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# A REVIEW ON FUZZY AND STOCHASTIC EXTENSIONS OF THE MULTI INDEX TRANSPORTATION PROBLEM

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**Abstract**: The classical transportation problem (having source and destination as indices) deals with the objective of minimizing a single criterion, i.e. cost of transporting a commodity. Additional indices such as commodities and modes of transport led to the Multi Index transportation problem. An additional fixed cost, independent of the units transported, led to the Multi Index Fixed Charge transportation problem. Criteria other than cost (such as time, profit etc.) led to the Multi Index Bi-criteria transportation problem. The application of fuzzy and stochastic concept in the above transportation problems would enable researchers not only to introduce real life uncertainties but also to obtain solutions of these transportation problems. The review article presents an organized study of the Multi Index transportation problem and its fuzzy and stochastic extensions till today, and aims to help researchers working with complex transportation problems.

**Keywords**: Multi Index Transportation Problem, Fixed Charge, Bi-Criteria, Fuzzy Numbers, Stochastic Concept.

**MSC:** 90B06, 03E72.

## 1. INTRODUCTION

The classical transportation problem (TP) with the objective of minimizing cost of transportation of a single commodity from m sources to n destinations is well studied in literature. The classical TP considered two indices, source and destination and a single criterion as transportation cost. The problem is extended into Multi Index transportation problem (MITP) by considering commodities and modes of transport as additional indices.

The MITP becomes Multi Index Bi-criteria transportation problem (MIBCTP) when there are two requirements such as minimizing transportation time in addition to minimizing transporting cost. In MITPs, the cost of transportation is directly proportional to the number of units transported, which is known as direct cost. However, many times, apart from direct cost, some fixed costs are associated with the activities such as storage cost, toll charges, landing cost at an airport etc. which, in some cases, are independent of the number of units transported. The TP which takes into account direct, as well as fixed charges, is called Multi Index Fixed Charge transportation problem (MIFCTP).

In spite of many uncertainties in real life, till early 1970s, most of the MITPs were solved using only crisp values of transportation parameters. The causes of uncertainty may be weather conditions, high information cost, measurement inaccuracy, data unavailability etc., and in such cases, getting an optimal solution is nearly impossible. Since 1993, many researchers have applied either the fuzzy or the stochastic concept to MITP to deal with uncertainty in transportation problem.

Kumar and Yadav [51] gave a survey on various solution procedures of MITP and its crisp and fuzzy extensions till 2012. However, it fails to provide a detailed analysis on the type of MITPs as well as its extensions. This review article presents a concise and updated survey of Multi Index Multi Criteria Fixed Charge transportation problems using both fuzzy and stochastic parameters. In Section 1, we deal with an overview of MITP and the related research. Section 2 outlines the work done in the extensions of MITP such as Bi-criteria and Fixed Charge Transportation Problems. In Section 3, the Fuzzy MITP is considered in some detail. In Section 4 ,the extensions of Fuzzy MITP are listed briefly. Section 5 explains the stochastic concept, and presents the work done on stochastic transportation problems. In Section 6, the stochastic MITP is discussed. Lastly, in Section 7, the extensions of stochastic MITP are listed briefly.

## 2. MULTI INDEX TRANSPORTATION PROBLEM

The transportation problem (TP) having three or more indices such as source, destination, commodity, and type of transport is called Multi Index transportation problem (MITP). The well known ways of representing the constraints of MITP (3-index) are three planar sums and three axial sums.

(i) Mathematical formulation of MITP (3-index) with three planar sums can be represented as

$$\text{Minimize } z = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} c_{ijk}x_{ijk} \tag{1}$$

subject to the constraints

$$\left.\begin{array}{l}\sum_{i=1}^{m} x_{ijk} = A_{jk} \quad (j = 1,2,...,n) \text{ and } (k = 1,2,..., p) \\ \sum_{j=1}^{n} x_{ijk} = B_{ki} \quad (k = 1,2,..., p) \text{ and } (i = 1,2,...,m) \\ \sum_{k=1}^{p} x_{ijk} = E_{ij} \quad (i = 1,2,...,m) \text{ and } (j = 1,2,...,n) \\ x_{ijk} \geq 0, \quad (i = 1,2,...,m);(j = 1,2,...,n);(k = 1,2,..., p)\end{array}\right\} \tag{2}$$

$$\sum_{j=1}^{n} A_{jk} = \sum_{i=1}^{m} B_{ki}, \sum_{k=1}^{p} B_{ki} = \sum_{j=1}^{n} E_{ij}, \sum_{i=1}^{m} E_{ij} = \sum_{k=1}^{p} A_{jk} \tag{3}$$

$$\sum_{j=1}^{n}\sum_{k=1}^{p} A_{jk} = \sum_{k=1}^{p}\sum_{i=1}^{m} B_{ki} = \sum_{i=1}^{m}\sum_{j=1}^{n} E_{ij} \tag{4}$$

where $i = 1, 2, 3,..., m$ are the number of sources,

$j = 1, 2, 3,..., n$ are the number of destinations,

$k = 1, 2, 3,..., p$ are the various types of commodities,

$x_{ijk}$ = quantity of the $k^{th}$ type of commodity transported from the $i^{th}$ origin to the $j^{th}$ destination,

$c_{ijk}$ = variable cost per unit quantity of $k^{th}$ type of commodity transported from the $i^{th}$ origin to the $j^{th}$ destination which is independent of the quantity of commodity transported so long as $x_{ijk} > 0$ ,

$A_{jk}$ = total quantity of the $k^{th}$ type of commodity to be sent to the $j^{th}$ destination,

$B_{kj}$ = total quantity of the $k^{th}$ type of commodity available at the $i^{th}$ origin,

$E_{ij}$ = total quantity to be sent from the $i^{th}$ origin to the $j^{th}$ destination.

(ii) Mathematical formulation of MITP (3- index) with three axial sums can be represented as

$$\text{Minimize } z = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} c_{ijk} x_{ijk} \tag{5}$$

subject to the constraints

$$\left.\begin{array}{l}\sum_{j=1}^{n}\sum_{k=1}^{p} x_{ijk} = a_i \quad (i = 1,2,...,m) \\ \sum_{i=1}^{m}\sum_{k=1}^{p} x_{ijk} = b_j \quad (j = 1,2,...,n)\end{array}\right\} \tag{6}$$

$$\begin{array}{l}\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} = e_k \quad (k = 1,2,..., p) \\ x_{ijk} \geq 0, \ (i = 1,2,...,m);(j = 1,2,...,n);(k = 1,2,..., p) \\ \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j = \sum_{k=1}^{p} e_k\end{array} \tag{7}$$

where $a_i$ = total supply at source $i$ .

$b_j$ = total demand at destination $j$ ,

$e_k$ = total capacity of conveyance $k$

The above formulation is of a balanced MITP problem. Unbalanced MITP can be converted to a balanced MITP in the same manner as in classical TP.

Haley [31] called the MITP as Solid transportation problem (STP) and after comparing it with all features of a Classical TP, he showed MITP to be an extension of the Classical TP. He further solved the problem by an extension of the MODI method. Haley [32] gave a necessary condition for the existence of a feasible solution of MITP as

$$\left.\begin{aligned} \sum_{i=1}^{m} M_{ijk}^{(r)} \le A_{jk} \le \sum_{i=1}^{m} m_{ijk}^{(r)} \\ \sum_{j=1}^{n} M_{ijk}^{(r)} \le B_{ki} \le \sum_{j=1}^{n} m_{ijk}^{(r)} \\ \sum_{k=1}^{p} M_{ijk}^{(r)} \le E_{ij} \le \sum_{k=1}^{p} m_{ijk}^{(r)} \end{aligned}\right\} \tag{8}$$

where $M_{ijk}^{(r)}$ and $m_{ijk}^{(r)}$ are monotonically increasing and decreasing series of lower and upper bound of $x_{ijk}$ respectively. He also showed that unbalanced MITP can be converted to a balanced one by adding a dummy variable.

Further Haley [33] gave a necessary and sufficient condition for the existence of a solution of MITP, i.e. there exists a solution to MITP if and only if $x_{ijk}$ is bounded above by $m_{ijk}^{(r)}$ and below by $M_{ijk}^{(r)}$ . Moravek and Vlach [63], [64] have found the necessary condition for the existence of a solution of MITP, and they proved that the sufficient condition holds if p is less than or equal to two. Smith [94] gave a procedure for finding a necessary and sufficient condition for the existence of a solution of a MITP having indices less than or equal to three. Vlach [98] gave a survey for existence condition of the solution of the three index planar TP. Korsnikov [47] has considered a class of balanced planar three index TP and gave a procedure to reduce it to Classical TP by the dominating index of one of $A_{jk}$ , $B_{kj}$ , $E_{ij}$ .

Junginger [43] has considered the problem of school timetabling as MITP where set of teachers, classes, and hours are taken as the three indices. In this paper MITP is represented in terms of a characteristic matrix and a characteristic hypergraph. The four index, axial, TP has been considered by Bulut and Bulut [19], who have shown that both axial and planar four index TPs have the same algebraic characterization.

Pandian and Anuradha [74] applied a new method, based on the zero point method, to solve an axial solid transportation problem which gave a direct optimal solution without verifying the condition of degeneracy, i.e. the number of occupied cells must be at least (m+n+p-2). Bandopadhyaya and Puri [11] have handled the case of impaired flow in an axial MITP wherein some warehouses are either forced to close down or made to operate below operational level.

The objective of most MITP's is to minimize the cost of transportation. However, there are some specific emergency situations in which minimizing transportation time holds more importance than minimizing the cost. Bhatia, Swarup and Puri [15] have solved the time minimization MITP. The mathematical formulation of the problem is given below.

$$Min\,[\max_{\substack{1\le i\le m\\1\le j\le n\\1\le k\le p}} t_{ijk} \mid x_{ijk} > 0]$$

subject to constraints (2) and $t_{ijk}$ is the time required to transport the $k^{th}$ commodity from the $i^{th}$ source to the $j^{th}$ destination.

Zitouni et.al [105], and Djamel et.al [24] considered the MITP as a capacitated axial MITP by considering the capacity of each mode of transport as the fourth index, which is the same as solid transportation problem. Pham and Dott [75] gave an exact method for the solution of the four index transportation problem with elimination of degeneracy. Sharma and Bansal [91] have solved the n-index axial MITP with a method based on the simplex method and the two phase potential method, following convergence and pivot principles. The highlights of the work done in the area of MITP's are summarized below, in Table 1.

**Table1:** Multi Index Transportation Problem

| Sr. No. | Author | Type of MITP | Contribution |
|---|---|---|---|
| 1. | Haley [31] | MITP | Procedure based on an extension of MODI method to solve an MITP . |
| 2. | Haley[32] | MITP | Necessary condition for the solution of MITP. |
| 3. | Haley [33] | MITP | Necessary condition given by Haley [32] is shown to be sufficient also. |
| 4. | Moravek and Vlash [63],[64] | MITP | Necessary condition for solution of MITP and also showed it to be sufficient if p is less than or equal to two. |
| 5. | Smith [94] | MITP | Procedure to find a necessary and sufficient condition for the solution of MITP and proved that the sufficient condition holds for entities in which index is less than or equal to three. |
| 6. | Bhatia, Swarup and Puri [15] | MITP | Solved the time minimization MITP. |
| 7. | Vlash [98] | MITP | Surveyed the existence condition of the solution of a three index planar transportation problem. |
| 8. | Korsnikov [47] | MITP | Procedure to reduce the three index transportation problem into the Classical transportation problem by dominating index. |
| 9. | Bandopadhyaya and Puri [11] | MITP | Investigated three axial MITP with Impaired Flow. |
| 10. | Junginger [43] | MITP | Formulated the school time table problem as a MITP |
| 11. | Bulut and Bulut [19] | MITP | Showed that the axial and planar four index transportation problems have the same algebraic characterizations. |
| 12. | Zitouni [105] | MITP | Investigated the capacitated four index transportation problem and solved it by an algorithm based on the simplex and potential methods. |
| 13. | Pandian and Anuradha [74] | MITP | Zero Point Method to solve the solid transportation problem. |
| 14. | Sharma and Bansal [91] | MITP | Solved the Capacitated n-index transportation problem based on simplex and two phase potential method. |
| 15. | Djamel [24] | MITP | Algorithm to solve the four-index transportation problem. |
| 16. | Pham and Dott [75] | MITP | Solved the four index transportation problem by an exact method. |

## 3. EXTENSIONS OF MULTI INDEX TRANSPORTATION PROBLEM

Some of the MITP's may have two or more criteria along with fixed charges in their objective functions. Thus, depending on the nature of the objective function, MITP can be extended to Multi Index Bi-Criteria (MIBCTP), Multi Index Fixed Charge (MIFCTP), and Multi Index Bi-Criteria Fixed Charge (MIBCFCTP).

### *3.1. Multi Index Bi-Criteria Transportation Problem*

The Multi Criteria approach is a practical and effective way to handle real life transportation problems. In a business scenario, a businessman wants to minimize cost along with time in order to maintain his business performance. So, the transportation problem has an objective of minimizing two criteria cost and time. The MITP having the objective function to minimize two criteria, cost and time, is called the Multi Index Bi-Criteria transportation problem (MIBCTP). An MITP, with the objective to minimize two criteria time and cost, and to maximize another criterion, i.e. profit is called the Multi Index Multi Criteria transportation problem (MIMCTP).

The Mathematical Formulation for MIBCTP is given as

$$\text{Minimize}\left(\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} c_{ijk}x_{ijk}, \max_{\substack{1\le i\le m\\ 1\le j\le n\\ 1\le k\le p}}\left(t_{ijk} : x_{ijk} \ge 0\right)\right) \tag{9}$$

subject to constraints (2) or constraints (6) and $t_{ijk} \ge 0$

where $t_{ijk}$ is the time required to transport a commodity from source **i** to destination **j** via the **k^th^** mode of transport, and other notations are the same as give in Section 2 of this article.

Bi-Criteria transportation problem was solved by Aneja and Nair [6] by finding non-dominated extreme points in the criteria space. Gen, Ida and Li [28] solved the Solid Bi-Criteria transportation problem by using a genetic algorithm. The proposed method was a combination of Aneja's [6] criteria approach and the genetic algorithm.

Basu, Pal and Kundu [13] solved the MIBCTP to find the Pareto Optimal cost-time pairs. The authors have considered the total transportation cost as the sum of linear and quadratic fractional costs. The objective function was taken as

$$MinZ = \left(\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} c_{ijk}x_{ijk} + \left(\frac{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} s_{ijk}x_{ijk}}{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} p_{ijk}x_{ijk}}\right)^2, \max_{\substack{1\le i\le m\\ 1\le j\le n\\ 1\le k\le p}}[t_{ijk} : x_{ijk} > 0]\right) \tag{10}$$

and the initial basic feasible solution is found by the North-West Corner Rule. For optimal solution, the authors have first minimized the total cost and then, they minimized the time. Bhatia and Puri [16] have solved the MIBCTP to find the Pareto Optimal time-cost pair, where time is minimized first and cost afterwards.

Bandopadhyaya [12] has considered the three axial sum MIBCTP with transportation cost and time as the two criteria. The author has converted the problem into a single criterion one by assigning suitable weights to the criteria. Equivalence has been established between the bi objective cost-time trade off pairs of the three axial problem and the single objective standard three axial sum problem.

Traditional transportation methods fail to handle large data sizes of the transportation parameters. Neural network approach has a large number of computing units-neurons and parallelism, the approach is useful to solve large data sized TPs. Li, Ida, Gen and Kobuchi [56] have solved the Multi Criteria Solid transportation problem by converting the multi criteria problem into a single criterion one using a global criteria method and applying neural network approach to solve it.

Sun and Lang [95] have considered the Multi-Modal transportation routing planning problem with an additional index i.e. the mode of transportation and the two Criteria i.e. cost and time. The authors have found the optimal route of transportation through multiple modes of transport.

Krile and Krile [49] have investigated the airline transportation problem, taking transportation cost and better utilization of the airport capacity as the criteria, and they found the sequence of passenger distribution between sources and destinations by using the network optimization methodology.

### *3.2. Multi Index Fixed Charge Transportation Problem*

Multi Index Fixed Charge transportation problem (MIFCTP) is an extension of MITP in which, apart from direct cost, some fixed cost is associated with the transportation related activity, such as storage cost, toll charge, landing cost etc., which are usually independent of the number of units transported.

The Mathematical Formulation of MIFCTP is

$$\text{Minimize } z = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} c_{ijk}x_{ijk} + \sum_{i=1}^{m}\sum_{k=1}^{p} F_{ik} \tag{11}$$

subject to constraints (2) or constraints (6) and $F_{\mathrm{ik}} \geq 0$ , where $F_{ik}$ is the fixed charge applicable when a commodity is transported from the $i^{th}$ source to the $j^{th}$ destination. In case the fixed charge depends on the quantity transported, the objective function behaves like a step function. Sandrock [87] and Kowalski [48] considered the step fixed charge in a Classical TP.

Basu, Pal and Kundu [14] have solved the MIFCTP by including fixed charge in the objective function. The authors have found the initial basic feasible solution by applying the North-West Corner Rule, and the optimal solution by considering fixed charge difference along with direct cost difference in the Modified Distribution (MODI) method, given by Haley [31] for MITP.

Sanei, Mahmoodirad, and Zavardehi [88] applied electromagnetism like algorithm (EM) to solve MIFCTP. The results, obtained by the proposed method, were compared with a simulated annealing algorithm (SA). SA is a metaheuristic algorithm for getting the solution of a combinatorial optimization problem. EM gave better optimal solution for large problem sizes. Also, for all problem sizes, EM shows a significant performance improvement over SA. Sanei et al. [89] have solved the step fixed charge multi index transportation problem. The authors have applied a Lagrangian relaxation heuristic approach.

### *3.3. Multi Index Fixed Charge Bicriteria Transportation Problem*

The MITP in which the objective function includes transportation cost, time, and fixed charges is called the Multi Index Fixed Charge Bi-criteria transportation problem (MIFCBCTP).

The Mathematical formulation of MIFCBCTP is as follows,

$$\text{Minimize} \left( \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} c_{ijk}x_{ijk} + \sum_{i=1}^{m}\sum_{k=1}^{p} F_{ik}, \quad \max_{\substack{1\le i\le m\\ 1\le j\le n\\ 1\le k\le p}} \left(t_{ijk} : x_{ijk} > 0\right) \right) \tag{12}$$

subject to constraints (2) or constraints (6) and $F_{ik}, t_{ijk} \ge 0.$

An exact method of finding the solution of MIFCBCTP is given by Ahuja and Arora [3]. The authors applied North West corner rule to find an initial basic feasible solution, and then an extended form of modified distribution (MODI) method to minimize the transportation cost. They found the corresponding maximum transportation time $T_1$ and the first cost-time trade off pair. For the second cost-time trade off pair, the authors modified transportation table by

$$C_{ijk} = \begin{cases} M, t_{ijk} \ge T_1 \\ c_{ijk}, otherwise \end{cases}$$

and continued the procedure till the infeasible solution occurred. Khurana and Adlakha [46] solved the problem by considering direct and fixed cost difference in the extended form of MODI method. The authors claimed that the cost obtained by their method is lower than that obtained by Ahuja's [3] method.

Arora and Khurana [7] considered the indefinite quadratic MIFCBCTP in which the objective function is quadratic in nature.

Mathematically, the problem is represented as

$$\text{Minimize} \left( \left(\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} c_{ijk}x_{ijk}\right)\left(\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} d_{ijk}x_{ijk}\right) + \sum_{i=1}^{m}\sum_{k=1}^{p} F_{ik}, \quad \max_{\substack{1\le i\le m\\ 1\le j\le n\\ 1\le k\le p}} \left(t_{ijk} : x_{ijk} > 0\right) \right) \tag{13}$$

subject to constraint (2) and $d_{ijk}, F_{ik}, t_{ijk} \ge 0$ where $d_{ijk}$ is per unit depreciation cost of $k^{th}$ commodity transported from $i^{th}$ source to $j^{th}$ destination. The authors minimize total cost, direct cost and fixed cost, to find first cost-time trade off pair. They modified transportation table to find cost-time trade-off pair by applying Ahuja’s [3] method.

Genetic algorithm is applied to solve Multi Index, Fixed Charge, Multi-Critreria transportation problem by Jin, Lee and Gen [42]. The authors take plants, distribution centers, customers, products, and time periods as the indices, while the objective function includes minimization of transportation cost, inventory cost, and fixed cost to open the distribution center.

The work done in the area of extensions of MITP is summarized in Table2 given below.

**Table 2:** Extensions of Multi Index Transportation Problem

| Sr. No. | Author | Type of MITP | Contribution |
|---|---|---|---|
| 1. | Bhatia and Puri [16] | MIBCTP | Algorithm to obtain the Pareto Optimal efficient solution as time-cost trade off pairs. |
| 2. | Basu, Pal and Kundu [13] | MIBCTP | Algorithm to obtain the Pareto Optimal efficient solution as cost-time trade off pairs. |
| 3. | Gen, Ida and Li [28] | MIBCTP | Applied Genetic Algorithm to solve the problem. |
| 4. | Bandopadhyaya [12] | MIBCTP | Solved the three axial sums problem with transportation cost and time as the two criteria by converting into a single criterion |
| 5. | Basu, Pal and Kundu [14] | MIFCTP | Optimal Solution procedure for MIFCTP. |
| 6. | Li, Gen Ida and Kobuchi [56] | MIBCTP | Applied Neural Network approach to solve the problem. |
| 7. | Ahuja and Arora [3] | MIBCFCTP | Applied a simple extension of MODI method to solve the problem. |
| 8. | Arora and Khurana [7] | MIBCFCTP | Applied a simple extension of MODI method to solve the problem having a quadratic objective function. |
| 9. | Jin, Lee and Gen [42] | MIBCFCTP | Applied Genetic Algorithm to solve the problem. |
| 10. | Sanei [88] | MIFCTP | Applied Electromagnetism(EM) algorithm. |
| 11. | Sun and Lang [95] | MIBCTP | Solved a multi-modal transportation routing planning problem. |
| 12. | Krile and Krile [49] | MIBCTP | Solved an airline transportation problem. |
| 13. | Khurana and Adlakha [46] | MIBCFCTP | Optimal solution procedure for the problem vis-à-vis existing method [3]. |
| 14. | Sanei et.al. [89] | MIFCTP | Solved the problem by Lagrangian relaxation heuristic approach. |

## 4. FUZZY CONCEPT AND FUZZY MULTI INDEX TRANSPORTATION PROBLEM

Many times a decision maker has no exact information about the parameters such as supplies, demands, capacities, and cost of transportation in a TP. This may be due to several factors such as unknown, or incorrect, or uncertain data entries. The fuzzy concept came into being to handle such uncertainties. A Fuzzy set, having imprecise boundaries, was introduced by Zadeh [103], as an extension of the classical (crisp) set. A Fuzzy number is described by a membership function which has a value in the real unit interval [0, 1], while a crisp number is described with membership value that is either 0 or 1. A crisp set has a unique membership function, while a fuzzy set is represented by an infinite membership function. In a fuzzy set, data is represented by different grades of membership function such as triangular, trapezoidal, and LR Fuzzy.

The membership function of a triangular fuzzy number A=(a b c) is as follows,

$$\mu_A(x) = \begin{cases} \dfrac{(x-a)}{(b-a)} & , \ a \le x < b \\ 1 & , \ x = b \\ \dfrac{(c-x)}{(c-b)} & , \ b < x \le c \end{cases} \tag{14}$$

The membership function of a trapezoidal fuzzy number B=(a b c d) is given as,

$$\mu_B(x) = \begin{cases} 0 & , \ x \le a, x \ge d \\ \dfrac{(x-a)}{(b-a)} & , \ a \le x \le b \\ 1 & , \ b \le x \le c \\ \dfrac{(d-x)}{(d-c)} & , \ c \le x \le d \end{cases} \tag{15}$$

The membership function of an LR fuzzy number C=(a b c d) is

$$\mu_C = \begin{cases} L\left(\dfrac{x-a}{b-a}\right) & , \ a \le x \le b \\ 1 & , \ b \le x \le c \\ R\left(\dfrac{d-x}{d-c}\right) & , \ c \le x \le d \end{cases} \tag{16}$$

where L and R are continuous, non increasing reference function, which define the left and right shapes, respectively of $\mu_C(x)$.

Unlike crisp numbers, fuzzy numbers are not comparable. For that, a fuzzy number must be converted into a crisp number by using different ranking functions.

A Fuzzy Multi Index transportation problem (FMITP) arises when a decision maker has unclear information about one or more parameters such as supplies, demands, capacities, and cost of transport. The mathematical formulation of FMITP is the same as the formulation of MITP with crisp parameters of MITP being replaced by fuzzy numbers such as triangular, trapezoidal, or LR fuzzy. The initial basic feasible solution (IBFS) of the problem can be found by North-West Corner rule [92], Matrix Minima method [92], or Vogel's Approximation method (VAM) [92].

Jimenez and Verdegay [40] have considered a Solid transportation problem with supplies, demands, and conveyance's capacities taken as trapezoidal fuzzy numbers. The auxiliary Parametric Solid transportation problem was solved by a method based on a genetic algorithm. The authors [41] have claimed that the method based on genetic algorithm gives better optimal solution to Parametric Solid transportation problem than the Nonlinear optimization method (Gradient method).

Senapati and Samanta [90] have considered MITP with budgetary restriction. In some cases, the consumer wants a product from a particular factory, or a factory owner wants to fulfill the demand at some destination. For this type of transportation, budgetary restriction concept is used. The authors have constructed fuzzy goal programming model with the destination's demand expressed as triangular fuzzy numbers. The model is solved by a linear programming algorithm. It was seen that by using this model, a factory owner would develop a good reputation in business but with transportation cost raised.

Narayanamoorthy and Anukokila [69] have considered a robust fuzzy Solid transportation problem with uncertainty in demand, and they solved two such models (with inequality and equality constraints in demand). Robust optimization deals with data uncertainty and does not require probability distribution of random data. It helps the decision maker to find optimal solution during uncertainty.

Baidya, Bera and Maiti [8] introduced the concept of safety factor in MITP. When different products are transported from sources to destinations by different modes of transport, uncertainty due to bad road conditions may occur. In that case, safety factor is considered as necessary in such transportation problem. The authors used trapezoidal fuzzy numbers to demonstrate uncertainties in TP.

Unbalanced MITP is considered by Kumar and Kaur [50] with all parameters as LR fuzzy numbers. The authors converted the problem into four crisp TPs and solved it by Haley's [31] method. The optimal solution is also in the form of LR fuzzy numbers. Further, they applied the proposed method to coal transportation problem without fixed charge and studied later by Yang and Liu [102].

Liu et.al.[57]considered the FMITP where indices are described by type-2 fuzzy variables. The variables in the problem were defuzzified by optimistic value criterion, pessimistic value criterion, and expected value criterion. The chance-constrained programming model with least expected transportation cost is formulated for the problem and solved by fuzzy simulation based tabu search algorithm. Kundu, Kar and Maiti [53] have solved the problem with restrictions on specific items; those which cannot be transported by a particular conveyance. The authors have taken supplies, demands, and transportation costs as type-2 triangular fuzzy variables and solved the problem by Generalized Reduced Gradient (GRG) method using LINGO software and genetic algorithm.

Fractional solid transportation problem has been studied by Narayanamoorthy and Anukokila [70], where the objective function is taken as a linear fractional function and solved by fractional programming method and duality optimality condition.

The work done in the area of FMITP is outlined in Table 3 below.

**Table 3:** Fuzzy Multi Index Transportation Problem

| Sr. No. | Author | Type of MITP | Contribution |
|---|---|---|---|
| 1. | Jimenez and Verdegay [40] | FMITP | Applied Genetic Algorithm to solve MITP having parameters in trapezoidal fuzzy numbers. |
| 2. | Jimenez and Verdegay [41] | FMITP | Applied Genetic Algorithm to solve MITP having parameters in fuzzy and interval numbers. |
| 3. | Senapati and Samanta [90] | FMITP | Applied triangular fuzzy numbers for budgetary restriction parameters and constructed a fuzzy goal programming model and solved it by linear programming method. |
| 4. | Narayanamoorthy and Anukokila [69] | FMITP | Investigated a Robust Fuzzy Solid transportation problem. |
| 5. | Baidya, Bera and Maiti [8] | FMITP | Introduced the concept of safety factor in MITP and used trapezoidal fuzzy numbers to solve it. |
| 6. | Kumar and Kaur [50] | FMITP | Solved an unbalanced MITP having parameters in LR fuzzy numbers. |
| 7. | Liu, Yang, Wang and Li [57] | FMITP | Solved MITP having supplies, demands and capacities, taken as type-2 fuzzy variables. |
| 8. | Narayanamoorthy and Anukokila [69] | FMITP | Solved the solid fuzzy transportation problem with fractional objective function. |
| 9. | Kundu, Kar and Maiti [53] | FMITP | Solved the MITP with Generalized Reduced Gradient (GRG) method and Genetic Algorithm using type-2 fuzzy numbers. |
| 10. | Narayanamoorthy and Anukokila [70] | FMITP | Solved the FMITP using fractional programming method and duality optimality condition. |

## 5. EXTENSIONS OF FUZZY MULTI INDEX TRANSPORTATION PROBLEM

Further extensions of the FMITP have been studied by several researchers, some details of which are given below.

### *5.1. Fuzzy Multi Index Multi Criteria Transportation Problem*

FMITP having two or more criteria such as fuzzy cost, fuzzy time, fuzzy product deterioration time, etc. is considered as Fuzzy Multi Index Multi Criteria transportation problem (FMIMCTP).

FMIMCTP with **p** objectives can be solved by Fuzzy Programming approach which gives **p** efficient solutions and an optimal compromise solution. Bit, Biswal and Alam [18] used the approach on FMIMCTP and developed a FORTRAN program based on fuzzy linear programming algorithm. Nagarajan and Jeyaraman [66] used the method to solve Solid Multi Criteria transportation problem having supplies, demands and conveyance capacities as stochastic interval. Chance Constrained programming method was used to convert the problem into Classical Multi Objective transportation problem. Further, the authors [67] have constructed the equivalent crisp model of the problem taking the expectation of random variables and solved it using the fuzzy programming approach. Kundu, Kar and Maiti [52] solved the FMIMCTP with budget constraints in each destination by fuzzy programming and gradient based optimization method i.e.the Generalized Reduced Gradient (GRG) method. The authors also considered the problem with random and hybrid transportation parameters and showed that the transportation problem having hybrid parameters gives the least optimum solution.

Tzeng, Teodorovic and Hwang [97] have considered the coal transportation problem as FMIBCTP. The authors have taken sources, destinations, types of coal and shipping vessels as indices and the objective function was to minimize the total shipping cost and the maximum satisfaction level of the schedule of coal transportation. The problem was solved by the method based on the reducing index method and the interactive fuzzy multiobjective linear programming technique. Jana and Roy [39] studied the FMIMCTP with mixed constraints which is formulated as

$$\text{Minimize } z = [Z_1, Z_2, Z_3, ..., Z_p]$$
$$\text{where } Z_p(x) = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p}\tilde{c}_{ijk}^{p} x_{ijk} \tag{17}$$

subject to

$$\begin{aligned}
&\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p}\tilde{t}_{ijk}x_{ijk} \le \tilde{T}\\
&\sum_{j=1}^{n}\sum_{k=1}^{p}x_{ijk} \ge,=,\le \tilde{a}_i\\
&\sum_{i=1}^{m}\sum_{k=1}^{p}x_{ijk} \ge,=,\le \tilde{b}_j\\
&\sum_{i=1}^{m}\sum_{j=1}^{n}x_{ijk} \ge,=,\le \tilde{e}_k\\
&x_{ijk} \ge 0
\end{aligned} \tag{18}$$

where $\tilde{t}_{ijk}$ is the fuzzy delivery time when the transportation occurs from the $i^{th}$ source to the $j^{th}$ destination by the $k^{th}$ conveyance for the $p^{th}$ criterion. The transportation cost, supplies, demands, and conveyance capacities are denoted by triangular fuzzy numbers. The proposed problem was solved by fuzzy decisive set method.

Ojha et.al. [71] considered FMIMCTP with entropy function in the objective function. Entropy function is used to determine distribution of trips among sources, destinations, and conveyances for the minimum transportation cost and the maximum entropy amount. Weighted average of the objectives is used to convert the multi criteria problem into the single criterion one, which is then solved by the reduced gradient method.

Lohgaonkar et.al. [58] solved FMIMCTP by fuzzy programming technique with linear, hyperbolic, and exponential membership function. They computed and compared optimal solutions with these membership functions. Kaur, Mukherjee and Basu [44] have considered the problem of transporting raw material to the sites of Durgapur Steel Plant as a FMIMCTP and solved it by fuzzy programming technique with linear, hyperbolic, and exponential membership functions. The authors have claimed that better results are obtained on using the exponential membership function.

Ritha and Vinotha [82] have included the environmental and congestion time cost in the objective function. The authors applied a priority based fuzzy goal programming method to solve the problem. Pramanik and Banerjee [76] applied the fuzzy goal programming method to solve the multi objective chance constrained capacitated transportation problem.

Chakraborty, Jana and Roy [20] solved the fuzzy inequality in FMIMCTP by Fuzzy Interactive Satisfied Method, Global Criterion Method, and Convex Combination Method by using MatLab and Lingo-11.0 software.

Kaur and Kumar [45] have solved the unbalanced problem with the minimal cost flow. In this problem, it may be required to store excess product in some node and to supply it later to the required destination. The authors have taken all parameters as LR fuzzy numbers and solved the problem by fuzzy programming method. Kundu, Kar and Maiti [54] have solved the problem with the condition that total supply and capacity may not fall below the total demand. All parameters are represented as trapezoidal fuzzy numbers and the problem is solved by the fuzzy programming method and global criterion method. However, an infeasible solution is obtained in some cases. Rani, Gulati and Kumar [81] have solved Kundu's [54] problem (having an infeasible solution) by converting the unbalanced problem into a balanced one and applying the Fuzzy Programming method to obtain a feasible solution.

Radhakrishnan and Anukokila [80] have proposed fractional goal programming approach for solving a FMIMCTP. The authors have used the hyperbolic membership function,in order to develop a non-linear model for FMIMCTP and solved it by LINGO software.

Ammar and Khalifa [5] have introduced the concept of α-fuzzy efficient solution of a FMIMCTP, which is an extension of the crisp efficient solution based on α-cut of fuzzy numbers. Sinha, Das and Bera [93] have considered the objectives of maximizing profit and minimizing transportation time in FMIMCTP and solved the problem by the Fuzzy Interactive Satisfied method and LINGO 13.0 software.

### *5.2. Fuzzy Multi Index Fixed Charge Transportation Problem*

The MITP having fuzzy parameters such as supplies, demands, capacities, unit transportation cost and fixed charge is known as the Fuzzy Multi Index Fixed Charge transportation problem (FMIFCTP).

Yang and Liu [102] have considered the coal transportation problem as the FMIFCTP with trapezoidal fuzzy numbers. Expected Value, Chance constrained, and Dependent Chance Programming models are constructed by authors using credibility theory. The models are further solved by hybrid intelligent algorithm based on the Fuzzy Simulation technique and the Tabu Search algorithm.

Ahuja and Arora [3] gave an exact method to solve the MIFCTP, while Adlakha and Kowalski [2] gave a heuristic method to solve the Fixed Charge transportation problem (FCTP). Their work was extended by Ritha and Vinotha [83], who presented a heuristic method to solve the FMIFCTP, by using trapezoidal fuzzy numbers for all the parameters in the problem. The optimal solution is obtained by LINDO software.

Zaverdehi et.al.[104] have considered the MIFCTP with total transportation cost as triangular fuzzy numbers. The authors applied simulated annealing (SA), variable neighborhood search (VNS), and hybrid VNS to solve the problem and showed that the hybrid VNS gave better results than the SA and VNS.

Giri, Maiti and Maiti [30] have solved balanced and unbalanced FMIFCTP with additional index as product with all parameters being taken as triangular fuzzy numbers and considered the objective to minimize transportation cost and fuzziness of the solution.

### *5.3. Fuzzy Multi Index Multi Criteria Fixed Charge Transportation Problem*

Ojha, Mondal and Maiti [73] investigated the Fuzzy Multi Index Multi criteria Fixed Charge transportation problem (FMIMCFCTP). They have considered the FMIMCFCTP with partial nonlinear transportation cost, which is inversely proportional to the transported quantity, fixed charge and small vehicle cost. They have used trapezoidal fuzzy numbers and converted them into equivalent nearest interval numbers in order to change the fuzzy problem into a crisp problem; they solved the problem by Fuzzy Interactive Programming technique and Fuzzy Generalized Reduced Gradient method.

The work done in these areas of fuzzy extensions of MITP's are summarized in Table 4 given below.

**Table 4:** Fuzzy Extension of Multi Index Transportation Problem

| Sr. No, | Author | Type of MITP | Contribution |
|---|---|---|---|
| 1. | Bit, Biswal and Alam [18] | FMIMCTP | Applied Fuzzy Programming approach to solve the problem. |
| 2. | Tzeng, Teodorovic and Hwang [97] | FMIBCTP | Considered the coal transportation problem as a FMIBCTP. |
| 3. | Jana and Roy [39] | FMIMCTP | Applied Fuzzy Decisive Set method to solve the problem. |
| 4. | Yang and Liu [102] | FMIFCTP | Considered the coal transportation problem as FMIFCTP |
| 5. | Ojha, Das and Maiti [71] | FMIMCTP | Applied Reduced Gradient Method using Entropy function. |
| 6. | Lohgoankar et.al. [58] | FMIMCTP | Applied Fuzzy Programming method with linear, hyperbolic and exponential membership function. |
| 7. | Nagarajan and Jeyaraman [67] | FMIMCTP | Applied Fuzzy Programming approach to solve the problem. |
| 8. | Ritha and Vinotha [82] | FMIMCTP | Applied Priority based Fuzzy Goal Programming method to solve the problem. |
| 9. | Ritha and Vinotha [83] | FMIFCTP | Applied a heuristic algorithm to solve the problem. |
| 10. | Pramanik and Banerjee [76] | FMIMCTP | Applied Fuzzy Programming method to the problem. |
| 10. | Kaur and Kumar [45] | FMIMCTP | Solved the unbalanced problem with minimal cost flow. |
| 11. | Kundu, Kar and Maiti [54] | FMIMCTP | Applied Fuzzy Programming approach and Global Criterion method to solve the problem. |
| 12. | Zaverdehi et.al. [104] | FMIFCTP | Applied Simulated Annealing algorithm (SA) and Variable Neighborhood Search (VNS) method to solve the problem. |
| 13. | Kundu, Kar and Maiti [52] | FMIMCTP | Applied Fuzzy Programming and gradient based optimization-Generalized Reduced Gradient (GRG)method. |
| 14. | Chakraborty, Jana and Roy [20] | FMIMCTP | Applied Fuzzy Interactive Satisfied Method, Global Criterion Method, Convex Combination Method to solve the problem. |
| 15. | Ojha, Mondal and Maiti [73] | FMIFCMCTP | Applied Interactive Fuzzy Goal Programming and Generalized Fuzzy Reduced Gradient Method to solve the problem. |
| 16. | Radhakrishnan and Anukokila [80] | FMIMCTP | Applied Fractional Goal Programming Approach to the problem. |
| 17. | Kaur, Mukharjee and Basu [44] | FMIMCTP | Solved a real life problem of Durgapur steel plant by Fuzzy Programming method using linear, hyperbolic and exponential membership function. |
| 18. | Ammar and Khalifa [5] | FMIMCTP | Introduced the concept of α-efficient fuzzy solution. |
| 19. | Giri, Maiti and Maiti [30] | FMIFCTP | Solved Balanced and Unbalanced FMIFCTP. |
| 20. | Rani, Gulati and Kumar [81] | FMIMCTP | Solved the Unbalanced FMIMCTP to get feasible solution. |
| 21. | Sinha, Das and Bera [93] | FMIMCTP | Solved the problem with objectives to maximize profit and minimize time. |

## 6. STOCHASTIC CONCEPT AND STOCHASTIC TRANSPORTATION PROBLEM

In the classical transportation problem, demand at destination is exactly known. But in reality, demand may depend upon many factors such as fuel cost, economic condition, political stability, natural calamity etc. Such a transportation problem having uncertain demands necessitates its description by random variables and is called the Stochastic Transportation Problem (STP). In STP, demand is measured from the penalties when the problem is unbalanced, i.e. demand is greater than the supply or supply is greater than the demand. For both the cases, penalties are paid for each unit, be it short or in excess, of the product. Thus, the objective function of STP includes minimizing expected penalty costs along with the transportation cost.

Elmaghraby [25] first considered the STP and was later followed by Williams [99], Szwarc [96], Cooper [22], Wilson [100] and Qi [79].

The Mathematical Formulation of STP is as follows.

$$\text{Minimize } Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} + \sum_{j=1}^{n} \underset{b_j \geq x_j}{E}[l_j^1(b_j - x_j)] + \sum_{j=1}^{n} \underset{b_j \leq x_j}{E}[l_j^2(x_j - b_j)] \tag{19}$$

subject to

$$\sum_{j=1}^{n} x_{ij} \leq a_i \quad (i = 1,2,3,...,m) \tag{20}$$

where

$c_{ij}$ =unit transportation cost when transportation occur between $i^{th}$ source to $j^{th}$ destination.

$x_{ij}$ =number of units transported from the $i^{th}$ source to the $j^{th}$ destination.

$a_i$ =supply at source $i$.

$b_j$ =demand at destination $j$.

$l_j^1$ =the penalty rate for unit shortage demand at the $j^{th}$ destination.

$l_j^2$ =the penalty rate for unit excess quantity received at the $j^{th}$ destination.

$\underset{b_j \geq x_j}{E}[l_j^1(b_j - x_j)]$ =the expected value of random variable in the domain $x_j \leq b_j$ .

$\underset{b_j \leq x_j}{E}[l_j^2(x_j - b_j)]$ =the expected value of random variable in the domain $x_j \geq b_j$ .

$$x_j = \sum_{i=1}^{m} x_{ij}$$

Elmaghraby [25] considered a STP with uncertain demand having a continuous distribution function and gave a necessary and sufficient condition for its optimal solution. The author further solved the problem by a finitely convergent iterative method using the above condition and he compared his results with problems having constant demand and discrete demand distribution function respectively.

Later, Williams [99] considered the STP and claimed that classical transportation problem is a special case of STP. He solved the problem by using a method which is a generalization of the linear programming method with certain (known) demand. Szwarc [96] converted the problem into linear programming form and solved it by replacing the original cost by its controlled linear approximation.

Wilson [100] has developed a linear approximation model of STP to find its integer solution. The problem is solved as a deterministic capacitated transportation problem using the transportation algorithm or the primal-dual algorithm.

STP can be solved by various other techniques such as Frank-Wolf (FW) method [26], Separable Programming [36], Mean Value Cross Decomposition [37], Forest Iteration [79], Cross Decomposition [35]. Cooper and LeBlanc [21] solved the STP by FW method. The authors claimed that the method is efficient for large scale problems. Qi [79] solved the STP by Forest Iteration method which is finitely convergent, as the method iterates from the optimal solution of a forest to the optimal solution of another forest, with strictly decreasing objective function value. Holmberg and Jornsten [35] solved the STP by an iterative method based on the cross decomposition method, proposed by Roy [86]. The authors compared their method by setting the desired accuracy and speed, with the FW and Separable Programming (SP) methods, concluding that their method gave better results. For high accuracy, FW method is very slow and SP method is fast. However, in the SP method one cannot determine accuracy before solving the problem, and it also requires a lot of computer memory. On the basis of decomposition techniques and linearization programming approach, Holmberg [37] compared the different methods for solving the STP and concluded that the best method is Mean Value Decomposition with Separable Programming. Daneva et.al.[23] presented a comparative study of solution methods for the STP and concluded that for high solution accuracy, Diagonalized Newton (DN) , Conjugate Frank–Wolfe (CFW), and Frank-Wolfe with Multi Dimensional search (FWMD) algorithms are better than FW algorithm and its heuristic variation, but DN is the best. Cooper [22] has investigated the STP to determine the amount shipped, along with the location of sources, and he developed an exact method for small size problems, on the basis of complete enumeration of all extreme points of the convex set of feasible solutions. But the exact method is not applicable to large scale problems, as the evaluated basic feasible solutions become unreasonably large with an increase in the sources and destinations. The author has also given a heuristic method for the large size problem.

Romeijn [84] has considered the STP in which the demand at each destination must be fulfilled from a single source, and he gave a branch-and-price method to solve it. Stochastic Bi-Criteria bottleneck transportation problem that included random transportation time and fuzzy supply and demand was considered by Ge and Ishii [27]. Bilevel stochastic transportation problem was considered by Akdemir and Tiryaki [4]. Mahapatra, Roy and Biswal [59] have considered the Multi-Objective Stochastic Transportation problem (MOSTP) with supplies and demand as random variables, while Roy and Mahapatra [85] have taken supplies and demands as log-normal random variables, and interval numbers are used for the coefficients of the objective function. Biswal and Samal [17] solved the problem by Goal Programming method where supply and demand are taken as multichoice Cauchy random variables. Acharya et.al. [1] investigated the MOSTP with fuzzy random supply and demand. Here the alpha-cut method has been used to defuzzify fuzzy number.

LeBlanc [55] considered the discrete stochastic transportation location problem as a generalization of the fixed charge transportation problem, with the construction cost at a location as fixed cost. He solved the problem by a heuristic method and determined the shipment, along with the possibility of constructing at each proposed site. Holmberg and Tuy [38] considered the STP with fixed charge, where the objective was to minimize the total transportation cost which includes direct and shortage cost along with the production cost. Later, Hinojosa et.al. [34] considered a two-stage stochastic fixed charge transportation problem; in the first stage, the distribution link was obtained, while in the next stage, the total flow was determined. Mahapatra, Roy and Biswal [60] solved the STP in which multi-choice type of cost coefficients are considered in the objective function. Maurya et.al. [61] claimed that the method given by Mahapatra, Roy and Biswal [60] is not heuristic, and also, not efficient with respect to time and has unnecessary multi-choice structure for transportation cost. The authors have given an analytical and a heuristic method to find the optimal solution of the problem.

Table 5 given below outlines the work done in the area of STP.

**Table 5:** Stochastic Transportation Problem

| Sr. No. | Author | Type of STP | Contribution |
|---|---|---|---|
| 1. | Elmaghraby [25] | STP | Investigated the problem for the first time. |
| 2. | Williams [99] | STP | The problem is considered as a generalization of classical transportation problem. |
| 3. | Szwarc [96] | STP | Solution procedure for the problem by replacing original cost by controlled linear approximation. |
| 4. | Wilson [100] | STP | Found the integer solution of the problem. |
| 5. | Cooper and LeBlanc [21] | STP | Solved the problem by FW method. |
| 6. | LeBlanc [55] | STP | Solved the problem with fixed charges. |
| 5. | Cooper [22] | STP | Investigated the problem to determine the amount shipped along with the location of sources. |
| 6. | Holmberg [36] | STP | Applied Separable Programming method to solve the problem. |
| 7. | Holmberg and Jornsten [35] | STP | Applied Cross Decomposition method to solve the problem. |
| 8. | Qi [79] | STP | Applied Forest Iteration method to solve the problem. |
| 9. | Holmberg [37] | STP | Comparative study between methods of solving STP. |
| 10. | Holmberg and Tuy [38] | STP | Investigated the problem with fixed charges. |
| 11. | Mahapatra, Roy and Biswal [59] | STP | Investigated the Multi Objective STP (MOSTP)with random supplies and demands. |
| 12. | Daneva [23] | STP | Introduced the Frank Wolfe(FW) method with Multi Dimensional search to solve the STP. |
| 13. | Romeijn [84] | STP | Solved the single source STP by branch and price method. |
| 14. | Ge and Ishii [27] | STP | Bi-Criteria bottleneck stochastic transportation problem is considered. |
| 15. | Roy and Mahapatra [85] | MOSTP | Investigated MOSTP with log-normal supplies and demand. |
| 16. | Akdemir and Tiryaki [4] | STP | Considered the bi-level stochastic transportation problem. |
| 17. | Mahaptra, Roy and Biswal[60] | STP | Solved the problem with Multi-Choice transportation cost. |
| 18. | Biswal and Samal [17] | MOSTP | Solved the problem by Goal Programming method. |
| 19. | Acharya et.al.[1] | MOSTP | Investigated MOSTP with fuzzy random supply and demand. |
| 20. | Hinojosa [34] | STP | Solved two stage STP with fixed charges. |
| 21. | Maurya et.al. [61] | STP | Solved multi-choice transportation problem. |

## 7. STOCHASTIC MULTI INDEX TRANSPORTATION PROBLEM

The STP can be extended to the Stochastic Multi Index Transportation Problem (SMITP) by considering additional indices as products and modes of transports. Since transportation cost is dependent upon fuel cost, labor charge, tax charge etc., which keeps on varying with time and the supply, demand and capacity of the mode of conveyance depends upon market conditions, manpower and raw-material availability etc., so the existing data may not be appropriate to find the minimum transportation cost. In this case, some or all transportation parameters such as supply, demand, capacity, transportation cost may be taken as random variables.

The Mathematical Formulation of the SMITP follows,

$$Min\, Z = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} E[c_{ijk}]x_{ijk} \tag{21}$$

subject to the constraints

$$\left.\begin{array}{ll} \sum_{j=1}^{n}\sum_{k=1}^{p} x_{ijk} \le \mathrm{E}[a_i], & i = 1,2,3,\dots,m \\ \sum_{i=1}^{m}\sum_{k=1}^{p} x_{ijk} \ge E[b_j], & j = 1,2,3,\dots,n \\ \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} \le E[e_k], & k = 1,2,3,\dots,p \\ x_{ijk} \ge 0, & \forall\ i,j,k \end{array}\right\} \tag{22}$$

Taking bad road conditions into account, Baidya, Bera and Maiti [9] have introduced stochastic safety factor in SMITP. The authors considered the safety factor constraint, as it is shown below.

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} \hat{s}_{ijk}\, y_{ijk} \ge \boldsymbol{B} \tag{23}$$

where $\hat{s}_{ijk}$ is stochastic safety factor, B is desired safety measure, and $y_{ijk}$ is the indicator variable given below.

$$y_{ijk} = \begin{cases} 1 & \text{if } x_{ijk} > 0 \\ 0 & \text{otherwise} \end{cases}$$

The other constraints are the same as the constraints of axial MITP, stated earlier in (6). The problem was solved by Generalized Reduced Gradient (GRG) Method and LINGO software.

## 8. EXTENSIONS OF STOCHASTIC MULTI INDEX TRANSPORTATION PROBLEM

The SMITP can also be extended to the Stochastic Multi Index Multi Criteria Transportation Problem (SMIMCTP), the Stochastic Multi Index Fixed Charge

Transportation Problem (SMIFCTP), and the Stochastic Multi Index Multi Criteria Fixed charge Transportation problem (SMIMCFCTP).

### *8.1. Stochastic Multi Index Multi Criteria Transportation Problem*

The SMITP having two or more criteria, such as cost, time or deterioration rate extends the problem to the Stochastic Multi Index Multi Criteria Transportation Problem (SMIMCTP).

Nagarajan and Jeyaraman [66], [67], [68] used the fuzzy programming approach to solve the SMIMCTP, in which all the transportation parameters such as transportation costs ,supplies, demands and capacities are stochastic intervals.

Ojha et.al. [72] have introduced the concept of breakability of items, which is dependent upon the conveyance type and transported amount in SMIMCTP. Because of breakable items, the completion of consumer's demand is stochastic in nature. Sometimes, company owners offer discount in unit transportation cost, which may be All Unit Discount (AUD), and Incremental Quantity Discount (IQD). The authors have introduced nested discounts (IQD within AUD) in the problem. The problem is converted into an equivalent crisp problem by the chance constrained method. The authors found Pareto optimal solutions by Multi-Objective Genetic Algorithm (MOGA), and the best solution by Analytical Hierarchy Process (AHP). Pramanik, Jana and Maiti [77] have introduced damageability of transported items in the problem. This factor is dependent upon the conveyance and the routes of conveyance. In the problem, transportation cost, supply, demand and capacity are all taken as fuzzy random numbers. The authors have claimed that for the first time, MIMCTP is formulated with fuzzy random parameters, and a random fuzzy interactive method is introduced to derive the fuzzy and stochastic model. The problem is further solved by the Generalized Reduced Gradient (GRG) method and LINGO -10 software.

### *8.2. Stochastic Multi Index Fixed Charge Transportation Problem*

For SMITP, if decision maker prefers minimizing expected total cost which includes direct and fixed charges then the problem is known as Stochastic Multi Index Fixed Charge Transportation Problem (SMIFCTP).

Yang and Liu [102] have considered SMIFCTP with stochastic transportation parameters. The authors have constructed chance-constrained model, dependent-chance constrained model and expected value model. The expected value model minimizes the expected objective function under the expected constraints. The models are solved by hybrid intelligent algorithm based on fuzzy simulation technique and Tabu search algorithm.

Baidya, Bera and Maiti [10] have introduced the concept of breakability and safety factor in SMIFCTP. The authors have considered a fuzzy and a hybrid model with fuzzy and hybrid (fuzzy random) variables and converted them into an equivalent crisp model using expected values. The problem is then solved by the Generalized Reduced Gradient (GRG) method using LINGO.13 software and genetic algorithm.

### *8.3. Stochastic Multi Index Multi Criteria Fixed Charge Transportation Problem*

SMITP can be extended to Stochastic Multi Index Fixed Charge Multi criteria Fixed Charge Transportation Problem (SMIMCFCTP) by including additional criteria along with transportation cost such as fixed charge in the objective function of the problem.

Yang and Feng [101] have investigated SMIMCFCTP with stochastic parameters. Mathematical models such as expected value goal programming model, chance-constrained goal programming model and dependent chance goal programming models are constructed for the problem and solved by hybrid algorithms based on random simulation and tabu search methods.

Nagarajan and Jeyaraman [65] have considered the SMIMCFCTP with all transportation parameters taken as stochastic variables. The authors have constructed expected value goal programming model, chance constrained goal programming model and dependent chance constrained goal programming model under stochastic environment. Giri, Maiti and Maiti [29] have considered the problem with random transportation parameters and expressed the constraints as fuzzy constraints. The problem is converted into corresponding crisp problem by derandomization and defuzzification methods, such as fuzzy goal programming, additive fuzzy goal programming, crisp and fuzzy weighted fuzzy goal programming methods, and then solved by Generalized Reduced Gradient (GRG) method.

Pramanik, Jana and Maiti [78] have solved the SMIMCFCTP with supplies, demands and conveyance capacity as bi-fuzzy number. Bi-fuzzy number is extensions of fuzzy number in which its membership function is also fuzzy number. Using the expected value of bi-fuzzy number, the problem is converted into an equivalent crisp form and solved by the multi-objective genetic approach (MOGA).

Midya and Roy [62] have considered the SMIMCFCTP with deterioration rate and underused capacity of transportation of raw material as additional criteria. The authors have considered all criteria in the form of random variables and converted them into an equivalent crisp form by stochastic programming approach. The crisp problem is further solved by the fuzzy programming technique.

The work done in the area of SMITP and extensions of SMITP is summarized briefly in Table 6 below.

**Table 6:** Stochastic Multi index and extensions of Stochastic Multi Index Transportation Problem

| Sr. No. | Author | Type of MITP | Contribution |
|---|---|---|---|
| 1. | Yang and Liu [102] | SMIFCTP | Applied hybrid intelligent algorithm based on fuzzy simulation and tabu search methods to solve the problem. |
| 2. | Yang and Feng [101] | SMIMCFCTP | Applied hybrid intelligent algorithm based on random simulation and tabu search methods to solve the problem. |
| 3. | Nagarajan and Jeyaraman [65] | SMIMCFCTP | Constructed the expected value goal programming model, chance constraint goal programming model and dependent chance constraint goal programming model under stochastic environment. |
| 4. | Nagarajan and Jeyaraman [66], [67], [68] | SMIMCTP | Applied fuzzy programming approach to solve the problem. |
| 5. | Ojha, Das, Mondal and Maiti [72] | SMIMCTP | Introduced breakability concept in the problem and obtained solutions by Multi-Objective Genetic Algorithm (MOGA) and Analytical Hierarchy Process (AHP). |
| 6. | Baidya, Bera and Maiti [9] | SMITP | Introduced safety factor in the problem and solved the problem by Generalized Reduced Gradient (GRG) method and Lingo software |

| | | | |
|---|---|---|---|
| 7. | Pramanik, Jana and Maiti [77] | SMIMCTP | Introduced damageability in the problem and solved it by GRG method and Lingo software. |
| 8. | Baidya, Bera and Maiti [10] | SMIFCTP | Introduced the concept of breakability and safety factor in the problem solved the problem by GRG method, Lingo software and Genetic algorithm. |
| 9. | Giri, Maiti and Maiti [29] | SMIMCFCTP | Solved the problem by the GRG method. |
| 10. | Pramanik, Jana and Maiti [78] | SMIMCFCTP | Solved the problem by Multi-Objective Genetic Approach (MOGA). |
| 11. | Midya and Roy [62] | SMIMCFCTP | Applied fuzzy programming approach to solve the problem. |

## 9. CONCLUSION

The article provides an in-depth review of the significant work done in the area of MITP and its extensions, summarized in Tables 1 and 2. The article also reviews work done on fuzzy and stochastic MITP and its extensions, summarized in Tables 3, 4, 5, and 6. Fuzzy set theory and stochastic concept have added a new dimension of uncertainty to transportation problems both theoretically and in applications. Complex real life transportation problems, so far considered unsolvable, can be simulated well by FMIMCFCTP and SMIMCFCTP. The article would be of immense use to researchers who attempt to solve these complex problems by the existing methods or by developing their possible variants.

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