INVENTORY MODELS FOR STOCK-DEPENDENT DEMAND AND TIME VARYING HOLDING COST UNDER DIFFERENT TRADE CREDITS

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Abstract: Deterioration, a continuous process of all items, may be low or high for different items. This paper develops an inventory model for stock-dependent demand and time varying holding cost under different trade credits, considering four different situations. The second order approximations are used for exponential terms. Optimal solutions are obtained using Mathematica 9.0 software. Numerical examples and sensitivity analysis are provided to illustrate the proposed model.

Keywords: Stock-dependent Demand, Deterioration, Credit, Cash Discount, Time-dependent Holding Cost.

MSC: 90BB05.

1. INTRODUCTION

It is generally considered that the demand rate depends on stock availability, price of items, etc. but it is observed that customers are more attracted by the products display on the shelf, its popularity and variety. In case of low stock in the shop, goods are generally treated as though they are not fresh. Levis et al. [1] established the model considering the fact that large amount of items displayed in a market attract customer to buy more. Baker and Urban [2] considered an inventory

model for a power-form inventory level dependent demand pattern. Datta and Pal [3] pointed out an EOQ model for instantaneous inventory level dependent demand. Urban and Baker [4] considered an inventory model for multivariate price, time and stock-induced demand. Pal and Chandra [5] studied a periodic review EOQ model with stock-dependent demand. Min et al. [6] developed a lot-sizing model for deteriorating items with a current stock-dependent demand and delay in payment. Soni and Shah [7] established the optimal ordering policy for retailer when demand is stock-dependent and the supplier offers two progressive credit periods. Soni [8] studied inventory model from two aspects (i) the demand rate is multivariate function of price and level of inventory and (ii) Delay in payment is permissible. Jiangtao et al. [9] pointed out a multi-item EOQ model for perishable item where the demand rate of the items are stock-dependent, two level trade credit is adopted and the restriction of inventory capacity is also discussed. Goyal and Chang [10] discussed an ordering-transfer inventory model to determine the retailers optimal order quantity and the number of transfer per order from the ware house to the display area. Some of the related articles in this direction are: Ray and Chaudhari [11], Wang and Gerchak [12], Dye and Ouyang [13], Chang [14], Cling [15], Urban [16], Teng et al. [17], Yang et al. [18], Chang et al. [19], Huang [20], Chung and Huang [21], Tripathi and Singh [22], Tripathi et al. [23] etc.

In several cases, the manufacturer allows the inventory owner a certain fixed period to settle his account. During this fixed period no interest is charged, but after this period manager has to pay an interest to the manufacturer. Goyal [24] pointed out the inventory model under condition of permissible delay in payment. Aggarwal and Jaggi [25] generalized Goyals model for deteriorating items. Ouyang et al. [26] considered an EOQ model for deteriorating items with partial backlogging and trade credits. Das et al. [27] considered an EOQ model with time varying demand under permissible delay in payments. Tripathi and Mishra [28] established an inventory model under trade credits. Bhunia et al. [29] considered an inventory model for single deteriorating item with two separate warehouses having different preserving facilities. Some of the articles related to trade credit can be found in Guchhail et al. [30], Chung et al. [31], Li et al. [32], Chan et al. [33], Teng et al. [34] and their references.

In the global economics we must consider the effect of inflation and time value of money. Aggarwal [35] considered a purchase inventory decision model for inflationary conditions. Chen [36] pointed out the EOQ model for deteriorating items with time proportion demand and shortage under inflation and time discounting. Sarkar et al. [37] considered retailers optimal replenishment decision under trade credit policy with the effects of inflation. Sarkar et al. [38] dealt with an economic manufacturing quantity model for the selling price and the time-dependent demand pattern. In this direction the work of Wu et al. [39], Tripathi [40, 41], Tripathi and Kumar [42] are worth mentioning.

The remaining part of the paper is framed as follows. Assumptions and notations are discussed in section 2. Mathematical formulation and optimal solution is obtained in section 3, followed by numerical examples. Sensitivity analysis is

presented in section 5. Conclusion and future research directions are mentioned in the last section.

2. ASSUMPTIONS AND NOTATIONS

The assumptions used in this manuscript are as follows:

1. The demand rate of items is stock-induced.

2. The time horizon is infinite and lead time is negligible.

3. The deterioration rate θ is constant and $0 < \theta < 1$.

4. The period of cash discount is less than the permissible delay period.

5. The holding cost varies with time.

The notations are as follows:

$I(t)$	:	Inventory level at time t
c	:	The purchase cost per unit
p	:	Selling price per unit
$D\{I(t)\} = \{\alpha + \beta I(t)\}$	:	Inventory induced demand, $\alpha > 0, 0 < \beta < 1$
θ	:	Deterioration rate, $0 < \theta < 1$
$h(t) = h.t$	:	Holding cost / item / time
s	:	Ordering cost per order
r	:	The cash discount $0 < r < 1$
I_c, I_d	:	The interest charged and earned per dollar / year, respectively
$M_1 \& M_2$	:	Cash discount and permissible delay periods, respectively, $M_2 > M_1$
T	:	Cycle time
$T_1^*, T_2^*, T_3^* \& T_4^*$	:	The optimal cycle time for cases I, II, III & IV, respectively
$\phi_1, \phi_2, \phi_3 \& \phi_4$	:	The total relevant cost per year for cases I, II, III & IV, respectively
$\phi_1^*, \phi_2^*, \phi_3^* \& \phi_4^*$	:	Optimal total relevant cost for cases I,II,III & IV, respectively
$Q_1^*, Q_2^*, Q_3^* \& Q_4^*$	:	The optimal order quantity for cases I, II, III & IV, respectively

3. MATHEMATICAL FORMULATION AND OPTIMAL SOLUTION

Any kind of inventory will decay either due to spoilage or demand of the items. So, the differential equation of inventory decay will be given by

$$\frac{dI(t)}{dt} + \theta I(t) = -\{\alpha + \beta I(t)\}, 0 \leq t \leq T, \tag{1}$$

under the condition $I(0) = Q$ (order quantity), $I(T) = 0$.
The total relevant cost/year consists of the following elements:

$$(i). \quad \text{Cost of placing order} \; = \frac{s}{T} \tag{2}$$

$$(ii). \quad \text{Cost of purchasing units} \; = \frac{cQ}{T} = c\alpha \left\{ 1 + \frac{(\theta + \beta)T}{2} \right\} \tag{3}$$

$$(iii). \quad \text{Holding cost} \; = \frac{h}{T} \int_0^T I(t)tdt = \frac{h\alpha T^2}{6} \left\{ 1 + \frac{(\theta + \beta)T}{4} \right\} \tag{4}$$

Four possible cases are discussed with respect to cash discount and interest charged and earned according to M_1 or M_2 and the length of T. For case I, the payment is paid at cash discount period to get a cash discount and $T \geq M_1$. For case II, the customer pays in full at M_1 but $T < M_1$. In the same manner, if the payments are paid at permissible delay periods to get the permissible delay and $T \geq M_2$, then it is case III. As to case IV, the customer pays in full at M_2 but $T < M_2$.

Case I: $T \geq M_1$

The discount saving per year by the customer is

$$\frac{rcQ}{T} = rc\alpha \left\{ 1 + \frac{(\theta + \beta)T}{2} \right\}. \tag{5}$$

According to assumption, the customer pays off all units ordered at time of cash discount to obtain profit. Consequently, the items in stock have to be financed (at interest rate I_c) after time M_1 and hence, the interest payable per year is

$$\frac{c(1 - r)I_c}{T} \int_{M_1}^T I(t)dt = \frac{c(1 - r)I_c\alpha(T - M_1)^2}{6T} \{3 + (\theta + \beta)(T - M_1)\}. \tag{6}$$

During $[0, M_1]$, the customer sells products and deposits the revenue into an account that earns I_d per dollar per year. Thus, interest earned per year is

$$\frac{pI_d}{T} \int_0^{M_1} \{\alpha + \beta I(t)\}tdt = \frac{\alpha pI_d M_1^2}{T} \left[\begin{array}{c} \left(\frac{1}{2} - \frac{\beta M_1}{3} + \frac{\beta(\theta+\beta)M_1^2}{8} \right) \\ +\beta \left\{ \frac{1}{2} + \frac{(\theta+\beta)M_1}{3} \right\} T + \frac{\beta(\theta+\beta)}{4} T^2 \end{array} \right]. \tag{7}$$

The total relevant cost per year ϕ_1 is given by

$$
\begin{aligned}
\phi_1 \quad = \quad & \frac{s}{T} + c\alpha(1 - r)\left\{ 1 + \frac{(\theta + \beta)T}{2} \right\} + \frac{h\alpha T^2}{6}\left\{ 1 + \frac{(\theta + \beta)T}{4} \right\} \\
& + \frac{c(1 - r)I_c\alpha(T - M_1)^2}{6T}\{3 + (\theta + \beta)(T - M_1)\} \\
& - \frac{\alpha pI_d M_1^2}{T}\left[\begin{array}{c} \left(\frac{1}{2} - \frac{\beta M_1}{3} + \frac{\beta(\theta+\beta)M_1^2}{8} \right) \\ +\beta\left\{ \frac{1}{2} + \frac{(\theta+\beta)M_1}{3} \right\}T + \frac{\beta(\theta+\beta)}{4}T^2 \end{array} \right].
\end{aligned} \tag{8}
$$

Case II: $T < M_1$

In this case the customer sells $\{\alpha + \beta I(T)\}T$ units in total at time T and has

$c(1 - r)\{\alpha + \beta I(T)\}T$ to pay the supplier in full at time M_1. Consequently, there is no interest payable, while the cash discount is the same as that in case I. The interest earned per year is

$$\frac{pI_d}{T} \int_0^T \{\alpha + \beta I(t)\}t\,dt + (M_1 - T) \int_0^T \{\alpha + \beta I(t)\} =$$

$$pI_d \left[\alpha M_1 - \frac{\alpha}{2}(1 - \beta M_1)T + \frac{\alpha\beta}{6}\{(\theta + \beta)M_1 - 2\}T^2 - \frac{\alpha\beta(\theta + \beta)}{8}T^3 \right]. \quad (9)$$

The total relevant cost per year ϕ_2 is given by

$$\phi_2 = \frac{s}{T} + c\alpha(1 - r)\left\{1 + \frac{(\theta + \beta)T}{2}\right\} + \frac{h\alpha T^2}{6}\left\{1 + \frac{(\theta + \beta)T}{4}\right\}$$

$$- pI_d \left[\alpha M_1 - \frac{\alpha}{2}(1 - \beta M_1)T + \frac{\alpha\beta}{6}\{(\theta + \beta)M_1 - 2\}T^2 - \frac{\alpha\beta(\theta + \beta)}{8}T^3 \right].$$

$$(10)$$

Case III: $T \geq M_2$

In this case, the payment is paid at time M_2, there is no cash discount. The interest payable per year is

$$\frac{c(1 - r)I_c}{T} \int_{M_2}^T I(t)\,dt = \frac{c(1 - r)I_c\alpha(T - M_2)^2}{6T}\{3 + (\theta + \beta)(T - M_2)\}. \quad (11)$$

The interest earned per year is

$$\frac{pI_d}{T} \int_0^{M_2} \{\alpha + \beta I(t)\}t\,dt = \frac{\alpha pI_d M_2^2}{T} \left[\begin{array}{l} \left(\frac{1}{2} - \frac{\beta M_2}{3} + \frac{\beta(\theta+\beta)M_2^2}{8}\right) \\ + \beta\left\{\frac{1}{2} - \frac{(\theta+\beta)M_2}{3}\right\}T + \frac{\beta(\theta+\beta)}{4}T^2 \end{array} \right]. \quad (12)$$

The total relevant cost per year ϕ_3 is given by

$$\begin{aligned} \phi_3 \quad = \quad & \frac{s}{T} + c\alpha\left\{1 + \frac{(\theta + \beta)T}{2}\right\} + \frac{h\alpha T^2}{6}\left\{1 + \frac{(\theta + \beta)T}{4}\right\} \\ & + \frac{cI_c\alpha(T - M_2)^2}{6T}\{3 + (\theta + \beta)(T - M_2)\} \\ & - \frac{\alpha pI_d M_1^2}{T} \left[\begin{array}{l} \left(\frac{1}{2} - \frac{\beta M_1}{3} + \frac{\beta(\theta+\beta)M_1^2}{8}\right) \\ + \beta\left\{\frac{1}{2} + \frac{(\theta+\beta)M_1}{3}\right\}T + \frac{\beta(\theta+\beta)}{4}T^2 \end{array} \right]. \end{aligned} \quad (13)$$

Case IV: $T < M_2$

In this case, there is no interest charged. The interest earned per year is

$$\frac{pI_d}{T} \int_0^T \{\alpha + \beta I(t)\}t\,dt + (M_2 - T) \int_0^T \{\alpha + \beta I(t)\} =$$

$$pI_d \left[\alpha M_2 - \frac{\alpha}{2}(1 - \beta M_2)T + \frac{\alpha\beta}{6}\{(\theta + \beta)M_2 - 2\}T^2 - \frac{\alpha\beta(\theta + \beta)}{8}T^3 \right]. \quad (14)$$

The total relevant cost per year ϕ_4 is given by

$$
\begin{aligned}
\phi_4 \quad = \quad & \frac{s}{T} + c\alpha \left\{ 1 + \frac{(\theta + \beta)T}{2} \right\} + \frac{h\alpha T^2}{6} \left\{ 1 + \frac{(\theta + \beta)T}{4} \right\} \\
& + \frac{cI_c\alpha(T - M_2)^2}{6T} \{ 3 + (\theta + \beta)(T - M_2) \} \\
& - pI_d \left[\begin{array}{l} \alpha M_2 - \frac{\alpha}{2}(1 - \beta M_2)T + \frac{\alpha\beta}{6}\{(\theta + \beta)M_2 - 2\}T^2 \\ - \frac{\alpha\beta(\theta + \beta)}{8}T^3 \end{array} \right].
\end{aligned} \tag{15}
$$

Differentiating the total cost equations (8), (10), (13) and (15) for cases I, II, III and IV, respectively and further equating them to zero to find the optimal solutions, we get the following results:

$$
\begin{aligned}
\frac{d\phi_1}{dT} = & -\frac{s}{T^2} + \frac{1}{2}c\alpha(1 - r)(\theta + \beta) + \frac{1}{6}h\alpha T \left\{ 2 + \frac{3}{4}(\theta + \beta)T \right\} \\
& + \frac{1}{6}c\alpha I_c(1 - r) \left\{ 3\left(1 - \frac{M_1^2}{T^2}\right) + (\theta + \beta)\left(2T + \frac{M_1^3}{T^2} - 3M_1\right) \right\} \\
& + \alpha pI_d M_1^2 \left\{ \left(\frac{1}{2} - \frac{1}{3}\beta M_1 + \frac{1}{8}\beta M_1^2(\theta + \beta) \right) \frac{1}{T_2} - \frac{1}{4}\beta(\theta + \beta) \right\} = 0
\end{aligned} \tag{16}
$$

$$
\begin{aligned}
\frac{d\phi_2}{dT} = & -\frac{s}{T^2} + \frac{1}{2}c\alpha(1 - r)(\theta + \beta) + \frac{1}{6}h\alpha T \left\{ 2 + \frac{3}{4}(\theta + \beta)T \right\} \\
& - pI_d \left[\frac{1}{3}\alpha\beta\{(\theta + \beta)M_1 - 2\}T - \frac{1}{2}\alpha(1 - \beta M_1) - \frac{3}{8}\alpha\beta(\theta + \beta)T^2 \right] = 0
\end{aligned} \tag{17}
$$

$$
\begin{aligned}
\frac{d\phi_3}{dT} = & -\frac{s}{T^2} + \frac{1}{2}c\alpha(\theta + \beta) + \frac{1}{6}h\alpha T \left\{ 2 + \frac{3}{4}(\theta + \beta)T \right\} \\
& + \frac{1}{6}c\alpha I_c \left\{ 3\left(1 - \frac{M_2^2}{T^2}\right) + (\theta + \beta)\left(2T + \frac{M_2^3}{T^2} - 3M_2\right) \right\} \\
& + \alpha pI_d M_2^2 \left\{ \left(\frac{1}{2} - \frac{1}{3}\beta M_2 + \frac{1}{8}\beta M_2^2(\theta + \beta) \right) \frac{1}{T^2} - \frac{1}{4}\beta(\theta + \beta) \right\} = 0
\end{aligned} \tag{18}
$$

$$
\begin{aligned}
\frac{d\phi_4}{dT} = & -\frac{s}{T^2} + \frac{1}{2}c\alpha(\theta + \beta) + \frac{1}{6}h\alpha T \left\{ 2 + \frac{3}{4}(\theta + \beta)T \right\} \\
& - pI_d \left[\frac{1}{3}\alpha\beta\{(\theta + \beta)M_2 - 2\}T - \frac{1}{2}\alpha(1 - \beta M_2) - \frac{3}{8}\alpha\beta(\theta + \beta)T^2 \right] = 0
\end{aligned} \tag{19}
$$

and

$$
\begin{aligned}
\frac{d^2\phi_1}{dT^2} = & \frac{2s}{T^3} + \frac{1}{12}h\alpha\{4 + 3(\theta + \beta)T\} \\
& + \frac{1}{3}c\alpha I_c(1 - r) \left\{ 3\left(\frac{M_1^2}{T^3} \right) + (\theta + \beta)\left(1 - \frac{M_1^3}{T^3}\right) \right\} \\
& - \frac{2\alpha pI_d M_1^2}{T^3} \left\{ \frac{1}{2} - \frac{1}{3}\beta M_1 + \frac{1}{8}\beta M_1^2(\theta + \beta) \right\}
\end{aligned} \tag{20}
$$

$$\frac{d^2\phi_2}{dT^2} = \frac{2s}{T^3} + \frac{1}{12}h\alpha\{4 + 3(\theta + \beta)T\} - \frac{1}{12}pI_d\alpha\beta\{(\theta + \beta)(4M_1 - T) - 8\}$$

(21)

$$\frac{d^2\phi_3}{dT^2} = \frac{2s}{T^3} + \frac{1}{12}h\alpha\{4 + 3(\theta + \beta)T\}$$
$$+ \frac{1}{3}c\alpha I_c \left\{ 2\left(\frac{M_2^2}{T^3}\right) + (\theta + \beta)\left(1 - \frac{M_2^3}{T^3}\right) \right\}$$
$$- \frac{2\alpha pI_d M_2^2}{T^3} \left\{ \frac{1}{2} - \frac{1}{3}\beta M_2^2(\theta + \beta) \right\}$$

(22)

$$\frac{d^2\phi_4}{dT^2} = \frac{2s}{T^3} + \frac{1}{12}h\alpha\{4 + 3(\theta + \beta)T\} - \frac{1}{12}pI_d\alpha\beta\{(\theta + \beta)(4M_2 - T) - 8\}$$

(23)

Since $\frac{d^2\phi_i}{dT^2} > 0, i = 1, 2, 3, 4$ therefore, the solutions obtained are global minimum. The second derivatives of all four cases are positive, which is shown in the last column of sensitivity analysis as well as in the Appendix section.

4. NUMERICAL EXAMPLES

In this section, the model is illustrated by numerical examples for all four different cases using Mathematica 9.0 software.

Let us consider the following parameters in appropriate units for all four different cases:

Example 1. *(Case I):*
$\alpha = 150, h = 15, I_c = 0.09, I_d = 0.06, c = 20, p = 35, \theta = 0.02, r = 0.02, M_1 = 0.03, s = 10, \beta = 0.2$. *Substituting these values in Eq. (16), we get $T_1^* = 0.134666$ year and the corresponding values of $Q_1^* = 20.4991$ units and $\phi_1^* = \$3074.45$.*

Example 2. *(Case II):*
$\alpha = 1000, h = 11, I_c = 0.15, I_d = 0.1, c = 20, p = 35, \theta = 0.02, r = 0.02, M_1 = 0.082 = 30$ *days* $, s = 5$, *and* $\beta = 0.2$. *Substituting these values in Eq. (17), we get $T_2^* = 0.0352779$ year and the corresponding values of $Q_2^* = 35.4148$ units and $\phi_2^* = \$19870.8$.*

Example 3. *(Case III):*
$\alpha = 1000, h = 4, I_c = 0.09, I_d = 0.06, c = 5, p = 70, \theta = 0.02, r = 0.02, M_2 = 0.027 = 10$ *days* $, s = 10$, *and* $\beta = 0.2$. *Substituting these values in Eq. (18), we get $T_3^* = 0.113036$ year and the corresponding values of $Q_3^* = 114.441$ units and $\phi_3^* = \$5186.9$.*

Example 4. *(Case IV):*
$\alpha = 1000, h = 30, I_c = 0.30, I_d = 0.15, c = 70, p = 100, \theta = 0.05, r = 0.02, M_2 = 0.041 = 15$ *days* $, s = 5$, *and* $\beta = 0.2$. *Substituting these values in Eq. (19), we get $T_4^* = 0.0174661$ year and the corresponding values of $Q_4^* = 17.5042$ units and $\phi_4^* = \$70247.3$.*

5. SENSITIVITY ANALYSIS

In this section sensitivity analysis is carried out for the present model. The following Tables 1-4 show the variation in the optimal inventory policy with change in a model parameter, when other parameters remain constant.

Table 1. Case I Variation of Q and Φ with s, I_c and r

Table 1.a

s	T_1(in years)	Q_1 units	ϕ_1^* in dollars	$\frac{d^2\phi_1}{dT^2}$
10	0.134666	20.4991	3074.45	8952.98
20	0.184352	28.2136	3137.00	7165.75
30	0.220684	33.9062	3186.33	6371.65
40	0.250278	38.5752	3228.77	5897.0
50	0.275648	42.6009	3266.79	5572.51

Table 1.b

I_c	T_1(in years)	Q_1 units	ϕ_1^* in dollars	$\frac{d^2\phi_1}{dT^2}$
0.09	0.134666	20.4991	3074.45	8952.98
0.08	0.136231	20.7409	3073.22	8662.35
0.07	0.137850	20.9910	3071.97	8375.00
0.06	0.139526	21.2501	3070.70	8093.94
0.05	0.141264	21.5189	3069.40	7815.00

Table 1.c

r	T_1(in years)	Q_1 units	ϕ_1^* in dollars	$\frac{d^2\phi_1}{dT^2}$
0.02	0.134666	20.4991	3074.45	8952.98
0.03	0.134518	20.5785	3043.89	8927.81
0.04	0.135699	20.6587	3013.33	8716.27
0.05	0.136224	20.7398	2982.77	8624.23
0.06	0.136755	20.8218	2952.21	8532.58

Table 2. Case II Variation of Q and Φ with s and r

Table 2.a

s	T_2 in years	Q_2 units	ϕ_2^* in dollars	$\frac{d^2\phi_2}{dT^2}$
5	0.0352779	35.4148	19870.8	231905
10	0.0495406	49.8106	20258.4	168641
15	0.0603549	60.7556	20614.6	140609
20	0.0693874	69.9170	20953.7	123897
25	0.0772831	77.9401	21281.0	112490

Table 2.b

r	T_2(in years)	Q_2 units	ϕ_2^* in dollars	$\frac{d^2\phi_2}{dT^2}$
0.02	0.0352779	35.4148	19870.8	231905
0.03	0.0354693	35.6077	19472.2	228238
0.04	0.0356638	35.8037	19070.6	224592
0.05	0.0358614	36.0029	18675.3	220968
0.06	0.0360623	36.2054	18277.0	217364

Table 3. Case III Variation of Q and Φ with s, I_c and r

Table 3.a

s	T_3 in years	Q_3 units	ϕ_3^* in dollars	$\frac{d^2\phi_3}{dT^2}$
10	0.113036	114.441	5186.90	13345.20
15	0.133132	135.082	5227.50	12948.00
20	0.149986	152.461	5262.81	12440.50
25	0.164701	167.685	5294.59	11977.75
30	0.177872	181.352	5295.66	11576.85

Table 3.b

I_c	T_3 (in years)	Q_3 units	ϕ_3^* in dollars	$\frac{d^2\phi_3}{dT^2}$
0.09	0.113036	114.441	5186.90	13345.20
0.08	0.114366	115.805	5185.31	12905.30
0.07	0.115745	117.219	5183.68	12471.84
0.06	0.117175	118.685	5182.02	12044.93
0.05	0.118659	120.208	5180.33	11624.74

Table 3.c

h	T_3 (in years)	Q_3 units	ϕ_3^* in dollars	$\frac{d^2\phi_3}{dT^2}$
4	0.113036	114.441	5186.90	13345.2
8	0.105907	107.141	5194.60	1829.67
12	0.100487	101.598	5201.44	2235.27
16	0.096142	97.1584	5207.65	2633.49
20	0.092534	93.4754	5213.37	3026.25

Table 4. Case IV Variation of Q and Φ with s and β

.

Table 4.a

s	T_4 in years	Q_4 units	ϕ_4^* in dollars	$\frac{d^2\phi_4}{dT^2}$
5	0.0174661	17.5042	70247.3	1888759.03
10	0.0246386	24.7145	70773.9	1349157.10
15	0.0301181	30.2315	71243.8	1110106.44
20	0.0347214	34.8721	71684.1	967609.00
25	0.038765	38.9528	72105.2	870359.00

Table 4.b

β	T_4 (in years)	Q_4 units	ϕ_4^* in dollars	$\frac{d^2\phi_4}{dT^2}$
0.2	0.0174661	17.5042	70247.3	1888759.03
0.3	0.0158686	15.9127	70374.0	2515544.77
0.4	0.0146415	14.6897	70475.0	3199967.75
0.5	0.0136608	13.7121	70559.0	3937560.18
0.6	0.0128538	12.9075	70632.2	4724722.26

All the observations discussed in the above tables can be summed up as follows:

(i) Increase of s causes increase in Q and Φ.

(ii) Increase of I_c results in slight increase in Q and slight decrease in Φ.

(iii) Increase of r will lead to slight increase in Q and decrease in Φ.

(iv) Increase of h leads to slight decrease in Q and increase in Φ.

(v) Increase of β results decrease in Q and increase in Φ.

6. CONCLUSION AND FUTURE WORK

This paper develops a periodic review EOQ model with inventory-induced demand under deterioration and cash discount. The period of cash discount is considered to be shorter than the period of permissible delay. Four different cases have been discussed. The optimal solutions are obtained by using second order approximation and differential calculus. The main objective of this study is to minimize the total cost. Through numerical study and sensitivity analysis, it is seen that high discount rate results in slight increase in order quantity and decrease in total cost.

This paper may be further generalized for allowing shortages. We could also extend the model by adding freight charges and advertisement costs.

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APPENDIX

From Eq. (20), we get

$$\frac{d^2\phi_1}{dT^2} = \frac{1}{T}\left[2s - \alpha p I_d M_1^2 \left\{1 - \beta M_1 \left(\frac{2}{3} - \frac{1}{4}M_1\theta - \frac{1}{4}M_1\beta\right)\right\}\right] + \frac{1}{12}h\alpha\{4 + 3(\theta+\beta)T\}$$

$$+ c\alpha I_c(1-r)\left\{\left(\frac{M_1^2}{T^3}\right) + \frac{(\theta+\beta)}{3}\left(1 - \frac{M_1^3}{T^3}\right)\right\} \qquad (A_1)$$

Since $I_d << 1, M_1 < 1, \beta < 1, r < 1$, thus $\left\{1 - \beta M_1 \left(\frac{2}{3} - \frac{1}{4}M_1\theta - \frac{1}{4}M_1\beta\right)\right\} < 1$. Again 2nd and 3rd terms of (A_1) are positive, therefore

$$\frac{d^2\phi_1}{dT^2} > 0, \text{ if } 2s - \alpha p I_d M_1^2 \left\{1 - \beta M_1 \left(\frac{2}{3} - \frac{1}{4}M_1\theta - \frac{1}{4}M_1\beta\right)\right\} > 0. \quad (A_2)$$

We have already shown in the sensitivity (in the last column). Eq. (21) can be written as

$$\frac{d^2\phi_2}{dT^2} = \frac{2s}{T^3} + \frac{1}{12}h\alpha\{4 + 3(\theta+\beta)T\} + \frac{p I_d \alpha \beta}{12}\{8 - (\theta+\beta)(4M_1 - T)\}.$$

Here $8 - (\theta + \beta)(4M_1 - T) > 0$, as $(\theta + \beta)(4M_1 - T) < 1$, therefore $\frac{d^2\phi_2}{dT^2} > 0$. Eq. (22) can be written as

$$\frac{d^2\phi_3}{dT^2} = \frac{1}{T^3}\left[2s - \alpha p I_d M_2^2\left\{1 - \beta M_2\left(\frac{2}{3} - \frac{1}{4}M_2\theta - \frac{1}{4}M_2\beta\right)\right\}\right]$$

$$+\frac{1}{12}h\alpha\{4 + 3(\theta + \beta)T\} + c\alpha I_c\left\{\left(\frac{M_2^2}{T^3}\right) + \frac{(\theta + \beta)}{3}\right\}\left(1 - \frac{M_2^3}{T^3}\right) \qquad (A_3)$$

Since $I_d << 1, M_2 < 1, \beta < 1$ and $\theta < 1, r < 1$, thus $\{1 - \beta M_2\left(\frac{2}{3} - \frac{1}{4}M_2\theta - \frac{1}{4}M_2\beta\right)\} < 1$. Again 2nd and 3rd terms of (A_3) are positive, therefore

$$\frac{d^2\phi_3}{dT^2} > 0, \text{ if } 2s - \alpha p I_d M_2^2\left\{1 - \beta M_2\left(\frac{2}{3} - \frac{1}{4}M_2\theta - \frac{1}{4}M_2\beta\right)\right\} > 0.$$

Eq. (23) can be written as

$$\frac{d^2\phi_4}{dT^2} = \frac{2s}{T^3} + \frac{1}{12}h\alpha\{4 + 3(\theta + \beta)T\}$$

$$+\frac{1}{12}pI_d\alpha\beta\{8 - (\theta + \beta)(4M_2 - T)\} \qquad (A_4)$$

Since $8 - (\theta + \beta)(4M_2 - T) > 0$, as $(\theta + \beta)(4M_2 - T) < 1$, therefore $\frac{d^2\phi_4}{dT^2} > 0$.