle 0.9001, 0.2581\rangle$	$\mathbb{Y}\langle 0.9321, 0.2224\rangle$	$\mathbb{Y}\langle 0.9359, 0.2210\rangle$	$\mathbb{Y}\langle 0.8400, 0.3021\rangle$	$\mathbb{Y}\langle 0.8562, 0.2754\rangle$
C_6	$\mathbb{Y}\langle 0.4400, 0.6603\rangle$	$\mathbb{Y}\langle 0.2817, 0.9378\rangle$	$\mathbb{Y}\langle 0.3687, 0.8222\rangle$	$\mathbb{Y}\langle 0.4282, 0.6911\rangle$	$\mathbb{Y}\langle 0.4282, 0.6721\rangle$
C_7	$\mathbb{Y}\langle 0.2420, 0.9001\rangle$	$\mathbb{Y}\langle 0.2224, 0.9321\rangle$	$\mathbb{Y}\langle 0.3107, 0.8316\rangle$	$\mathbb{Y}\langle 0.2707, 0.8423\rangle$	$\mathbb{Y}\langle 0.3267, 0.7871\rangle$
C_8	$\mathbb{Y}\langle 0.2420, 0.9001\rangle$	$\mathbb{Y}\langle 0.2817, 0.8263\rangle$	$\mathbb{Y}\langle 0.3107, 0.8314\rangle$	$\mathbb{Y}\langle 0.2705, 0.8421\rangle$	$\mathbb{Y}\langle 0.3267, 0.7871\rangle$
C_9	$\mathbb{Y}\langle 0.5501, 0.5001\rangle$	$\mathbb{Y}\langle 0.3783, 0.7916\rangle$	$\mathbb{Y}\langle 0.4284, 0.6723\rangle$	$\mathbb{Y}\langle 0.5298, 0.5832\rangle$	$\mathbb{Y}\langle 0.6039, 0.4653\rangle$
C_{10}	$\mathbb{Y}\langle 0.4833, 0.5762\rangle$	$\mathbb{Y}\langle 0.8126, 0.3859\rangle$	$\mathbb{Y}\langle 0.6913, 0.4093\rangle$	$\mathbb{Y}\langle 0.4221, 0.6684\rangle$	$\mathbb{Y}\langle 0.4833, 0.5762\rangle$
C_{11}	$\mathbb{Y}\langle 0.4676, 0.6266\rangle$	$\mathbb{Y}\langle 0.3957, 0.7797\rangle$	$\mathbb{Y}\langle 0.4048, 0.7736\rangle$	$\mathbb{Y}\langle 0.4572, 0.6325\rangle$	$\mathbb{Y}\langle 0.4895, 0.5961\rangle$
C_{12}	$\mathbb{Y}\langle 0.5021, 0.5857\rangle$	$\mathbb{Y}\langle 0.6706, 0.4300\rangle$	$\mathbb{Y}\langle 0.6741, 0.4666\rangle$	$\mathbb{Y}\langle 0.5963, 0.4897\rangle$	$\mathbb{Y}\langle 0.5355, 0.5292\rangle$

Table 11: Evaluating the closeness coefficient and ranking the Disease

Alternatives	$\Gamma(z_{ij}, \mathbb{N}^{p+})$	$\Gamma(z_{ij}, \mathbb{N}^{p-})$	$\mathbb{R}_i$	**Ranking**
D_1	0.0249	0.0488	0.6621	1
D_2	0.0526	0.0451	0.4616	4
D_3	0.0332	0.0531	0.6153	2
D_4	0.0322	0.0497	0.6068	3
D_5	0.0566	0.0309	0.3531	5

their relative proximity to the ideal solution. As shown in Table 11, alternative D_1 (solar energy) achieved the highest closeness coefficient, indicating that it is the most favorable among the evaluated renewable energy alternatives. The final preference ranking is as follows: $D_1 > D_3 > D_4 > D_2 > D_5$. This ranking highlights the practical applicability of the proposed PF-SWARA–TOPSIS framework in addressing complex decision problems under uncertainty. The top-ranked alternative, solar energy, outperformed others across key criteria such as long-term sustainability, minimal environmental impact, and broad regional applicability. These characteristics, along with moderate cost and increasing technological advancement, make solar energy a reliable and scalable solution in contemporary energy planning. The second-ranked alternative, D_3 (e.g., wind energy), also performed well but may have been affected by region-specific variability or higher initial infrastructure costs, slightly reducing its overall score. Biomass and hydroelectric energy (D_4 and D_2) ranked in the middle, possibly due to trade-offs in environmental or economic performance. The lowest-ranked alternative, D_5 (e.g., geothermal energy), may have been influenced by geographical constraints or relatively lower importance assigned to its attributes by experts. These results also demonstrate the flexibility and robustness of the proposed distance measure, particularly in distinguishing between alternatives with close or uncertain evaluations. Traditional methods may struggle to differentiate such cases, but the enhanced Pythagorean fuzzy-based model ensures more consistent and interpretable outcomes. From a practical standpoint, the proposed decision-making framework offers a structured, expert-informed, and mathematically sound tool for policymakers, regional planners, and energy stakeholders. By effectively integrating expert judgments and managing imprecise information through the PFS framework, the model supports transparent, data-driven, and sustainable energy decision-making.

11. COMPARATIVE STUDY

To evaluate the credibility of our proposed method, we conducted a thorough comparative analysis against established approaches, as outlined in Table 12. This section explores three distinct scenarios: one without subjective weighting, another incorporating subjective weighting using various ranking methods, and a third employing a different ranking approach. By extending our investigation to include PF-TOPSIS [6] and introducing random criteria weights without considering expert input, we observed variations in rankings compared to those generated by our proposed method. Similarly, when employing different ranking methods without evaluating expert weights and assuming criteria weights, such as the PF-VIKOR Method [53], disparities in rankings become evident. Furthermore, by applying the PF-Entropy-SWARA-MARCOS Method [52], which employs the Entropy measure for objective weights, the SWARA technique for deriving subjective weights, and the MARCOS method for ranking, we uncovered additional discrepancies. Our literature review indicates that TOPSIS provides the best ranking compared to other methods. These instances underscore the robustness of our proposed approach; neglecting any one of these considerations results in divergent rankings. However, all existing techniques identify that D_1 (Solar Power) has the best RERs compared to others.

Table 12: The comparative study of the existing vs proposed technique

Methods	Standard	Experts	Criteria	Ranking
[6]	TOPSIS	No	Assumed	$D_1 > D_3 > D_4 > D_5 > D_2$
[53]	VIKOR	No	Assumed	$D_1 > D_3 > D_5 > D_4 > D_2$
[52]	Entropy-SWARA-MARCOS	Yes	Combined	$D_1 > D_3 > D_2 > D_4 > D_5$
Proposed	SWARA- TOPSIS	Yes	Subjective	$D_1 > D_3 > D_4 > D_2 > D_5$

According to Table 12, there are multiple methods to perform an evaluation and each one has its own strengths and weaknesses. What makes this approach so special is the fact that it takes into account expert opinion when calculating the importance of different criteria through assigning them weights subjectively. In such way a D_1 alternative consistently becomes a leader under all methods. Therefore, the authors show their strong preference towards D_1, meaning that it best meets selected standards or performs well against them. This kind of detailed comparisons help us understand how effective each approach can be in relation to specific requirements and serve as great tools for decision-making processes within given field.

Note: The choice of solar energy can be wise in several real-life circumstances. It is an inexhaustible energy source that is environmentally friendly, cutting down on fossil fuels and thus mitigating climate change. Solar panels can be installed on rooftops or open spaces, generating clean electricity for homes, industries, or villages. The efficiency of solar power has increased while the cost has decreased with time due to technological advancements that have made it affordable to many people. Moreover, in most cases where there are solar systems, they may also have some form of battery storage system attached, which makes them independent from the grid, providing reliable power, especially during blackouts or areas with no access to electricity. Therefore, adopting solar energy leads to sustainability and economic benefits and energy security.

12. CONCLUSION

In conclusion, the intricacy inherent in selecting the most appropriate renewable energy source in real-world scenarios underscores the necessity for advanced methodologies. Pythagorean fuzzy sets, when combined with tailored distance metrics, emerge as potent tools for navigating this intricate landscape. This study introduces a novel distance measure, specifically designed to address the limitations of existing metrics, and explores its mathematical properties. Our benchmark analysis highlights the promise of this measure, demonstrating its potential to improve decision making. Furthermore, our integrated approach to selecting renewable energy sources was demonstrated through numerical testing, in which solar power emerged as the optimal choice among the alternatives . The ranking derived from our methodology underscores its effectiveness: $D_1 > D_3 > D_4 > D_2 > D_5$. To enhance the reliability and trustworthiness of the proposed method, we conducted an in-depth comparative analysis using established methods, confirming its effectiveness. In the future, the applicability of this research will expand to many higher Pythagorean fuzzy sets, promising diverse perspectives and results. As we look to the future, different techniques applied to these problems will yield different results, paving the way for continued exploration and refinement.

List of Abbreviations

Abbreviation	Full form
PF	Pythagorean fuzzy
MCDM	Multicriteria decision making
SWARA	Stepwise weight assessment ratio analysis
TOPSIS	Technique for order of preference by similarity to the ideal solution
PFS	Pythagorean fuzzy sets
COCOSO	Combined compromise solution
COPRAS	Complex proportional assessment
MULTIMOORA	Multiple objective optimization on the basis of ratio analysis
ARAS	Ascending reticular activating system
PFN	Pythagorean fuzzy number
DEs	Decision experts
RERs	Renewable energy resources
APFD	Aggregated Pythagorean fuzzy decision
PFWA	Pythagorean fuzzy weighted averaging

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