MAX-MIN SOLUTION APPROACH FOR MULTI-OBJECTIVE MATRIX GAME WITH FUZZY GOALS

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Abstract: In this paper, we consider a multi-objective two person zero-sum matrix game with fuzzy goals, assuming that each player has a fuzzy goal for each of the payoffs. The max-min solution is formulated for this multi-objective game model, in which the optimization problem for each player is a linear programming problem. Every developed model for each player is demonstrated through a numerical example.

Keywords: Multi-objective matrix game, Fuzzy goal, Max-min solution.

MSC: 90C29, 91A05.

1. INTRODUCTION

Game theory is concerned with decision making problem where two or more autonomous decision makers have conflicting interests. They are usually referred to as players who act strategically to find out a compromise solution. On the other hand, in multi-objective optimization problems, a single decision maker optimizes the solution among the conflicting objectives. Multi-objective matrix games are capable of dealing with both types of conflicts. When interest of one player is completely against the interest of others, matrix game is determined as two person zero-sum matrix game.
Fuzziness in game problems may occur in goals and payoffs. Such types of game were first studied by Campos [8]. His approach was based on ranking of fuzzy numbers. Afterwards, Sakawa and Nishizaki [17] studied single and multi-objective matrix games with fuzzy goals and fuzzy payoffs by using max-min principle of game theory. Bector et al. [5, 6], and Vijay et al. [18] proved that a two person zero-sum matrix game with fuzzy goals and fuzzy payoffs is equivalent to a pair of linear programming problems, which are dual to each other in fuzzy sense. Their methodology was based on fuzzy duality

theory [3, 11, 16]. Vijay et al. [19] solved fuzzy matrix game by using a ranking function and fuzzy relation approach. Chen and Larbani [10] used two person zero-sum game approach to solve fuzzy multiple attributes decision making problem. They define fuzzy matrix with triangular membership function and proved that two person zero-sum game with fuzzy payoff matrices is equivalent to two linear programming problems. Çevikel and Ahlatçıoglu [9] explained new concepts of solutions for multi-objective two person zero-sum games with fuzzy goals and fuzzy payoffs using linear membership functions. Pandey and Kumar [15] introduced modified approach to solve multi-objective matrix game with vague payoffs using a new order function. Nan et al. [14] proposed a lexicographic methodology to determine the solution of matrix game with payoffs of triangular intuitionistic fuzzy numbers for both players. Li and Hong [12] gave an approach for solving constrained matrix games with payoffs of triangular fuzzy numbers. In their approach, they introduced the concepts of alpha constrained matrix games. Bandyopadhyay et al. [2] introduced a matrix game with payoff as triangular intuitionistic fuzzy number. They developed a score function to defuzzify the pay-off matrix for solving the matrix game. Bandyopadhyay and Nayak [1] introduced an approach for solving a matrix game whose payoffs are symmetric trapezoidal fuzzy number. In their approach, they transformed symmetric trapezoidal fuzzy numbers to interval fuzzy numbers so that lengths of all intervals were different.

The outlay of this paper is as follows. In section 2, main definitions are given. Main features of single objective two person zero-sum matrix game model and its solution concepts are introduced in section 3. Section 4 proposes a method to solve multi-objective matrix game. In section 5 a numerical example is given.

2. PRELIMINARIES

Definition 2.1 (Matrix game with fuzzy goals): *A two person zero-sum matrix game with fuzzy goals (FG) is defined as*

$$FG = (S^m, S^n, A, \overline{v}, \underset{\sim}{\geq}, \underline{v}, \underset{\sim}{\leq})$$

where S^m, S^n are strategy space for Player I and II, respectively and A is a payoff matrix for Player I. Here $\overline{v}$ and $\underline{v}$ are scalars representing the aspiration levels of Player I and Player II, and symbols "$\underset{\sim}{\geq}$" and "$\underset{\sim}{\leq}$" are fuzzified versions of usual "$\geq$" and "$\leq$" respectively.

A two person zero-sum multi-objective matrix game with fuzzy goals (MOFG) is represented by multi-payoff matrices $A^1, A^2, \ldots, A^r$ where it is assumed that each player has the same r objectives. .

Definition 2.2 (Fuzzy goal): *Let $D^k \subseteq \mathbb{R}$ be the domain of kth payoff for Player I, then a fuzzy goal $\tilde{g}_1^k$ with respect to k^{th} payoff for Player I is a fuzzy set on D^k, characterized by the membership function*

$$\mu_{\tilde{g}_1^k} : D^k \to [0,1]. \tag{2.1}$$

Similarly, a fuzzy goal $\tilde{g}_{II}^k$ for Player II is also a fuzzy set on D^k, characterized by the membership function

$$\mu_{\tilde{g}_{II}^k} : D^k \rightarrow [0,1] \qquad (2.2)$$

A value of membership function for a fuzzy goal can be interpreted as the degree of attainment of fuzzy goal for the payoff. Therefore, when a player has two different payoffs, he prefers the payoff possessing higher membership function value. It means that Player I aims to maximize his degree of attainment.

Definition 2.3 (Max-min value): The max-min value with respect to the degree of attainment of an aggregated fuzzy goal to Player I is

$$\max_{x \in S^m} \min_{y \in S^n} \min_k \ \{\mu_{\tilde{g}_I^k}(x^T A^k y)\} \qquad (2.3)$$

Similarly, the max-min value with respect to degree of attainment of an aggregated fuzzy goal to Player II is

$$\max_{y \in S^n} \min_{x \in S^m} \min_k \ \{\mu_{\tilde{g}_{II}^k}(x^T A^k y)\} . \qquad (2.4)$$

Definition 2.4. (Bellman and Zadeh's Decision Making Principle): *Suppose that there is a set of goals* $G_i(i=1,2,...,m)$ *along with a set of constraints* $C_j(j=1,2,...,n)$ *in a space of alternatives* X, *each of which is characterized by a fuzzy set on* X. *Bellman and Zadeh [7] proposed that a fuzzy decision is determined by an appropriate aggregation of the fuzzy sets* $G_i(i=1,2,...,m)$ *and* $C_j(j=1,2,...n)$. *They suggested the aggregation operator to be fuzzy intersection. Therefore, a fuzzy decision D which is a fuzzy set resulting from intersections of* $G_i(i=1,2,...,m)$ *and* $C_j(j=1,2,...,n)$, *i.e.* $\mu_D : X \rightarrow [0,1]$ *given by* $\mu_D(x) = \min_{i,j}(\mu_{G_i}(x), \mu_{C_j}(x))$.

The optimal decision $x^* \in X$ *can be expressed as* $\mu_D(x^*) = \max_x \mu_D(x)$.

3. SINGLE OBJECTIVE MATRIX GAME WITH FUZZY GOAL

In this section, first, we analyze single objective matrix game with fuzzy goal, and then extend it to multi-objective matrix game with fuzzy goals.

A two person zero-sum matrix game with fuzzy goal is defined as

$$FG = (S^m, S^n, A, \overline{v}, \underset{\sim}{>}, \underline{v}, \underset{\sim}{<})$$

where all symbols in FG have the same meaning as given in Definition 2.1.

Optimization problem for Player I

The degree of attainment of the assigned fuzzy goal $\tilde{g}_I$ for Player I is

$$\max_{x \in S^m} \min_{y \in S^n} \ \mu_{\tilde{g}_I}(x^T Ay)$$

The membership function for fuzzy goal $\tilde{g}_I$ for Player I is given as:

$$\mu_{\tilde{g}_I}(x^T Ay) = \begin{cases} 0 & , \ x^T Ay \leq \underline{v}, \\ 1 - \dfrac{\overline{v} - x^T Ay}{\overline{v} - \underline{v}} & , \ \underline{v} < x^T Ay \leq \overline{v}, \\ 1 & , \ \overline{v} < x^T Ay, \end{cases} \tag{3.1}$$

where $\underline{v}$ and $\overline{v}$ are the payoffs for which degree of attainment to Player I is 0 and 1. The values of $\underline{v}$ and $\overline{v}$ can be obtained as [4].

$$\underline{v} = \min_x \min_y x^T Ay = \min_i \min_j a_{ij}, \tag{3.2}$$

$$\overline{v} = \max_x \max_y x^T Ay = \max_i \max_j a_{ij}, \tag{3.3}$$

where $i \in (1,2,...,m) = I$ is pure strategy of Player I, and $j \in (1,2,...,n) = J$ is pure strategy of Player II.

We shall make use of the following theorem [4].

Theorem 3.1. *For a two person zero-sum game with fuzzy goal FG , let the membership function of a fuzzy goal for Player I be linear as shown in (3.1). Then Player I's max-min solution with respect to degree of attainment of the fuzzy goal*

$$\max_{x \in S^m} \min_{y \in S^n} \mu_{\tilde{g}_I} \ (\mathrm{x}^T Ay) \tag{3.4}$$

is equal to optimal solution of the following linear programming problem

$$\max \quad \lambda$$

subject to,

$$\sum_{i=1}^m \frac{a_{ij}}{\overline{v} - \underline{v}} x_i - \frac{\underline{v}}{\overline{v} - \underline{v}} \geq \lambda, \quad (j = 1,...,n),$$

$$\sum_{i=1}^m x_i = 1, \ \lambda \leq 1, \ x, \lambda \geq 0. \tag{3.5}$$

Optimization problem for Player II

Next, we consider the optimization problem for Player II. The degree of attainment of the assigned fuzzy goal to Player II is

$$\max_{y \in S^n} \min_{x \in S^m} \ \mu_{\tilde{g}_{II}} \ (\mathrm{x}^T Ay)$$

Here, the membership function $\mu_{\tilde{g}_{II}} \ (\mathrm{x}^T Ay)$ for fuzzy goal $\tilde{g}_{II}$ to Player II is given as:

$$\mu_{\tilde{g}_{II}} \ (\mathrm{x}^T Ay) = \begin{cases} 1 & , \ \mathrm{x}^T Ay \leq \underline{v}, \\ 1 - \dfrac{\mathrm{x}^T Ay - \underline{v}}{\overline{v} - \underline{v}}, & \underline{v} \leq \mathrm{x}^T Ay \leq \overline{v}, \\ 0 & , \ \overline{v} < \mathrm{x}^T Ay. \end{cases} \tag{3.6}$$

We use the following theorem [4] for Player II.

Theorem 3.2. *For a two person zero-sum matrix game with fuzzy goal, let the membership function of a fuzzy goal for Player II be linear as shown in (3.6). Then the max-min solution with respect to degree of attainment of the fuzzy goal for Player II is*

$$\max_{y \in S^n} \min_{x \in S^m} \mu_{\tilde{g}_{II}}(\mathbf{x}^{\mathrm{T}}\mathbf{A}\mathbf{y}) \tag{3.7}$$

The LP for Player II can be given as following:

$$\max \quad \eta$$

subject to,

$$\sum_{j=1}^{n} \frac{a_{ij}}{\overline{v} - \underline{v}} y_j - \frac{\underline{v}}{\overline{v} - \underline{v}} \le 1 - \eta, \quad (i=1,\ldots,m),$$

$$\sum_{j=1}^{n} y_j = 1, \ \eta \le 1, \ y, \eta \ge 0, \ \text{where } \eta = 1 - \lambda. \tag{3.8}$$

4. MULTI-OBJECTIVE MATRIX GAME WITH FUZZY GOALS

Using Definition 2.1., a MOFG is represented by fuzzy multiple payoff matrices A^k, $k=1,2,\ldots,r$, and fuzzy goals to each objective is $\overline{v}^k(\underline{v}^k)$, $k=1,2,\ldots,r$ to Player I (Player II). In this section, we proposed linear models for optimization problem to Player I and Player II, respectively as follows:

Optimization problem for Player I

Let the membership function of the fuzzy goal for k^{th} objective of Player I be denoted by $\mu_{\tilde{g}_I^k}(x^T A y)$. Assuming the membership function $\mu_{\tilde{g}_I^k}(x^T A y)$ to be linear, it can be represented as

$$\mu_{\tilde{g}_I^k}(x^T A^k y) = \begin{cases} 0 & , \quad x^T A^k y \le \underline{v}^k, \\ 1 - \dfrac{\overline{v}^k - x^T A^k y}{\overline{v}^k - \underline{v}^k} & , \quad \underline{v}^k < x^T A^k y \le \overline{v}^k, \\ 1 & , \quad \overline{v}^k < x^T A^k y. \end{cases} \tag{4.1}$$

The membership function for aggregated fuzzy goal $\tilde{g}_I$ to Player I can be constructed by using Bellmen and Zadeh's decision making principle [7, 13] for fuzzy environment. Accordingly, the membership function for aggregated fuzzy goal is

$$\min_{k} \ \{\mu_{\tilde{g}_I^k}(x^T A^k y)\} \tag{4.2}$$

The degree of attainment of an aggregated fuzzy goal $\tilde{g}_I$ to Player I is

$$\max_{x \in S^m} \min_{y \in S^n} \min_{k} \ \{\mu_{\tilde{g}_I^k}(x^T A^k y)\}$$

Theorem 4.1. *For a two person zero-sum multi-objective matrix game with fuzzy goals, let the membership function of an aggregated fuzzy goal for Player I be linear and obtained as (4.2). Then max-min solution with respect to degree of attainment of an aggregated fuzzy goal for Player I is given by*

$$\max_{x \in S^m} \min_{y \in S^n} \min_{k} \quad \{\mu_{\tilde{g}_I^k}(x^T A^k y)\}. \tag{4.3}$$

The optimal solution for the above game problem can be obtained through the following linear programming problem.

$$\max \quad \lambda$$

subject to,

$$\sum_{i=1}^{m} \frac{a_{i1}^k}{\overline{v}^k - \underline{v}^k} x_i - \frac{\underline{v}^k}{\overline{v}^k - \underline{v}^k} \geq \lambda, \quad (k = 1, \ldots, r),$$

$$\sum_{i=1}^{m} \frac{a_{i2}^k}{\overline{v}^k - \underline{v}^k} x_i - \frac{\underline{v}^k}{\overline{v}^k - \underline{v}^k} \geq \lambda, \quad (k = 1, \ldots, r),$$

$$\ldots \qquad \ldots \qquad \ldots$$

$$\sum_{i=1}^{m} \frac{a_{in}^k}{\overline{v}^k - \underline{v}^k} x_i - \frac{\underline{v}^k}{\overline{v}^k - \underline{v}^k} \geq \lambda, \quad (k = 1, \ldots, r),$$

$$\sum_{i=1}^{m} x_i = 1, \quad \lambda \leq 1,$$

$$x, \lambda \geq 0 \tag{4.4}$$

Proof: The max-min problem for Player I is

$$\max_{x \in S^m} \min_{y \in S^n} \min_{k} \quad \{\mu_{\tilde{g}_I^k}(x^T A^k y)\},$$

which can be transformed to

$$\max_{x \in S^m} \min_{y \in S^n} \min_{k} \quad \left(1 - \frac{\overline{v}^k - x^T A^k y}{\overline{v}^k - \underline{v}^k}\right) = \max_{x \in S^m} \min_{y \in S^n} \min_{k} \quad \left(\sum_{i=1}^{m} \sum_{j=1}^{n} \overline{a}_{ij}^k x_i y_j + c^k\right)$$

$$= \max_{x \in S^m} \min_{k} \min_{y \in S^n} \quad \sum_{j=1}^{n} \left(\sum_{i=1}^{m} \overline{a}_{ij}^k x_i + c^k\right) y_j$$

$$= \max_{x \in S^m} \min_{k} \min_{j \in J} \quad \left(\sum_{i=1}^{m} \overline{a}_{ij}^k x_i + c^k\right),$$

where $\overline{a}_{ij}^k = \dfrac{a_{ij}^k}{\overline{v}^k - \underline{v}^k}$ and $c^k = \dfrac{\underline{v}^k}{\overline{v}^k - \underline{v}^k}$.

Let $\displaystyle\min_{j \in J} \left(\sum_{i=1}^{m} \bar{a}_{ij}^{k} x_i + c^k \right) = \lambda_k$, and further let $\displaystyle\min_{k} \lambda_k = \lambda$, the max-min problem for Player I reduces to the linear programming problem (4.4).

Equivalent FLP for linear programming problem (4.4) is

Find $x \in \mathbb{R}^m$ such that

$$\sum_{i=1}^{m} a_{i1}^{k} x_i \gtrsim_{p^k} \bar{v}^k, \quad (k=1,2,...,r),$$

$$\sum_{i=1}^{m} a_{i2}^{k} x_i \gtrsim_{p^k} \bar{v}^k, \quad (k=1,2,...,r),$$

$$\cdots \qquad\qquad \cdots \qquad\qquad \cdots$$

$$\sum_{i=1}^{m} a_{in}^{k} x_i \gtrsim_{p^k} \bar{v}^k, \quad (k=1,2,...,r),$$

$$\sum_{i=1}^{m} x_i = 1, \ x \geq 0, \tag{4.5}$$

where $p^k = \bar{v}^k - \underline{v}^k$ is the tolerance value for k^{th} objective of Player I. The constraint $\displaystyle\sum_{i=1}^{m} a_{ij}^{k} x_i \gtrsim_{p^k} \bar{v}^k$ means that $\displaystyle\sum_{i=1}^{m} a_{ij}^{k} x_i$ is essentially greater than or equal to $\bar{v}^k$ with tolerance error p^k , see [5].

Optimization problem for Player II

Similarly for Player II, the membership function $\mu_{\tilde{g}_{II}^k}(x^T A^k y)$ for k^{th} fuzzy goal is linear and can be represented as

$$\mu_{\tilde{g}_{II}^k}(x^T A^k y) = \begin{cases} 1 & , \ x^T A^k y \leq \underline{v}^k, \\ 1 - \dfrac{x^T A^k y - \underline{v}^k}{\bar{v}^k - \underline{v}^k} & , \ \underline{v}^k \leq x^T A^k y \leq \bar{v}^k, \\ 0 & , \ \bar{v}^k < x^T A^k y. \end{cases} \tag{4.6}$$

The membership function of an aggregated fuzzy goal $\tilde{g}_{II}$ to Player II can be obtained as

$$\min_{k} \ \{\mu_{\tilde{g}_{II}^k}(x^T A^k y)\} \tag{4.7}$$

The degree of attainment of the aggregated fuzzy goal to Player II is

$$\max_{y \in S^n} \min_{x \in S^m} \min_{k} \ \{\mu_{\tilde{g}_{II}^k}(x^T A^k y)\}.$$

Theorem 4.2. *For a two person zero-sum multi-objective matrix game with fuzzy goals, let the membership function $\mu_{\tilde{g}_{II}}$ of an aggregated fuzzy goal for Player II be linear and*

obtained as (4.7). Then max-min solution with respect to degree of attainment of an aggregated fuzzy goal for Player II is

$$\max_{y \in S^n} \min_{x \in S^m} \min_k \ \{\mu_{\tilde{g}_{II}^k}(x^T A^k y)\} \tag{4.8}$$

Equivalent LP, FLP for the above game problem can be written in a manner similar to Theorem 4.1. This is given in expressions (4.9), (4.10).

max η

subject to,

$$\sum_{j=1}^{n} \frac{a_{1j}^k}{\overline{v}^k - \underline{v}^k} y_j - \frac{\underline{v}^k}{\overline{v}^k - \underline{v}^k} \leq 1 - \eta, \quad (k = 1, \ldots, r),$$

$$\sum_{j=1}^{n} \frac{a_{2j}^k}{\overline{v}^k - \underline{v}^k} y_j - \frac{\underline{v}^k}{\overline{v}^k - \underline{v}^k} \leq 1 - \eta, \quad (k = 1, \ldots, r),$$

$$\ldots \qquad \ldots \qquad \ldots$$

$$\sum_{j=1}^{n} \frac{a_{mj}^k}{\overline{v}^k - \underline{v}^k} y_j - \frac{\underline{v}^k}{\overline{v}^k - \underline{v}^k} \leq 1 - \eta, \quad (k = 1, \ldots, r),$$

$$\sum_{j=1}^{n} y_j = 1, \ \eta \leq 1, \ y, \eta \geq 0, \tag{4.9}$$

Find $y \in \mathbb{R}^n$ such that

$$\sum_{j=1}^{n} a_{1j}^k y_j \lesssim_{q^k} \underline{v}^k, \ (k = 1, 2, \ldots, r),$$

$$\sum_{j=1}^{n} a_{2j}^k y_j \lesssim_{q^k} \underline{v}^k, \ (k = 1, 2, \ldots, r)$$

$$\ldots \qquad \ldots \qquad \ldots$$

$$\sum_{j=1}^{n} a_{mj}^k y_j \lesssim_{q^k} \underline{v}^k, \ (k = 1, 2, \ldots, r)$$

$$\sum_{j=1}^{n} y_j = 1,$$

$$y \geq 0, \tag{4.10}$$

where $q^k = \overline{v}^k - \underline{v}^k$ is tolerance value for k^{th} objective to Player II.   The   constraint $\sum_{j=1}^{n} a_{ij}^k y_j \lesssim_{q^k} \underline{v}^k$ means that $\sum_{j=1}^{n} a_{ij}^k y_j$ is essentially less than or equal to $\underline{v}^k$ with tolerance error q^k, see [5].

5. NUMERICAL EXAMPLE

Consider the following numerical problem of two person zero-sum multi-objective game with fuzzy goals based on an example given in [20].

Suppose there are two companies, I and II, aiming to enhance the sales amount and market share of a product in a targeted market. Under the circumstance that the demand amount of the product in the targeted market is basically fixed, the sales amount and market share of one company increases, following the decrease of the sales amount and market share of another company, but the sales amount is not certain to be proportional to the market share. The two companies are considering the three strategies to increase the sales amount and market share:

x_1: Advertisement; x_2: reduce the price; x_3: improve the package.

This problem is a multi-objective two person zero-sum matrix game. Further, let Company I be Player I, adopting the strategy (x_1, x_2, x_3), Company II be Player II, adopting strategy (y_1, y_2, y_3). Under the three strategies, the payoff matrices A^1, A^2 of targeted sales quantity f_1 (million) and market share f_2 (percentage) are separately indicated as:

$$A^1 = \begin{pmatrix} 180 & 350 & 575 \\ 255 & 430 & 180 \\ 90 & 156 & 125 \end{pmatrix}, \quad A^2 = \begin{pmatrix} 25 & 35 & 42 \\ 32 & 22 & 29 \\ 15 & 10 & 25 \end{pmatrix}$$

Here, using (3.2) & (3.3), we get

$\underline{v}^1 = 90$, $\overline{v}^1 = 575$ and $\overline{v}^1 - \underline{v}^1 = 485$; $\underline{v}^2 = 10$, $\overline{v}^2 = 42$ and $\overline{v}^2 - \underline{v}^2 = 32$. In this case, model (4.4) can be written as following:

max $\quad \lambda$

subject to,

$$-485\lambda + 180x_1 + 255x_2 + 90x_3 \geq 90, \ -485\lambda + 350x_1 + 430x_2 + 156x_3 \geq 90,$$
$$-485\lambda + 575x_1 + 180x_2 + 125x_3 \geq 90, \ -32\lambda + 25x_1 + 32x_2 + 15x_3 \geq 10,$$
$$-32\lambda + 35x_1 + 22x_2 + 10x_3 \geq 10, \ -32\lambda + 42x_1 + 29x_2 + 25x_3 \geq 10,$$
$$x_1 + x_2 + x_3 = 1, \lambda \leq 1, \ \lambda, x_1, x_2, x_3 \geq 0,$$

Using TORA, the optimal solution of the problems is obtained as;

$$\left(x^* = (0.1596, 0.8404, 0)^T, \lambda^* = 0.3155 \right)$$

Similar results for Player II are obtained by using Model (4.9).

The optimal solution to Player II is $(y^* = (0.6500, 0.3500, 0)^T, \eta^* = 0.4219)$.

6. CONCLUSIONS

We have considered a multi-objective two person zero-sum matrix game with fuzzy goals. The proposed method is shown as generalization of that used to solve a single objective matrix game with fuzzy goals and it is based on the max-min solution approach. Every developed model for each player is demonstrated through a numerical example.

REFERENCES

[1] Bandyopadhyay, S. and Nayak, P.K., "Matrix games with trapezoidal fuzzy pay offs", *International Journal of Engineering Research and Development,* 5 (2013) 21-29.

[2] Bandyopadhyay, S., Nayak, P.K. and Pal, M., "Solution of matrix game with triangular intuitionistic fuzzy pay-off using score function", *Open Journal of Optimization*, 2 (2013) 9-15.

[3] Bector, C.R. and Chandra, S.,"On duality in linear programming under fuzzy environment", *Fuzzy Sets and Systems*, 125(2002) 317-325.

[4] Bector, C.R. and Chandra, S., *Fuzzy Mathematical Programming and Fuzzy Matrix Games.*,Springer, Berlin, Heidelberg, 2004.

[5] Bector, C.R., Chandra, S. and Vijay, V.,"Duality in linear programming with fuzzy parameters and matrix games with fuzzy payoffs", *Fuzzy Sets and Systems*, 146 (2004) 253-269.

[6] Bector, C.R., Chandra, S. and Vijay, V.,"Matrix games with fuzzy goals and fuzzy linear programming duality", *Fuzzy Optimization and Decision Making*, 3 (2004) 255-269.

[7] Bellman, R.E. and Zadeh, L.A.,"Decision-making in a fuzzy environment", *Management Science*, 17 (4) (1970) 141 -164.

[8] Campos, L.,"Fuzzy linear programming models to solve fuzzy matrix games", *Fuzzy Sets and Systems*, 32(1989) 275-289.

[9] Çevikel, A.C. and Ahlatçıoglu, M.,"A linear interactive solution concept for fuzzy multiobjective games", *European Journal of Pure and Applied Mathematics*, 35 (2010) 107-117.

[10] Chen,Y-W. and Larbani, M., "Two-person zero-sum game approach for fuzzy multiple attribute decision making problems", *Fuzzy Sets and Systems*, 157 (2006) 34-51.

[11] Gupta, P. and Mehlawat, M.K.,"Bector-Chandra type duality in fuzzy linear programming with exponential membership functions", *Fuzzy Sets and Systems*, 160 (2009) 3290-3308.

[12] Li, D.F. and Hong, F.X.,"Solving constrained matrix games with payoffs of triangular fuzzy numbers", Computers & Mathematics with Applications, 64 (2012) 432-448.

[13] Lin, R., "Note on fuzzy sets", *Yugoslav Journal of Operations Research*, 23 (2013) 1-5.

[14] Nan, J.X.**,** Li, D.F. and Zhang, M.J.,"A lexicographic method for matrix games with payoffs of triangular intuitionistic fuzzy numbers", International Journal of Computational Intelligence Systems, 3 (2010) 280-289.

[15] Pandey, D., and Kumar, S., "Modified approach to multiobjective matrix game with vague payoffs", *Journal of International Academy of Physical Sciences*, 14 (2) (2010) 149-157.

[16] Pandey, D., and Kumar, S., "Fuzzy Optimization of Primal-Dual Pair Using Piecewise Linear Membership Functions", *Yugoslav Journal of Operations Research*, 22 (2) (2012) 97-106.

[17] Sakawa, M. and Nishizaki, I.,"Max-min solutions for fuzzy multiobjective matrix games", *Fuzzy Sets and Systems*, 67 (1994) 53-69.

[18] Vijay, V., Chandra, S. and Bector, C.R.,"Matrix games with fuzzy goals and fuzzy payoffs", *Omega*, 33 (2005) 425-429.

[19] Vijay, V., Mehra, A., Chandra, S. and Bector, C.R.,"Fuzzy matrix games via a fuzzy relation approach", *Fuzzy Optimization and Decision Making*, 6 (2007) 299-314.

[20] Zhou, X., Song, Y., Zhang, Q. and Gao, X., "Multiobjective matrix game with vague payoffs", *ICFIE*, ASC 40, 543-550.