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RETAILER'S OPTIMAL PRICING AND REPLENISHMENT POLICY FOR NEW PRODUCT AND OPTIMAL TAKE-BACK QUANTITY OF USED PRODUCT

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Abstract: In the world of limited resources, recovery of used products for reselling or recycling is a critical issue from the economic and environmental point of view. In this paper, we have assumed that a retailer sells the new product to customers as well as collects and sells the used products. We adopt a price dependent quadratic demand function, and the return of used product as a price and time-dependent linear function. The proposed problem is formulated as a profit maximization problem for the retailer. The objective is to find the optimal selling price, the optimal ordering quantity for the new product, and the optimal quantity of used product simultaneously such that the retailers total profit is maximized. The model is validated by a numerical example and sensitivity analysis is performed for the key parameters.

Keywords: Inventory, Replenishment quantity, Price-dependent demand, Used product.

MSC: 90B05.

1. INTRODUCTION

In todays world, one of the greatest challenges is to preserve our limited natural resources and reduce the waste material. Due to growing environmental concern among customers, the manufacturing sectors are also promoting environmentfriendly methods to attract new customers. Customers also prefer to buy from the companies with green image. That is why recovery of used materials and items has got more attention in past decade. Earlier, recycling and reusing were limited to commonly used items like metal and glass. In recent times, items like print cartridges, one-time-use cameras, carpet material etc. are the examples where recycling and product recovery are widely used.

Fleischmann *et* al. [4] considered a generic facility location model to discuss the effects of return flows on logistic networks. Koh *et* al. [9] have discussed reusable items with the simple recovery process. In their paper, the demand is fulfilled by a new product and recycled old products. Kannan *et* al. [8] have developed a multi-echelon closed loop supply chain network model for a multi-period and multi-products. They have studied the case of battery recycling where old battery material is used in the production of new batteries. Govindan *et* al. [6] have provided a very nice review of the recent research papers in which they reviewed a total of 382 papers to construct a good framework of past, and they identified the gaps where future work was required. Recently, Chen *et* al. [2] have developed models for the retailer, assuming that the retailer sells the new products as well as collects the used products. Other motivating work in the area of recycling and reusing are paper recycling, Pati *et* al. [11], Glass recycling, Gonzlez-Torre and Adenso-Daz [5], electronic waste recycling, Nagurney and Toyasaki [10], batteries recycling, Daniel *et* al. [3] etc.

In recent time, pricing is an important strategic issue because it directly affects the demand. The demand is said to be elastic if, when the price goes up, the generated revenue goes down. Thus, this is a very important decision for any manager to make. Whitin [15] was the first to study price-dependent demand. Shah *et* al. [12] studied an inventory model for price and frequency of advertisement dependent demand. They have provided a general model by using general-type of deterioration and holding cost rates. Jaggi *et* al. [7] also used selling price dependent demand in their study. Their model features a two ware-house inventory model with non-instantaneous deterioration and under the effects of trade credit. Wu *et* al. [16], Shastri *et* al. [14], Shah *et* al. [13] also considered price dependent demand in their study.

In this study, we have assumed that a retailer sells the new product to the customers as well as collects and sells the used products. The optimal pricing, the ordering quantity for a new product and the optimal quantity of a used product are discussed, where customer demand is sensitive to time and the retail price. The total profit is maximized with respect to selling price and cycle time.

The rest of the paper is structured as follows. Section-1 contains a brief literature review of recent papers. Model assumptions and notation are provided in Section-2. Section-3 provides model formulation. In section-4, numerical example and sensitivity analysis are provided followed by conclusion in Section-5.

2. ASSUMPTIONS AND NOTATION

2.1. NOTATION

A	Ordering cost for retailer ($/order)
C	Purchase cost per item (constant), ($/order)
h	Inventory holding cost per unit item for new product ($/unit)
h_u	Inventory holding cost per unit item for used product ($/unit)
Q	The replenishment quantity for new product
Q_u	The quantity of used product
T	The replenishment time (a decision variable) (years)
τ	The point of time when collection of used products starts (years)
p	Selling price per item (a decision variable) ($/unit)
$R(p,t)$	Demand rate for new product at $t \geq 0$ (units)
$R_u(p,t)$	Demand rate for used product at $t \geq \tau$ (units)
$I(t)$	Inventory level at time $t \geq 0$ for new product (units)
$I_u(t)$	Inventory level at time $t \geq \tau$ for used product (units)
$\pi\,(p,T)$	Total profit of the retailer during cycle time (in $)

2.2. ASSUMPTIONS

1. It is a single item inventory system.
2. The replenishment is instantaneous and planning horizon is infinite.
3. The Lead time is negligible or zero and shortages are not allowed.
4. The Demand rate of new product $R(p,t)$ is considered as, $R(p,t) = \alpha(1+\alpha_1 t - \alpha_2 t^2)) - \beta p$ where, $\alpha > 0$ denotes the scale demand and $\alpha_1, \alpha_2 > 0$. The parameter $\beta > 0$ denotes the price elasticity.
5. The return rate of used product is considered as $R_u\,(p,t) = a\,(1-bt) - p\,(1-p_0)$ where, $a, b > 0$ and p_0 are the parameters associated with price for the used product.

3. MATHEMATICAL MODEL

In this section, we present the general formulations and solutions to the inventory models for a new product as well as for the used product. For the new product the inventory is consumed due to time and price dependent demand. Suppose Q is the ordering quantity to be sold during cycle time $[0,T]$, then the governing differential equation for inventory level $I(t)$ at any time t , where $0 \leq t \leq T$, is given by

$$\frac{dI\,(t)}{dt} = -R\,(p,t)\,, \qquad 0 \leq t \leq T \tag{1}$$

with $I(t) = 0$ and $I(0) = Q$. The solution of the differential equation (1) is given by

$$I\,(t) = -\alpha\left(t + \frac{\alpha_1 t^2}{2} - \frac{\alpha_2 t^3}{3}\right) + \beta p t + Q \tag{2}$$

By using the boundary condition $I(t) = 0$, the inventory level $I(t)$ and the ordering quantity Q are given by

$$I(t) = \alpha\left((T - t) + \frac{\alpha_1\left(T^2 - t^2\right)}{2} - \frac{\alpha_2\left(T^3 - t^3\right)}{3}\right) - \beta p(T - t) \tag{3}$$

$$Q = \alpha\left(T + \frac{\alpha_1 T^2}{2} - \frac{\alpha_2 T^3}{3}\right) - \beta pT \tag{4}$$

Now, for the used product during the period $[\tau, T]$, the inventory level is affected by the return rate of the used product so the governing differential equation for inventory level $I_u(t)$ at any time t, where $\tau \le t \le T$, is given by

$$\frac{dI_u(t)}{dt} = -R_u(p,t), \quad \tau \le t \le T \tag{5}$$

with $I_u(\tau) = Q_u$ and $I_u(T) = 0$. The solution of the differential equation (5) is given by

$$I_u(t) = a\left[(T - t) - \frac{b}{2}\left(T^2 - t^2\right)\right] - p(1 - p_0)(T - t) \tag{6}$$

From the boundary conditions, the quantity of used product Q_u is given by

$$Q_u = -a\left[(T - \tau) - \frac{b}{2}\left(T^2 - \tau^2\right)\right] + p(1 - p_0)(T - \tau) \tag{7}$$

Now to calculate total profit, we calculate all the components for both new product and used product. The components of profit function of the inventory system for new product are as follows.

$$SR_n = \text{Sales revenue from new product} = \frac{1}{T}\int_0^T [p \cdot R(p,t)]\,dt \tag{8}$$

$$PC_n = \text{Purchase cost} = \frac{CQ}{T} \tag{9}$$

$$OC_n = \text{Ordering cost} = \frac{A}{T} \tag{10}$$

$$HC_n = \text{Holding cost} = \frac{1}{T}\int_0^T [h \cdot I(t)]\,dt \tag{11}$$

The components of profit function for the used product are as below.

$$SR_u = \text{Sales revenue from new product} = \frac{1}{T}\int_\tau^T [p(1 - p_0) \cdot R_u(p,t)]\,dt \tag{12}$$

$$PC_n = \text{Purchase cost} = \left(\frac{C \cdot (1-d) \cdot Q_u}{T-\tau}\right) \tag{13}$$

$$HC_n = \text{Holding cost} = \frac{1}{T}\int_{\tau}^{T} \left[h_u \cdot I_u(t)\right] dt \tag{14}$$

Therefore, the total profit is given by

$$\pi(p,T) = (SR_n - HC_n - PC_n - OC_n) + (SR_u - HC_u - PC_u) \tag{15}$$

$$\pi(p,T) = \frac{1}{T}\int_{0}^{T} \left[p \cdot R(p,t) - h \cdot I(t)\right] dt - \left(\frac{CQ+A}{T}\right) + \frac{1}{T}\int_{\tau}^{T} \left[p \cdot (1-p_0) \cdot R_u(p,t) - h_u \cdot I_u(t)\right] dt - \left(\frac{C \cdot (1-d) \cdot Q_u}{T-\tau}\right) \tag{16}$$

The total profit is a function of two variables p and T. Using the classical optimization technique, we calculate maximum profit for the numerical example provided in the next section.

4. NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS

In this section, the model is validated by numerical example using Maple 18 software and sensitivity analysis is performed for the developed model.

Example:: We consider an inventory system with the following parameters in appropriate units:

$\alpha = 200$, $\alpha_1 = 4\%$, $\alpha_2 = 14\%$, $\beta = 2$, $a = 100$, $b = 0.4$, $C = \$45$, $A = \$100$, $h = \$0.5$, $h_u = \$0.2$, $d = 0.15$, $\tau = \frac{30}{365}$ and $p_0 = 0.4$.

Using Maple 18 software, the optimal values of decision variables are obtained as $(p^*,\ T^*) = (83.37,\ 1.91)$. The optimum quantity of fresh and used products are obtained as $Q^* = 144.11$ units and ${Q_u}^* = 18.62$ units. The maximum profit gained is $\pi_{\max} = \$2906.55$. The concavity of the profit function is shown in Figure 1.

Now, for the data used in example 1, we perform sensitivity analysis to observe the effects of inventory parameters on profit and decision variables of the model. We consider parameter variation from -20% to 20%. The results are shown in Figures 2 – 4.

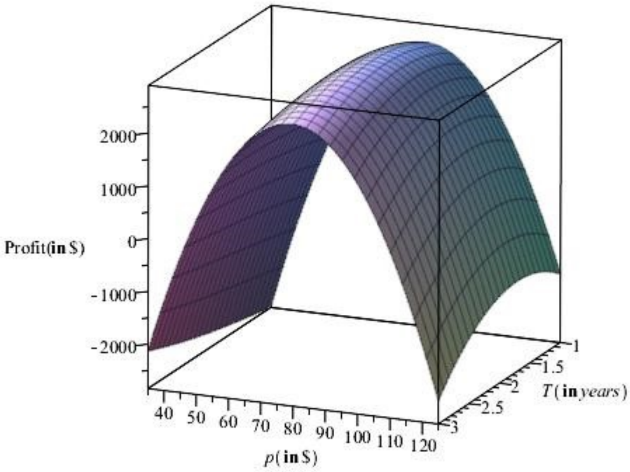

Figure 1: Concavity of profit function

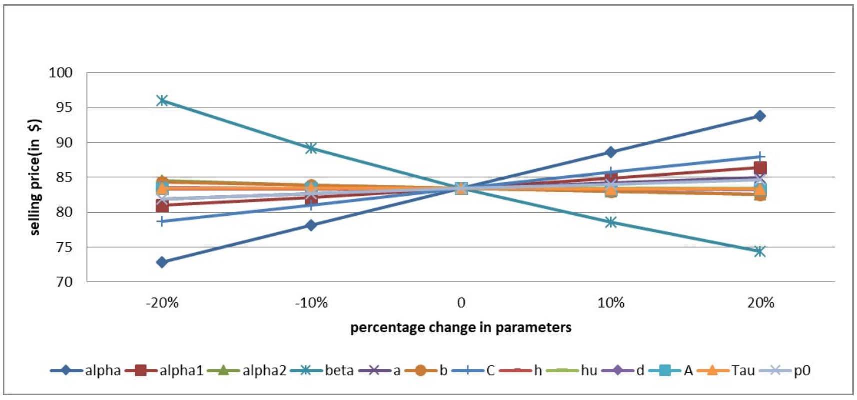

Figure 2: Effect on Selling price w.r.t. inventory parameters

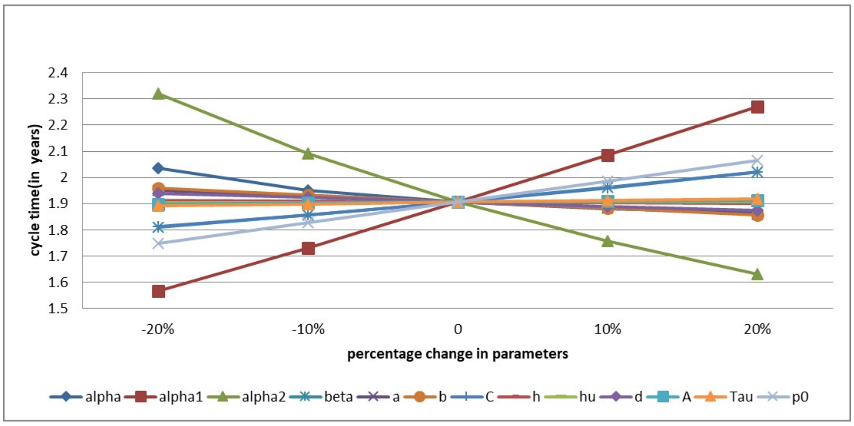

Figure 3: Effect on Cycle time w.r.t. inventory parameters

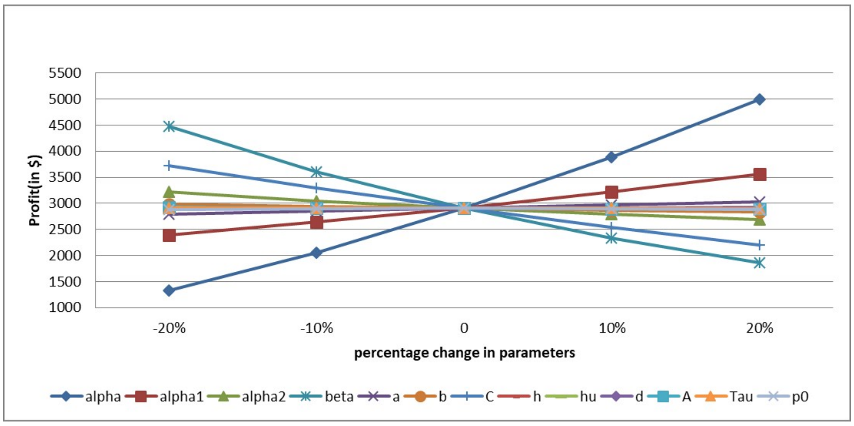

Figure 4: Effect on Profit w.r.t. inventory parameters

From figures 2, 3, and 4, we can observe that

The scale demand α and parameter α_1 have great positive influence on selling price and total profit. This finding implies that a higher scale demand motivates a retailer to set a high selling price and grabs the opportunity of greater investment. Also as shown in Figure 3, an increasing scale demand will lead to heavy decrease in cycle time T.

Figures 2–4, show that an increasing purchase cost C leads to increasing selling price p and cycle time T, while the total profit π will decrease. This finding implies that a high purchase cost reduces the retailers profit. So, the retailer should decrease his ordering quantity and increase the selling price in order to reduce the loss.

As the price elasticity β increase, the cycle time T increases heavily, while the selling price p and profit π decrease significantly. The price elasticity β reduces the opportunity for the retailer to set a high selling price.

When the holding cost h per unit time increases, the selling price increases marginally, while the cycle time and total profit decrease slightly. Similar to holding cost, other parameters have very minor effects on decision variables and the total profit.

5. CONCLUSION

In this study, we developed an inventory model under the assumption that a retailer sells the new product as well as collets used products from the customers. The demand rate of new products and return rate of used products both are linked to selling price. We established a Mathematical model to maximize the profit of the retailer. The optimal selling price, replenishment time, ordering quantity of new product, and optimal quantity of used product are determined using classical optimization. Finally, a numerical example is given to validate the model. Some important managerial insights are provided through sensitivity analysis.

The research can be further extended in several directions. For example, a similar model can be developed for a manufacturer to see effects of remanufacturing process. A possible extension is to study behaviour of each player of the supply chain from the perspectives of game theory.

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