Yugoslav Journal of Operations Research
27 (2017), Number 4, 451–462
DOI: 10.2298/YJOR161028002C

MEASUREMENT ERROR EFFECT ON THE POWER OF CONTROL CHART FOR ZERO TRUNCATED NEGATIVE BINOMIAL DISTRIBUTION (ZTNBD)

A. B. CHAKRABORTY
Department of Statistics, St. Anthony's College, Shillong, Meghalaya, India
abc_sac@rediffmail.com

A. KHURSHID
Department of Mathematical and Physical Sciences, College of Arts and Sciences, University of Nizwa, Birkat Al Mouz, Oman
anwer@unizwa.edu.om, anwer_khurshid@yahoo.com

R. ACHARJEE
Department of Statistics, St. Anthony's College, Shillong, Meghalaya, India
rituparna26acharjee@gmail.com

Received: September 2016 / Accepted: March 2017

Abstract: In the present article measurement error effect on the power of control chart for ZTNBD is investigated based on standardized normal variate.Numerical calculations are presented so as to enable an appreciation of the consequences of measurement errors on the power curve. To examine the sensitivity of the monitoring procedure, average run length is also considered.

Keywords: Power, Zero-truncated Negative Binomial Distribution (ZTNBD), Measurement Error, Average Run Length (ARL).

MSC: 62P30.

1. INTRODUCTION

Count data which comprises of non-negative integer values that record the number of discrete events frequently linked to explanatory values are encountered in statistical research [10]. The Poisson distribution is extensively used in studying count data but the constraint for Poisson distribution so that its mean and variance are identical is not fulfilled at all times in real life. Thus, the Negative Binomial Distribution (NBD), which can manage overdispersion, is used [11]. There are widespread applications of NBDs in a variety of substantive fields including accident statistics, econometrics, quality control, biometrics, pharmacokinetics, and pharmacodynamics [24] etc. For detailed description consult Johnson et al. [12], Khurshid et al. [15], Ryan [26], and Krishnamoorthy [19], among others.

In industries, a conventional inspecting tool is to construct control charts to realize whether a process is in control or not [9]. A control chart is a statistical system developed with the objective of inspection after which,the statistical stability of a process is checked. The traditional tool for this purpose is the Shewhart and Cumulative sum control charts. While there is a vast literature on the construction of these control charts for continuous distributions (Mittag and Rinne [22], Wadsworth et al. [29]), much less research has been focused on discrete distributions. The literature on the control charts for the NBD is scanty (Kaminsky et al. [13], Ma and Zhang [20], Xie and Goh [30], Hoffman [11], and Schwertman [28]).

In several situations, however, the complete distribution of counts is not observed. Zero-truncated models are those where the number of individuals falling into zero class cannot be defined, or the observational apparatus becomes operational only when at least one event happens. Chakraborty and Kakoty [3] and Chakraborty and Bhattacharya [1,2] have constructed CUSUM charts for zero-truncated Poisson distribution, doubly truncated geometric distribution, and doubly truncated binomial distribution, respectively. Chakraborty and Singh [8] constructed Shewhart control charts for zero-truncated Poisson distribution where average length and operating characteristic function were obtained. Chakraborty and Khurshid [4,5] have constructed CUSUM charts for zero-truncated binomial distribution and doubly truncated binomial distribution, respectively. Recently, Khurshid and Chakraborty [16, 18] have constructed CUSUM, and Shewhart control charts for ZTNBD, respectively.

In the present article, measurement error effect on the power of control chart for ZTNBD is investigated based on standardized normal variate. Numerical calculations are presented as a means of appreciating the consequences of measurement errors on the power curve. To examine the sensitivity of the monitoring procedure, average run length (*ARL*) is also considered.

2. MATERIALS AND METHODS

2.1. Zero-Truncated Negative Binomial Distribution (ZTNBD)

A negative binomial distribution (NBD) arises in the following circumstances. Assume a box contain np non-defective items and nq defective items. Items are drawn at random with replacement. Now the probability that exactly $(x+k)$ trials are required to produce k non-defective items is $\frac{(x+k-1)!}{(k-1)!x!}p^k q^x$.

Thus, a random variable X is said to have a NBD with parameters k and p if its probability mass function is given by

$$P(X=x)=\begin{cases}\binom{k+x-1}{x}p^kq^x, & x=0,1,2,\cdots\\ 0, & \text{otherwise}\end{cases} \tag{1}$$

where the parameters satisfy $0<p<1$ and $k=1,2,3,\cdots$.

The distribution (Eq. 1) even remains meaningful when k is not an integer. When k is an integer, the distribution is sometimes called a Pascal distribution, or a discrete waiting time distribution. For $k=1$ the distribution reduces to geometric distribution.

The statistical literature shows that most of the probability distributions can be parameterized in numerous ways, the NBD being no exemption. A commonly used parameterization of the NBD can be achieved from the expansion of $(Q-P)^{-k}$, where $Q=1+P$, k is positive real and $P>0$ with P not to be in $(0,1)$. Under this parameterization, the probability mass function of NBD, given in Eq. 1, reduces to [31, 25]

$$P(X=x)=\binom{k+x-1}{x}\left(\frac{P}{Q}\right)^k\left(1-\frac{P}{Q}\right)^x, \tag{2}$$

where $x=0,1,2,\cdots$.

We consider a negative binomial distribution truncated at $x=0$. The zero-truncated form of Eq. 2

$$f(x;k,p)=\binom{k+x-1}{x}\left(1-Q^{-k}\right)^{-1}\left(\frac{P}{Q}\right)^k\left(1-\frac{P}{Q}\right)^x,\ \ x=1,2,3,\cdots \tag{3}$$

which is probability mass function of the ZTNBD (Khurshid and Chakraborty Khurshid2013).

The mean and variance of ZTNBD are given as

$E(X)=\frac{kP}{1-Q^{-k}}$ and $V(X)=\frac{kPQ}{1-Q^{-k}}\left[1-k\left(\frac{P}{Q}\right)\right]\{(1-Q^{-k})^{-1}-1\}$.

The significance of ZTNBD is illustrated by Johnson et al. [12] with real-life applications.

3. MEASUREMENT ERROR

Measurement errors which are frequently observed in practice, may significantly affect the performance of control charts [26, 21]. The sources of error may be due to natural variability of the process, and the error due to measurement instrument. The efficiency and the ability of the control chart to observe the shift of the process level will be affected if the measurement error is largely associated to the process variability [6]. Sankle et al. [27] studied the cumulative sum control charts for the truncated normal distribution under measurement error. Chakraborty and Khurshid [7], as well as Khurshid and Chakraborty [17] investigated measurement error effect on the power of control charts for various truncated distributions. For the consequences of measurement error on the actual functioning of various control charts see [6] and references therein.

4. ASSUMPTIONS AND NOTATIONS

In this article, we evaluate power of control chart for standardized ZTNBD under the following assumptions and notations:

(i) The measurement of items is considered to determine the magnitude of the attribute characteristics in the lot;

(ii) The process has ZTNBD with mean μ_p and variance σ_p^2;

(iii) The applied measurement process (which is independent of the manufacturing process) has a variance σ_m^2. Thus, the complete variability is given by $\sigma^2 = \sigma_p^2 + \sigma_m^2$;

(iv) Measurements of the items are taken to classify the produced units into defective and non-defective ones;

(v) The process is in a state of statistical control at the time of determining the control limits and the same measuring instrument is used for future measurements;

(vi) When the process parameter changes, the data still comes from ZTNBD, however, with mean $\mu_{p'}$ and variance $(\sigma_{p'}^2 + \sigma_m^2)$, where $\sigma_{p'}^2$ is the process variance when the process parameter shifts (For details see Chakraborty and Khurshid [6, 7]).

Thus, considering the above assumptions, Shewhart control limits will be $\mu_p \pm K\sqrt{(\sigma_p^2 + \sigma_m^2)/n}$. Typically, we select $K = 3$ as it will give no false alarm with probability of at least 99.73% [23] and where n is the size of the sample. The power of detecting the change of the process parameter is given by

$$P_d = P\{\overline{X} \geq \mu_p + 3\sqrt{(\sigma_p^2 + \sigma_m^2)/n}\} + P\{\overline{X} \leq \mu_p - 3\sqrt{(\sigma_p^2 + \sigma_m^2)/n}\}. \quad (4)$$

5. POWER OF CONTROL CHART FOR STANDARDIZED ZTBD

Under standardization procedure, Eq. 4 can be expressed in terms of standardized normal variable Z (when sample size is large and varies):

$$Z \Big| \{(\mu_{p'}, \sigma_{p'}^2, \sigma_m^2, n)\} = \frac{\overline{X} - \mu_{p'}}{\sqrt{((\sigma_{p'}^2 + \sigma_m^2)/n)}}. \quad (5)$$

Now, following Kanazuka [14], Chakraborty and Khurshid [6] and using Eq. 5, when the process parameter changes from μ_p to $\mu_{p'}$, the power of the control chart for ZTNBD equation is

$$
\begin{aligned}
P_{\overline{X}|\{(\mu_p,\mu_{p'},\sigma_p^2,\sigma_{p'}^2,\sigma_m^2,n)\}} &= P_d \\
&= P\left\{\left(\overline{X}\left|\frac{\overline{X}-\mu_{p'}}{\sqrt{((\sigma_{p'}^2+\sigma_m^2)/n)}} \geq \frac{\mu_p-\mu_{p'}}{\sqrt{((\sigma_{p'}^2+\sigma_m^2)/n)}} + 3\frac{\sqrt{(\sigma_p^2+\sigma_m^2)}}{\sqrt{(\sigma_{p'}^2+\sigma_m^2)}}\right.\right)\right\} \\
&+ P\left\{\left(\overline{X}\left|\frac{\overline{X}-\mu_{p'}}{\sqrt{((\sigma_{p'}^2+\sigma_m^2)/n)}} \leq \frac{\mu_p-\mu_{p'}}{\sqrt{((\sigma_{p'}^2+\sigma_m^2)/n)}} - 3\frac{\sqrt{(\sigma_p^2+\sigma_m^2)}}{\sqrt{(\sigma_{p'}^2+\sigma_m^2)}}\right.\right)\right\} \\
&= P\left\{\left(Z\left|Z \geq \frac{(\frac{\mu_p-\mu_{p'}}{\sigma_p})\sqrt{n}}{\sqrt{(\sigma_{p'}^2/\sigma_p^2)+(\sigma_m^2/\sigma_p^2)}} + 3\sqrt{\frac{1+(\sigma_m^2/\sigma_p^2)}{(\sigma_{p'}^2/\sigma_p^2)+(\sigma_m^2/\sigma_p^2)}}\right.\right)\right\} \\
&+ P\left\{\left(Z\left|Z \leq \frac{(\frac{\mu_p-\mu_{p'}}{\sigma_p})\sqrt{n}}{\sqrt{(\sigma_{p'}^2/\sigma_p^2)+(\sigma_m^2/\sigma_p^2)}} - 3\sqrt{\frac{1+(\sigma_m^2/\sigma_p^2)}{(\sigma_{p'}^2/\sigma_p^2)+(\sigma_m^2/\sigma_p^2)}}\right.\right)\right\} \\
&= P\left\{\left(Z\left|Z \geq \frac{-d\sqrt{n}}{\sqrt{(S^2+R^2)}} + 3\frac{\sqrt{1+R^2}}{\sqrt{(S^2+R^2)}}\right.\right)\right\} \\
&+ P\left\{\left(Z\left|Z \leq \frac{-d\sqrt{n}}{\sqrt{(S^2+R^2)}} - 3\frac{\sqrt{1+R^2}}{\sqrt{(S^2+R^2)}}\right.\right)\right\} \\
&= P\left\{\left(Z\left|Z \geq \sqrt{\frac{1+R^2}{(S^2+R^2)}}\left[3-\frac{d\sqrt{n}}{\sqrt{(1+R^2)}}\right]\right.\right)\right\} \\
&+ P\left\{\left(Z\left|Z \leq \sqrt{\frac{1+R^2}{(S^2+R^2)}}\left[-3-\frac{d\sqrt{n}}{\sqrt{(1+R^2)}}\right]\right.\right)\right\} \\
&= \Phi\left\{\sqrt{\frac{1+R^2}{(S^2+R^2)}}\left(-3+\frac{d\sqrt{n}}{\sqrt{(1+R^2)}}\right)\right\} + \Phi\left\{\sqrt{\frac{1+R^2}{(S^2+R^2)}}\left(-3-\frac{d\sqrt{n}}{\sqrt{(1+R^2)}}\right)\right\} \\
&= \Phi(M) + \Phi(V),
\end{aligned} \tag{6}
$$

where $d = \frac{\mu_p-\mu_{p'}}{\sigma_p}$, $S^2 = (\sigma_{p'}^2/\sigma_p^2)$, $R^2 = (\sigma_m^2/\sigma_p^2)$,
$M = \left\{\sqrt{\frac{1+R^2}{(K^2+R^2)}}\left(-3+\frac{d\sqrt{n}}{\sqrt{(1+R^2)}}\right)\right\}$, $N = \left\{\sqrt{\frac{1+R^2}{(S^2+R^2)}}\left(-3-\frac{d\sqrt{n}}{\sqrt{(1+R^2)}}\right)\right\}$ and
$\Phi = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{z} e^{-(u^2/2)}du$.

Using Eq. 6, the power of the control chart P_d can be found simply by solving $\Phi(z)$ for various combinations of d, R^2 and S^2, as shown in Tables 1 - 11.

6. AVERAGE RUN LENGTH (*ARL*) FOR ZTNBD UNDER MEASUREMENT ERROR

To explore the sensitivity of the monitoring procedure, one can also study *ARL*, the average number of points that must be plotted before a point shows an out of control condition(Khurshid and Chakraborty [17]).

For any Shewhart control chart, the $ARL = [P]^{-1}$ where P is the probability of a false alarm that a single point exceeds control limits.Thus *ARL* of ZTNBD under measurement error, just by reversing Eq. 6, is

$$ARL = \left[\Phi\left\{\sqrt{\tfrac{1+R^2}{(S^2+R^2)}}\left(-3+\tfrac{d\sqrt{n}}{\sqrt{(1+R^2)}}\right)\right\}+\Phi\left\{\sqrt{\tfrac{1+R^2}{(S^2+R^2)}}\left(-3-\tfrac{d\sqrt{n}}{\sqrt{(1+R^2)}}\right)\right\}\right]^{-1} \quad (7)$$

The values of ARL are shown in Table 12 .

7. CONCLUDING REMARKS

The effects of truncation as well as measurement errors on the power of detecting the changes in the process parameters by 3σ control limits with the control chart for ZTNBD are shown in Tables 1 - 11.

It has been observed, from Table 1, that as we go on increasing the shift of the process parameter μ_p to $\mu_{p'}$, there is an increasing trend in the power of control chart P_d for fixed values of $K, p, n, \mu_p, \sigma_p, \sigma_m$. It can also be concluded that as the ratio between μ_p and $\mu_{p'}$ decreases, there is an increasing trend in the values of P_d, the power of control chart.

It has also been observed from the Tables 1, 2 and 3, for fixed p, n, and $3\sigma^2$, that if there is a change in the values of K, the corresponding values of μ_p, σ_p and hence, R^2 change accordingly. As we go on increasing the values of K, there is a decreasing trend in the values of R^2 and the corresponding changes, observed, in the values of P_d.

For Tables 2 and 4, we observe that for fixed K and n, as we increase the value of p, there is an increasing trend in the values of R^2 and the corresponding values of P_d increase, too.

Tables 4 and 5 depict an increasing trend in the values of P_d for fixed K and p when the size of sample n is increased.

There is also an increasing trend in the values of R^2, and hence, the corresponding values of P_d decrease when the value of σ_m increases fixed K, p, and n; this can be observed from Tables 5 and 6.

When $K = 1$, Eq. 3 becomes zero truncated geometric distribution and trend of the values of P_d can be understood from the Tables 7 and 11.

Table 12 shows the values of ARL. It has been observed from the table that ARL values decrease as there is an increase in the size of sample for fixed K, p, and n, but they increase for fixed K, p, and n when the values of σ_m decrease. There is also a decreasing trend in the values of ARL for fixed n, p, and σ_m when there is an increasing trend in the values of K.

Thus, we observe that the larger the measurement error, the smaller the detecting power. However, this can be overcomed by increasing the sample size n and the process average deviation d.

Acknowledgements: The authors are grateful to referees and Branka Mladenovic for their helpful comments and suggestions. Also the authors would like to thank Dr. Khizar Hayat for his help.

REFERENCES

[1] Chakraborty, A. B., and Bhattacharya, S. K., "CUSUM control charts for doubly truncated geometric and Poisson distributions", *Proceedings of Quality for Progress and Development, Asian Congress on Quality and Reliability*, Wiley Eastern Limited, 1989, 509–512.

[2] Chakraborty, A. B., and Bhattacharya, S. K., "Cumulative sum control chart for a doubly truncated binomial distribution" *The Egyptian Statistical Journal*, 35 (1991) 119-124.

[3] Chakraborty, A. B., and Kakoty, S., "Cumulative sum control charts for zero truncated Poisson distribution" *IAPQR Transacrions*, 12 (1987) 17-25.

[4] Chakraborty, A. B., and Khurshid, A., "One-sided cumulative sum (CUSUM) control charts for the zero-truncated binomial distribution", *Economic Quality Control*, 26 (2011) 41-51.

[5] Chakraborty, A. B., and Khurshid, A., "Control charts for the doubly-truncated binomial distribution", *Economic Quality Control*, 27 (2012) 187–194.

[6] Chakraborty, A. B., and Khurshid, A., "Measurement error effect on the power of control chart for the ratio of two Poisson distributions", *Economic Quality Control*, 28 (2013) 15–21.

[7] Chakraborty, A. B., and Khurshid A., "Measurement error effect on the power of control chart for zero-truncated Poisson distribution", *International Journal for Quality Research*, 7 (2013) 411-419.

[8] Chakraborty, A. B., and Singh, B. P., "Shewhart control chart for ZTPD", *Proc. Quality for Progress and Development, Asian Congress on Quality and Reliability NIQR*, Trivandrum, India, 1990, 18–24.

[9] Dou, Y., and Sa P., "One-sided control charts for the mean of positively skewed distributions", *Total Quality Management*, 13 (2002) 1021-1033.

[10] Hilbe, J., M., Count Data, Cambridge University Press, Cambridge, New York, 2014.

[11] Hoffman, D., "Negative binomial control limits for count data with extra-Poisson variation", *Pharmaceutical Statistics*, 2 (2003) 127-132.

[12] Johnson, N. L., Kotz, S., and Kemp, A. W., *Univariate Discrete Distributions*, Third Edition, Wiley-Interscience, Hoboken, New Jersey, 2005.

[13] Kaminsky, F. C., Banneyan, J. C., Davis, R. D., and Burke, R. J., "Statistical control Charts based on a geometric distribution", *Journal of Quality Technology*, 24 (1992) 63-69.

[14] Kanazuka, T., "The effects of measurement error on the power of $\bar{X}$-R charts", *Journal of Quality Technology*, 18 (1986) 91-95.

[15] Khurshid, A., Ageel, M. I., and Lodhi, R. A., "On confidence intervals for the negative binomial distribution", *Revista Investigacion Operacional*, 26 (2005) 59-70.

[16] Khurshid A., Chakraborty, A. B., "CUSUM control charts for zero-truncated negative binomial and geometric distributions", *Revista Investigacion Operacional*, 34 (2013) 195-204.

[17] Khurshid A., Chakraborty, A. B., "Measurement error effect on the power of control chart for zero-truncated binomial distribution under standardization procedure", *International Journal for Quality Research*, 8 (2014) 495-504.

[18] Khurshid A., Chakraborty, A. B., "On Shewhart control charts for zero-truncated negative binomial distributions", *Pakistan Journal of Engineering, Technology and Science*, 4 (2014) 1-12.

[19] Krishnamoorthy, K., *Handbook of Statistical Distributions, Second Edition*, *Taylor and Francis*, Boca Raton, 2016.

[20] Ma, Y., and Zhang, Y., "Q control charts for negative binomial distribution" *Computers and Industrial Engineering*, 31 (1995) 813-816.

[21] Maravelakis, P. E., "Measurement error effect on the CUSUM control chart" *Journal of Applied Statistics*, 39 (2012) 323-336.

[22] Mittag, H. J., and Rinne, H., *Statistical Methods of Quality Assurance*, Chapman & Hall, London, New York, 1993.

[23] Montgomery, D. C., *Introduction to Statistical Quality control*, Seventh Edition, John Wiley, Chichester, New York, 2013.

[24] Plan, E. L., Maloney A., Troconiz, I. F., and Karlsson, M. O., "Performance in population models for count data, part I: maximum likelihood approximations" *Journal of Pharmacokinetics and Pharmacodynamics*, 36 (2009) 353-366.

[25] Promislow, S. D., *Fundamentals of Actuarial Mathematics, Second Edition*, *John Wiley*, New York, 2004.

[26] Ryan, T. P., *Statistical Methods for Quality Improvement*, Third Edition, John Wiley, Chichester, New York, 2011.

[27] Sankle, R., Singh, J. R., and Mangal, I. K., ”Cumulative sum control charts for truncated normal distribution under measurement error”, *Statistics in Transition*, 13 (2012) 95-106.
[28] Schwertman, N. C., ”Designing accurate control sharts based on the geometric and negative binomial distributions”, *Quality and Reliability Engeneering International*, 21 (8) (2005) 743-756.
[29] Wadsworth, H. M., Stephens, K. S., and Godfrey, A. B., *Modern Methods for Quality Control and Improvement: The Statistics of Quality Assurance*, Second Edition, John Wiley, Chichester, New York, 2002.
[30] Xie, M., and Goh, T. N., ”The use of probability limits for process control based on geomtric distribution” *International Journal of Quality and Reliability and Management*, 16 (1997) 64-73.
[31] Zelterman, D., *Discrete Distributions: Applications in the Health Sciences*, John Wiley, New York, 2004.

APPENDIX

Table 1: Values of P_d for controlling the parameter λ.

When $K = 1,\ p = 0.15,\ n = 5,\ \mu_p = 6.67,\ \sigma_p = 6.146,\ \sigma_m = 0.5,\ R^2 = 0.006617$						
$\mu_{p'}$	$\sigma_{p'}$	$d = (\mu_{t'} - \mu_t)/\sigma_p$	S^2	$\Phi(M)$	$\Phi(N)$	P_d
6.70	6.16	0.005423261	1.0004442	0.001430	0.00132	0.0027608
6.74	6.24	0.011931175	1.0307010	0.001699	0.00143	0.0031342
6.79	6.29	0.020066067	1.0472850	0.001937	0.00146	0.0033990
6.89	6.34	0.036335852	1.1064001	0.002324	0.00141	0.0037307
7.00	6.40	0.054232614	1.0842350	0.002839	0.00136	0.0041987

Table 2: Values of P_d for controlling the parameter λ.

When $K = 2,\ p = 0.15,\ n = 5,\ \mu_p = 11.59,\ \sigma_p = 8.61,\ \sigma_m = 0.5,\ R^2 = 0.003366$						
$\mu_{p'}$	$\sigma_{p'}$	$d = (\mu_{t'} - \mu_t)/\sigma_p$	S^2	$\Phi(M)$	$\Phi(N)$	P_d
11.60	8.70	1.019118	1.019118	0.001487	0.001473	0.0029604
11.68	8.72	1.023809	1.023809	0.001409	0.001409	0.0030336
11.70	8.79	1.040312	1.040312	0.001780	0.001497	0.0032775
11.76	8.84	1.052181	1.052181	0.001969	0.001505	0.0034739
11.84	8.90	1.066512	1.066512	0.002230	0.001504	0.0037340

Table 3: Values of P_d for controlling the parameter λ.

When $K = 3,\ p = 0.15,\ n = 5,\ \mu_p = 17.06,\ \sigma_p = 10.62,\ \sigma_m = 0.5,\ R^2 = 0.002217$						
$\mu_{p'}$	$\sigma_{p'}$	$d = (\mu_{t'} - \mu_t)/\sigma_p$	S^2	$\Phi(M)$	$\Phi(N)$	P_d
17.10	10.70	0.00399	1.015566	0.001498	0.001414	0.0029612
17.16	10.74	0.00964	1.023173	0.001617	0.001407	0.0030245
17.22	10.79	0.01529	1.032722	0.001758	0.001414	0.0031723
17.29	10.84	0.02185	1.042316	0.001922	0.001410	0.0033328
17.35	10.90	0.02754	1.053886	0.002101	0.001430	0.0035317

Table 4: Values of P_d for controlling the parameter λ.

When $K = 2,\ p = 0.2,\ n = 5,\ \mu_p = 8.33,\ \sigma_p = 6.24,\ \sigma_m = 0.5,\ R^2 = 0.006428$						
$\mu_{p'}$	$\sigma_{p'}$	$d = (\mu_{t'} - \mu_t)/\sigma_p$	S^2	$\Phi(M)$	$\Phi(N)$	P_d
8.40	6.30	0.0106	1.0206	0.0016	0.00137	0.00298
8.46	6.37	0.0203	1.0434	0.0019	0.00143	0.00334
8.50	6.42	0.0267	1.0598	0.00214	0.00147	0.00362
8.59	6.48	0.0411	1.0797	0.00255	0.00145	0.00402
8.64	6.52	0.0491	1.0931	0.00284	0.00146	0.00431

Table 5: Values of P_d for controlling the parameter λ.

When $K = 2,\ p = 0.2,\ n = 8,\ \mu_p = 8.33,\ \sigma_p = 6.24,\ \sigma_m = 0.5,\ R^2 = 0.006428$						
$\mu_{p'}$	$\sigma_{p'}$	$d = (\mu_{t'} - \mu_t)/\sigma_p$	S^2	$\Phi(M)$	$\Phi(N)$	P_d
8.40	6.30	0.0106	1.0206	0.00164	0.00135	0.00299
8.46	6.37	0.0203	1.0434	0.00198	0.00139	0.00335
8.50	6.42	0.0267	1.0598	0.00224	0.00140	0.00365
8.59	6.48	0.0411	1.0797	0.00275	0.00135	0.00401
8.64	6.52	0.0491	1.0931	0.00309	0.00134	0.00443

Table 6: Values of P_d for controlling the parameter λ.

When $K = 2,\ p = 0.2,\ n = 8,\ \mu_p = 8.33,\ \sigma_p = 6.24,\ \sigma_m = 1.5,\ R^2 = 0.0578$						
$\mu_{p'}$	$\sigma_{p'}$	$d = (\mu_{t'} - \mu_t)/\sigma_p$	S^2	$\Phi(M)$	$\Phi(N)$	P_d
8.40	6.30	0.0106	1.0206	0.00163	0.00134	0.00297
8.46	6.37	0.0203	1.0434	0.00195	0.00137	0.003325
8.50	6.42	0.0267	1.0598	0.00224	0.00139	0.00360
8.59	6.48	0.0411	1.0797	0.00268	0.00134	0.00400
8.64	6.52	0.0491	1.0931	0.00300	0.00132	0.00437

Table 7: Values of P_d for controlling the parameter λ.

When $K = 1,\ p = 0.15,\ n = 5,\ \mu_p = 6.67,\ \sigma_p = 6.146,\ \sigma_m = 0.5,\ R^2 = 0.0595$						
$\mu_{p'}$	$\sigma_{p'}$	$d = (\mu_{t'} - \mu_t)/\sigma_p$	S^2	$\Phi(M)$	$\Phi(N)$	P_d
6.70	6.18	0.0054	1.01	0.00147	0.00136	0.00284
6.74	6.24	0.0119	1.03	0.00168	0.00142	0.0031
6.79	6.29	0.0200	1.047	0.00191	0.00145	0.00336
6.82	6.32	0.0249	1.057	0.0020	0.00146	0.003523
6.89	6.39	0.0363	1.080	0.00243	0.00149	0.00393

Table 8: Values of P_d for controlling the parameter λ.

When $K = 1,\ p = 0.2,\ n = 5,\ \mu_p = 5,\ \sigma_p = 4.47,\ \sigma_m = 1.5,\ R^2 = 0.1125$						
$\mu_{p'}$	$\sigma_{p'}$	$d = (\mu_{t'} - \mu_t)/\sigma_p$	S^2	$\Phi(M)$	$\Phi(N)$	P_d
5.2	4.5	0.0447	1.0125	0.0019	1.043×10^{-3}	0.00297
5.8	4.56	0.178	1.03968	0.005	4.49×10^{-4}	0.00545
6.2	4.59	0.268	1.0534	0.0087	2.453×10^{-4}	0.009
6.7	4.63	0.38	1.0718	0.0167	1.12×10^{-4}	0.0168
6.9	4.69	0.424	1.0998	0.0221	9.32×10^{-5}	0.02225

Table 9: Values of P_d for controlling the parameter λ.

When $K = 1,\ p = 0.2,\ n = 10,\ \mu_p = 5,\ \sigma_p = 4.47,\ \sigma_m = 1.5,\ R^2 = 0.1125$						
$\mu_{p'}$	$\sigma_{p'}$	$d = (\mu_{t'} - \mu_t)/\sigma_p$	S^2	$\Phi(M)$	$\Phi(N)$	P_d
5.2	4.5	0.0447	1.0125	0.00218	9.14×10^{-4}	0.0031
5.8	4.56	0.1788	1.03968	0.00774	2.55×10^{-4}	0.0079
6.2	4.59	0.2683	1.0534	0.01599	1.01×10^{-4}	0.016
6.7	4.63	0.3801	1.0718	0.03569	3.008×10^{-5}	0.035
6.9	4.69	0.4248	1.0998	0.049	2.119×10^{-5}	0.049

Table 10: Values of P_d for controlling the parameter λ.

When $K = 2,\ p = 0.2,\ n = 5,\ \mu_p = 8.33,\ \sigma_p = 6.24,\ \sigma_m = 1.5,\ R^2 = 0.05785$							
$\mu_{p'}$	$\sigma_{p'}$	$d = (\mu_{t'} - \mu_t)/\sigma_p$	S^2	$\Phi(M)$	$\Phi(N)$	P_d
8.43	6.30	0.0155	1.0206	0.0017	0.00127	0.00299
8.48	6.39	0.0235	1.0499	0.0021	0.00134	0.003345
8.53	6.43	0.0315	1.0631	0.0024	0.00131	0.00371
8.59	6.48	0.0411	1.0797	0.00279	0.00128	0.00408
8.65	6.54	0.0507	1.0998	0.003279	0.00127	0.00455

Table 11: Values of P_d for controlling the parameter λ.

When $K = 1,\ p = 0.2,\ n = 10,\ \mu_p = 5,\ \sigma_p = 4.47,\ \sigma_m = 0.5,\ R^2 = 0.1125$						
$\mu_{p'}$	$\sigma_{p'}$	$d = (\mu_{t'} - \mu_t)/\sigma_p$	S^2	$\Phi(M)$	$\Phi(N)$	P_d
5.2	4.5	0.0447	1.0125	0.0024	9.001×10^{-4}	0.00314
5.8	4.56	0.1788	1.03968	0.00839	2.375×10^{-4}	0.008629
6.2	4.59	0.2683	1.0534	0.0177	8.99×10^{-5}	0.017866
6.7	4.63	0.3801	1.0718	0.0405	2.525×10^{-5}	0.040558
6.9	4.69	0.4248	1.0998	0.0561	1.76×10^{-5}	0.056118

Table 12: Values of ARL.

$\mu_{p'}$	$\sigma_{p'}$	K	R^2	n	p	σ_m	P_d	ARL
5.2	4.5	1	0.1125	5	0.2	1.5	0.00297	336.7
5.2	4.5	1	0.1125	10	0.2	1.5	0.0031	322.58
6.7	6.16	1	0.006617	5	0.15	0.5	0.00276	362.2
6.7	6.18	1	0.0595	5	0.15	1.5	0.00284	352.1
8.4	6.3	2	0.006428	5	0.2	0.5	0.00298	335.57
8.4	6.3	2	0.0064205	8	0.2	0.5	0.00299	334.11
8.4	6.3	2	0.0578	8	0.2	1.5	0.00297	336.7
11.6	8.7	2	0.00337	5	0.15	0.5	0.00296	337.84
17.1	10.7	3	0.002217	5	0.15	0.5	0.00291	343.38