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ALLOCATION OF WEIGHTS USING SIMULTANEOUS OPTIMIZATION OF INPUTS AND OUTPUTS CONTRIBUTION IN CROSS-EFFICIENCY EVALUATION OF DEA

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Abstract: Cross-efficiency evaluation, an extension of the data envelopment analysis (DEA), has found an appropriate function in ranking decision making units (DMU). However, DEA suffers from a potential flaw, that is, the existence of multiple optimal solutions. Different methods have been proposed to obtain a unique solution (based on a specific criterion). In this paper, we refer to Wang's method for ranking DMUs but argue that his way of selecting the weights is not the appropriate one. Namely, in the cross-efficiency evaluation of DMUs, we always search for the weights which use minimum resources to increase the production. Therefore, we suggest that the selection of weights among the multiple weights should be determined by decreasing the contribution of inputs in the use of resources, and increasing the contribution of outputs in the production, which should overtly prevent the selection of zero solutions to the extent possible. To this end, some examples are given to illustrate differences and advantages of our method compared to those usually used.

Keywords: Data Envelopment Analysis, Cross-efficiency, Contribution of Inputs and Outputs, Non-zero weights, Ranking.

MSC: 90B50, 90C05.

1. INTRODUCTION

Data envelopment analysis is a nonparametric method based on linear programming in relative efficiency evolution of a series of homogeneous decision making units (DMUs) with multiple input and outputs. The relative measure of each DMU was first introduced by Farell [11], but got interest among researches by Charnes et al. [2]. One of the appealing features of DEA is that it does not need accurate and greater values of the weights. DEA aims to estimate the unit under evaluation in its best state possible. On the one hand, the flexibility of DEA in the allocation of weights often leads to unreal solutions (especially zero weights), hence, undesirable. Therefore, some researchers suggested the use of weight control factor to prevent the occurrence of unreal weights in efficiency evaluation. On the other hand, most of DMUs are evaluated as efficient and are given the same efficiency score. Therefore, we cannot distinguish their performances theoretically. This problem led the researchers to propose methods to distinguish efficiency of the given DMUs and thus, provide a complete ranking for all DMUs. Different methods have been proposed to rank DMUs, each of which uses specific qualities as the ranking criteria. For more information regarding the new ranking methods, readers can refer to Hosseinzadeh Lotfi et al. [14], Nasseri et al. [20], Ruiz and Sirvent [24], Nasseri and Kiaei [21], Emrouznejada et al. [9], Jahanshahloo et al. [13].

The cross-efficiency evaluation method is one of the most popular ranking methods. Sexton et al [25] are among the pioneer users of this method. The overall objective of cross-efficiency evaluation is to evaluate each DMU from the perspective of all others, which is generally referred to as peer evaluation. Unlike DEA that uses self-evaluation, this method offers two advantages: provide a ranking for each DMU, and also avoid using the pattern of unreal weights in the efficiency evaluation of DMUs. Due to its high distinguishing power in the ranking of DMUs, the cross-efficiency evaluation has a wide range of applications in such studies as Sexton et al. [25], Doyle and Green [8] , Ertay and Ruan [10], Cooper et al. [4], Lim et al. [18] and Cui and Li [5].

Nevertheless, there are some problems in cross-efficiency evaluation. Perhaps the main problem is the existence of multiple optimal solutions for the weights resulting from the DEA model that leads to various efficiency scores (depending on the selection of weights). To solve this problem, secondary goal was first introduced by Sexton et al. [25] in 1986, and generalized by Doyle and Green [7] in 1994. This goal is a potential adjustment which avoids deduction of the cross-efficiency advantages. In most papers, this idea is used for all DMUs with the some conditions on cross-efficiency results. Thus, these conditions are referred to aggressive and benevolent formulas. A benevolent (aggressive) model searches the optimal weights that not only preserve the efficiency score of the unit under evaluation, but also increase (decrease) the efficiency score of the other DMUs. Liang et al. [17], in their attempt to expand Doyle and Green's [7] models, suggested three various secondary goals from a benevolent perspective. Wang and Chin [29] extended Liang et al.'s [17] models by describing the true ideal points.

Liang et al. [16] generalized cross-efficiency concepts to the game cross-efficiency and achieved the convergence of the repetitive algorithm by deduction from the balanced point. Wu et al. [39] exhibited a mixed integer programming model for cross-efficiency evaluation and for determining the best ranking arrangement for each DMU. Lam [15] proposed the development of a new, improved method for selecting more suitable weights to be used in cross-evaluation. Wang et al. [33] suggested three alternative approaches to determine relative importance of weights for cross efficiency aggregation. Soltanifar et al. [27], by extending some of the previous researchers' ideas in the same area, set the goal constituted three new secondary goals and by selecting the best among them and using the voting model, they suggested a new assessment. Wu et al. [37], demonstrated inefficiency of the conventional aggressive and benevolent model, and exhibited better secondary goals. Wu et al. [36] suggested a DEA cross-efficiency evaluation based on Pareto improvement.

The review of the related articles makes clear that the available cross-efficiencies (except for game cross-efficiency) are all computed either as aggressive or benevolent. Yet, there is no warranty that the aggressive and benevolent formulas can give the same results in ranking. Therefore, a neutral model is introduced, which determines the weights of the inputs and outputs for the unit under evaluation, without examining the aggressiveness and benevolence of other DMUs. This model helps the decision maker to evade the problem of selecting between aggressive and benevolent model. Wang and Chin [30] suggested a neutral model DEA for cross-efficiency evaluation and its generalization for evaluation with cross-weights. They maximized the minimum of relative efficiency of each output, and as a result, they considerably decreased the number of zero weights for the outputs. Ramon et al. [22] exhibited another model, operating concurrent with assessment, whose secondary goal was to select profiles weights. So, high differences in the weights related to inputs and outputs, to the extent possible were avoided. Also, Ramon et al. [23] selected a peer-restricted method, disregarding the weights of a specific inefficient DMU in computing cross-efficiency. Wang et al. [34] presented some neutral DEA models to minimize the virtual disparity in the cross-efficiency evaluation. Wang et al. [32] suggested a neutral model whose goal was to decrease the number of zero weights both for the inputs and outputs. Wang et al. [31], using ideal and anti-ideal virtual DMUs, exhibited four neutral models for cross-efficiency evaluation from the view of multiple criteria decision analysis (MCDA). Jahanshahloo et al. [12] suggested the selection of symmetric weights as a secondary goal in cross-efficiency evaluation. Wu et al. [40] introduced a weight balanced DEA model whose goal was to decrease the number of zero weights. Lin et al. [19] presented an iterative method for assigning weights in cross-efficiency evaluation, which not only certifies a unique weight set for positive input and output data, but also decreases the number of zero weights maximally without imposing any prior weight limitation.

In this paper, we seek to allocate weights among the multiple optimal weights based on Wang et al. [32], so that the consumption contribution of each input and the production contribution of each output in the efficiency evaluation of DMU under study decreases and increases respectively. However, we avoid the simultaneous selection of zero weights in both inputs and outputs. In other words, in addition to allocating the optimal contribution of the inputs and outputs in the consumption of resources and production, and retaining the efficiency of the unit under evaluation, we try to have all the inputs and outputs of other DMUs participate in the cross-efficiency evaluation, to the extent possible. We believe that such a selection of weights is more logical than the selection method introduced by Wang et al. [32].

The advantage of the proposed method over the Wangs is that the neutral weights are selected for each unit based on increasing outputs share and reducing inputs share, simultaneously. This type of weight selection appears to be more appropriate in terms of performance evaluation viewpoint, since in performance evaluation, it is always sought to increase the total outputs share and reduce the total inputs share, simultaneously. Although the proposed method may cause zero weight assignment to inputs, and thus reduce the number of inputs in the performance evaluation of units, these weights can be avoided by providing a two-phase process.

The rest of this article is organized as follows: In section 2, we present a concise point out of the cross-efficiency evaluation, its main formulations, and the secondary goals. In Section 3, the proposed method is introduced and illustrated by numerical examples in Section 4. Finally, Section 5 is assigned to the conclusions.

2. CROSS-EFFICIENCY

2.1. Cross-efficiency evaluation

Suppose that there is a set of n Decision Making Units (DMUs), and each $DMU_j (j = 1, \ldots, n)$,using different m inputs, produces different s outputs which are respectively determined by x_{ij} $(i = 1, \ldots, m)$ and y_{rj} $(r = 1, \ldots, s)$. To assess each DMU_k $(k = 1, \ldots, n)$, the efficiency score E_{kk} performance can be calculated by the input-oriented CCR multiplier model as follows :

$$\begin{aligned}
\max \quad & E_{kk} = \frac{\sum_{r=1}^{s} u_{rk} y_{rk}}{\sum_{i=1}^{m} \nu_{ik} x_{ik}} \\
s.t. \quad & E_{kj} = \frac{\sum_{r=1}^{s} u_{rk} y_{rj}}{\sum_{i=1}^{m} \nu_{ik} x_{ij}} \leq 1, \quad j = 1, \ldots, n, \\
& \nu_{ik} \geq 0, \quad i = 1, \ldots, m, \\
& u_{rk} \geq 0, \quad r = 1, \ldots, s.
\end{aligned} \tag{1}$$

where ν_{ik} and u_{rk} represent ith input and rth output weights for DMU_k. By using the Charnes and Cooper [1] Conversion, we changed it to a linear model as follows:

$$
\begin{aligned}
\max \quad & E_{kk} = \sum_{r=1}^{s} u_{rk} y_{rk} \\
s.t. \quad & \sum_{i=1}^{m} \nu_{ik} x_{ik} = 1, \\
& \sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} \nu_{ik} x_{ij} \leq 0, \quad j = 1, \ldots, n, \\
& \nu_{ik} \geq 0, \quad i = 1, \ldots, m, \\
& u_{rk} \geq 0, \quad r = 1, \ldots, s.
\end{aligned} \tag{2}
$$

Suppose that $\nu^*_{ik}(i = 1, \ldots, m)$ and $u^*_{rk}(r = 1, \ldots, s)$ are the optimal solutions above of LP model, then $E^*_{kk} = \sum_{r=1}^{s} u^*_{rk} y_{rk}$ shows the CCR-efficiency of DMU_k, which resulted from self-evaluation. The cross-efficiency of DMU_j, resulting from peer-evaluation using DMU_k, can be obtained as follows:

$$
E^*_{kj} = \frac{\sum_{r=1}^{s} u^*_{rk} y_{rj}}{\sum_{i=1}^{m} \nu^*_{ik} x_{ij}}, \quad j = 1, \ldots, n; j \neq k, \tag{3}
$$

The corresponding model for each DMU is solved, and as a result, n series of input and output weights for n DMU are computed. Each DMU has $(n-1)$ cross-efficiency in addition to one CCR efficiency. These efficiencies constitute an $n \times n$ matrix, called the cross-efficiency matrix, where E_{kj} is an entry in row k and in column j .

Doyle and Green [7] described the cross-efficiency score as a cross efficiency average DMU_j with optimal weights of the other DMUs, as follows:

$$
E_j = \frac{1}{n} \sum_{k=1}^{n} E^*_{kj} \tag{4}
$$

Optimal solutions resulting from Model (2) are not often unique, but, a desirable cross-efficiency was gained. This is related to a specific software that ideally selects optimal solutions (Despotis, [6]). To overcome this problem, secondary goals in cross-efficiency evaluation are exhibited.

Secondary goals to solve the problem of multiple optimal weights were introduced to examine one solution among the multiple optimal solutions on the basis of a given criterion. For the first time, Sexton et al. [25] discussed the benevolent and aggressive models. Doyle and Green[7] exhibited another form of benevolent and aggressive formulas, which are used more frequently in practice.

$$\begin{aligned}
\min \quad & \sum_{r=1}^{s} u_{rk}(\sum_{j=1,j\neq k}^{n} y_{rk}) \\
s.t. \quad & \sum_{i=1}^{m} \nu_{ik}(\sum_{j=1,j\neq k}^{n} x_{ik}) = 1, \\
& \sum_{r=1}^{s} u_{rk}y_{rk} - E_{kk}^{*}\sum_{i=1}^{m} \nu_{ik}x_{ik} = 0, \\
& \sum_{r=1}^{s} u_{rk}y_{rj} - \sum_{i=1}^{m} \nu_{ik}x_{ij} \leq 0, \quad j = 1,\dots,n, \\
& \nu_{ik} \geq 0, \quad i = 1,\dots,m, \\
& u_{rk} \geq 0, \quad r = 1,\dots,s.
\end{aligned} \tag{5}$$

and

$$\begin{aligned}
\max \quad & \sum_{r=1}^{s} u_{rk}(\sum_{j=1,j\neq k}^{n} y_{rk}) \\
s.t. \quad & \sum_{i=1}^{m} \nu_{ik}(\sum_{j=1,j\neq k}^{n} x_{ik}) = 1, \\
& \sum_{r=1}^{s} u_{rk}y_{rk} - E_{kk}^{*}\sum_{i=1}^{m} \nu_{ik}x_{ik} = 0, \\
& \sum_{r=1}^{s} u_{rk}y_{rj} - \sum_{i=1}^{m} \nu_{ik}x_{ij} \leq 0, \quad j = 1,\dots,n, \\
& \nu_{ik} \geq 0, \quad i = 1,\dots,m, \\
& u_{rk} \geq 0, \quad r = 1,\dots,s.
\end{aligned} \tag{6}$$

Model (5) is known as the aggressive formulation for cross-efficiency evaluation, which aims at minimizing the cross-efficiencies of the other DMUs in some way, whereas model (6) is known as the benevolent formulation for cross-efficiency evaluation, which aims at maximizing the cross-efficiencies of the other DMUs to some extent(Wang and Chin [29]). The two models choose the optimal weights from two different viewpoints, whereby two different ranking methods are achieved in the cross-efficiency evaluation, while the decision maker may seek to choose a neutral method being neither benevolent nor aggressive. So, Wang and Chain [30] introduced a neutral model of DEA in the cross-efficiency evaluation as follows:

$$\begin{aligned}
\max \quad & \delta \\
s.t. \quad & \sum_{i=1}^{m} \nu_{ik} x_{ik} = 1, \\
& \sum_{r=1}^{s} u_{rk} y_{rk} = E_{kk}^{*}, \\
& \sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} \nu_{ik} x_{ij} \leq 0, \quad j = 1, \ldots, n, \\
& u_{rk} y_{rk} - \delta \geq 0, \quad r = 1, \ldots, s, \\
& \nu_{ik}, u_{rk}, \delta \geq 0, \quad i = 1, \ldots, m, \quad r = 1, \ldots, s.
\end{aligned} \tag{7}$$

Among multiple optimal weights, Model (7) selects such optimal weights which maximize the comparative efficiency of each output. In this method, the number of zero weights of the output decreases effectively. Wang et al. [30] stated: "The economic meaning of the model (7) can be interpreted as DMU_k searches for a set of input and output weights to maximize its efficiency as a whole and at the same time to make each of its output be as efficient as possible to produce sufficient efficiency as an individual." If we have only one output component, then model (7) does not necessarily yield a unique optimal solution. In this case, model (7) and model (2) have the same result. Wang et al. [32] proposed the following model, by generalizing the neutral model of Wang et al. [30], which simultaneously reduces the number of the weights of zero input and output.

$$\begin{aligned}
\max \quad & \alpha.\delta + \beta.\gamma \\
s.t. \quad & \sum_{i=1}^{m} \nu_{ik} x_{ik} = 1, \\
& \sum_{r=1}^{s} u_{rk} y_{rk} = E_{kk}^{*}, \\
& \sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} \nu_{ik} x_{ij} \leq 0, \quad j = 1, \ldots, n, \\
& u_{rk} y_{rk} - \delta \geq 0, \quad r = 1, \ldots, s, \\
& \nu_{ik} x_{ik} - \gamma \geq 0, \quad i = 1, \ldots, m, \\
& \nu_{ik}, u_{rk}, \delta, \gamma \geq 0, \quad i = 1, \ldots, m, \quad r = 1, \ldots, s.
\end{aligned} \tag{8}$$

Where α and β are parameters that hold in $\alpha + \beta = 1$ condition.

Wang et al. [32] said:" the economic meaning of model (8) can be interpreted as seeking for a set of input and output weights for DMU_k to make each of its output and input as efficient or sufficiently important as possible while keeping its CCR-efficiency unchanged such that each output can produce sufficient efficiency as an individual and every input can be sufficiently utilized."

3. THE PROPOSED MODEL

Wang and Chin [30] proposed a neutral model for cross-efficiency evaluation to evade the problem of selecting between aggressive or benevolent formulas. They suggested a new strategy to select the weights so as not to increase or decrease the other DMUs. Likewise, Ramon et al. [22] followed the same strategy. They also suggested the participation of all inputs and outputs' weights of DMUs in the efficiency evaluation of the unit under evaluation to avoid the selection of zero weights. In the present paper, by combining the two ideas above, we intend to propose a model that considers the selected weights in the cross-efficiency evaluation.

Wang et al. [32] commented that among the multiple optimal weights the one should be selected which yields an improved relative efficiency for each input and output, and to the extent possible, covertly prevents the selection of zero solutions in cross-efficiency evaluation. We agree with them in this regard. However, in their model, Wang et al. [32] actually seek to select weights that increase the contribution of input consumption while it seems that this contradicts the optimization of the relative efficiency of each input. We believe that such selection of weights is not appropriate because it worsens the relative efficiency of each input though the relative efficiency of each output is improved. Therefore, we intend to propose a model that improves the relative efficiency of each input and output. To this end, we first propose the following model:

$$
\begin{aligned}
\max \quad & \delta - \gamma \\
s.t. \quad & \sum_{i=1}^{m} \nu_{ik} x_{ik} = 1, \\
& \sum_{r=1}^{s} u_{rk} y_{rk} = E^*_{kk}, \\
& \sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} \nu_{ik} x_{ij} \leq 0, \quad j = 1, \ldots, n, \\
& u_{rk} y_{rk} - \delta \geq 0, \quad r = 1, \ldots, s, \\
& \nu_{ik} x_{ik} - \gamma \geq 0, \quad i = 1, \ldots, m, \\
& \nu_{ik}, u_{rk}, \delta, \gamma \geq 0, \quad i = 1, \ldots, m, \quad r = 1, \ldots, s.
\end{aligned}
\tag{9}
$$

In model (9), in the cross-efficiency evaluation, those weights are selected that, in addition to retaining the efficiency of the unit under evaluation, they optimize the relative efficiency of each input and output. In other words, this allows the consumption contribution of each input to reduce while the production contribution of each output increases. However, in doing, so in model (9), the zero weights are allocated to each input to the extent possible, but for the outputs a non-zero is obtained. To avoid zero weights, to the extent possible, we propose the following model:

$$\begin{aligned}
\max \quad & \min\{\nu_{1k},\dots,\nu_{mk},u_{1k},\dots,u_{sk}\} \\
s.t. \quad & \sum_{i=1}^{m} \nu_{ik}x_{ik} = 1, \\
& \sum_{r=1}^{s} u_{rk}y_{rk} = E^*_{kk}, \\
& \sum_{r=1}^{s} u_{rk}y_{rj} - \sum_{i=1}^{m} \nu_{ik}x_{ij} \le 0, \quad j=1,\dots,n, \\
& \nu_{ik},u_{rk} \ge 0, \quad i=1,\dots,m, \quad r=1,\dots,s.
\end{aligned} \tag{10}$$

To minimize the weights, this model uses the optimal solutions of model(2), so it is a non-linear model defined by $\epsilon = \min\{\nu_{1k},\dots,\nu_{mk},u_{1k},\dots,u_{sk}\}$, that converts to the following linear model, which is the same as the phase one model.

$$\begin{aligned}
\max \quad & \epsilon \\
s.t. \quad & \sum_{i=1}^{m} \nu_{ik}x_{ik} = 1, \\
& \sum_{r=1}^{s} u_{rk}y_{rk} = E^*_{kk}, \\
& \sum_{r=1}^{s} u_{rk}y_{rj} - \sum_{i=1}^{m} \nu_{ik}x_{ij} \le 0, \quad j=1,\dots,n, \\
& \nu_{ik},u_{rk} \ge \epsilon, \quad i=1,\dots,m, \quad r=1,\dots,s.
\end{aligned} \tag{11}$$

The limitations applied to the weights do not make model (11) infeasible because here ϵ is a variable not a constant value with no impact on the infeasibility. Due to the objective function of the fourth and fifth restrictions, the non-zero weights are selected among the multiple optimal weights. In fact, the model assigns the non-zero weights to efficient units and inefficient units whose images were on the efficient frontier.

If the model (2) has been used instead of the first phase(model (11)), there are two problems: the existence of multiple optimal solutions and the existence of zero weights. In the method, it is attempted to solve the mentioned problems as far as possible. Hence, in the first phase, model (11) has been applied which included two advantages: 1) it drives the weights towards selecting unique weights (it does not necessarily yield the unique result); 2) they are obtained the weights being against zero, as far as possible.

Now, in order to achieve the weights of the inputs and outputs of the unit under evaluation, we propose the following model:

$$
\begin{aligned}
\max \quad & \delta - \gamma \\
s.t. \quad & \sum_{i=1}^{m} \nu_{ik} x_{ik} = 1, \\
& \sum_{r=1}^{s} u_{rk} y_{rk} = E_{kk}^{*}, \\
& \sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} \nu_{ik} x_{ij} \leq 0, \quad j = 1, \ldots, n, \\
& u_{rk} y_{rk} - \delta \geq 0, \quad r = 1, \ldots, s, \\
& \nu_{ik} x_{ik} - \gamma \geq 0, \quad i = 1, \ldots, m, \\
& \nu_{ik}, u_{rk} \geq \epsilon^{*}, \quad i = 1, \ldots, m, \quad r = 1, \ldots, s, \\
& \delta, \gamma \geq 0.
\end{aligned} \tag{12}
$$

Where ϵ^* is the solution resulted from model (11).

In this method, a neutral secondary goal is suggested that prevents the occurrence of multiple optimal solutions. In fact, model (12) selects a solution among the multiple optimal solutions that decreases the consumption contribution of each input but increases the production contribution of each output, in addition to ensuring the non-zero weights to the extent possible. Assume that model (12) is solved n times and $\nu_{ik}^*, u_{rk}^*(\forall i, r, k)$ are derived the weights, then the cross-efficiency of DMUs, from the perspective of other DMUs, is calculated by equation (3). Also, the cross-efficiency scores for DMUs are calculated by equation (4).

Remark: Model (2) has multiple optimal solutions when the unit under evaluation is efficient or its projection falls on the efficient frontier of production possibility set, model (12) produces non-zero solutions for such DMUs. Therefore, the zero weights resulting from model (12) are due to a unique solution in model (2) for the unit under evaluation.

To clarify the suggested ranking method in the evaluation of cross-efficiency in comparison with other methods, we will exhibit two examples in the next section.

4. NUMERICAL EXAMPLES

In this section, we provide two numerical examples so to demonstrate the incongruity between the orderings achieved by the aggressive and benevolent formulations for cross-efficiency evaluation and the applications of the proposed neutral DEA model in cross-efficiency evaluation.

4.1. Example 1.

Seven academic departments (DMUs) in a university (Wong and Beasley, [35]) are evaluated based on three inputs and three outputs, given below, and their

Table 1: Data for seven academic departments in a university.

DMU	x_1	x_2	x_3	y_1	y_2	y_3	CCR Efficiency
1	12	400	20	60	35	17	1
2	19	750	70	139	41	40	1
3	42	1500	70	225	68	75	1
4	15	600	100	90	12	17	0.8197
5	45	2000	250	253	145	130	1
6	19	730	50	132	45	45	1
7	41	2350	600	305	159	97	1

Table 2: Cross-efficiencies score of the seven academic departments in the university and their rankings

Department (DMU)	Model(5)	Model(6)	Model(7)	Model(8)	Model(12)
1	0.8081 (2)	0.9442 (2)	0.8499(3)	0.8647(3)	0.9282(2)
2	0.7191 (4)	0.9326 (3)	0.8612(2)	0.8835(2)	0.9271(3)
3	0.7669 (3)	0.7952 (6)	0.7963(5)	0.8252(4)	0.8180(5)
4	0.3904 (7)	0.5793 (7)	0.5244(7)	0.5143(7)	0.5688(7)
5	0.6576 (5)	0.9100 (4)	0.8256(4)	0.8092(5)	0.8387(4)
6	0.8424 (1)	0.9929 (1)	0.9453(1)	0.9829(1)	0.9929(1)
7	0.5264 (6)	0.8963 (5)	0.7679(6)	0.6709(6)	0.7349(6)

input and output, data are shown in Table 1 along with the CCR efficiencies of the seven academic departments.

x_1: Number of academic staff.
x_2: Academic staff salaries in thousands of pounds.
x_3: Support staff salaries in thousands of pounds.
y_1: Number of undergraduate students.
y_2: Number of postgraduate students.
y_3: Number of research papers.

Since the CCR-efficiencies evaluate six out of seven academic departments as efficient and cannot distinguish them any further, the cross-efficiencies are computed.

In the first phase of the proposed method of solving model (11), the optimal target functions for units 1 through 7 are 0.002315, 0.000949, 0.000356, 0.000000, 0.000427, 0.001252 and 0.000190, respectively. We put these values in the second phase in (12) model to solve the optimal solutions, resulting in cross-efficiency and score cross efficiency.

The cross-efficiency scores of the seven academic departments in the university, adopted from Models (5), (6), (7), (8), and (12) are presented in Table 2. As can be seen, DMU_4 and DMU_5 have the best and worst performance respectively among the different DMUs in the models mentioned above . The results of the

Table 3: Input and output weights for the seven academic departments produced by aggressive model (5).

DMU	Input 1	Input 2	Input 3	Output 1	Output 2	Output 3
1	0	0	0.0008772	0	0.0005013	0
2	0	0.0001142	0.0001232	0.0006783	0	0
3	0	0	0.0009174	0.0002854	0	0
4	0.0053885	0.0000053	0	0.0007651	0	0
5	0.0042872	0	0.0004016	0	0	0.0022564
6	0.0009669	0	0.0007493	0	0	0.0012408
7	0.0065789	0	0	0	0.0016965	0

Table 4: Input and output weights for the seven academic departments produced by benevolent model (6).

DMU	Input 1	Input 2	Input 3	Output 1	Output 2	Output 3
1	0.0019853	0.0000808	0	0.0003273	0.0009051	0.0002835
2	0.0028663	0.0000661	0	0.0005581	0.0006460	0
3	0	0.0000287	0.0007376	0	0.0002436	0.0010415
4	0.0053885	0.0000053	0	0.0007651	0	0
5	0.0024655	0.0001003	0	0.0004065	0.0011241	0.0003521
6	0.0020692	0.0000842	0	0.0003412	0.0009434	0.0002955
7	0.0025294	0.0001029	0	0.0004170	0.0011532	0.0003613

ranking of DMUs in the proposed model is more similar to the neutral model (7) with only the ranks of DMU_1 and DMU_2 being exchanged.

In Tables 3-8, the weights obtained from different models indicate that the proposed model included the most non-zero weights in the cross-efficiency evaluation. In other words, every efficient DMU participates in cross-efficiency evaluation of other DMUs. However, the other models discussed above do not have such a feature. This again implies the superiority of the proposed model to the ones mentioned above. Also, it can be seen that this model is able to rank every individual DMU. It can be seen in Tables 6 and 7 that Wang et al. [32] also intended to decrease the number of zero weights of the input and output by changing the parameters α and β. However, in comparison with the proposed method (Table 8), this strategy failed to achieve this goal.

In Table (9), the weights obtained from the model (9) are given without regard to phase 1. In Table (8), we observed that the number of zero weights obtained from model (12) reduced compared to the model (9) in Table (9), which was our goal in phase 2. The results differ when model (12) is used in the first phase compared to model (9). Although the application of the proposed two-phase method seems more complicated, its idea is to prevent the selection of zero weights in the performance evaluation of the units, because from managerial viewpoint, managers expect all units to participate in performance evaluation. Model (12) chooses a larger number of non-zero weights than model (9) to look for an optimal share of

Table 5: Input and output weights for the seven academic departments produced by neutral DEA model (7).

DMU	Input 1	Input 2	Input 3	Output 1	Output 2	Output 3
1	0	0.0016879	0.0162413	0.0055556	0.0095238	0.0196078
2	0.0370936	0.0003936	0	0.0053032	0.0032055	0.0032856
3	0	0	0.0142857	0.0008000	0.0026471	0.0085333
4	0.0641504	0.0000629	0	0.0091082	0	0
5	0.0206555	0	0.0002820	0.0013175	0.0022989	0.0025641
6	0	0.0013699	0	0.0025253	0.0074074	0.0074074
7	0.0243902	0	0	0.0010929	0.0020964	0.0034364

Table 6: Input and output weights for the seven academic departments produced by neutral DEA model (8) [$\alpha = 0.5, \beta = 0.5$].

DMU	Input 1	Input 2	Input 3	Output 1	Output 2	Output 3
1	0.0277778	0.0008333	0.0166667	0.0050076	0.0114024	0.0176740
2	0.0356618	0.0002149	0.0023030	0.0071942	0	0
3	0.0051643	0.0001446	0.0080886	0.0007440	0.0024619	0.0088691
4	0.0641504	0.0000629	0	0.0091082	0	0
5	0.0175341	0.0000527	0.0004219	0.0013175	0.0022989	0.0025641
6	0.0175439	0.0004566	0.0066667	0.0023633	0.0069323	0.0083575
7	0.0206904	0.0000323	0.0001264	0.0010929	0.0020964	0.0034364

Table 7: Input and output weights for the seven academic departments produced by neutral DEA model (8) [$\alpha = 0.8, \beta = 0.2$].

DMU	Input 1	Input 2	Input 3	Output 1	Output 2	Output 3
1	0.0238609	0.0007158	0.0213670	0.0055556	0.0095238	0.0196078
2	0.0370936	0.0003936	0	0.0053032	0.0032055	0.0032856
3	0.0051643	0.0001446	0.0080886	0.0007440	0.0024619	0.0088691
4	0.0641504	0.0000629	0	0.0091082	0	0
5	0.0175341	0.0000527	0.0004219	0.0013175	0.0022989	0.0025641
6	0.0209475	0.0004123	0.0060200	0.0025253	0.0074074	0.0074074
7	0.0206904	0.0000323	0.0001264	0.0010929	0.0020964	0.0034364

Table 8: Input and output weights for the seven academic departments produced by neutral DEA model (12).

DMU	Input 1	Input 2	Input 3	Output 1	Output 2	Output 3
1	0.0023148	0.0023148	0.0023148	0.0041294	0.0144134	0.0145744
2	0.0116848	0.0009488	0.0009488	0.0066414	0.0009488	0.0009488
3	0.0003559	0.0003559	0.0064447	0.0042182	0.0003559	0.0003559
4	0.0641504	0.0000629	0	0.0091082	0	0
5	0.0008897	0.0004267	0.0004267	0.0004267	0.0029689	0.0035505
6	0.0012516	0.0012516	0.0012516	0.0025253	0.0074074	0.0074074
7	0.0106952	0.0001903	0.0001903	0.0012375	0.0034939	0.0006911

Table 9: Input and output weights for the seven academic departments produced by model (9).

DMU	Input 1	Input 2	Input 3	Output 1	Output 2	Output 3
1	0.0277778	0.0008333	0.0166667	0.0050076	0.0114024	0.0176740
2	0.0263158	0.0006667	0	0.0055463	0.0027935	0.0028633
3	0	0.0003131	0.0075772	0.0006788	0.0022461	0.0092604
4	0.0641504	0.0000629	0	0.0091082	0	0
5	0.0110615	0.0002489	0.0000179	0.0013175	0.0022989	0.0025641
6	0.0175439	0.0004566	0.0066667	0.0023633	0.0069323	0.0083575
7	0.0121951	0.0002128	0	0.0013795	0.0022179	0.0023362

input and output weights in evaluating cross-efficiency in an impartial view.

4.2. Example 2.

Fifteen US cities (Chen, [3]) were assessed with respect to three inputs and three outputs that are defined as follows:

x_1: highend housing price (1,000 US$).
x_2: lower-end housing monthly rental (US$).
x_3: number of violent crimes.
y_1: median household income (US$).
y_2: number of bachelor's degrees (million) held by persons in the population.
y_1: number of doctors (thousand).

Table 10 shows the input and output data of the 15 US cities together with their CCR-efficiencies which assess seven out of the 15 US cities as DEA efficient but cannot distinguish them any further.

Table 11 shows the efficiency scores resulting from cross evaluation of 15 US cities in Models (5), (6), (7), (8), and (12). It can be seen that the proposed model assigns a different rank to each unit and DMU_6 and DMU_{12}, respectively have the best and the worst performance among the different DMUs in the models mentioned above. The results obtained from the ranking of models show that the ranking performance of the proposed model is most similar to that of the neutral model (7). In Table 12, all the weights of efficient DMUs are non-zero. Therefore, it can be concluded that every input and output has participated in the cross-efficiency evaluation of other DMUs.

Table 10: Data and CCR efficiency scores.

DMU	City	x_1	x_2	x_3	y_1	y_2	y_3	CCR-efficiency
1	Seattle	586	581	1193.06	46928	0.6534	9.878	1
2	Denver	475	558	1131.64	42879	0.5529	5.301	0.9766
3	Philadelphia	201	600	3468	43576	1.135	18.2	1
4	Minneapolis	299	609	1340.55	45673	0.729	7.209	1
5	Raleigh	318	613	634.7	40990	0.319	4.94	1
6	StLouis	265	558	657.5	39079	0.515	8.5	1
7	Cincinnati	467	580	882.4	38455	0.3184	4.48	0.8930
8	Washington	583	625	3286.7	54291	1.7158	15.41	1
9	Pittsburgh	347	535	917.04	34534	0.4512	8.784	0.9573
10	Dallas	296	650	3714.3	41984	1.2195	8.82	0.9092
11	Atlanta	600	740	2963.1	43249	0.9205	7.805	0.7068
12	Baltimore	575	775	3240.75	43291	0.5825	10.05	0.6826
13	Boston	351	888	2197.12	46444	1.04	18.208	1
14	Milwaukee	283	727	778.35	41841	0.321	4.665	0.9941
15	Nashville	431	695	1245.75	40221	0.2365	3.575	0.7747

Table 11: Cross-efficiencies score of the 15 city by different models and their rankings.

DMU	Model(5)	Model(6)	Model(7)	Model(8)	Model(12)
1	0.7185(5)	0.9134(3)	0.8211(5)	0.8000(5)	0.817364(5)
2	0.6497(9)	0.8444(6)	0.7443(8)	0.7365(9)	0.785380(7)
3	0.7969(2)	0.8889(5)	0.8256(4)	0.8648(3)	0.827730(4)
4	0.7494(3)	0.9149(2)	0.8328(2)	0.8650(2)	0.892982(2)
5	0.6993(6)	0.8437(7)	0.7845(6)	0.7700(7)	0.853188(3)
6	0.8528(1)	0.9567(1)	0.9319(1)	0.9397(1)	0.952825(1)
7	0.5766(12)	0.7494(10)	0.6591(11)	0.6397(12)	0.710512(11)
8	0.7466(4)	0.9050(4)	0.8323(3)	0.8501(4)	0.785154(8)
9	0.6878(8)	0.8242(8)	0.7642(7)	0.7665(8)	0.774302(9)
10	0.5933(11)	0.7037(12)	0.6420(12)	0.6736(11)	0.656515(12)
11	0.4782(14)	0.6097(14)	0.5392(14)	0.5488(13)	0.550340(14)
12	0.4456(15)	0.5879(15)	0.5029(15)	0.5042(15)	0.531258(15)
13	0.6901(7)	0.7463(11)	0.7272(9)	0.7717(6)	0.715834(10)
14	0.6284(10)	0.7496(9)	0.6983(10)	0.7013(10)	0.785382(6)
15	0.4833(13)	0.6500(13)	0.5515(13)	0.5466(14)	0.636357(13)

Table 12: Input and output weights for the fifteen city produced by neutral DEA model (12).

DMU	Input 1	Input 2	Input 3	Output 1	Output 2	Output 3
1	0.0000213	0.0011895	0.0002484	0.0000213	0.0000213	0.0000213
2	0.0002843	0.0013351	0.0001060	0.0000228	0	0
3	0.0019023	0.0006373	0.0000678	0.0000229	0.0000229	0.0000229
4	0.0014866	0.0007299	0.0000828	0.0000219	0.0000219	0.0000219
5	0.0004415	0.0007011	0.0006772	0.0000244	0.0000244	0.0000244
6	0.0012579	0.0005974	0.0005070	0.0000256	0.0000256	0.0000256
7	0	0.0012907	0.0002849	0.0000232	0	0
8	0.0003678	0.0011601	0.0000184	0.0000184	0.0000184	0.0000184
9	0	0.0013607	0.0002966	0.0000049	0	0.0895484
10	0.0010646	0.0010537	0	0	0.7455351	0
11	0.0001925	0.0010527	0.0000356	0.0000163	0	0
12	0.0003809	0.0009819	0.0000062	0.0000158	0	0
13	0.0016737	0.0000059	0.0001854	0.0000059	0.0000059	0.0399867
14	0.0032269	0	0.0001115	0.0000238	0	0
15	0.0002404	0.0011290	0.0000897	0.0000193	0	0

5. CONCLUSIONS

Cross-efficiency evaluation is an effective method to rank DMUs in DEA. The major problem, however, is that DEA suffers from a potential flaw, i.e., the existence of multiple optimal solutions, which results in different efficiency scores, thus in different ranking for DMUs. In the present article, we proposed a model that can decrease the consumption contribution of each input while increasing the production contribution of each output to select an appropriate solution among the multiple optimal solutions. The proposed method seems to be more logical compared to the previous methods because it adopts the same strategy to the evaluation of the performance DMUs in DEA.

The factor that seems to be appealing in cross-efficiency evaluation is to include all the inputs and outputs of other DMUs in the cross-efficiency evaluation of a given DMU to the extent possible. To do so, it is strongly recommended to avoid the selection of non-zero weights, which our model provides to the extent possible. This further indicates the advantage of the proposed method over the existing methods. Our method, considered as a neutral model, is able to offer different ranking in the efficiency evaluation of DMUs. According to Wang and Chin [30], basically, by a neutral model the problem of selecting between aggressive and benevolent formulas can be evaded.

In the paper, they were presented two examples to compare the proposed method with the other methods. In the proposed method, in the first phase, the optimal value of the objective function in model (11) has been primarily computed. These values were used in the second phase(model (12)) so that the unique optimal weights and as much as possible non-zero weights were obtained. We have found that the weights obtained from the proposed method show fewer numbers of zero weights than the other methods, as a result of which more inputs and outputs have contributed to the evaluation of the performance of each unit. Also, the choice of optimal weights among multiple optimized weights is based on reducing the input share and increasing the share of outputs for each unit independently in evaluating unit performance.

Therefore, the advantages of this method, compared with other methods, are the reduction of the number of zero weights and the selection of optimal weights based on the share of the optimal use of resources and the production of outputs in the performance evaluation, regardless of how other units perform. A problem that is not addressed in Wang's way is the use of the optimal share (decrease) of inputs in addition to increasing the share of outputs that we have achieved by modifying their model.

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