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A MODEL OF THE SUPPLIER INVOLVEMENT IN THE PRODUCT INNOVATION

Manoj KUMAR
International Engineering Services
H.No.- 87A, RZI – Block, West Sagarpur, India
kumarm1968@rediffmail.com
Jyoti RAMAN
International Engineering Services
H.No.- 87A, RZI – Block, West Sagarpur, India
jyotiraman2004@rediffmail.com
Priya RAMAN
International Engineering Services
H.No.- 87A, RZI – Block, West Sagarpur, India.
priyaraman2006@rediffmail.com

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Abstract: In this paper we examine the product innovation in a supply chain by a supplier and derive a model for a supplier's product innovation policy. The product innovation of a supplier can contribute to the long-term competitiveness for the supply chain, and as it is for many supply chains a major factor, it should be considered in the development of strategies for a supplier. Here, we evaluate the effectiveness of supplier product innovation as a strategic tool to enhance the competitiveness and viability of supply chain. This paper explores the dynamic research performance of a supplier with endogenous time preference under a given arrangement of product innovation. We find that the optimal effort level and the achieved product innovation obey a saddle point path, or show tremendous fluctuations even without introducing the stochastic nature of product innovative activity. We also find that the fluctuation frequency is largely dependent both on the supplier's characteristics such as supplier's product innovative ability and on the nature of product innovation process per se. Short-run analyses are also made on the effect of supply chain cooperation in the product innovation process.

Keywords: Supplier Performance, Technical Change, Product Innovation.

MSC: 91B42.

1. INTRODUCTION

The development of new products and improvement of existing products, are considered to be critical to the survival of a supply chain facing tough competition and globalization. Due to an increase in competition and rapid advancement of technology, product innovation and new product development are becoming the essential strategies for the management of the suppliers to survive in the world [15]. Product innovation is an effective tool that a supplier can utilize to maintain its competitive position in the market. The position of the supplier in the market can be influenced by the frequency of releasing new or improved products [16]. As a result of increased buyer communications in many markets, buyers are flooded with many product choices, resulting in shorter product life cycles and rapid changes in buyer preferences. Product dynamism refers to the continuous change in the product which is characterized by speed and magnitude of technological change in the product. To cope up with competition and dynamic change in market demand many suppliers are adopting open product innovation which involves collaboration with external entities, like suppliers, universities, research organizations and competitors [23]. Many suppliers are switching towards open product innovation models, from traditional closed product innovation models, to increase their level of product innovation and retain competitive position in the supply chain. Open product innovation involves interaction of suppliers with several external entities to generate new ideas to improve existing products and develop new products. In recent years, the literature on product innovation has made some important issues of discussion. Of all the different types of methodologies adopted for innovation of the product, suppliers' product innovation has the most significant impact on the performance and success of the supply chain [14]. Supplier increases the technology knowledge available to a supply chain, which helps it reduce the product development time by identifying the potential problems beforehand [6]. Ragatz *et al.* (2002) found that involvement of suppliers in product innovation process result in benefits such cost reduction, improved quality and sales increase. Peterson *et al.* (2003) argued that the integration of suppliers into new product development activities can reduce the risk involved with technology which is at its formative stage. Supplier product innovation reduces potential problems during the early stages of development [29]. Although significant benefits can be achieved through the use of supplier, for a company seeking to leverage the product innovation of its supplier through collaboration, it is necessary that the correct attributes are considered in the design of supplier collaboration relationships. This paper is regarded as the benchmark in this area. The objective of our research is to gain a better understanding of the factors that a supplier must consider in product innovation. In this study, we target on developing the optimal model for a supplier involvement for product innovation. Therefore it is very helpful to see how a supplier, who is the most important contributor to the technological progress for product development, behaves in the different situations. What determines their effort decisions and their product innovation achievements? How supplier's knowledge for product innovation is per se produced? If we have a thorough understanding in these questions, then we will be more confident to provide some suggestions as to how to enhance a supplier's product innovation achievement and ultimately boost the technical progress of the new product development. Surprisingly, however, this important issue has by far received inadequate attention. Yet most of the above literatures only treat the product innovation within a macro context with little effort by far devoted to the analysis of supplier's performance to discover how product innovation is produced. Actually in most cases, the R&D and other product innovation activities are just implicitly assumed to take place automatically so long as supplier is input into the “product innovation generator”. There are several reasons that might explain why this problem has not been studied sufficiently. First, the product innovation is conducted mainly in the institutions affiliated with profit-maximizing firms, which tends to induce the economists to treat product innovation process in the same way as physical production. But actually these two kinds of processes are fundamentally different in many aspects. First of all, the knowledge and innovation are partially non-excludable and non-competitive, which is dissimilar to the common products, therefore the accumulation of knowledge is essentially different from accumulation of physical capital; Secondly, the product innovation process is intrinsically more independent, harder to monitor, more risky and vulnerable to failure, and with a more obvious accumulating nature which means today’s product innovation relies heavily on yesterday’s achievement. Thirdly, the profits for the suppliers is greatly different from that for the traditional production activities, the former is based on the non-market priority right or the market-oriented patent system, and it is more common that knowledge is undersupplied because the supplier is often not fully compensated duo to externality of knowledge. Fourthly, these two kinds of processes also take place in the different institutions. Another reason why a supplier’s performance hasn’t been intensively examined is possibly that it is quite difficult to make the analysis beyond the existent frameworks such as game-theoretic teamwork theory, principle-agent theory, and other topics on mechanism design etc. Admittedly, these frameworks of analysis could be important in dealing with this problem, but clearly they are not proper in some essential aspects. For instance, the analysis should be compatible in a dynamic general equilibrium setting, which means that the horizon should be the same with other agents such as a buyer; the supplier’s time preference is endogenously determined. The task of this paper is to make a preliminary analysis on a utility-maximizing supplier’s performance by developing a new model compatible with the dynamic general equilibrium setting, which aims to provide a more primitive negotiating foundation for the technical change. We will see later that this model might even provide a foundation for the technical fluctuation assumption assumed universally.

The remainder of this paper is arranged as follows: In the next section we review some relevant papers on this topic that have different contributions. Models are developed in section 3, a general dynamic model, supplier’s utility, product innovation function, the steady state analysis, dynamics of the product innovation achievement and the supplier, supply chain cooperation in product innovation. Where we find that the optimal effort level adjusts dynamically in a diversifying way and the research achievement also undergoes variation accordingly, in some circumstances following a saddle-point stable equilibrium path while in others showing a cyclical change. The frequency of the fluctuation is usually determined by the nature of the research process or the supplier’s own characteristics such as supplier’s product innovative ability. The short-run analysis of the effect of the supply chain cooperation is also conducted in the product innovation activity. The concluding remarks are in the last section.

2. LITERATURE REVIEW

The first paper that tries to model this phenomenon was carried out by Krugman (1979). He developed an exogenous product innovation rate of new product, g, and an exogenous rate of technology migration. This exogenous process ensures that the share of the supplier product measure in the whole measurement is constant. In his analysis, he got that the technological lag gives rise to exporting new products and importing old products. This one is really suggestive for product innovation effect, but also suffers from the causes for this technology transfer. Also, the assumptions are simple enough and lose some kind of generality. The level of supplier integration in product development ranges from simple consultation in the design of products, to the independent development of entire modules by suppliers [25]. Research shows that supplier involvement in new product development enhances the customer firm's performance along various dimensions. Supplier involvement in the product development reduces development cost, development time, and cost of the product to the customer firm [25]. Supplier involvement in innovation helps firms penetrate at a faster rate into the new markets, share risks among suppliers and increase the competency level. Park and Oduntan (2010) identified the following key attributes for innovative suppliers.

- Innovative suppliers are responsible for improving the attributes related to the module they innovate.
- The modules supplied by innovative supplier have unique attributes which can be perceived by the buyer.
- Innovative suppliers identify the features and attributes of the module as desired by the buyers and work for improving the attributes.
- Innovations in the modules supplied by innovative suppliers can influence the buying decision of the buyer.
- Clark and Fujimoto (1991) classified the parts related to auto industry into three types based on the extent of involvement of suppliers in their development:
- "Detailed control parts" in which development is performed entirely by the buyer i.e., the auto manufacturer
- "Black-box" parts in which specifications and interface requirements are given by buyer. "Supplier proprietary" parts whose development is performed entirely by the supplier [3].

Fredrik *et al*. (2002) identified nine factors which are crucial for the success of new product development based on the case study of an auto manufacturer. Proactive role of the supplier, role of auto manufacturer as coordinator, linkage between production and development, supplier's support to other auto manufacturers are the four key factors which need to be investigated before making the decision to involve suppliers in new product development [4]. Liker *et al*. (1996) identified several variables, including tier structure, degree of responsibility, inter-company communication, intellectual property agreements and supplier membership which play key role in supplier integration and the success of new product development projects. Primo *et al*. (2002) found that factors such as current technological capabilities and product innovation level required by the customer firm are critical for analyzing the level of involvement of suppliers in the product innovation. Wagner *et al.* (2006) categorized the critical factors for the success of new product innovation projects with the involvement of suppliers into two domains such as factors related to organizational level and management of suppliers in the project. The architecture of the product design and interaction with suppliers during product innovation process must be in coherence [2]. Modular architecture of the product allows one-to-one mapping of the functional requirements to the physical components and allows standardized interfaces between modules [22]. Modular product architecture enables easy upgrade and substitution of components allowing the customer firm to divide the design and development activities to the suppliers efficiently [19, 24]. With the integral product architecture there is more than one physical component which performs a single functional requirement [22], which makes the task of dividing the design and development of components complex because a change in one functional requirement necessitates changes in more than one physical components [19, 24]. Supplier involvement strategies depend on product architecture, design and interfaces with suppliers ranging from “none” and “white box” to “grey box” and “black box” supplier integration [13]. Henderson and Clark (1990) distinguished the R &D capabilities of the suppliers as architectural and component knowledge. Component knowledge refers to the capability of the supplier to design and manufacture the component for the final product, but not the final product itself. Architectural knowledge refers to the ability of the supplier to integrate and coordinate the knowledge between other suppliers and customer firm. Supplier assessment based on their manufacturing, assembly and logistic expertise is vital for supplier selection in the new product development process [13]. Mabert *et al*. (1992) found that early involvement of suppliers in the innovation process reduces the development time. Wagner and Hoegl (2006) categorized the variables influencing supplier selection into “hard” and “soft” criteria categories. “Hard” criteria involve supplier potential to innovate new products for the customer, and a “soft” criterion involves openness, mutual support and reliability between the supplier and the customer firms. Although much research supports the theory that supplier product innovation is beneficial to the new product development performance of a firm, there is some research that suggests supplier product innovation might not have a significant positive influence on a firm's product development performance. Littler *et al*. (1998) through their study of UK firms in communication sector concluded that the involvement of suppliers increases the cost of product as the complexity involved in the management of collaborative projects increases. As the differentiation among the firms involved in new product development increases, challenges to achieving common goals also increase [20]. A major obstacle for supplier integration comes from unwillingness to share the internal design information and not invented here culture which prevents engineers from relinquishing product development responsibilities to suppliers [18]. Johnson (1999) emphasizes the need for the implementation of standard procedures to involve suppliers in new product development processes. Rapid generation of new technology creates technology turbulence in the business environment in which the firms operate. Technology turbulence reduces the life cycle of the product as new products with new technology emerge at faster rate [21]. The firms should increase research and development activities and create advanced products from the new technologies to capture market and to retain competitive position [12]. To cope with challenges created by the technology turbulence, the firms must continuously strive to introduce new products at faster rate to sustain competitive advantage [16]. Adopting supplier product innovation strategy can reduce the development time of the product as the suppliers are more knowledgeable about their products which enable the manufacturer to release new products to sustain competitive advantage. By utilizing supplier product innovation strategy, manufacturers can invest more resources in developing the core competency while outsourcing the product innovation activities related to non-core competency. Following our review of literature we categorized the variables influencing supplier product innovation into the following categories (Table 1): environmental characteristics; supplier attributes; product characteristics; quality and management of relationship; and duration of partnership.

Table 1: Classification of characteristics for the supplier involvement in the new product development.

Category	**Variable Investigated in Prior Studies**
Firm Characteristics	• Willingness to accept external ideas
Nature of Enterprise Supplier Relationship	Role of the supplier (Clark,1991) and enterprise Supplier's responsibility Frequency of communications (Wasti and Liker, 1999) Nature of relationship (what aspect?) Birou and Fawcett (1994) Newness of partnership - Gerwin and Ferrari (2004) Distribution skills - Gerwin and Ferrari (2004) Degree of differentiation (sushman and ray, 1999) Coordination (Gerwin, 2004 Timing for the involvement of suppliers (wasti and liker 1997)
Supplier Characteristics	• Uniqueness (Park *et al*. 2010) • Technical expertise • Innovativeness (Wagner and Hoegl, 2006) • Component and architectural knowledge (Henderson,1990) • Supplier support to competitors • Downstream customer orientation(Wagner, 2010) Trust and reliability (Wagner and Hoegl, 2006) Supplier Innovation rate
Business Environment	• Technology Turbulence (Tushman*et al*. 1986) • Number of suppliers (Swan *et al*. 2003)
Product Characteristics	• Integral product • Modular design (Schrader and Göpfert, 1997) • Number of modules • Product Dynamism (Swan *et al*. 2003)

3. THE MODEL

Assumptions

(i) Products can be innovated
(ii) Supplier has the ability to innovate the product.

(iii) The quality level associated with a new variety of products created at time t in supplier is linked to the average level of weighted qualities at time t.
(iv) Supplier's time preference factor is not constant.
(v) Product innovation becomes more difficult as products improve in quality.
(vi) Product innovation process is deterministic.
(vii) Innovation takes place the form of improvements in the quality of products.

Notations:

t Time
$I(t)$ Product innovation at time t
$m(t)$ Effort level made by supplier at time t
u Utility function of supplier
β Supplier's time preference factor
$\beta(I)$ Supplier's time preference function
f Product innovation function
δ Depreciation rate
γ Delaying of the supplier
σ Risk aversion coefficient of the supplier
φ Intrinsic patience of the supplier
A Capital input of the supplier
α Elasticity of the newly achieved product innovation to the effort level

3.1. Supplier's utility

The utility of the supplier can be influenced by the monetary return for a specific product innovation or product invention, etc. and of course the effort level. Actually, the utility function is best characterized by the detailed treatment according to the entire monetary return system. Here for simplicity, we only select the two most fundamental factors, namely the achieved product innovation $I(t)$ and the effort $m(t)$, where t represents the time. The utility function $u(I(t), m(t))$ is assumed to be twice differentiable and satisfies that $\frac{\partial u}{\partial I} > 0$, $\frac{\partial^2 u}{\partial I^2} < 0$, $\frac{\partial u}{\partial m} < 0$.

The supplier's time preference factor, β, is not constant in this paper, instead, it's determined by the product innovation achieved by the supplier. One reason for this assumption is the generally observed fact that the rule of diminishing marginal return also applies for the effect of achievement on the patience. In other words, the larger the achieved product innovation, the more enduring the supplier becomes; but the marginal increase of endurance is decreasing. Assuming the time preference function $\beta(I)$ is differentiable, we then have $\beta'(I) < 0$ and $\beta''(I) > 0$. By contrast, the assumption of the endogenous time preference, as we can see later, provides us with a tremendously satisfying explaining power as to the various kinds of dynamic evolution of the supplier's characteristics as well as supplier's innovation achievements. As a common practice, the supplier is assumed to live infinitely. This is particularly helpful when we analyze the long run dynamics. And it is more compatible with most of the dynamic general equilibrium models in literature. The supplier's product innovation activity is highly independent and harder to monitor, so given the related incentive system, the supplier's characteristics is mainly determined by supplier's own will. Thus the supplier is to maximize the following inter-temporal utility:

$$\int_0^\infty u(I(t), m(t)) \exp\{-\int_0^t \beta(I(s))ds\}dt \quad (1)$$

The product innovation generating process is intrinsically very risky. However, here to keep the analysis as simple as possible, we temporarily assume that the process is deterministic. This can be partially justified by noting that the present paper is to describe a representative agent model, which could be seen as the average level over a very large sample. According to the strong large number theorem in the probability theory, the average level reflects the most regular behavior.

In this paper, the product innovation function for a supplier has two arguments, namely effort level $m(t)$ and the past product innovation achievement $I(t)$. Please note that the past product innovation can also serve as the proxy for the supplier's "useful knowledge stock" which is essential for the product innovation. We have not explicitly adopted the variable of physical capital input partly because no consensus has been satisfyingly achieved as for its effect on product innovation, and more importantly, we herein focus mainly on the supplier's characteristic per se. So for analytical convenience, all the other possible factors exogenous to the supplier such as the environment including capital input are simply taken as given. The product innovation function $f(m(t), I(t))$ is assumed to be twice differentiable with $\frac{\partial f}{\partial m} > 0$, $\frac{\partial^2 f}{\partial m^2} < 0$ and $\frac{\partial f}{\partial I} > 0$, $\frac{\partial^2 f}{\partial I^2} < 0$. We also know that the newly product innovation might render some past product innovations obsolete, so the net increase of product innovation equals to the remaining part after subtracting some depreciation of the past product innovation achievement from the newly product innovation, that is,

$$\dot{I} = f(m, I) - \delta I \quad (2)$$

Where δ is the constant depreciation rate.

The supplier's problem is to maximize (1) subject to (2) with the initial innovation $I(0)$ as given. $I(0)$ is determined before the supplier enters the in the product innovation process. Since the supplier usually has the ability to innovate the product, hence we can reasonably assume that $I(0) > 0$.

We define $\Delta_t \equiv \int_0^t \beta(I(s))ds$, then we have $d\Delta_t = \beta(I)dt$. The optimization problem faced by the supplier can be restated as follows:

$$Max \int_0^\infty \frac{u(I, m)}{\beta(I)} e^{-\Delta_t} d\Delta_t$$

$$\text{s.t.} \frac{dI}{d\Delta_t} = \frac{f(m, I) - \delta I}{\beta(I)} \quad (3)$$

Construct the Hamiltonian as follows:

$$H = \frac{u(I,m)}{\beta(I)} + \lambda \frac{f(m,I) - \delta I}{\beta(I)} \tag{4}$$

The first order conditions are:

$$u_m(I,m) + \lambda f_m(m,I) = 0 \tag{5}$$

and

$$\frac{d\lambda}{dt} = \lambda[\beta(I) - (f_I - \delta) + \frac{(f - \delta I)}{\beta(I)} \beta'(I)] - [u_I - \frac{u\beta'(I)}{\beta(I)}] \tag{6}$$

and the transversality condition:

$$\lim_{\Delta_t \to \infty} \lambda I e^{-\Delta_t} = 0$$

Equation (5) indicates that the marginal contribution (in current value) of net product innovation increase to the supplier's utility equals to the ratio between the marginal disutility from effort and the marginal productivity of effort. To further sharpen our insight and enrich the implications from the general model above, I will adopt as an example a set of more specific forms of the utility function $u(I(t), m(t))$, the product innovation function $f(m(t), I(t))$, and the function of time preference $\beta(I)$ in the following analysis. Although this may unavoidably cause some loss of generality, we can be still reasonably confident that most of our conclusions derived from this specific set of assumptions are robust when alternative assumptions are adopted, so long as these assumptions satisfy the basic conditions listed above.

3.2. Product innovation function

We assume that the utility function takes the form, When $\sigma = 1$, we define $u(I,m) = \frac{(\frac{I}{m^\gamma})^{1-\sigma} - 1}{1-\sigma}$, which is equivalent to $u(I,m) = \ln(I) - \gamma \ln(m)$. Most of our results remain true for this special case, so we just ignore it.

$$u\,(\mathrm{I},\mathrm{m}) = \frac{(\frac{I}{m^\gamma})^{1-\sigma}}{1-\sigma} \tag{7}$$

Where $\sigma \neq 1$ is the relative risk aversion coefficient of the supplier, and the coefficient $\gamma > 0$ measures the delaying of the supplier, less γ means more diligence. The simplest form of the time preference function that has the required properties is

$$\beta(I) = \frac{\varphi}{I}, \tag{8}$$

where the positive coefficient φ stands for the intrinsic patience of the supplier.

Since both the effort level and the knowledge for product innovation are essential factors, we have the product innovation function take the Cobb-Douglass form as follows:

$$f(m(t), I(t)) = Am^{\alpha} I^{\theta} \tag{9}$$

where coefficient A can be interpreted in many ways, for example, it could reflect the capital input of the supplier or any form of support from the outside. The positive coefficient α is the elasticity of the newly achieved product innovation to the effort level, which indicates the product innovative ability of the supplier. $\theta \geq 0$ measures the reliance of the product innovative activity on the earlier product innovation, if it equals zero, it means the new achievement is independent from the history. Note that we place no restriction of $\alpha + \theta = 1$, since there exists no convincing evidence supporting constant return to scale in the knowledge production.
From (5), (6), (7), (8), (9), we derive

$$\frac{\dot{I}}{I} = Am^{\alpha} I^{\theta-1} - \delta \tag{10}$$

and

$$[-\gamma(1-\sigma)-\alpha]\frac{\dot{m}}{m} = \frac{\varphi}{I} + \delta(3-\sigma-\theta) - [2-\sigma+\frac{\alpha(2-\sigma)}{\gamma(1-\sigma)}]Am^{\alpha} I^{\theta-1} \tag{11}$$

3.3. The steady state analysis

We can derive the steady state immediately from the above two equations (10) and (11):

$$m^* = \left[\frac{\delta^{\theta}[\theta-1+\frac{\alpha(2-\sigma)}{\gamma(1-\sigma)}]^{\theta-1}}{A\varphi^{\theta-1}}\right]^{\frac{1}{\alpha}} \tag{12}$$

and

$$I^* = \frac{\varphi}{\delta[\theta-1+\frac{\alpha(2-\sigma)}{\gamma(1-\sigma)}]} \tag{13}$$

Please note that in order to make the analysis more relevant, throughout the paper we simply deal with the case when the following condition is satisfied:

$$\theta-1+\frac{\alpha(2-\sigma)}{\gamma(1-\sigma)} > 0 \tag{14}$$

Obviously, when $\sigma \in (1,2)$, equation (14) means that $\theta > 1$. From now on, we always assume that $\theta > 1$ holds unless explicitly stated otherwise.

We can easily prove the following two propositions.

Propositions 1: *In the steady state, the ultimate optimal effort level* m^* *is positively determined by the depreciation rate of the exiting knowledge* δ *and is negatively determined by the outside support to the innovation* A *and the intrinsic impatience* φ *if* $\theta > 1$. *But the impact of the delaying degree* γ *on* m^* *is determined by the supplier's risk attitude* σ *and the dependence of the product innovation activity* θ. *When the risk-aversion coefficient* $\sigma \notin [1,2]$ *(for example, if the supplier is risk-preferred or risk-neutral), the larger* γ *leads to less* m^* *if* $\theta > 1$*; when* $\sigma \in (1,2)$*, the result is reversed.*

Proof. *According to the equation (12),* $\frac{\partial m^*}{\partial \delta} > 0$, $\frac{\partial m^*}{\partial A} < 0$*;*

$$\frac{\partial m^*}{\partial \varphi} < 0 \text{ if } \theta > 1; \frac{\partial m^*}{\partial \varphi} > 0 \text{ if } \theta < 1,$$

$$\frac{\partial m^*}{\partial \gamma} > 0, \text{ when } \sigma \notin [1,2], \frac{\partial m^*}{\partial \gamma} < 0 \text{ ,otherwise, if } \theta > 1$$

The increase in the depreciation rate of the existing knowledge threatens the supplier's sustainable ability to achieve future product innovations, thus forces the supplier to exert a greater effort to guarantee the product innovation output to avoid the plumage of the utility. The most puzzling result might be the negative role that the outside support plays on the long run characteristic of the supplier. The better equipment for product innovation, or easier access to the success of product innovation actually induces the supplier to shirk instead of stimulating supplier enthusiasm for harder work. This is particularly relevant for the supplier's policy-making when they consider the projects of upgrading the instruments and improving the working conditions of the supplier. These projects should be carried out with some caution not only to save the product innovation expenditure, but also to guarantee a satisfying level of effort taken by the supplier, although these projects are indispensable sometimes for the research.

This proposition tells us another novel conclusion that in some cases the intrinsically supplier has to work more diligently. This is because the supplier has to achieve greater product innovation in order to offset the relatively more disutility from a given level of effort. Of course, when the risk aversion coefficient of the supplier is moderately low but not too low, the supplier would rather shirk because the disgust for the greater effort exceeds the utility gain from achievement of more product innovation due to the more effort input.

Corollary 1: *If the risk-aversion coefficient of the supplier happens to be that* $\sigma = 2$*, then the delaying degree* γ *has no impact on the effort level,* m^**, and the enhancement of innovative ability,* α *, definitely causes the reduction of optimal effort.*

This corollary gives the critical value of the supplier's attitude toward risk at which the delaying degree has impact on the ultimate research, for the tendency to offer more effort in order to achieve more product innovation, and the tendency to reduce the effort level to decrease the disutility from the more effort happen to cancel out each other.

Propositions 2: *In the steady state, the ultimate product innovation* I^* *of the supplier is negatively correlated with the depreciating rate* δ *and the dependence of new product innovation on the past achievement* θ*,but is positively correlated with the supplier's* *intrinsic impatience degree* φ *;However, the impact of the supplier's product innovative ability* α *and the delaying degree* γ *on* I^* *is determined by the supplier's risk attitude. More exactly, when the risk-aversion coefficient* $\sigma \notin [1,2]$ *(for example, if the supplier is risk-preferred or risk-neutral), larger* γ *and/or lower* α *causes larger* I^**; when* $\sigma \in (1,2)$*, the result is reversed.*

Proof: *Using the equation (13) and the results of the first proposition.*

This proposition expatiate the different factors that influence the long-run product innovation level. It shows that faster depreciation of the product innovation compels the supplier to exert a greater effort (this has been proved in the first proposition), which is, however, less than offset the depreciation effect. This also implies that there should be some worry about the possible shrinkage of the knowledge stock due to the accelerating depreciation of existent knowledge. This result is consistent with some recent researches which have indicated that the increasing difficulty of product innovation leads to the decline of aggregate product innovation speed. Actually we have also pointed out that the product innovation difficulty may be partly due to the accelerating depreciation of existent knowledge. Another surprising result is that the more contribution share of the past product innovation to the new achievement reduces the ultimate product innovation level. This phenomenon, however, can be explained as that the existing product innovation might induce the supplier to work less diligently, just waiting passively for the product innovation to emerge automatically, and this effort-substitution effect dominates its positive role in the product innovation activity.

It is intuitive that an intrinsically more impatient supplier might attach more value to the short run product innovation achievement, which in turn boosts the future product innovation because of the strong dependence nature of product innovation activity (note $\theta > 1$), therefore, the long-run optimal effort level declines due to the substitution effect.

The proposition 2 also highlights the importance of the supplier's attitude toward risk. We know that when the risk-aversion coefficient does not fall onto the interval [1,2], the higher product innovative ability can increase the short run product innovation more effectively which makes the supplier more and more patient with the product innovation achievement because of the endogenous time preference. The over-patience has prevented the supplier from exerting a sufficient effort, which decreases the final product innovation level. This proposition proves that when $\sigma \in (1,2)$, the result is reversed.

We also have the counterpart for the corollary 1.

Corollary 2: *When the risk aversion coefficient* σ *equals to two, the delaying degree* γ *has no effect on the ultimate product innovation level. It's noteworthy saying that both the product innovative ability* α *and the outside support for the product innovation activity* A *have no influence on the ultimate achievement of product innovation since they ruin some effort supply, although the product innovation capacity is strengthened.*

3.4. Dynamics of the product innovation achievement and the supplier

It seems not very hopeful to work out the analytical solutions to the nonlinear differential system constituted by equations (10) and (11), so we have to make the local stability analysis near the steady state characterized by the equations (12) and (13). After taking the linear approximation by Taylor expansion, we have the following linear system:

Define $D \equiv \begin{pmatrix} (\theta-1)\delta & \alpha\delta\dfrac{I*}{m*} \\ \dfrac{\frac{m*}{I*}}{-\gamma(1-\sigma)-\alpha}[-\dfrac{\varphi}{I*}-\delta(\theta-1)(2-\sigma+\dfrac{\alpha(2-\sigma)}{\gamma(1-\sigma)})] & \dfrac{\alpha\delta(2-\sigma)}{\gamma(1-\sigma)} \end{pmatrix}$,

Where $I*$ and $m*$ are given by equations (12) and (13).
We have

$$\begin{pmatrix} \dot{I}_\varepsilon(t,\varepsilon) \\ \dot{m}_\varepsilon(t,\varepsilon) \end{pmatrix} = D\begin{pmatrix} I_\varepsilon(t,\varepsilon) \\ m_\varepsilon(t,\varepsilon) \end{pmatrix} + \begin{pmatrix} z(t) \\ 0 \end{pmatrix} \tag{15}$$

We can work out the trace of the matrix D, which is

$$\mathrm{Tr}(D) = \delta[\theta-1+\frac{\alpha(2-\sigma)}{\gamma(1-\sigma)}] > 0, \tag{16}$$

And the determinant of the matrix is

$$\mathrm{Det}\,(D) = -\frac{\alpha\delta^2[\theta-1+\frac{\alpha(2-\sigma)}{\gamma(1-\sigma)}]}{\gamma(1-\sigma)+\alpha}\ .$$

Define $\Delta = Tr(D)^2 - 4Det(D)$.

Case I: $Det(D) < 0$, that is, $-\gamma(1-\sigma)-\alpha < 0$,

The two eigenvalues of the matrix D must be real, one positive and the other negative. It implies that the point $E(I*, m*)$ is a saddle in the $I*$-$m*$ space. To put it more formally, we have the following proposition:

Proposition 3: *When $Det(D) < 0$, typically, if the supplier is risk-neutral or even prefers the risk, the optimal effort input and the product innovation achievement of the supplier will converge to the long run steady state $E(I*, m*)$ along a unique saddle path.*

To be more concrete, let us see an example by adding some more conditions. Suppose that we further assume $3-\sigma-\theta \geq 0$, then it is easy to derive the following phase diagram from equation (11)

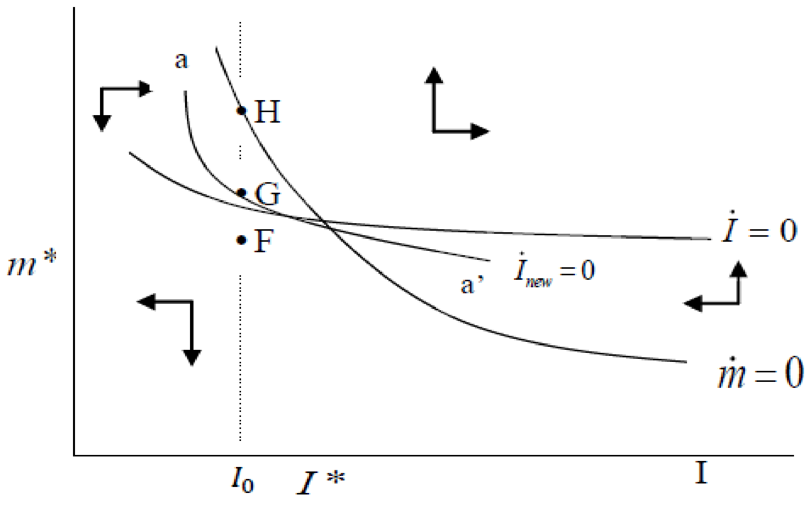

Figure 1: The optimal effort level obeys a saddle-stable path (curve aa') when $Det(D) < 0$, $3 - \sigma - \theta \geq 0$, $\theta > 1$ together with $\theta - 1 + \dfrac{\alpha(2-\sigma)}{\gamma(1-\sigma)} > 0$.

Suppose the initial innovation level is I_0 in *Figure 1,* if the supplier chooses too small effort, say, at F, then both the product innovation and effort levels approach zero, which case has been precluded earlier in this paper because the supplier with zero product innovation and complete shirking cannot stay in the research for a long time. If the supplier chooses too high an effort level, say, at the point H, then the effort tends to be infinite, which is clearly infeasible. Only when the effort level at point G is selected on the converging path aa' , can the supplier arrive at the feasible and sustainable steady state at the point E. If the initial product innovation level exceeds the steady state level, the optimal product innovation level gradually declines due to the large absolute amount of the depreciation of the existing knowledge while the effort level increases monotonically in order to innovate more for the compensation. The above analysis also indicates that when the product innovation level is lower than the I^*, the optimal effort will gradually come down with the product innovation approaching the steady state E. It can be proved that this result also obtains in a much broader case.

Case II: $Det(D) > 0$*, that is,* $-\gamma(1-\sigma) - \alpha > 0$.

In this case, the matrix D *has two eigenvalues with positive parts, which means that the steady state E is a source.*

Sub-case 1*:* $\Delta > 0$*, that is* $\theta > 1 - \dfrac{\alpha(2-\sigma)}{\gamma(1-\sigma)} - \dfrac{4\alpha}{\gamma(1-\sigma)+\alpha}$.

Point E is an unstable node. Therefore, there exists no convergent path toward the equilibrium point. The effort level tends to be infinite or zero dependent on the initial conditions.

Sub-case 2: $\Delta = 0$, *that is* $\theta = 1 - \frac{\alpha(2-\sigma)}{\gamma(1-\sigma)} - \frac{4\alpha}{\gamma(1-\sigma)+\alpha}$.

Point E is an unstable focus. The implication is similar to sub-case 1.

Sub-case 3: $\Delta < 0$, *that is* $1 < \theta < 1 - \frac{\alpha(2-\sigma)}{\gamma(1-\sigma)} - \frac{4\alpha}{\gamma(1-\sigma)+\alpha}$.

The two eigenvalues are imaginary with positive real parts. Therefore the dynamic path is periodically divergent. Please see the following phase portrait:

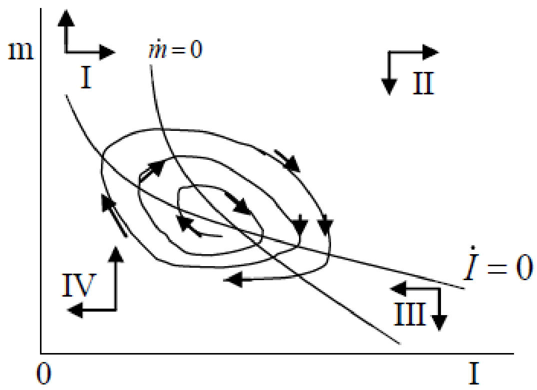

Figure 2: The optimal effort level obeys a periodical divergent path

when $\Delta < 0$, $\theta > 1$ *together with* $\theta - 1 + \frac{\alpha(2-\sigma)}{\gamma(1-\sigma)} > 0$.

The above Figure 2 shows that in the neighborhood of the steady state, the supplier's performance is diverging periodically: When the product innovation level is below the steady state level I^* and the effort level is not very high, say in the area IV, then the product innovation achievement is diminishing, which pushes the supplier to increase his effort with an accelerating speed; When supplier's effort level is sufficiently high, supplier enters the area I, the product innovation achievement starts to bump back more and more quickly thanks to the relative high input of effort. The supplier is probably satisfied with this achievement, and more product innovation experience accumulated in this process substitutes part of supplier's effort level, so supplier's increasing speed of the effort supply is declining. However, the supplier is too optimistic about supplier's future achievement and begins to reduce supplier's effort when he climbs over the peak of supplier's effort and enters the area II. The product innovation speed declines although the absolute level of the product innovation is still moving upward due to the relative abundant product innovative experience and the relative great effort input in terms of the absolute amount. Of course, the existing product innovation stock cannot be relied on forever without adequate effort input. We see that when the supplier comes into the area III, with the depreciation of the past product innovation knowledge and the diminishing input of the effort, the product innovation level finally declines with an accelerating speed .The disutility from the insufficient achievement brings the supplier to fully realize how essential supplier’s own effort is for the generating of product innovation, hence the supplier returns to the area I, and resumes supplier’s passion for the product innovative work and devotes more and more effort to supplier’s research. Thus a cyclical movement emerges. This periodical research performance continues forever without converging.

As we have seen, theoretically if the dynamic system is unstable, the research performance and the related product innovation achievement of the supplier tend to infinity. It is necessary, however, to modify this conclusion from a practical standpoint, since the effort level of an individual supplier cannot be without limit, for example, once the dynamic trajectory first meets, say, the upper border of effort, the effort can no longer grow higher, hence supplier’s product innovation achievement cannot be infinitely large either. But this constrained optimal control problem is much more complicated, here we shall not pursue in this direction. Therefore, we can by far only conclude that within the border of feasible effort supply, the supplier behaves as we have analyzed above. Another meaningful problem that we want to ask is what determines the period of this divergent cycle, the answer lies in the following proposition, which also summarizes the foregoing analysis:

Proposition 4: *When* $\Delta < 0$, *that is* $1 < \theta < 1 - \frac{\alpha(2-\sigma)}{\gamma(1-\sigma)} - \frac{4\alpha}{\gamma(1-\sigma)+\alpha}$, *the supplier’s performance and supplier’s corresponding product innovation achievement near the steady state vary periodically and divergently without halt. The period T of the behavioral cycle is negatively determined by the depreciation rate of past product innovative knowledge* δ *; but it is independent from the intrinsic impatience degree* φ. *The period T prolongs with the increase of the dependence of the product innovative activity* θ *when it is fairly large shortens when it is relatively small.*

Proof: *It can be worked out that the two eigen values of the coefficient matrix* D *is*

$$\frac{1}{2}\delta[\theta-1+\frac{\alpha(2-\sigma)}{\gamma(1-\sigma)}] \pm i \bullet \frac{1}{2}\sqrt{-\delta^2[\theta-1+\frac{\alpha(2-\sigma)}{\gamma(1-\sigma)}]\{[\theta-1+\frac{\alpha(2-\sigma)}{\gamma(1-\sigma)}]+\frac{4\alpha}{\gamma(1-\sigma)+\alpha}\}}$$

So the dynamical system constituted by matrix equation (15) can be transformed into the following standard form:

$$\begin{pmatrix} \dot{\xi} \\ \dot{\eta} \end{pmatrix} = \tilde{D} \begin{pmatrix} \xi \\ \eta \end{pmatrix},$$

where

$$\begin{pmatrix} \xi \\ \eta \end{pmatrix} = \begin{pmatrix} \phi_{11} & \phi_{12} \\ 0 & \phi_{22} \end{pmatrix} \begin{pmatrix} I - I^* \\ m - m^* \end{pmatrix}$$

$$\phi_{11} = \frac{\frac{m^*}{I^*}}{\gamma(1-\sigma)+\alpha}[-\frac{\varphi}{I^*}-\delta(\theta-1)(2-\sigma+\frac{\alpha(2-\sigma)}{\gamma(1-\sigma)})]$$

$$\phi_{12} = \frac{\delta}{2}[(\theta-1)-\frac{\alpha(2-\sigma)}{\gamma(1-\sigma)}]$$

$$\phi_{22} = \frac{\delta}{2}\sqrt{-\{[\theta-1+\frac{\alpha(2-\sigma)}{\gamma(1-\sigma)}]^2+[\theta-1+\frac{\alpha(2-\sigma)}{\gamma(1-\sigma)}]\frac{4\alpha}{\gamma(1-\sigma)+\alpha}\}}$$

$$\tilde{D} = \begin{pmatrix} \frac{1}{2}\delta[\theta-1+\frac{\alpha(2-\sigma)}{\gamma(1-\sigma)}] & \phi_{22} \\ -\phi_{22} & \frac{1}{2}\delta[\theta-1+\frac{\alpha(2-\sigma)}{\gamma(1-\sigma)}] \end{pmatrix},$$

and $e^{t\tilde{D}} = e^{\frac{1}{2}\delta[\theta-1+\frac{\alpha(2-\sigma)}{\gamma(1-\sigma)}]t}\begin{pmatrix} \cos(t\phi_{22}) & \sin(t\phi_{22}) \\ -\sin(t\phi_{22}) & \cos(t\phi_{22}) \end{pmatrix},$

Therefore, it is clear that the period of the behavioral cycle

$$T = \frac{4\pi}{\sqrt{\delta^2[\theta-1+\frac{\alpha(2-\sigma)}{\gamma(1-\sigma)}]\{-[\theta-1+\frac{\alpha(2-\sigma)}{\gamma(1-\sigma)}]+\frac{-4\alpha}{\gamma(1-\sigma)+\alpha}\}}}, \text{ and } \frac{\partial T}{\partial \delta} > 0; \frac{\partial T}{\partial \varphi} = 0;$$

$$\frac{\partial T}{\partial \theta} > 0 \text{ , when } 1-\frac{\alpha(2-\sigma)}{\gamma(1-\sigma)}-\frac{2\alpha}{\gamma(1-\sigma)+\alpha} < \theta < 1-\frac{\alpha(2-\sigma)}{\gamma(1-\sigma)}-\frac{4\alpha}{\gamma(1-\sigma)+\alpha};$$

$$\frac{\partial T}{\partial \theta} < 0 \text{ , when } 1 < \theta < 1-\frac{\alpha(2-\sigma)}{\gamma(1-\sigma)}-\frac{2\alpha}{\gamma(1-\sigma)+\alpha}$$

This proposition tells us how the nature of product innovation process and the supplier's characteristics influence the variation frequency of the supplier's performance and supplier's corresponding product innovation achievement. Larger depreciation rate of the product innovative knowledge leads to the larger variation frequency of the supplier's performance; this is because quicker depreciation makes the existent stock of product innovative knowledge less dependable in the product innovation process, which compels the supplier to adjust his effort level more frequently in order to buffet the quicker change of the stock of product innovative knowledge. Instead, when the depreciation rate is small, the supplier only needs to finely calibrate supplier's effort level according to the relative stability of the stock of product innovative knowledge. It's surprising that the supplier's intrinsic impatience φ has no impact on the adjustment frequency. Intuitively speaking, when φ becomes larger, the supplier is more likely to make supplier's effort supply decision primarily based on the present situation, that is, the supplier will give more weight to the current utility. However, the tendency to shirk to decrease the disutility from effort exertion happens to cancel out the tendency to increase product innovation by working harder. This proposition also indicates that the increase in the dependence of product innovative activity might either prolong the period or shorten it. When θ is relatively small, larger θ only means that product innovation is becoming more automatic, then the marginal utility from the product innovation increase has a fairly large substitution effect on the effort decision, therefore the dynamic change of optimal effort is more frequent. By contrast, when θ is relatively large, the supplier's effort level will change less frequently because the marginal utility from more product innovation is decreasing, only making the effort level more inert. We have also seen that the period is also related with the supplier's product innovative capacity α and supplier's intrinsic delaying γ, but the effect is indeterminate, dependent on the substitution effect between the effort and the product innovation achievement

3.5. Supply chain cooperation in product innovation

We have proved in part B that permanent change in the coefficient A (which represents any of the other essential factors for knowledge production except effort and past product innovation) cannot change the ultimate product innovation level in the steady state. Then what if the inessential factor changes? Particularly, what effect will supply chain cooperation in product innovation have on the optimal effort level and initial as well as the long –run product innovation? Since cooperation in supply chain is becoming more and more popular, we believe this is an increasingly important question in the study of product innovation activity. Now suppose that the supplier is in supplier's steady state at time t=0, when the cooperation begins.
Then, we have the following two equations:

$$\dot{I}(t,\varepsilon) = Am^{\alpha} I^{\theta} - \delta I + \varepsilon z(t) \tag{16}$$

$$\dot{m}(t,\varepsilon) = \frac{\dfrac{\varphi}{I} + \delta(3-\sigma-\theta) - [2-\sigma+\dfrac{\alpha(2-\sigma)}{\gamma(1-\sigma)}]Am^{\alpha} I^{\theta-1}}{-\gamma(1-\sigma)-\alpha} m \text{ ,} \tag{17}$$

Where ε is a parameter; function z(t) indicates the inter-temporal change of various parameters due to the supply chain cooperation. Here z(t) can be regarded as a parabola when $t \in [0,T]$ in this paper, because the benefit from supply chain cooperation usually first increases and then decline. After time T, the supply chain cooperation ends, therefore z(t)=0. Differentiating with respect to ε at ε =0 in equations (16) and (17), we have

$$\begin{pmatrix} \dot{I}_{\varepsilon}(t,\varepsilon) \\ \dot{m}_{\varepsilon}(t,\varepsilon) \end{pmatrix} = D \begin{pmatrix} I_{\varepsilon}(t,\varepsilon) \\ m_{\varepsilon}(t,\varepsilon) \end{pmatrix} + \begin{pmatrix} z(t) \\ 0 \end{pmatrix}, \tag{18}$$

where $D \equiv \begin{pmatrix} (\theta-1)\delta & \alpha\delta \dfrac{I^*}{m^*} \\ \dfrac{\frac{m^*}{I^*}}{-\gamma(1-\sigma)-\alpha}[-\dfrac{\varphi}{I^*} - \delta(\theta-1)(2-\sigma+\dfrac{\alpha(2-\sigma)}{\gamma(1-\sigma)})] & \dfrac{\alpha\delta(2-\sigma)}{\gamma(1-\sigma)} \end{pmatrix}$.

We will denote the Laplace transformations as follows:

$$I_\varepsilon(s,\varepsilon) = \int_0^\infty I_\varepsilon(t,\varepsilon)e^{-st}dt \quad ,$$

and $$Z(s) = \int_0^\infty z(t)e^{-st}dt \tag{19}$$

Following the above examples, we make the Laplace transformation in equation (18), we get

$$s\begin{pmatrix} I_\varepsilon(s,\varepsilon) \\ m_\varepsilon(s,\varepsilon) \end{pmatrix} = D\begin{pmatrix} I_\varepsilon(s,\varepsilon) \\ m_\varepsilon(s,\varepsilon) \end{pmatrix} + \begin{pmatrix} Z(s)+I_\varepsilon(0,\varepsilon) \\ m_\varepsilon(0,\varepsilon) \end{pmatrix} \tag{20}$$

Clearly, the product innovation level is the state variable, which cannot jump at the beginning, therefore, we have $I_\varepsilon(0,\varepsilon)$ =0 .Please note that here we only treat the case when the steady state is a saddle point.(We have proved that in this case $Det(D) < 0$, that is, $-\gamma(1-\sigma)-\alpha < 0$).We know that both $I_\varepsilon(s,\varepsilon)$ and $m_\varepsilon(s,\varepsilon)$ are bounded when $s = \mu$ (μ is the positive eigenvalue of the matrix D), but in this case the coefficient matrix of the linear equation (20) is singular. Therefore, by Cramer's rule, in order to maintain the finiteness of $I_\varepsilon(s,\varepsilon)$ and $m_\varepsilon(s,\varepsilon)$, the determinants of both of the following two matrices must be zero:

$$\begin{pmatrix} Z(s)+I_\varepsilon(0,\varepsilon) & -\alpha\delta\dfrac{I^*}{m^*} \\ m_\varepsilon(0,\varepsilon) & \mu - \dfrac{\alpha\delta(2-\sigma)}{\gamma(1-\sigma)} \end{pmatrix} \text{and}$$

$$\begin{pmatrix} \mu-(\theta-1)\delta & Z(s)+I_\varepsilon(0,\varepsilon) \\ -\dfrac{\frac{m^*}{I^*}}{-\gamma(1-\sigma)-\alpha}[-\dfrac{\varphi}{I^*}-\delta(\theta-1)(2-\sigma+\dfrac{\alpha(2-\sigma)}{\gamma(1-\sigma)})] & m_\varepsilon(0,\varepsilon) \end{pmatrix}.$$

We can easily derive

$$m_\varepsilon(0,\varepsilon) = \frac{[\mu - \dfrac{\alpha\delta(2-\sigma)}{\gamma(1-\sigma)}]}{-\alpha\delta\dfrac{I^*}{m^*}} Z(\mu) < 0, \tag{21}$$

Substituting $I_\varepsilon(0,\varepsilon)$ =0 and (21) back into equation (18) at time t=0:

$$\begin{pmatrix} \dot{I}_\varepsilon(0,\varepsilon) \\ \dot{m}_\varepsilon(0,\varepsilon) \end{pmatrix} = D\begin{pmatrix} I_\varepsilon(0,\varepsilon) \\ m_\varepsilon(0,\varepsilon) \end{pmatrix} + \begin{pmatrix} z(0) \\ 0 \end{pmatrix} \tag{22}$$

Proposition 5: *An initial increase in supply chain cooperation intensity of product innovation activity will boost the initial net product innovation achievement, but it has no impact on the initial change rate of effort.*

Proof: *According to (22)* $\frac{d}{dz(0)}(\dot{I}_{\varepsilon}(0,\varepsilon)) = 1 > 0$

$$\frac{d}{dz(0)}(\dot{m}_{\varepsilon}(0,\varepsilon)) = 0$$

This is because the initial increase of supply chain cooperative intensity can enhance the supplier's product innovation capacity immediately although the initial product innovation stock cannot jump at once. The initial effort change rate depends on the initial product innovation stock and the future product innovation expectation, but the only initial increase of supply chain cooperative intensity can alter none of them, therefore the decision of initial effort change rate remains unchanged.

Next, let's examine the effects of supply chain cooperation which lasts for a longer period of time. Suppose

$$z(t) = \begin{cases} at^2 + bt + c, & t \in [0,T] \\ 0, & otherwise \end{cases}, \text{ where } a < 0, b > 0, c > 0. \tag{23}$$

Then we have

$$Z(\mu) = -\frac{aT^2 e^{-\mu T}}{\mu} + Te^{-\mu T}(-\frac{2a}{\mu^2} - \frac{b}{\mu}) - e^{-\mu T}(\frac{2a}{\mu^3} + \frac{b}{\mu^2} + \frac{c}{\mu}) + (\frac{2a}{\mu^3} + \frac{b}{\mu^2} + \frac{c}{\mu}) \tag{24}$$

Please note that when we set a=b=0 and c=1, then function z(t) is the normal step function as is usually employed in such analyses ,in which case the $Z(\mu) = \frac{1-e^{-\mu T}}{\mu}$.

Proposition 6: *A temporal increase in the supply chain cooperation intensity will decrease the initial supplier's effort level and decrease the initial net product innovation achievement.*

Proof: *According to equations (21) and (22), we have*

$$\frac{d(m_{\varepsilon}(0,\varepsilon))}{d(Z(\mu))} = \frac{[\mu - \frac{\alpha\delta(2-\sigma)}{\gamma(1-\sigma)}]}{-\alpha\delta\frac{I^*}{m^*}} < 0 \quad \text{, and}$$

$$\frac{d}{dZ(\mu)}(\dot{I}_{\varepsilon}(0,\varepsilon)) = \alpha\delta\frac{I^*}{m^*}\frac{d(\dot{m}_{\varepsilon}(0,\varepsilon))}{d(Z(\mu))} < 0$$

Proposition 6 indicates that the supplier will reduce supplier's initial effort level with the temporal increase in the supply chain cooperation intensity in a joint knowledge production because the temporal supply chain cooperation program will have an inter-temporal substitution effect on the effort supply decision at each time. Please recall, however, that only an initial increase of supply chain cooperation intensity will not affect the initial change rate of the effort level (See Proposition 5). Since the supplier's initial effort level is reduced while the initial stock of product innovation remains unchanged, it is very natural that the initial net product innovation achievement will decline because the enhanced capacity of product innovation through supply chain cooperation cannot sufficiently compensate the effort-reduction effect. Obviously, when we extend T to infinity in the form of the function z(t) ,which means that the supply chain cooperation is permanent, we can also analyze the long-run effect of permanent supply chain cooperation. Suppose that $\lim_{t\to\infty} z(t) = \hat{z} > 0$, then we can easily prove that the new steady state E** is

$$I^{**} = \frac{[2-\sigma+\dfrac{\alpha(2-\sigma)}{\gamma(1-\sigma)}]\varepsilon\hat{z}+\varphi}{\delta[\theta-1+\dfrac{\alpha(2-\sigma)}{\gamma(1-\sigma)}]} \tag{25}$$

$$m^{**} = \left[\frac{\delta^{\theta}[\theta-1+\dfrac{\alpha(2-\sigma)}{\gamma(1-\sigma)}]^{\theta-1}}{A\{[2-\sigma+\dfrac{\alpha(2-\sigma)}{\gamma(1-\sigma)}]\varepsilon\hat{z}+\varphi\}^{\theta-1}}\right]^{\frac{1}{\alpha}} \tag{26}$$

Proposition 7: *When* $2-\sigma+\dfrac{\alpha(2-\sigma)}{\gamma(1-\sigma)}$ *>0 and equation (14) holds, the permanent supply chain cooperation will increase the ultimate product innovation level but effectively reduce the supplier's effort level simultaneously.*
Proof: *It can be easily seen from equations (12), (13), (25), and (26).*

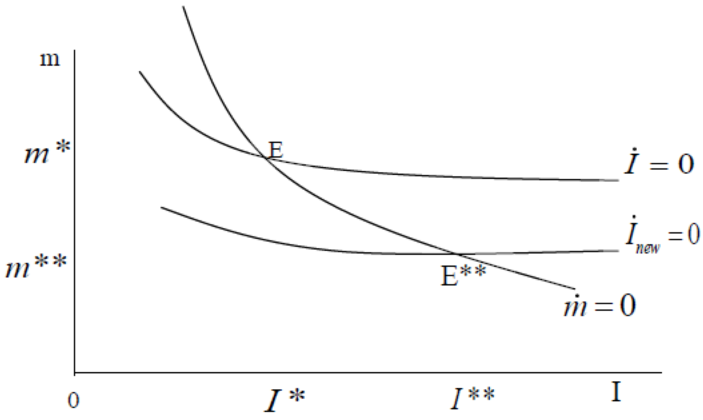

Figure 3: The long-run effect of permanent cooperation in innovation activity on the equilibrium effort and innovation levels when $\sigma < \min\{3-\theta, 1\}$, and $\theta > 1$

The above Figure 3 intuitively demonstrates the above proposition. This proposition can partly explain why it's becoming more and more popular in the supply chain to keep cooperating with other partners of the supply chain in the knowledge production process for a very long time. It can also illuminate the importance of "clustering effect" in the product innovation activity, because if a supplier is working in a place with a large number of suppliers, chances are that the permanent supply chain cooperation can be sustained much easily. Of course, we will be able to explain this phenomenon better if we interpret z(t) as many other factors that will affect the product innovation such as the working condition for information collection ,etc.

4. NUMERICAL EXAMPLE

To draw more managerial insights from Proposition 7, we examine numerical examples. In this section, we report key numerical results using parameter values in Table 2.

Table 2: Parameter values for numerical analysis

t	u	β	γ	σ	φ	A	α
100	2	4	50	1.5	0.3	10	5

Since we know how the supplier's effort level affects the product innovation, we focus on the numerical examples in which equations (12), (13), (25), and (26) is satisfied with varying by the depreciation rate of the exiting knowledge δ and the dependence of the product innovation activity θ. Figure 4 shows the ultimate optimal effort level $m*$ is positively determined by the depreciation rate of the exiting knowledge δ and the dependence of the product innovation activity θ. The increase in the depreciation rate of the existing knowledge warns the supplier's ability to achieve product innovations. The better equipment for product innovation or easier access to the success of product innovation is determined. This is particularly relevant for the supplier's policy-making when they consider the projects of upgrading the instruments and improving the working conditions of the supplier. Figure 4 shows another conclusion that, in some cases, the intrinsically supplier has to work more diligently. This is because the supplier has to achieve greater product innovation in order to offset the relatively more disutility from a given level of effort.

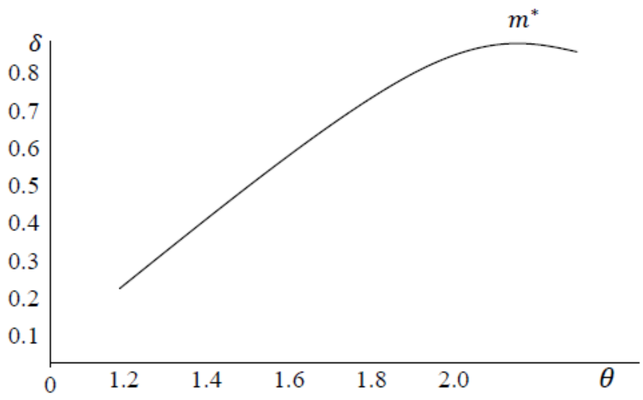

Figure 4: The ultimate effort level of the supplier is positively related to depreciating rate.

Figure 5 shows the effect of increase in the depreciation rate of the existing knowledge on the ultimate product innovation I^* of the supplier is negatively correlated with the depreciating rate δ and the dependence of new product innovation on the past achievement θ. Figure 5 expatiates how the depreciating rate has influence the long-run product innovation level. It shows that faster depreciation of the product innovation compels the supplier to exert a greater effort. This also explains that there should be some possible shrinkage of the knowledge stock due to the accelerating depreciation of existent knowledge. This result is consistent with some recent researches which have indicated that the increasing difficulty of product innovation leads to the decline of aggregate product innovation speed. Actually, we have also pointed out that the product innovation difficulty may be partly due to the accelerating depreciation of existent knowledge. This phenomenon, however, can be explained as that the existing product innovation might induce the supplier to work less carefully. It is intuitive that an intrinsically more impatient supplier might attach more value to the short run product innovation achievement. Figure 5 also highlights the importance of the supplier's attitude toward risk. The higher product innovative ability can increase the short run product innovation more effectively.

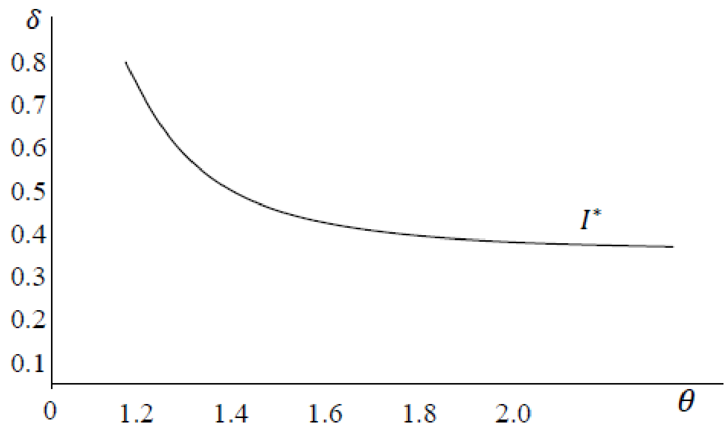

Figure 5: The ultimate product innovation of the supplier is negatively related to depreciating rate

Figure 6 shows that the effect of increase in the depreciation rate of the existing knowledge on the ultimate product innovation on increase of supply chain cooperation I^{**} of the supplier is negatively correlated with the depreciating rate δ and the dependence of new product innovation on the past achievement θ Figure 6 indicates that the temporal increase in the supply chain cooperation intensity will decrease the initial supplier's effort level and decrease the initial net product innovation achievement. Figure 6 indicates that the supplier will reduce supplier's initial effort level with the temporal increase in the supply chain cooperation intensity in a joint knowledge production. The supplier's initial effort level is reduced while the initial stock of product innovation remains unchanged. The initial net product innovation achievement will decline because the enhanced capacity of product innovation through supply chain cooperation cannot sufficiently compensate the effort-reduction effect.

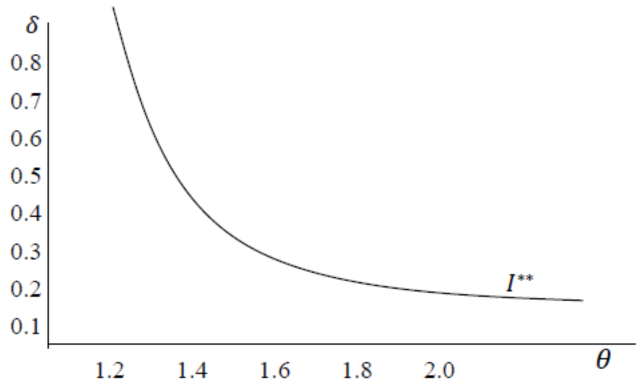

Figure 6: The ultimate product innovation in permanent supply chain cooperation of the supplier is negatively related to depreciating rate.

Figure 7 shows that the ultimate optimal effort level in the supply chain cooperation m^* is positively determined by the depreciation rate of the exiting knowledge δ and the dependence of the product innovation activity θ. The permanent supply chain cooperation will reduce the supplier's effort level simultaneously. The above Figure 7 intuitively demonstrates the effect on optimal effort level. This Figure 7 can partly explain why it's becoming more popular in the supply chain to keep cooperating with other partners.

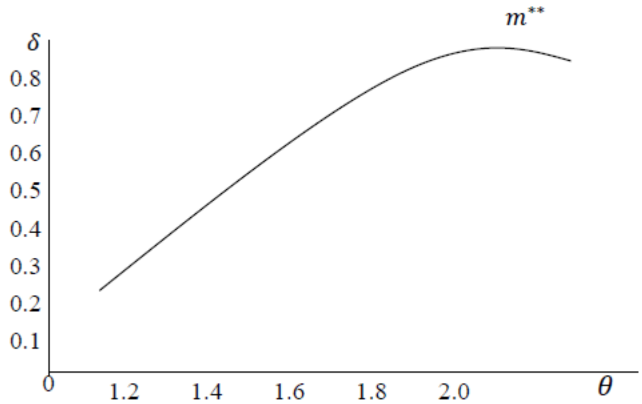

Figure 7: The supplier's effort level in permanent supply chain cooperation is positively related to depreciating rate.

We can ask what kind of industry can most benefit from this supplier's participation for product innovation. Based on the above observations, we can infer that it might be an industry for product innovation. We can use the analysis model to investigate other important dynamics associated with different parameters as well as variables.

5. CONCLUSION

This paper develops a dynamic model to explore supplier's intentional research characteristics, which aims to open the technical innovation assumed universally. Since the product innovation characteristics is a highly complicated production activity with great distinction from the traditional production process, we believe that the effort invested in this research is meaningful for the better understanding of the most primitive developmental micro-foundation for the endogenous technical change and fluctuations. Simultaneously, we hope to provide a more unified and tractable framework of analysis for the various factors that might boost or undermine the knowledge-creating activity, which is directly responsible for the sustained product development. For those suppliers with less advanced technology, the technical change may largely depend on the imitation through all sorts of channels, but the imitated technologies or the imported supplier equipment containing new technology have to be identified, chosen properly and adapted to the local use, which is also a creative research activity similar in nature to the new knowledge production that we analyze. We find that the fluctuations of the supplier's effort input and the related product innovation achievement result mainly from supplier optimal calculation even in the absence of uncertainty in the product innovative process. This discovery amazingly has the flavor of the essential proposition that the technology fluctuation is the best response to the change in technical environment, or particularly, the technological shock, but we go still further to prove that even the seemingly exogenous technical disturbance, to a great extent, is actually the rational response to the constraints imposed by the supplier's own characteristics and the nature of product innovation activity such as the attitude toward the risk, the product innovative ability, the depreciation rate and so on. It is clear that the aggregate technological level of a supplier is the outcome of a summation of the individual supplier's achievement with some modifications to the adding method.

Limitations

We acknowledge a number of limitations to this study. First, this study was limited to focus on the suppliers' characteristics and role of variables on supplier involvement outcomes only; these were selected based on the fact that managers mentioned them frequently during our field interviews. Further, results suggest that they plan an important role in exploring the variance in project outcomes. Second, quantified performance outcomes measures were not included, nor were there quantified measures of supplier involvement. We did, however, examine the correlations between subjective measures of performance and actual performance outcomes. Third, the effects of multiple industry respondents were not controlled for, and future studies should assess whether industry in fact does contribute to the variance in our results. Fourth, we relied on only two or three items for each construct, which is unlikely to capture the full domain of these concepts. We recognize this limitation to the study likely is due to the fact that the measures were new and that no previous measures of supplier involvement had been tested formally for convergent validity. We recognize that using new measures is always a risk, but we felt it was important to capture the key variables that emerged from the case studies and had to rely on developing new scales. Researchers pursuing work in this area may use our scales as a starting point for developing a stronger set of measurement items.

Future Research Directions

The results obtained from this study clearly show the importance of supplier involvement but also generate a set of new and interesting research questions. A set of additional issues that build on our empirical approach include the following:

- What are the key dimensions of explicit and tacit capabilities of suppliers that can predict the likelihood of a successful supplier involvement effort? For example, how should managers weigh the relative strengths and weaknesses of various suppliers in areas such as technological knowledge, manufacturing capabilities, and length of relationship with the supplier, degree of trust, and alignment of technology roadmaps with future products?
- What are the key variables a new product development team should consider in measuring the relative alignment of a supplier's technology roadmap with its customer's product requirements?
- What are the means to gain access to and to assess the degree of alignment of supplier technology roadmaps across multiple industries on a global basis? This type of research would involve understanding the nature of product development organizational structures, as well as an assessment of the role of advanced technology groups and technology boundary spanners.
- In cases when a future required product/process technology does not exist in the market, what are the key variables to consider whether to redirect a supplier's technology roadmap (outsource technology development) or to undertake the development of a new technology in-house (in source technology development)? This line of thinking takes the concept of technology uncertainty a step further by proposing that supplier involvement always may not be the best solution, as a firm may outsource a critical technology to an external party.
- How to differentiate the general purpose technology (GPT) from ordinary product innovation achievement? What if one's research achievement happens to be similar to another's? How to explicitly melt the research paralleling to the supplier's decision into a unified general equilibrium framework with the demand side of product innovation simultaneously treated? Quantitatively, how much of the technical fluctuation is due to the supplier's subjective adjustment and how much can be accounted by the indeterminate nature of the product innovation process per se? All these significant and challenging questions have not received adequate treatment in this paper by far, for this paper is only the preliminary step toward the deeper exploration in this area, presumably still more problems are waiting ahead for our effort

These questions represent challenges for the next decade; we believe they can benefit from prior insights derived from the literature on purchasing management, engineering management, and marketing. In order to understand fully how supplier involvement will unfold, we further believe that the focus should not be limited to single buying/supplying organizational units but should extend both up and down the supply chain. This framework would represent better the vision of the future, wherein entire supply chains of customers and suppliers will compete against similarly aligned chains, with the objective of creating the maximum value up and down the chain.

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