Yugoslav Journal of Operations Research 25 (2015), Number 3, 413-423 DOI: 10.2298/YJOR130706028O

SUPPLIER SELECTION USING DIFFERENT METRIC FUNCTIONS

Sunday. E. OMOSIGHO

Department of Mathematics University of Benin, Benin City, Nigeria seomosigho@uniben.edu Dickson E. A. OMOROGBE

Institute of Education University of Benin, Benin City, Nigeria erhaativie.omorogbe@uniben.edu

Received: July 2013 / Accepted: August 2014

Abstract: Supplier selection is an important component of supply chain management in today's global competitive environment. Hence, the evaluation and selection of suppliers have received considerable attention in the literature. Many attributes of suppliers, other than cost, are considered in the evaluation and selection process. Therefore, the process of evaluation and selection of suppliers is a multi-criteria decision making process. The methodology adopted to solve the supplier selection problem is intuitionistic fuzzy TOPSIS (Technique for Order Preference by Similarity to the Ideal Solution). Generally, TOPSIS is based on the concept of minimum distance from the positive ideal solution and maximum distance from the negative ideal solution. We examine the deficiencies of using only one metric function in TOPSIS, and propose the use of spherical metric function in addition to the commonly used metric functions. For empirical supplier selection problems, more than one metric function should be used.

Keywords: Supplier selection (a), TOPSIS (b), Metric functions (c), Spherical metric (d).

MSC: 62C86.

1. INTRODUCTION

Supplier evaluation and selection is a multi-criteria decision making problem involving many suppliers that have the potential to meet the need of an organization. But the suppliers are not the same in many respects. For example: one supplier may deliver on time but the items are costly. A supplier requires longer time to deliver but the items are cheaper than those of a supplier that requires a shorter time to deliver. There are other attributes, or factors that can be taken into consideration in choosing a supplier order than cost (Izadikhah, [9]). Some of the attributes used in the evaluation of suppliers are quantitative while others are qualitative. An important issue in the selection of suppliers is the fact that it is almost impossible to find a supplier that excels in all the possible criteria identified by an organization or decision makers. Hence, many approaches for the evaluation and selection of suppliers have been considered.

Ho et al [8] provides a review of some of the methods suggested for solving the supplier selection problem. The methods reviewed include: Data envelopment analysis(DEA), Mathematical programming, Analytic hierarchy process (AHP), Casebased reasoning (CBR), Fuzzy set theory, Simple multi-attribute rating technique (SMART), Genetic algorithm (GA). They also considered hybrid methods, combining some of the foregoing methods and their variations. For example, under mathematical programming, the following variations were considered: Linear programming, Binary integer linear programming, Mixed integer linear programming, Mixed integer nonlinear programming, Goal programming, and Multi-objective programming. Soeini et al [14] also reviewed some articles on the supplier selection problem. Theseauthors however ignored intuitionistic fuzzy TOPSIS(Technique for Order Preference by Similarity to the Ideal Solution). Amindoustet al [2] provide some information on intuitionistic fuzzy TOPSIS but ignored the problem associated with the use of several metric for the same problem. Hence, in this paper we re-examine the approach for supplier selection based on intuitionistic fuzzy TOPSIS.We identify some problems associated with the use of this technique and propose possible solution. The rest of the paper is arranged as follows. In section 2, we present the basic algorithm for TOPSIS. The use of different metric functions in TOPSIS is considered in section 3. Also in section 3, we present arguments for proposing a change in the commonly used metric. We illustrate the application of the proposed metric in section 4. Our conclusion is in section 5.

2. INTUITIONISTIC FUZZY TOPSIS

The algorithm for intuitionistic fuzzy TOPSIS as used by Izadikah [9] is as follows:

Step 1. Determine the most important criteria.

Step 2. Determine the weights of decision makers

Step 3. Construct the aggregated intuitionistic fuzzy decision matrix.

Step 4. Determine the weights of criteria

Step 5. Determine the weighted intuitionistic fuzzy decision matrix.

Step 6. Determine the positive ideal solution (PIS) and negative ideal solution (NIS).

Step 7. Construct the separation measures (distance from PIS and distance from NIS) for each supplier.

Step 8. Calculate the closeness coefficient for each supplier using the results obtained in step 7.

Step 9. Rank the alternatives supplier using the closeness coefficients.

In order to incorporate qualitative attributes in the evaluation and selection of suppliers, several versions of TOPSIS have been introduced. In particular, in intuitionistic fuzzy TOPSIS, the weights of criteria, the decision matrix (rating of the

alternative suppliers) are initially obtained in linguistic terms. The linguistic terms are transformed to intuitionistic fuzzy numbers, [3,5,9]. There are different versions of linguistic variables. In the literature, a five point scale, or a seven-point scale, or a tenpoint scale for the same concept can be found [5,9, 12, 17]. Details of how to manipulate the linguistic variables to obtain the aggregated weighted intuitionistic fuzzy decision matrix, intuitionistic fuzzy positive ideal solution (PIS) and intuitionistic fuzzy negative ideal solution (NIS) are however the same. The manipulation is based on the concept of interval arithmetic [3,5,9], and references therein. The PIS is a matrix containing the best ratings for all criteria, and the NIS is a matrix containing the worst ratings for criteria. The idea of the TOPSIS is that the selected supplier should be closest to the PIS and farthest from the NIS. To achieve this, the distances (S⁺ and S⁻) of each supplier from the PIS and NIS, respectively, are calculated based on a chosen metric function and used to calculate a closeness coefficient. The closeness coefficient is given by S⁻[(S⁺ + S⁻).

3. METRIC FUNCTIONS FORINTUITIONISTIC TOPSIS

A major problem with TOPSIS is the construction of the separation measures (S⁺ and S⁻) and calculation of the closeness coefficients. Although the calculation of the closeness coefficient is very simple, it is influenced by the separation measures adopted. Several metric functions are available for the construction of the separation measures in the literature [6,15]. However, the commonly used metric functions are Hamming and Euclidean metric functions, and it is common for only one metric function to be adopted when implementing TOPSIS. Givenany $U = \{u_1, u_2, ..., u_n\}$, two intuitionistic fuzzy subsets $A = \{\langle u_i, \mu_A(u_i), v_A(u_i), \tau_A(u_i) \rangle\}$ and $B = \{\langle u_i, \mu_B(u_i), v_B(u_i), \tau_B(u_i) \rangle\}$ of the universe of discourse and using the 3D representation, the following metric functions are well known, [19].

a. Hamming distance H(A,B)

$$H(A,B) = \frac{1}{2} \sum_{i=1}^{n} [|\mu_{A}(u_{i}) - \mu_{B}(u_{i})| + |\nu_{A}(u_{i}) - \nu_{B}(u_{i})| + |\tau_{A}(u_{i}) - \tau_{B}(u_{i})|]$$
(1)

b. Euclidean distance E(A, B)

$$E(A,B) = \left(\frac{1}{2}\sum_{i=1}^{n} \left[\left(\mu_{A}(u_{i}) - \mu_{B}(u_{i})\right)^{2} + \left(\nu_{A}(u_{i}) - \nu_{B}(u_{i})\right)^{2} + \left(\tau_{A}(u_{i}) - \tau_{B}(u_{i})\right)^{2}\right]\right)^{0.5}$$
(2)

c. Spherical distance S(A,B)

$$S(A,B) = \frac{2}{\pi} \sum_{i=1}^{n} ar \cos(\sqrt{\mu_A(u_i)\mu_B(u_i)} + \sqrt{\nu_A(u_i)\nu_B(u_i)} + \sqrt{\tau_A(u_i)\tau_B(u_i)})$$
(3)

The commonly used metric functions for the calculation of the separation measure are Euclidean metric and the Hamming metric and their variations. Table 1 shows an example of some authors that have applied these metric functions and their variations. Jorge et al[10] applied the Malahanobis metric.

| Tuble I. Examples of autors who used the indicated metric function | | | | | |
|--|--|--|--|--|--|
| Hamming Metric Function | Euclidean Metric Function | | | | |
| Chen and Tsao [6]); Boran[4]; Izadikah [9] | Chen and Tsao[6]; Boran et al[5], Jahanshahloo et al[11], Gerogiannis et al[7]; Shemshadi et al[13]; Alinezhad and Amini [1], Wu and Chuang[18], Wen et al [17] | | | | |
| | | | | | |

Table 1: Examples of authors who used the indicated metric function

Nevertheless, the application of more than one of these metric functions for the same supplier selection problem produces contradictory ranking of the suppliers[6]. Indeed, Chen and Tsao[6] performed a comparative study of TOPSIS technique using different metrics. The metrics considered were two different definitions of the Hamming metric, and three different definitions of the Euclidean metric. In their study, they concluded that "in a decision problem, the interval-valued fuzzy TOPSIS methods using the different distance definitions may yield distinct preference orders when the number of alternatives is greater than 5. Second, the best alternative suggested by the interval-valued fuzzy TOPSIS methods using different distance definitions might be contradictory in some degree. As the number of alternatives increases, there is greater chance that the most preferred alternatives based on distinct distances will differ substantially." Hence, it is important to examine the metric functions adopted in the application of intuitionistic fuzzy TOPSIS for the supplier selection problem.

The conclusion of Chen and Tsao[6] is not surprising based on the following argument. Szmidt and Kacprzyk[15] demonstrated that the representation of two intuitionistic fuzzy subsets affects the distance between them. Further, Yang and Chiclana[19] noted that the Hamming and Euclidean metrics are based on the linear representation of intuitionistic fuzzy sets. This is in sharp contrast to semantic differences which is not linear in nature. Zadeh[20]had earlier examined the concept of linguistic variables and their transformation to fuzzy numbers. Essentially, the argument proposed by Zadeh[20] shows that fuzzy numbers representing linguistic variables are not linear. He used the example of "young" and presented a non-linear graph to show the transformation of age in years to fuzzy numbers in the interval [0,1] (see Fig 1 in Zadeh, [20]. So, the use of metric functions based on linear representation of intuitionistic fuzzy sets is an error. Yang and Chiclana19] observed that the use of "distances based on the linear representation of intuitionistic fuzzy sets... might not seem to be the most appropriate ones. In such cases, nonlinear distances between intuitionistic fuzzy sets may be more adequate to capture the semantic difference." Further, Yang and Chiclana[19]gave examples to show that there are cases where semantic difference is significant but linear metric will not reflect the difference. Therefore, they proposed that nonlinear metric may be more appropriate in capturing the semantic difference reflected in intuitionistic fuzzy subsets, and they suggested that spherical metric should be used to compute the distance between intuitionistic fuzzy numbers. We recommend that spherical metric should be the choice metric function in the implementation of TOPSIS for supplier selection involving linguistics variables transformed into intuitionistic fuzzy numbers.

It is very easy to compute the Hamming distance and the Euclidean distance for any two intuitionistic fuzzy sets. But the computation of the spherical distance requires $\mu_A(u_i) + v_A(u_i) + \tau_A(u_i) = 1$ and $\mu_B(u_i) + v_B(u_i) + \tau_B(u_i) = 1$. Although this requirement is part of the definition of intuitionistic fuzzy numbers, numerical computation may

introduce some truncation errors in the process of obtaining the aggregated decision matrix, as we shall show in the next section. Hence, it is important to verify the requirement $\mu_A(u_i) + v_A(u_i) + \tau_A(u_i) = 1$ and $\mu_B(u_i) + v_B(u_i) + \tau_B(u_i) = 1$ before applying spherical metric. This is to ensure that there is no bug in the computational procedure.

4. APPLICATION OF SPHERICAL METRIC IN INTUITIONISTIC TOPSIS

We illustrate the application of spherical metric to the data shown in Table 2. Table 2 shows the aggregated weighted intuitionistic fuzzy decision matrix for five suppliers, the positive ideal solution (PIS and the negative ideal solution (NIS) for the selection process considered by Boran et al [5]. In Table 2, there are seven rows. The first five rows are the aggregated weighted intuitionistic fuzzy decision matrix for the five suppliers, indicated as A1, A2, A3, A4, and A5 respectively. There are only four criteria used in the selection process. Each criterion corresponds to three columns in Table 2. The first three columns, the second three columns, the third three columns and the fourth three columns correspond to the first, second, third, and fourth criterion, respectively. PIS and NIS correspond to the positive ideal solution and negative ideal solution, respectively.

Boran et al. [5] calculated the separation measures using Euclidean metric function. In what follows, we shall use the spherical metric given by equation (3) to calculate the distance(S^+) of each supplier from the positive ideal solution (PIS), and the distance (S^-) for each supplier from the negative ideal solution for each alternative supplier. From these values, we calculate the closeness coefficient, (S^- |($S^+ + S^-$)).

Table 2: Decision Matrix, PIS and NIS (Source: Boran et al. 2009)

| | | 10101111 | | is and it | 10 (00 a | Det Doi | an et an | 1 000) | | | | |
|-----|--------------|--------------|--------------|--------------|--------------|--------------|--------------|---------------|--------------|-------|-------|-------|
| | | B1 | | | B2 | | | B3 | | | B4 | |
| A1 | 0.627 | 0.276 | 0.097 | 0.47 | 0.418 | 0.112 | 0.53 | 0.353 | 0.117 | 0.403 | 0.497 | 0.1 |
| A2 | 0.513 | 0.391 | 0.096 | 0.454 | 0.434 | 0.112 | 0.438 | 0.453 | 0.109 | 0.333 | 0.573 | 0.094 |
| A3 | <u>0.731</u> | <u>0.215</u> | <u>0.054</u> | <u>0.585</u> | <u>0.294</u> | <u>0.121</u> | 0.523 | 0.361 | 0.116 | 0.443 | 0.452 | 0.105 |
| A4 | 0.571 | 0.334 | 0.095 | 0.404 | 0.489 | 0.107 | 0.507 | 0.378 | 0.115 | 0.371 | 0.531 | 0.098 |
| A5 | 0.484 | 0.422 | 0.094 | 0.347 | 0.55 | 0.103 | 0.454 | 0.436 | 0.11 | 0.303 | 0.606 | 0.091 |
| PIS | <u>0.731</u> | 0.215 | 0.054 | <u>0.585</u> | 0.294 | <u>0.121</u> | 0.53 | 0.353 | 0.117 | 0.303 | 0.606 | 0.091 |
| NIS | 0.484 | 0.422 | 0.094 | 0.347 | 0.55 | 0.103 | <u>0.438</u> | <u>0.453</u> | <u>0.109</u> | 0.443 | 0.452 | 0.105 |

Table 3: Separation measures based on Spherical metric, closeness coefficients, and ranks of suppliers

| \mathbf{S}^+ | S | $ClosenessCoefficientS^{-} (S^{+}+S^{-})$ | Supplier Rank |
|----------------|----------|---|---------------|
| 0.231154 | 0.442903 | 0.65707 | A1 2 |
| 0.327138 | 0.275151 | 0.456843 | A2 4 |
| 0.106103 | 0.619904 | 0.853854 | A3 1 |
| 0.254956 | 0.314754 | 0.552481 | A4 3 |
| 0.388637 | 0.175912 | 0.311598 | A5 5 |

Table 3 shows the distance from the PIS, distance from the NIS, and closeness coefficient for each alternative supplier using spherical metric. Based on the spherical

metric, the suppliers are ordered as follows A3>A1>A4>A2>A5. The ranking produces by Boran et al [5] using the normalized Euclidean metric is A3>A1>A2>A4>A5. Observe the change in the ranking of supplier A2 and A4. Supplier A2 is characterised by the negative ideal attribute for criterion 3. Supplier A4 is not characterised by any negative ideal attribute. This semantic difference between supplier A2 and A4 is captured by the spherical metric but not captured by the linear Euclidean metric.

The next example is taken from Wen et al [17]. Table 4 shows the aggregated decision matrix, the positive ideal solution, and the negative ideal solution for a supplier selection problem with three suppliers (A1, A2, A3), and five criteria (B1, B2, B3, B4, B5). This problem is very instructive because it illustrates some of the problems that may arise in the implementation of the spherical matrix. For criterion B2, the intuitionistic fuzzy score for all suppliers according to Wen et al[17] is <0.600, 0.278,0.123>. This is not an intuitionistic fuzzy number because $\mu + \nu + \tau > 1$. Attempts to apply spherical metric to this number (for example finding its spherical distance from it) will give rise to error. The score is adjusted to <0.599, 0.278,0.123>. The slight error in the representation of this number may be due to truncation error. Also, an error was detected in the score for supplier A3 in criterion B4 using the intuitionistic fuzzy condition $\mu + \nu + \tau = 1$. The score is given as <0.406,0.499,0.95> in Table 12 of Wen et al [17]. A comparison with the score for other suppliers in criterion B4 indicates that the score should be <0.406,0.499,0.095>. The adjusted and corrected scores are shown in Table 4. Using spherical metric, the separation distances from the positive ideal solution and negative ideal solution for each supplier are shown in Table 5. Also, in Table 5, the closeness coefficient and rank for each supplier areshown. The ranks produced for this example, using the spherical metric, are the same as that obtained by Wen et al [17].

| | | B1 | | | B2 | | | B3 | | | B4 | | | B5 | |
|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| A1 | 0.653 | 0.249 | 0.098 | 0.599 | 0.278 | 0.123 | 0.622 | 0.266 | 0.112 | 0.609 | 0.29 | 0.101 | 0.356 | 0.539 | 0.105 |
| A2 | 0.653 | 0.249 | 0.098 | 0.599 | 0.278 | 0.123 | 0.622 | 0.266 | 0.112 | 0.406 | 0.499 | 0.095 | 0.356 | 0.539 | 0.105 |
| A3 | 0.435 | 0.47 | 0.095 | 0.599 | 0.278 | 0.123 | 0.622 | 0.266 | 0.112 | 0.406 | 0.499 | 0.095 | 0.534 | 0.347 | 0.119 |
| PIS | 0.653 | 0.249 | 0.098 | 0.599 | 0.278 | 0.123 | 0.622 | 0.266 | 0.112 | 0.406 | 0.499 | 0.095 | 0.534 | 0.347 | 0.119 |
| NIS | 0.435 | 0.47 | 0.095 | 0.599 | 0.278 | 0.123 | 0.622 | 0.266 | 0.112 | 0.609 | 0.29 | 0.101 | 0.356 | 0.539 | 0.105 |

Table 4: Aggregated decision matrix, PIS and NIS (Source: Wen et al [17] Table 12 and Table 13)

| Table 5: Distances from $PIS(S^{+})$, NIS (S ⁻) closeness coefficient and ran |
|---|
|---|

| Supplier | \mathbf{S}^+ | S | Closeness coefficient | Rank |
|----------|----------------|-------|--------------------------|------|
| A1 | 0.420 | 0.239 | 0.362 | 3 |
| A2 | 0.199 | 0.460 | 0.698 | 1 |
| A3 | 0.239 | 0.420 | 0.638 | 2 |

5. COMPUTATIONAL ANALYSIS

In this section, we present results on some examples to show the advantages of using more than one metric function in the approach to solve supplier selection problem using TOPSIS. In the proposal of TOPSIS presented by Tzeng and Huang [16], they stated that

"the separation values can be measured using the Euclidean distance". This attitude of using only one metric function to calculate the separation measure has dominated the use of TOPSIS technique in the literature. Although Chen and Tsao[6] observed that the use of more than one metric function can give rise to contradictory ranking of suppliers, they stated that "if the number of decision alternatives is small, one may not have to concern on which distance definition to use." This conclusion can be misleading in practice. The following example shows that even in extreme situation where we have just three decision alternatives, the distance function used can affect the choice made. These examples illustrate the advantage of not using just one distance function to calculate the separation measure in supplier selection problem.

Table 6: Three suppliers and three criteria including the PIS and NIS

| D3 |
|------------------|
| 01 0.3604 0.4395 |
| 4 0.0859 0.4697 |
| 9 0.4459 0.4492 |
| 4 0.0859 0.4697 |
| 9 0.4459 0.4492 |
| |

Table 6 shows the decision matrix for a situation where we have three suppliers to choose from, and only three criteria together with the PIS and NIS. Here the first criterion is cost criteria while the other two criteria are benefit criteria. Table 7 shows the ranking of the suppliers using Hamming, Euclidean and Spherical metrics. This example shows that even in a situation where we have just three alternatives for the suppliers, there is contradiction in the ranking of the suppliers when Hamming and Euclidean metric functions are applied. Here the same supplier, (A2), is identified as the best using the three metric functions, but only Hamming metric and the Spherical metric identified supplier A1 as the second best, whereas the Euclidean metric placed supplier A1 as the third in its ranking.

| | 6 | 11 | | |
|---|-----------|---------------------------------|-----------------------------------|-----------------------------------|
| | suppliers | Ranking using Hamming Metric | Ranking using Euclidean Metric | Ranking using Spherical Metric |
| | A1 | 2 | 3 | 2 |
| 1 | A2 | 1 | 1 | 1 |
| | A3 | 3 | 2 | 3 |

Table 7: Ranking of the Suppliers in Table 6.

This example also shows that even for a low number of criteria, here we have three criteria, there can also be contradiction in the ranking of the suppliers when more than one metric function is adopted in TOPSIS. Thus the claim of Chen and Tsao [6] who stated that "the influence of the number of attributes on consistency rates does not seem to be important" can also be misleading in practice. If we interpret his conclusion to mean that one needs not be concerned with the metric used considering the number of attributes, we can arrive at misleading ranking of suppliers. Here we have an example which shows that even for three attributes contradiction in the ranking of suppliers can arise.

Table 8 shows the decision matrix for suppliers A1 and A3 as recorded in Table 6 together with the PIS and NIS using the values for A1 and A3 only. Clearly, supplier A1 is better than supplier A3 by considering their contributions to the PIS and NIS. Thus in this case, using only the Euclidean Metric willgive rise to a misleading conclusion.

Table 8: Decision matrix for Suppliers A1 and A3 from Table 6. PIS and NIS are included.

| | B1 | B2 | B3 |
|-----|----------------------|----------------------|----------------------|
| A1 | 0.1005 0.0478 0.8517 | 0.3202 0.0497 0.6301 | 0.2001 0.3604 0.4395 |
| A3 | 0.4380 0.4551 0.1069 | 0.2857 0.3773 0.3370 | 0.1049 0.4459 0.4492 |
| PIS | 0.1005 0.0478 0.2729 | 0.3202 0.0497 0.6301 | 0.4444 0.0859 0.4697 |
| NIS | 0.4380 0.0478 0.5142 | 0.04840.4363 0.5153 | 0.1049 0.4459 0.4492 |
| | | | |

The next example is very instructive. It shows one way the use of one metric function can easily produce a spurious result in practice. Table 9 shows decision matrix for a supplier selection problem with three suppliers and four criteria together with both the Positive Ideal Solution (PIS) and the Negative Ideal Solution (NIS). The first criterion is a cost criterion, (less is better), while the other three criteria are benefit criteria (more is better).

Table 9: Decision matrix for three suppliers and four criteria including the PIS and NIS

| | | B1 | | | B2 | | | B3 | | | B4 | |
|-----|--------|--------|--------|--------|-----------|--------|--------|-----------|--------|--------|-----------|--------|
| | | (cost) | | | (benefit) | | | (benefit) | | | (benefit) | |
| A1 | 0.441 | 0.4903 | 0.0687 | 0.3202 | 0.0497 | 0.6301 | 0.3291 | 0.3986 | 0.2723 | 0.3218 | 0.5378 | 0.1404 |
| A2 | 0.5558 | 0.3848 | 0.0594 | 0.0484 | 0.4363 | 0.5153 | 0.0196 | 0.4669 | 0.5135 | 0.6957 | 0.0314 | 0.2729 |
| A3 | 0.1562 | 0.3067 | 0.5371 | 0.2857 | 0.3773 | 0.337 | 0.3115 | 0.3476 | 0.3409 | 0.0478 | 0.1005 | 0.8518 |
| PIS | 0.1562 | 0.4903 | 0.3535 | 0.3202 | 0.0497 | 0.6301 | 0.3291 | 0.3476 | 0.3233 | 0.6957 | 0.0314 | 0.2729 |
| NIS | 0.5558 | 0.3067 | 0.1375 | 0.0484 | 0.4363 | 0.5153 | 0.0196 | 0.4669 | 0.5135 | 0.0478 | 0.5378 | 0.4144 |

| Lusie Lot Running of Suppliers in Fuele , | Table 10: | Ranking | of Supp | liers in | Table | 9 |
|--|-----------|---------|---------|----------|-------|---|
|--|-----------|---------|---------|----------|-------|---|

| suppliers | Ranking using Hamming | Ranking using Euclidean | Ranking using Spherical |
|-----------|-----------------------|-------------------------|-------------------------|
| | Metric | Metric | Metric |
| A1 | 1 | 2 | 1 |
| A2 | 3 | 1 | 3 |
| A3 | 2 | 3 | 2 |

Table 10 shows the ranking of the suppliers in Table 9. Suppose that the only metric function adopted in the study is Euclidean then, the best supplier is A2. But both Hamming and Spherical metric functions identify A1 as the best supplier. This is an example of contradiction in the best alternative when more than one metric function is adopted. Thus, for empirical problems, it is advisable to use more than one metric function. In this particular example, it is not difficult to see that supplier A2 is the closest to the negative ideal solution compared with supplier A1. This can be seen by comparing the scores of supplier A2 with the values in the NIS. A2 alone provides more that 50% of the values in the NIS. Indeed, Table 11 shows the decision matrix for selecting a supplier between supplier A1 and supplier A2. Table 11 also contains the Positive Ideal Solution and the Negative Ideal Solution, assuming that only supplier A1 and supplier A2 are in the Universal set of suppliers to choose from. Criterion B1 is still a cost criterion while

420

the other criteria are benefit criteria. Using the Hamming, Euclidean and Spherical metric functions, Supplier A1 is better than Supplier A2. Notice the contribution of both Suppliers to the Positive Ideal Solution and the Negative Ideal Solution. While supplier A1 contributes more values to the Positive Ideal solution, Supplier A2 contributes more values to the Negative Ideal Solution.

| | | B1 | | | B2 | | | B3 | | | B4 | |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| A1 | 0.441 | 0.4903 | 0.0687 | 0.3202 | 0.0497 | 0.6301 | 0.3291 | 0.3986 | 0.2728 | 0.3218 | 0.5378 | 0.1404 |
| A2 | 0.5558 | 0.3848 | 0.0594 | 0.0484 | 0.4363 | 0.5153 | 0.0196 | 0.4669 | 0.5135 | 0.6957 | 0.0314 | 0.2729 |
| PIS | 0.441 | 0.4903 | 0.0687 | 0.3202 | 0.0497 | 0.6301 | 0.3291 | 0.3986 | 0.2723 | 0.6957 | 0.0314 | 0.2729 |
| NIS | 0.5558 | 0.3848 | 0.0594 | 0.0484 | 0.4363 | 0.5153 | 0.0196 | 0.4669 | 0.5135 | 0.3218 | 0.5378 | 0.1404 |

Table 11: Comparison of suppliers A1 and A2 from Table 9. PIS and NIS are included

This example also illustrates how the spherical metric function can provide a way out of resolving problems associated with contradictory ranking of suppliers.

Table 12 shows the decision matrix for two suppliers, A2 and A4, taken from Table 2 in this paper. In the paper by Boran et al. [5], supplier A2 was ranked better than supplier A4. In Table 12, both PIS and NIS are stated based on the assumption that only the two suppliers are available for the selection process. Using Hamming and the Euclidean metrics for the calculation of the separation measure, supplier A4 is better than supplier A2. This is also the ranking provided by the spherical metric in Table 3 for A4 and A2.

Table 12: Decision Matrix, PIS, NIS for supplier A2 and A4 taken from Table 2 above.

| | B1 | | | B2 | | | В3 | | | B4 | | |
|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| A2 | 0.513 | 0.391 | 0.096 | 0.454 | 0.434 | 0.112 | 0.438 | 0.453 | 0.109 | 0.333 | 0.573 | 0.094 |
| A4 | 0.571 | 0.334 | 0.095 | 0.404 | 0.489 | 0.107 | 0.507 | 0.378 | 0.115 | 0.371 | 0.531 | 0.098 |
| PIS | 0.571 | 0.334 | 0.095 | 0.454 | 0.434 | 0.112 | 0.507 | 0.378 | 0.115 | 0.371 | 0.531 | 0.098 |
| NIS | 0.513 | 0.391 | 0.096 | 0.404 | 0.489 | 0.107 | 0.438 | 0.453 | 0.109 | 0.333 | 0.573 | 0.094 |

Indeed, the use of only one metric function in TOPSIScan lead to misleading conclusions in supplier selection problems. This is due to contradiction in ranking of the suppliers when two or more metric functions are applied. For practical problems, it is proposed that more than one metrics should be used to calculate the separation measure. When there is contradiction in the ranking provided by Hamming and Euclidean metrics, the Spherical metric should be used as a decider. It is possible that there may be some pathological examples in which the Spherical metric will contradict the recommendation of both Hamming and Euclidean metrics, a pair wise comparison as indicated in this paper can be used to settle the matter.

6. CONCLUSION

The examples presented in this paper show that once we have the aggregated decision matrix, the positive ideal solution (PIS), and the negative ideal solution (NIS), the ranking of the suppliers can be produced. But the produced ranking may be affected by the metric function adopted. Hence, it is recommended that more than one metric

function should be adopted to solve empirical supplier selection problem. The spherical metric should be included in the metric function adopted because it captures the nonlinear property of intuitionistic fuzzy numbers. Therefore, any supplier multi-agent develop in the near future should state clearly the metric adopted, and if the spherical metric is used, efforts should be made to verify that $\mu + \nu + \tau = 1$. Violation of this condition can lead to error in the computation of the separation measures.

REFERENCES

- [1] Alinezhad, A., and Amini, A., "Sensitivity Analysis of TOPSIS Technique: The Results of Change in the Weight of One Attribute on the Final Ranking of Alternatives", *Journal of Optimization in Industrial Engineering*, 7 (2011) 23-28.
- [2] Amindoust, A., Ahmed, S., and Saghafinia, A., "A taxonomy and review on supplier selection methods under uncertainty", *International Journal of Information Technology and Business Management*, 7 (1) (2012) 33-43.
- [3] Amindoust, A., Ahmed, S., Saghafinia, A., and Bahreininejad, A., "Sustainable supplier selection: A ranking model based on fuzzy inference system", *Applied Soft Computing*, 12 (2012) 1668–1677.
- [4] Boran, F. E. "An integrated intuitionistic fuzzy multi criteria decision making method for facility location selection", *Mathematical and Computational Applications*, 16 (2) (2011) 487-496.
- [5] Boran, F., E., Genc, S., Kurt, M., and Akay, D., "A multi-criteria intuitionistic fuzzy group decision making for supplier selection with TOPSIS method", *Expert Systems with Applications*, 36 (2009) 11363–11368.
- [6] Chen, Ting-Yu, Tsao, Chueh-Yung, "The interval-valued fuzzy TOPSIS method and experimental analysis", *Fuzzy Sets and Systems* 159, (2008), 1410 1428.
- [7] Gerogiannis, V. C., Fitsilis, P., and Kameas, A. D., "Using a Combined Intuitionistic Fuzzy Set-TOPSIS Method for Evaluating Project and Portfolio Management Information Systems": In Iliadis L. et al. (eds) EANN/AIAI 2011, Part II, IFIPAICT, 364 (2011) 67–81.
- [8] Ho, W., Xu, X., and Dey, P. K., "Multi-criteria decision making approaches for supplier evaluation and selection: A literature review", *European Journal of Operational Research* 202 (2010) 16–24.
- [9] Izadikhah, M., Group decision making process for supplier selection with topsis method under interval-valued intuitionistic fuzzy numbers. Advances in Fuzzy Systems Volume 2012, Article ID 407942, 14 pages doi:10.1155/2012/407942
- [10] Jorge, L., García, A., Gabriel, I. M., and Lázaro, R. P., "Improvement of TOPSIS Technique Through Integration of Malahanobis Distance: A Case Study", *Proceedings of the 14th Annual International Conference on Industrial Engineering Theory, Applications and Practice* Anaheim, California October 18-21, 2009.
- [11] Jahanshahloo, G. R., Lotfi, F. H., and Davoodi, A. R., "Extension of TOPSIS for decisionmaking problems with interval data: Interval efficiency", *Mathematical and Computer Modelling* 49 (2009) 1137-1142.
- [12] Saghafian, S., and Hejazi, S. R., "Multi-criteria Group Decision Making Using A Modified Fuzzy TOPSIS Procedure", Proceedings of the 2005 International Conference on Computational Intelligence for Modelling, Control and Automation, and International Conference on Intelligent Agents, Web Technologies and Internet Commerce (CIMCA-IAWTIC'05).
- [13] Shemshadi, A., Toreihi, M., Shirazi, H. and Tarokh, M. J., "Supplier selection based on supplier risk: AnANP and fuzzy TOPSIS approach", *The Journal of Mathematics and Computer Science*, 2 (1) (2011) 111-121.

- [14] Soeini, R. A. et al "Supplier selection based on multiple criteria", International Journal on New Computer Architectures and Their Applications (IJNCAA), 2 (1) (2012) 258-273.
- [15] Szmidt, E. and Kacprzyk, J., "Distances between intuitionistic fuzzy sets", Fuzzy Sets and Systems, 114 (2000) 505-518.
- [16] Tzeng, Gwo-Hshiung and Huang, Jih-Jeng, Multiple Attribute Decision Making Methods and Applications, CRC Press Taylor & Francis Group London, 2011.
- [17] Wen, L., Wang, R., and Zhao, W., "Supplier Selection Based on Intuitionistic Fuzzy Sets Group Decision Making", *Research Journal of Applied Sciences, Engineering and Technology*, 5 (3) (2013) 950-956.
- [18] Wu, Feng-Yi, and Chuang, Chung-Chu, "The optimal relationship between buyer and seller obtained using TOPSIS method", *Journal of Advanced Management Science*, 1 (1) (2013) 133-135.
- [19] Yang, Y. and Chiclana, F., "Intuitionistic Fuzzy Sets: Spherical Representation and Distances", *International Journal of Intelligent Systems*, 24 (2009) 399–420.
- [20] Zadeh, L. A., "The concept of a linguistic variable and its application to approximate reasoning-l", *Information Sciences* 8 (1975) 199-249.