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DECISION MAKING WITH CONSONANT BELIEF FUNCTIONS: DISCREPANCY RESULTING WITH THE PROBABILITY TRANSFORMATION METHOD USED

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Abstract: Dempster–Shafer belief function theory can address a wider class of uncertainty than the standard probability theory does, and this fact appeals the researchers in operations research society for potential application areas. However, the lack of a decision theory of belief functions gives rise to the need to use the probability transformation methods for decision making. For representation of statistical evidence, the class of consonant belief functions is used which is not closed under Dempster's rule of combination but is closed under Walley's rule of combination. In this research, it is shown that the outcomes obtained using both Dempster's and Walley's rules do result in different probability distributions when pignistic transformation is used. However, when plausibility transformation is used, they do result in the same probability distribution. This result shows that the choice of the combination rule and probability transformation method may have a significant effect on decision making since it may change the choice of the decision alternative selected. This result is illustrated via an example of missile type identification.

Keywords: Belief functions, Consonant belief functions, Plausibility transformation, Pignistic transformation.

MSC: 68T37.

1. INTRODUCTION

Belief functions represent ignorance, and a wider class of uncertainty than the standard probability theory, which creates a flexible framework for any sort of application where information is gathered from semi-reliable resources. For that reason belief functions establish a major appeal to operation researchers for potential application areas wherein uncertainty is involved. Belief functions has been used in a wide range of applications such as target identification (Delmotte and Smets 2004), data fusion (Appriou 1997), auditing (Srivastava *et al*. 2011), and data mining (Wickramaratna *et al*. 2009).

Belief functions theory originates to the early works of Dempster (1967 & 1968) on upper and lower limits of probability. This work was developed by Shafer (1976) and thus, belief functions are also named as Dempster-Shafer theory of belief functions, or theory of evidence.

For combination of independent belief functions, Dempster's rule of combination is the classical one, and the most widely used rule in Dempster-Shafer (D-S) theory. However, the rule has been criticized on various matters. One of the criticisms made at DS combination rule is the fact that consonant belief functions are not closed under Dempster's rule of combination. As a remedy to this problem, Walley (1987) proposed an alternative rule for combination of belief function representations of statistical evidence. Consonant belief functions are closed under Walley's rule of combination; however, the drawback of Walley's rule is that it is only defined for partially consonant belief functions. A detailed review and comparison of these two combination rules is done by Cinicioglu and Shenoy (2006). In their work they also showed that when plausibility transformation is applied to outcomes obtained by Dempster's and Walley's combination rule, they do result in the same probability distribution function. The fact that there is no decision theory of belief functions gives rise to the need to transform belief functions into probability distributions. Consequently, according to the result obtained by Cinicioglu and Shenoy (2006), a decision made with the expected utility theory would be indifferent of the combination rule (Dempster or Walley's rule) used. On the other hand, though plausibility transformation is one of the available methods of probability transformation, it is not the only one. In this research, both pignistic and the plausibility transformation are used to transform the outcomes of both of the combination rules into a probability distribution. It is shown that when pignistic transformation is used on the outcomes obtained from both of the combination rules, the result is different probability distributions. So, the choice of the combination rule, and the resulting discrepancy, depending on the transformation method used, may have significant effect on decisions.

The remainder of the paper is as follows: In section 2, the basics of belief functions theory and consonant belief functions are introduced. Additionally, two combination rules, the classical rule of Dempster and Walley's combination rule for consonant belief functions are demonstrated via an example of Missile Fall Down. In section 3, the need for probability transformation is explained, and the two transformation methods, plausibility and pignistic transformation are applied to the results of the combinations. The discrepancy resulting from the choice of the combination and/or transformation method is demonstrated. Finally, in section 4, we summarize and conclude.

2. EVIDENCE THEORY

2.1. Belief Functions

Belief functions theory is also called the theory of evidence since it deals with weights of evidence and with numerical degrees of support based on evidence (Shafer 1976). The main advantage of belief functions lies in their ability to represent ignorance and ambiguity. As shown by Ellsberg's paradox (1961), the probability theory is unable to distinguish between a situation of complete ignorance and a situation where we have complete knowledge (Srivastava, 1997). Dempster-Shafer theory allows the description of partial or complete ignorance, since the belief not accorded to a proposition does not have to be accorded to the negation of that proposition (Cattaneo, 2011).

There are several equivalent ways of representing a belief function, namely basic probability assignment, belief function, plausibility function, and a commonality function.

Consider a set of mutually exclusive and exhaustive propositions, $\Theta = \{\theta_1, \theta_2, \ldots, \theta_K\}$, referred to as frame of discernment. A proposition θ_i states the lowest level of discernible information. Any proposition that is not singleton, e.g. $\{\theta_1, \theta_2\}$, is referred to as a composite.

A *basic probability assignment* (bpa) m for Θ is a function m: $2^\Theta \to [0, 1]$ such that

$m(\varnothing) = 0$ and $\Sigma\{m(A) \mid A \subseteq \Theta\} = 1$ (1)

$m(A)$ is a measure of the belief that is committed exactly to A. If $m(A) > 0$, then A is called a *focal* element of m. Note that if $\bar{A}$ is the complement of A, then $m(A) + m(\bar{A}) \leq 1$. Basic probability assignment differ from a probability function in that they can assign a measure of belief to a subset of the state space without assigning any belief to its elements. If all the focal elements are singletons, a belief function is reduced to a Bayesian probability function (Shafer 1976). Consequently, belief function calculus is a generalization of probability calculus, and any Bayesian model of uncertainty is also a belief function model (Shafer and Srivastava 1990)

There are three important functions in DS theory, belief functions, plausibility functions, and commonality functions. They can all be defined in terms of the basic probability assignments.

A *belief function Bel* corresponding to a bpa*m* is a function

Bel: $2^\Theta \to [0, 1]$ such that $Bel(A) = \Sigma\{m(B) \mid B \subseteq A\}$ for all $A \subseteq \Theta$ (2)

$Bel(A)$ can be interpreted as the probability of obtaining a set observation that implies the occurrence of A.

A *plausibility function Pl* corresponding to a bpa*m* is a function

Pl: $2^\Theta \to [0, 1]$ such that $Pl(A) = \Sigma\{m(B) \mid B \cap A \neq \varnothing\}$ for all $A \subseteq \Theta$ (3)

$Pl(A)$ can be interpreted as the probability of obtaining a set observation that is consistent with some element of A.

A *commonality function Q* corresponding to bpa*m* is a function

Q: $2^\Theta \to [0, 1]$ such that $Q(A) = \Sigma\{m(B) \mid B \supseteq A\}$ for all $A \subseteq \Theta$ (4)

$Q(A)$ can be interpreted as the probability of obtaining a set observation that is consistent with every element of A. Since the m-values add to one, commonality functions have the property:

$$\sum_{A\neq\varnothing}(-1)^{|A|+1}Q(A)=1 \tag{5}$$

Notice that for singleton subsets {θ}, the definitions of plausibility and commonality functions coincide, i.e., $Q(\{\theta\}) = Pl(\{\theta\})$ for all $\theta \in \Theta$.

A famous example named “Betty’s testimony”, given by Shafer & Srivastava (1990), demonstrates the fact that belief functions base the beliefs on the evidence. Suppose that I have a friend called Betty, and according to my subjective probability, Betty is reliable 90% of time, and she is unreliable 10% of time.

Accordingly, *P*(Betty = reliable) = 0.9, *P*(Betty = unreliable) = 0.1.

Betty tells me a tree limb fell on my car. Betty's statement must be true if she is reliable, but it is not necessarily false if she is unreliable. So, representing this situation under a belief function framework, the following results are obtained:

Bel(limb fell) = 0.9 Bel(no limb fell) = 0

It does not mean that I am sure that no limb fell on my car, as a zero probability would. This zero value only means that Betty's testimony gives me no reason to believe that no limb fell on my car.

In the following section, consonant belief functions are introduced, and the use of consonant belief functions for representation of statistical evidence is demonstrated.

2.2. Consonant Belief Functions

A belief function is said to be consonant if its focal elements are nested, meaning that each is contained in the following one (Shafer, 1976). The nested structure of consonant belief functions restricts the number of focal elements the belief function may have. In a belief function which is not consonant, depending on the number of elements n, the belief function may have up to $2^n - 1$ focal elements. This property of consonant belief functions makes it preferable for representation of statistical evidence (Shafer 1976). However, when Dempster’s rule is applied for combination of consonant belief functions, then the resulting belief function is not consonant any more. For that reason, an alternative rule was proposed by Walley (1987) for combination of partially consonant belief functions.

An example of a consonant bpa m with the frame of discernment $\{x, y, z\}$, and the focal elements $\{x\}$, $\{x, z\}$ and $\{x, y, z\}$ is as follows: $m(\{x\}) = 0.5$, $\{x, z\} = 0.1$ and $\{x, y, z\} = 0.4$. The corresponding belief, plausibility, and commonality functions are given in Table 1.

Table 1: Different representations of a consonant belief function

A	*m*(*A*)	*Bel*(*A*)	*Pl*(*A*)	*Q*(*A*)
$\{x\}$	0.5	0.5	1.0	1.0
$\{y\}$			0.4	0.4
$\{z\}$			0.5	0.5
$\{x, y\}$		0.5	1.0	0.4
$\{x, z\}$	0.1	0.6	1.0	0.5
$\{y, z\}$			0.5	0.4
$\{x, y, z\}$	0.4	1.0	1.0	0.4

A type of consonant belief functions is partially consonant belief functions, where the state space is partitioned and with focal elements nested within each element of partition, (Walley, 1987). Partially consonant belief functions are the only class of DS belief functions that are consistent with the likelihood principle of statistics. A decision theory for partially consonant belief functions is proposed by Giang and Shenoy (2011).

An example for representation of statistical evidence that use consonant belief functions is provided below as "Missile Fall Down".

Example: Missile Fall Down

Suppose that an attack has been placed, and three foe missiles are thrown to the country of Neverland. The missile defense system of Neverland is able to shut down two of the three missiles. Neverland Security Defense Deputy has the information that all three foe missiles are of the same type, either type *X*309, type *Y*118, or type *Z*127. However, they do not know which type these three foe missiles do belong. These three types use different technologies and hence have different likelihoods for being shut down by the missile defense system of Neverland. Let MT denote the missile type and let the state space of MT be denoted by $\Omega_{MT} = \{X309, Y118, Z127\}$. Let *F* denote the result of Neverland Missile Defense Systems' response, *f* indicate the foe missile fell down, *nf* indicate it did not fell down (it was missed by the missile defense system of Neverland) $\Omega_F = \{f, nf\}$. The probabilistic likelihoods for each type of foe missile for being shut down by Neverland forces are as follows:

$P(f| X309) = 7/10 \quad P(nf| X309) = 3/10$
$P(f| Y118) = 5/10 \quad P(nf| Y118) = 5/10$
$P(f| Z127) = 3/10 \quad P(nf| Z127) = 7/10$

In this example, for representation of evidences observed (either the missile fall down or did not), a consonant belief function framework seems reasonable, since the number of parameters needed for a consonant belief function representation is the same as the number of likelihoods available, three. A belief function which is not consonant would require 2^3-1 parameters instead. Additionally, notice that the likelihood of *X*309 is greater than the likelihood of *Y*118 (for the fall down), and the likelihood of *Y*118 is greater than the likelihood of *Z*127. Following this intuition, an evidence *x* should lend the plausibility to a singleton $\{\theta\}\subset\Theta$ in strict proportion to the chance that q_θ assigns *x*, i.e., that *x* should determine a plausibility function Pl_x obeying

$$Pl_x(\{\theta\}) = cq_\theta(x) \text{ for all } \theta \in \Theta \qquad (6)$$

for all $\theta \in \Theta$, where the constant c does not depend on θ (Shafer, 1976). Together with the assumption of consonance, a plausibility function is completely determined Pl_x: $2^\Theta \rightarrow [0,1]$, since the plausibility of the most likely singleton is 1, the constant c is determined as the reciprocal of the largest likelihood. Consequently, it may be concluded, having observed that foe missile fell down, that the foe missile of type *X*309 is more plausible than *Y*118, and that *Y*118 is more plausible than *X*309. Conversely, observing that Neverland's defense missiles were not able to hit the foe missiles, it may be concluded that the foe missile of type *Z*127 is more plausible than the type *Y*118, and *Y*118 is more plausible than *X*309. Thus, the plausibilities *Pl* for the singleton subsets of MT, based on the evidence observed (*f*, *nf*), can be identified as follows:

$Pl_f(\{X309\}) = (10/7)(7/10) = 1$ $\quad$ $Pl_{nf}(\{X309\}) = (10/7)(3/10) = 3/7$
$Pl_f(\{Y118\}) = (10/7)(5/10) = 5/7$ $\quad$ $Pl_{nf}(\{Y118\}) = (10/7)(5/10) = 5/7$
$Pl_f(\{Z127\}) = (10/7)(3/10) = 3/7$ $\quad$ $Pl_{nf}(\{Z127\}) = (10/7)(7/10) = 1$

The corresponding bpa of these consonant belief functions can be found as follows:
$m_f(\{X309\}) = 1 - 5/7 = 2/7$ $\quad$ $m_{nf}(\{Z127\}) = 1 - 5/7 = 2/7$
$m_f(\{X309, Y118\}) = 5/7 - 3/7 = 2/7$ $\quad$ $m_{nf}(\{Z127, Y118\}) = 5/7 - 3/7 = 2/7$
$m_f(\{X309, Y118, Z127\}) = 3/7$ $\quad$ $m_{nf}(\{Z127, Y118, X309\}) = 3/7$

The next section describes Dempster's and Walley's combination rules.

2.3. Combination Rules

2.3.1 Dempster's Rule of Combination

The classical combination rule for combining independent belief functions is called Dempster's rule. Let $m_1 \oplus m_2$ denote the joint bpa resulting from the combination of two independent bpa's m_1 and m_2, where $\oplus$ represents the operator of combination, then
$(m_1 \oplus m_2)(A) = K^{-1} \sum\{m_1(B)\{m_2(C) | B, C \subseteq \Theta, B \cap C = A\}\}$ for all $A \subseteq \Theta$, $A \neq \emptyset$ (7)
where K is a normalization constant given by $K = \sum\{m_1(B) m_2(C) | B \cap C \neq \emptyset\}$
for $K > 0$

If $K = 0$, this means the two bpa's are totally conflicting and cannot be combined. Dempster's rule in terms of bpa's consists of assigning the product of the masses to the intersection of the focal elements followed by normalization. Zadeh (1986) claimed that this normalization involves counter-intuitive behaviours (Lefevre *et al.* 2002). Many alternative combination rules have been proposed in order to solve the problem of conflict management (Yager 1987, Dubois and Prade 1998, Smets 1990, Murphy 2000). Another point where Dempster's rule of combination is criticized is that the class of partially consonant belief functions is not closed under Dempster's rule of combination. For that reason, Walley (1987) proposed an alternative rule, defined for partially consonant belief functions (this rule is introduced in the next section).

Dempster's rule of combination may also be expressed by commonality functions. Let Q_1, Q_2, and $Q_1 \oplus Q_2$ denote commonality functions corresponding to m_1, m_2, and $m_1 \oplus m_2$, respectively. Then,
$(Q_1 \oplus Q_2)(A) = K^{-1} Q_1(A) Q_2(A)$ for all non-empty $A \subseteq \Theta$, (8)
where K is given as follows: $K = \sum_{A \neq \emptyset} (-1)^{|A|+1} Q_1(A)\, Q_2(A)$

Dempster's rule in terms of commonality functions is essentially pointwise multiplication of the commonality functions followed by normalization.

2.3.2 Walley's Rule of Combination

Walley's combination rule for partially consonant commonality functions Q_1 and Q_2 is defined as follows:

$(Q_1 \boxplus Q_2)(A) = 0$ if $Q_1(A)Q_2(A) = 0$

$(Q_1 \boxplus Q_2)(A) = K^{-1} min\{Q_1(\{\theta\})Q_2(\{\theta\}) \mid \theta \in A\}$ otherwise (9)

for all non-empty sets A, where $K > 0$ is uniquely determined by (5) so that $Q_1 \boxplus Q_2$ is a commonality function. $Q_1 \boxplus Q_2$ is well defined provided $Q_1(\{\theta\})Q_2(\{\theta\}) > 0$ for some $\theta \in \Theta$.

The class of partially consonant commonality functions are closed under $\boxplus$. However, Walley's combination rule cannot be used to combine arbitrary commonality functions, since if combined under this rule, they may not be commonality function.

In Table 2 the combination of observations *f, f, nf* represented by commonality functions $\boldsymbol{Q_f}$ $\boldsymbol{Q_f}$ $\boldsymbol{Q_{nf}}$ using both Dempster and Walley's rule of combination is illustrated. Notice that for the case of singletons, the two rules do agree before normalization since both are pointwise multiplication of commonality functions. The next section describes the probability transformation methods, pignistic and plausibility transformation.

Table 2: Missile Fall Down Example continued: Combination of observations *f, f, nf* using both Dempster and Walley's rule of combination

						normalized	
	Q_f	Q_f	Q_{nf}	Dempster	Walley	Dempster	Walley
{*X*309}	1.0000	1.0000	0.4286	0.4286	0.4286	0.6837	1.0000
{*Y*118}	0.7143	0.7143	0.7143	0.3644	0.3644	0.5814	0.8503
{*Z*127}	0.4286	0.4286	1.0000	0.1837	0.1837	0.2930	0.4286
{*X*309, *Y*118}	0.7143	0.7143	0.4286	0.2187	0.3644	0.3488	0.8503
{*X*309, *Z*127}	0.4286	0.4286	0.4286	0.0787	0.1837	0.1256	0.4286
{*Y*118, *Z*127}	0.4286	0.4286	0.7143	0.1312	0.1837	0.2093	0.4286
{*X*309, *Y*118, *Z*127}	0.4286	0.4286	0.4286	0.0787	0.1837	0.1256	0.4286
K_D	0.6268						
K_W	0.4286						

3. DISCREPANCY OF PROBABILITY DISTRIBUTIONS

3.1. Probability Transformation Methods

As there is no decision theory of belief functions, the researchers are forced to use probability transformation methods. In sections 3.1.1 and 3.1.2, two famous probability transformation methods, pignistic and plausibility transformation are introduced. In section 3.1.3, using both of the probability transformation methods introduced, the combination results of the Missile Fall Down example are transformed into probability distributions.

3.1.1 Pignistic Transformation

The most commonly used transformation method in DS theory is the pignistic transformation method. The basic idea of the pignistic transformation consists in transferring the positive belief of each compound (or nonspecific) element onto the singletons involved in that element split by the cardinality of the proposition when working with normalized basic probability assignments (Han et al. 2010).

Suppose that $\Theta = \{\theta_1, \theta_2, ..., \theta_3\}$ is the frame of discernment. The pignistic probability transformation for the singletons is illustrated as follows (Smets and Kennes 1994):

$$BetP_m(\theta_i) = \sum_{\theta_i \in B, B \subseteq 2^\Theta} \frac{m(B)}{|B|} \tag{10}$$

where 2^Θ is the power set of frame of discernment.

3.1.2 Plausibility Transformation

Cobb and Shenoy (2006) argue that the pignistic transformation method produces results that appear to be inconsistent with Dempster's rule of combination. Claiming this, they proposed the plausibility transformation which is defined as follows:

Suppose m is a subset for s. Let Pl_m denote the plausibility function for s corresponding to bpa$m$. Let Pl_P_m denote the probability function that is obtained from m using the plausibility transformation method. Let Pl_P_m denote the probability function that is obtained from m using the plausibility transformation method. Pl_P_m is defined as follows:

$Pl\_P_m(x) = K^{-1} Pl_m(\{x\}$ for all $x \in \Omega_s$,

where$K = \sum\{Pl_m(\{x\}) | x \in \Omega_s\}$ is a normalization constant (11)

3.2. Discrepancy Resulting from the Transformation Method Used

In the preceding sections, going back to the Example Missile Fall Down, we used two combination rules, Dempster's and Walley's rule, for combining the observations represented under a consonant belief function framework to identify the type of the missile thrown at the country of Neverland. When the results of these two combination rules are compared, identical results before normalization are obtained, so both rules are the same for the case of singletons, as illustrated in Table 2. Investigating the m-values after combination given in Table 4, it can be seen that the resulting belief function by Walley's rule of combination is consonant after combination. Looking to the results obtained by Dempster's rule of combination, it is clear that the class of consonant belief functions is not closed under Dempster's rule of combination.

As given in Table 3, when plausibility transformation method is used on the results of the two combination rules, they do end in exactly the same probability distribution. The reason is that the plausibility and commonality values are the same for singletons and that the two combination rules do agree for singletons. However, when pignistic transformation is applied, the probability distribution functions obtained do differ, though the ordinal ranking is the same. Both of the probability distributions estimate the highest probability for the missile type *X*309, and the lowest for *Z*127. The results are illustrated in Table 4. Notice that these two different probability distributions also differ from the one found by the plausibility transformation method given in Table 3.

Table 3: Transformation of results using plausibility transformation

	Plausibility-values after combination		Plausibility Transformation	
	Dempster	Walley	Dempster	Walley
{*X*309}	0.68372	1.0	0.43881	0.43881
{*Y*118}	0.58140	0.85034	0.37313	0.37313
{*Z*127}	0.29302	0.42857	0.18806	0.18806
{*X*309, *Y*118}	0.69302	0.57823		
{*X*309, *Z*127}	0.85116	1.0		
{*Y*118, *Z*127}	0.66512	0.85034		
{*X*309, *Y*118, *Z*127}	1.0	1.0		

Table 4: Transformation of results using pignistic transformation

	m-values after combination		Pignistic transformation	
	Dempster's rule	Walley's rule	Dempster	Walley
{*X*309}	0.33488	0.14966	0.48837	0.50340
{*Y*118}	0.14884	0	0.34419	0.35374
{*Z*127}	0.08372	0	0.16744	0.14286
{*X*309, *Y*118}	0.22326	0.42177		
{*X*309, *Z*127}	0	0		
{*Y*118, *Z*127}	0.08372	0		
{*X*309, *Y*118, *Z*127}	0.12558	0.42857		

The above illustrated example and the following results show that the choice of the combination rule and/or probability transformation method may have a significant effect on our decisions. The ordinal ranking of the probability distributions may agree, but depending on different outcomes of the decision alternatives, and as a consequence of the expected utility theory, that would not assure to end up in the same decision alternative. To illustrate this, suppose that if the missile is of type *X*309, than its correct identification would save the country of Neverland \$1000 (in thousand); if it is of type *Y*118, then \$1500; and if it is of type *Z*127, then the saving is \$3500. Consequently, as demonstrated in Table 5, considering the expected utility maximization, the s of the decision alternative do differ: *Z*127 according to the plausibility transformation method and also according to the pignistic transformation for Walley's combination rule, *Y*118 according to pignistic transformation for Dempster's combination rule.

Table 5: Expected utilities obtained with three probability distributions

		Probabilities			Expected Utilities		
		Dempster, Walley	Dempster	Walley	Dempster, Walley	Dempster	Walley
Missile type	Utilities $ (in thousand)	Plausibility	Pignistic	Pignistic	Plausibility	Pignistic	Pignistic
*X*309	1000	0.4388	0.4884	0.5034	438.8060	503.4014	488.3721
*Y*118	1500	0.3731	0.3442	0.3537	559.7015	**530.6122**	516.2791
*Z*127	3500	0.1881	0.1674	0.1429	**658.2090**	500.0000	**586.0465**

4. CONCLUSIONS

Dempster−Shafer belief function theory can address a wider class of uncertainty than the standard probability theory, and this fact appeals the researchers in operations research society for potential application areas. However, the lack of a decision theory of belief functions gives rise to the need to use the probability transformation methods for decision making.

In this work first an example of statistical evidence is represented using the consonant belief function framework. Then, the observations are combined using the classical Dempster's rule of combination and the newly proposed Walley's rule of combination for consonant belief functions. The combination results demonstrate that the class of consonant belief functions is closed under Walley's rule of combination, which is not the case for Dempster's rule. For decision making purposes, both of the combination results are transformed into probability distributions using the pignistic, and the plausibility transformation methods. In this work, it is shown that although the two combination rules do result in the same probability distribution function when plausibility transformation is used, they end up with different probability distributions when pignistic transformation is applied. Consequently, it is demonstrated that the choice of the combination rule and/or the probability transformation method may have a significant effect on decision making, on the choice of the decision alternative.

In the Missile Fall Down example used in this study although the probability distributions differ up to 7 %, the ordinal ranking of all the three probability distributions remained the same. For further research, it can be investigated whether the ordinal ranking of the probability distributions would differ if the likelihoods of the statistical evidence used also differ.

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