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A BILEVEL APPROACH TO OPTIMIZE ELECTRICITY PRICES

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Abstract: To meet unbalanced demand, an energy provider has to include costly generation technologies, which in turn results in high residential electricity prices. Our work is devoted to the application of a bilevel optimisation, a challenging class of optimisation problems, in electricity market. We propose an original demand-side management model, adapt a solution approach based on complementary slackness conditions, and provide the computational results on illustrative and real data. The goal is to optimise hourly electricity prices, taking into account consumers' behaviour and minimizing energy generation costs. By choosing new pricing policy and shifting electricity consumption from peak to off-peaks hours, the consumers might decrease their electricity payments and eventually, decrease the energy generation costs.

Keywords: Demand-Side Management, Electricity Market, Bilevel Optimization, Complementary Slackness Conditions.

MSC: 90-06, 90C46, 90C90, 91A12.

1. INTRODUCTION

Increase in the power consumption might reach limits of the generated power. Moreover, some power generating units can take long time or may be very expensive to operate at full power. Thus, any load serving entity, for example, EDF - "Electricity of France", seeks tools to meet customers' demand with the minimum costs, and to adjust the power demand instead of altering the supply in real-time. Demand side management (DSM) commonly refers to programs implemented by utility companies. It is about modifying the electricity consumption at the customer side [14, 15] to achieve energy efficiency and reduce peaks. Demand Response (DR) and Energy Efficiency programs are the economic tools for DSM implementation. DR and DSM are designed to encourage end-users to make, respectively, short-term and long-term reductions in energy demand in response to the price. DSM is more about planning issues related to deployment of improved technologies and changes in end-users behaviour. DR addresses operational issues. This paper deals with changing the residential customers behaviour to make it more energy efficient and to measure its impact on the power generation costs; also, with helping EDF and other energy providers to improve tariff offer by proposing a new DSM pricing model as a decision making tool.

This work allows us to determine the electricity prices for residential customers to optimize EDF profit and reduce customer's payments. By a customer segment, we mean a group of customers having the same behaviour with respect to the same signal price and consumption habits. We focus on modeling pricing incentives in residential load management programs to shift some consumption from peak to off-peak hours so that a power producer could meet peak demands with less production costs. Our model intends to determine new electricity prices that will be in competition with the existing ones. The combination of both tariffs should make a shift of some customers to a new tariff, resulting in the decrease of energy generation costs and less consumers' payments. An important characteristic of our model is that the hourly demand distribution is obtained by solving a mathematical problem.

We use bilevel optimization as a modeling tool. In economics, bilevel problems are known under various names, such as Stackelberg-Nash games, envy-free pricing, and principal-agent problem [2, 9]. They can also be seen as a supply-demand equilibrium problem where the demand is obtained by solving a second level problem. Bilevel problems have arisen from the real world application in market economy, military defense, and political science [2, 5, 9]. Although a wide range of applications fits the bilevel programming framework [8], there is a lack of efficient algorithms for tackling large-scale problems. In bilevel problems there are two agents, called a leader and a follower, interacting at two levels of a hierarchical structure. Both of them deal with the same resources but have different goals. The leader is trying to achieve his goal depending on the behaviour of the follower, who acts in its own interests. As the leader has no guarantee that the follower will always act in the leader's best interests, thus, he has to take into account the follower's behaviour.

In our case, the leader is an electricity provider that produces energy and has a duty to satisfy the demands of residential customers (or consumers). Thus, the follower corresponds to the customers who made a contract with the provider to consume the "resources" produced by the leader. Being determined by the preferences and the needs of the consumers, current hourly consumption is often unbalanced. Thus, it results in the peaks and, therefore, in involving costly power generation technologies (nuclear energy generator, coal energy, gas, hydroelectric, wind generators etc.). To this end, the leader has to regulate the hourly consumption by pricing strategies, which are to force consumers to react on a new pricing policy that can result in the rational usage of energy. However, when a leader offers new prices for "resources" consumers are not sure whether they are preferable to the existing prices, or the leader wants only to increase his profits. So, the consumers react according to their own interests, minimizing their electricity payments and inconveniences. Thus, it is important to clearly explain to consumers how to gain from rationally scheduled electricity loads.

The leader's problem (or upper level problem) is to define new prices (or pricing policy or tariff) so to maximize profit of the company, which is the difference between customers' electricity payments and the energy production costs. The consumers' problem (or lower level problem) is to choose between the existing and the new pricing policy to satisfy their own electricity demands, with minimal payments and inconveniences, caused by changing the time intervals of electricity usage. To properly model the consumers' behaviour is a challenge. We assume that customers minimize a dis-utility function, which is the sum of their costs (or electricity bills) and unwillingness to change consumption habits, which may differ depending on the category of customers. Fig. 1 presents a bilevel structure of the problem studied in this work.

Leader's problem:
maximize Profit=(Sales - Costs)
subject to:
    Power generation capacities
    Customers' problem:
    minimize (Electricity bills + Inconvenience of changes)
    subject to:
        Pricing policy restrictions
        Demand satisfaction

Figure 1: Bilevel model.

Bilevel programming is a fairly recent branch of optimization. Its major feature is that it includes a lower level problem in a part of the constraints of an upper level to build the hierarchical relations [3, 6]. Each problem has its own variables and constraints. The leader controls only a subset of all decision variables. The remaining variables fall under the control of the follower. Depending on the type of variables and constraints, bilevel problems may be intrinsically difficult, because a feasible region of an upper level problem might be non-convex, disconnected or empty in presence of upper level constraints and / or discrete variables.

The paper is organised as follows: section 2 presents relevant related works with similar approaches; section 3 introduces a bilevel pricing model, explains its main components, and presents the mathematical programs for the leader's and follower's problems; section 4 reports a solution approach based on duality theory. We provide the proposed mathematical programs in sections 3.3, 4.1, 4.2, and 4.3. In section 5, we test viability of the model, analyse different parameter settings, and discuss the numerical results. Finally, some conclusions and future perspectives are given in section 6.

2. STATE-OF-THE-ART

Electricity markets is evolving in recent years due to increase in power consumption and deployment of renewable energy resources. This results in new actors appearing in the electricity markets, the development of new energy reducing tools and, finally, in the consumers' behavior changes. Sooner or later, the households will have to start consuming centralized generated energy in a more efficient way to reduce their bills to the energy provider. These new circumstances require adequate mathematical structures and approaches to help producers and consumers at electricity market to make optimal decisions. Thus, lots of papers have recently been published about these issues.

In [11] the authors provide a large review of optimization models in energy markets starting with the energy planning model from the 1970's and ending with the current-days complex equilibrium problems. They underline the advantages of the models based on complementarity problems that generalize mathematical (linear, nonlinear, spatial price equilibria, and others) programs applying the Karush-Kuhn-Tucker (KKT) optimality conditions.

In [17] the central economic trends in electricity market modeling are highlighted, and the classification of the existing mathematical structures and approaches, dictated by these trends, is done. Three types of mathematical models are mainly distinguished: optimization, equilibrium (or Cournot, Stackelberg game theoretical model), and simulation models. Optimization models focus on maximizing or minimizing the objective function(s) for one decision maker (often it is an electricity producer that maximizes his profit) under a set of technical and economic constraints. Equilibrium models represent hierarchical relations of a market and suit for modeling competition among its participants. Simulation models are an alternative to the equilibrium ones. They are not burdened with strict, entirely mathematical formulation of a problem, rather using its semantic description, based on a set of rules. Simulation models allow implementing calculations to almost any kind of strategic behaviour.

In [18], an equilibrium model for electricity retailer in a demand response market environment is proposed. The aim is to determine dynamic hourly price to reduce the retailer energy procurement costs and modify end-customers consumption schedule according to the price signal sent by the retailer. The end-customers minimize their electricity consumption costs subject to constraints that guaranty comfort indoor temperature.

In [16], the authors consider the load serving entity (LSE). It procures energy from various sources including the main grid, battery, dispatchable distributed generators, etc., and can buy from wind/solar farms to manage and to guarantee electricity supply to several DR aggregators (customers) in a small geographic area. The authors propose a DR strategy (or pricing scheme) that the LSE may use to attract flexible load customers to participate in it. According to this strategy, the LSE charges inflexible loads (those that the LSE must serve) with the regular retail price, and flexible loads with dynamic tariffs that are always lower or equal to the retail price in each hour. The goal is to find optimal pricing tariffs and to schedule flexible loads so to bring advantages to the DR aggregators. The authors apply bilevel programming as a decision-making framework, then they convert the bilevel optimization problem into an equivalent mixed integer linear program (MILP) problem and replace each follower problem with its corresponding KKT optimality conditions. Through extensive numerical results, the authors show that the proposed scheme provides a winning solution for both the LSE and its customers.

In [13], the authors propose the demand side bidding mechanism that enables consumers to participate in electricity pool-market trading by offering to change their normal patterns of consumption. Consumers' payments are minimized at the upper level, they include energy payments and costs to switch on/off energy generation units. The social welfare is maximized at the lower level. Bilevel programming framework, linearization scheme based on duality theory of linear programming and KKT optimality conditions are used to model and solve the problem.

In [7], a bilevel model to compute tariffs and users' distributed generation investments in Photovoltaic modules under a net-metering regulator is proposed.

We propose a deterministic equilibrium model to optimize the electricity tariffs. We share a similar idea with [18]: to benefit from the flexible end-customers consumption in order to shift it to the low-price hours in accordance with the price signal. The difference is that our interest is to analyze the impact of dynamic residential tariffs on the existing tariffs, to define the bonuses and the impact on the electricity production technologies, being involved subject to demand-supply constraints. We adapt a solution scheme, mentioned in the cited articles above. It is based on reformulating bi-level program as a single-level MILP, then writing down the equivalent system of Karush-Kuhn-Tucker conditions, and then applying the Fortuny-Amat and McCarl linearization [10]. As it is common in the literature, among multiple lower-level optimal solutions, the one that yields the best profit for the upper level is selected. This solution is called the optimistic or strong Stackelberg solution. We let for future research studying the case of pessimistic solution.

3. BILEVEL ELECTRICITY PRICING MODEL

To write down a mathematical program, we introduce the following notations.

All consumers are divided into segments according to their electricity consumption habits, let $\mathbb{S}$ be a set of these segments. The subscribers to the existing tariff may pass to a new one without changing the total amount of consumed electricity. The passage might allow them to decrease their electricity bills by changing the time periods of electricity consumption.

Let $\mathbb{H}$ be a time horizon. We assume that all consumers are subscribed for Off-Peak / Peak tariff contract as it is one of the most subscribed contracts of EDF. Nevertheless, our model might be generalized for a case with multiple tariffs. Thus, the time horizon is divided into peak and off-peak hours. Let HP be a set of peak hours and HC a set of off-peak hours, $HP \cup HC = \mathbb{H}$. Peak hours are more loaded and, therefore, they are more expensive than off-peak ones under the existing tariff.

Let D_{sh} (kWh) be the electricity demand of consumers of segment s at hour h. To avoid overconsumption, the company limits consumption by $\bar{C}_{sh}$ (W) for each segment s per each hour h under new pricing policy. Usually, this value is quite high, and a typical consumer does not reach this limit.

Let W be a parameter expressed in monetary units that represents the unwillingness of customers to choose new pricing policy. The smaller the parameter value is, the lower the customers' unwillingness is.

Let P_h (cents per kWh) be the existing price for hour h. To encourage the consumers to shift some consumption to less loaded hours, the company awards monetary bonus B with those consumers who pass to a new tariff and shift some loads from the peak to off-peak hours.

To cover all customer demands, the company has a set of available power generation technologies $\mathbb{T}$. Each technology t has a certain power capacity C_t (MW) and its associated unit production costs F_t (euro). The technologies are involved successively at each moment of time: if the total electricity demand is less than C_1, then it is totally covered by the first technology; if the hourly demand is higher than C_1, then the second technology is involved to cover the deficient amount of energy, that is $\min(C_2, D_{sh} - C_1)$, and so on. The production costs are calculated on the basis of a piecewise linear function of the power production.

3.1. Leader's problem

The goal of the company is to propose a new price (p_h) to maximize its profit, which is the total sales under the existing and new prices minus the total expenses associated with the electricity generation costs and the bonuses awarded to the customers. It is expressed in the following objective function:

$$\max \sum_{s \in \mathbb{S}} \sum_{h \in \mathbb{H}} (P_h D_{sh} r_s^* + y_{sh}^* p_h) - Prod.Costs - Bq^*, \tag{1}$$

where (p_h) (cents per kWh) is a set of nonnegative variables controlled by the leader that represent the new price per each hour h, (r^*, y^*, q^*) is an optimal solution to a *customers' problem*, term $Prod.Costs$ corresponds to the electricity generation costs, and term Bq^* means the total amount of bonuses awarded to the consumers who shift the loads from the HP to HC hours.

Electricity generation costs are among the main parameters that influence on the prices. Here, we model these costs as a piecewise linear function (shown in Fig. 2) in amount of produced energy.

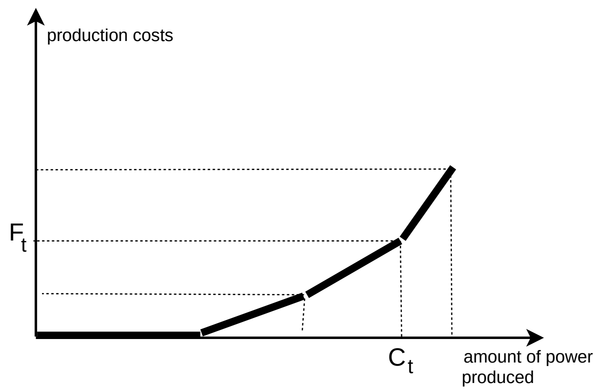

Figure 2: Piecewise linear generation costs function

We use the standard mathematical programming way to model a piecewise linear function. Let us define the auxiliary variables:

$$z_{th} = \begin{cases} 1 & \text{if technology } t \text{ is used for hour } h \\ 0, & \text{otherwise,} \end{cases}$$

and nonnegative variables a_{th} for each $t \in \mathbb{T}, h \in \mathbb{H}$ that mean the amount of generated power using technology t at hour h.

The following constraints define the slope to which the generated amount of energy belongs, taking into account the capacity limits:

$$(C_{t+1} - C_t)z_{(t+1)h} \leq a_{th} \leq (C_{t+1} - C_t)z_{th} \qquad h \in \mathbb{H}, t = 0, \ldots, |\mathbb{T}| - 2 \quad (2)$$

$$a_{th} \leq (C_{t+1} - C_t)z_{th} \qquad h \in \mathbb{H}, t = |\mathbb{T}| - 1, \quad (3)$$

where

$$C_0 = 0. \quad (4)$$

If $z_{th} = 1$, then $z_{(t+1)h}$ might be either equal to 1 or 0. If technology t is used with its full capacity, then a_{th} must be equal to $(C_{t+1} - C_t)$ and $z_{(t+1)h} = 1$, which is guaranteed by inequalities (2), otherwise $z_{(t+1)h} = 0$, and inequalities (2) are verified. The total amount of generated energy is a sum over all technologies involved to cover the total demand, that is

$$\sum_{s\in\mathbb{S}}(y^*_{sh} + D_{sh}r^*_s) = \sum_{t=0}^{|\mathbb{T}|-1} a_{th} \qquad h \in \mathbb{H}, \tag{5}$$

where

$$a_{0h} = 0 \qquad h \in \mathbb{H}. \tag{6}$$

Thus, the total electricity generation costs, that is term *Prod.Costs* in the objective function (1), are

$$\sum_{t=0}^{|\mathbb{T}|-1} F_{t+1} \sum_{h\in\mathbb{H}} a_{th}. \tag{7}$$

3.2. Consumers' problem

Given the existing and new prices, (P_h) and (p_h) respectively, the goal of the consumers is to minimize their electricity payments without changing the total amount of consumed energy but changing the time periods of consumption. To this end, the lower level objective function consists of the following components:

- payments of the customers who stay with the existing tariff after the new pricing policy is launched;
- payments of the customers who pass to the new pricing policy;
- total amount of monetary bonus reimbursed to the customers who shift some loads from the peak to the off-peak hours passing to the new tariff;
- and reluctance of customers expressed in monetary units to consumption changes,

it is written down as

$$\min_{y,r,q} \sum_{s\in\mathbb{S}} \sum_{h\in\mathbb{H}} (P_h D_{sh} r_s + p_h y_{sh}) - Bq + Wq \tag{8}$$

where $(y_{sh}), (r_s)$, and q are the lower level decision variables controlled by the consumers:

y_{sh} (kWh) is consumption at hour h under the existing tariff for customers belonging to segment s charged with the new pricing policy;

r_s is the ratio of so-called conservative customers, who do not choose the new pricing policy, $0 \leq r_s \leq 1$;

q (kW) is the total amount of consumption shifted from the peak to the off-peak hours.

Consumers' problem has to verify the following constraints:

all demand must be totally satisfied that is

$$\sum_{h \in HC} y_{sh} + r_s \sum_{h \in HC} D_{sh} - q = \sum_{h \in HC} D_{sh}, \qquad s \in \mathbb{S} \tag{9}$$

and

$$\sum_{h \in HP} y_{sh} + r_s \sum_{h \in HP} D_{sh} + q = \sum_{h \in HP} D_{sh}, \qquad s \in \mathbb{S}; \tag{10}$$

and the amount of electricity consumed at each hour by each segment s is capped by the provider at $\bar{C}_{sh}$:

$$y_{sh} + D_{sh} r_s \leq \bar{C}_{sh} \qquad s \in \mathbb{S}, h \in \mathbb{H}. \tag{11}$$

Knowing an optimal solution (y^*, r^*, q^*) to the customers' problem (8)–(11) the leader can calculate its total costs and sales.

3.3. Bilevel program

The full bilevel program of the proposed pricing model is the following:

$$\max_{p,a,z} \sum_{s \in \mathbb{S}} (\sum_{h \in \mathbb{H}} P_h D_{sh}) r_s^* + \sum_{s \in \mathbb{S}} \sum_{h \in \mathbb{H}} (y_{sh}^* p_h) - \sum_{t=0}^{|\mathbb{T}|-1} F_{t+1} \sum_{h \in \mathbb{H}} a_{th} - Bq^*,$$

subject to

$$(C_{t+1} - C_t) z_{t+1h} \leq a_{th} \leq (C_{t+1} - C_t) z_{th} \qquad h \in \mathbb{H}, t = 0, \ldots, |\mathbb{T}| - 2$$

$$a_{th} \leq (C_{t+1} - C_t) z_{th} \qquad h \in \mathbb{H}, t = |\mathbb{T}| - 1,$$

$$\sum_{s \in \mathbb{S}} (y_{sh}^* + r_s^* D_{sh}) = \sum_{t=0}^{|\mathbb{T}|-1} a_{th} \qquad h \in \mathbb{H},$$

$$C_0 = 0, a_{0h} = 0 \qquad h \in \mathbb{H},$$

$p_h \geq 0, a_{th} \geq 0, z_{th} \in \{0, 1\} \qquad h \in \mathbb{H}, t = 0, \ldots, |\mathbb{T}|$
(y^*, r^*, q^*) is an optimal solution to the *consumers' problem*:

$$\min_{y,r,q} \sum_{s \in \mathbb{S}, h \in \mathbb{H}} (P_h D_{sh}) r_s + \sum_{s \in \mathbb{S}, h \in \mathbb{H}} p_h y_{sh} - Bq + Wq$$

$$\sum_{h \in HC} y_{sh} + r_s \sum_{h \in HC} D_{sh} - q = \sum_{h \in HC} D_{sh}, \qquad s \in \mathbb{S}$$

$$\sum_{h \in HP} y_{sh} + r_s \sum_{h \in HP} D_{sh} + q = \sum_{h \in HP} D_{sh}, \qquad s \in \mathbb{S}$$

$$y_{sh} + r_s D_{sh} \leq \bar{C}_{sh} \qquad s \in \mathbb{S}, h \in \mathbb{H};$$

$$r_s \leq 1 \qquad s \in \mathbb{S};$$

$$y_{sh} \geq 0, r_s \geq 0, q \geq 0 \qquad s \in \mathbb{S}, h \in \mathbb{H}.$$

4. SINGLE LEVEL REFORMULATION BASED ON COMPLEMENTARY SLACKNESS CONDITIONS

When dealing with a bilevel problem, it is important to be sure that a lower level problem has the uniquely determined solution, otherwise the bilevel problem becomes ill-posed. In literature, the most common way to escape ill-posed situation is to consider pessimistic or optimistic solution concept [9]. Considering the pessimistic point of view, the leader tries to bound the damage resulting from the worst possible selection of the follower. We imply that the residential consumers are unselfish and support the company in rational production of energy, in other words, we consider the optimistic concept. Thus, among the optimal solutions to the lower level problem, providing the same values of the lower level objective function (8), consumers choose the solution which is the best for the company.

Different approaches are adopted to solve the problem, depending on type of variables and constraints [1, 9, 8]. In this work we deal with a continuous case: all the variables have real values and the lower level problem is a linear problem since all functions are linear. In such a way, we apply a solution approach developed for the linear mathematical programs. This solution approach is common under the optimistic concept [3, 4, 9]. Namely, we reformulate the bilevel problem as a mixed integer single level problem applying duality theory, Karush-Kuhn-Tucker conditions (or complementary slackness conditions) [3, 4], and the Fortuny-Amat and McCarl linearization [10]. A general scheme to provide a mixed-integer single level reformulation is as follows:

Algorithm : Building of a single-level reformulation

Step 1: Write down a dual program to the consumers' problem (8)–(11).

Step 2: Write down the complementary slackness conditions.

Step 3: Linearize the complementary slackness conditions.

Step 4: Add upper level constraints (2)–(6).

The dual problem, complementary slackness conditions and their linearization are in the following subsections 4.1, 4.2, and 4.3, respectively.

The obtained single level reformulation is solved by a ready to use optimization solver (for example, CPLEX via GAMS [12]).

4.1. Dual program to the consumers' problem

Let μ_s^{HC}, μ_s^{HP}, $\delta_{sh} \geq 0$, $\lambda_s \geq 0$, $s \in \mathbb{S}$, $h \in \mathbb{H}$ be the dual variables. Then the dual program is as follows:

objective function

$$\max \sum_{s\in\mathbb{S}} (\mu_s^{HC} \sum_{h\in HC} D_{sh} + \mu_s^{HP} \sum_{h\in HP} D_{sh}) - \sum_{s\in\mathbb{S}, h\in\mathbb{H}} \delta_{sh} \bar{C}_{sh} - \sum_{s\in\mathbb{S}} \lambda_s$$

subject to:

$$\mu_s^{HC} - \delta_{sh} \leq p_h \qquad s \in \mathbb{S}, h \in HC;$$

$$\mu_s^{HP} - \delta_{sh} \leq p_h \qquad s \in \mathbb{S}, h \in HP;$$

$$\mu_s^{HC} \sum_{h\in HC} D_{sh} + \mu_s^{HP} \sum_{h\in HP} D_{sh} - \delta_{sh} \sum_{h\in\mathbb{H}} D_{sh} - \lambda_s \leq \sum_{h\in\mathbb{H}} D_{sh} P_h \qquad s \in \mathbb{S};$$

$$\sum_{s\in\mathbb{S}} \mu_s^{HP} - \sum_{s\in\mathbb{S}} \mu_s^{HC} \leq W - B;$$

$$\delta_{sh} \geq 0, \lambda_s \geq 0, s \in \mathbb{S}, h \in \mathbb{H}.$$

4.2. Complementary slackness conditions for the consumers' problem

According to the strong duality theorem, a solution (y, r, q) is optimal to the consumers' problem (8)–(11) if and only if there exists a vector $(\mu^{HC}, \mu^{HP}, \delta, \lambda)$ such that the primal constraints, the dual constraints, and the complementary slackness conditions of the consumers' problem are satisfied, that is

$$\sum_{h\in HC} y_{sh} + r_s \sum_{h\in HC} D_{sh} - q = \sum_{h\in HC} D_{sh}, \qquad s \in \mathbb{S}$$

$$\sum_{h \in HP} y_{sh} + r_s \sum_{h \in HP} D_{sh} + q = \sum_{h \in HP} D_{sh}, \qquad s \in \mathbb{S}$$

$$y_{sh} + r_s D_{sh} \leq \bar{C}_{sh} \qquad s \in \mathbb{S}, h \in \mathbb{H};$$

$$r_s \leq 1 \qquad s \in \mathbb{S};$$

$$y_{sh} \geq 0, r_s \geq 0, q \geq 0 \qquad s \in \mathbb{S}, h \in \mathbb{H}$$

$$\mu_s^{HC} - \delta_{sh} \leq p_h \qquad s \in \mathbb{S}, h \in HC;$$

$$\mu_s^{HP} - \delta_{sh} \leq p_h \qquad s \in \mathbb{S}, h \in HP;$$

$$\mu_s^{HC} \sum_{h \in HC} D_{sh} + \mu_s^{HP} \sum_{h \in HP} D_{sh} - \delta_{sh} \sum_{h \in \mathbb{H}} D_{sh} - \lambda_s \leq \sum_{h \in \mathbb{H}} D_{sh} P_h \qquad s \in \mathbb{S};$$

$$\sum_{s \in \mathbb{S}} \mu_s^{HP} - \sum_{s \in \mathbb{S}} \mu_s^{HC} \leq W - B;$$

$$\delta_{sh} \geq 0, \lambda_s \geq 0 \qquad s \in \mathbb{S}, h \in \mathbb{H};$$

$$\delta_{sh}(\bar{C}_{sh} - y_{sh} - r_s D_{sh}) = 0 \qquad s \in \mathbb{S}, h \in \mathbb{H};$$

$$\lambda_s(1 - r_s) = 0 \qquad s \in \mathbb{S};$$

$$y_{sh}(\mu_s^{HC} - \delta_{sh} - p_h) = 0 \qquad s \in \mathbb{S}, h \in HC;$$

$$y_{sh}(\mu_s^{HP} - \delta_{sh} - p_h) = 0 \qquad s \in \mathbb{S}, h \in HP;$$

$$r_s(\mu_s^{HC}\sum_{h\in HC} D_{sh}+\mu_s^{HP}\sum_{h\in HP} D_{sh}-\delta_{sh}\sum_{h\in\mathbb{H}} D_{sh}-\lambda_s-\sum_{h\in\mathbb{H}} D_{sh}P_h)=0 \qquad s\in\mathbb{S};$$

$$q(\sum_{s\in\mathbb{S}}\mu_s^{HP}-\sum_{s\in\mathbb{S}}\mu_s^{HC}-W+B)=0.$$

Notice that the consumers' problem (8)–(11) is a linear problem with a nonempty and bounded feasible set. Indeed, $r_s=1$, $y_{sh}=0$, for each s,h, and $q=0$ is its feasible solution. Thus the duality theorem is verified. These conditions provide a single-level formulation presented in section 4.3.

4.3. Single level reformulation

Here we write down the full single level reformulation applying the Fortuny-Amat and McCarl linearization [10].

$$\max\sum_{s\in\mathbb{S}}(\mu_s^{HC}\sum_{h\in HC} D_{sh}+\mu_s^{HP}\sum_{h\in HP} D_{sh})-\sum_{s\in\mathbb{S},h\in\mathbb{H}}\delta_{sh}\bar{C}_{sh}-\sum_{s\in}$$
$$\mathbb{S}\lambda_s-Wq-\sum_{t=0}^{|\mathbb{T}|-1}F_{t+1}\sum_{h\in\mathbb{H}}a_{th}$$

Upper level constraints:

$$(C_{t+1}-C_t)z_{t+1h}\le a_{th}\le (C_{t+1}-C_t)z_{th} \qquad h\in\mathbb{H}, t=0,\dots,|\mathbb{T}|-2$$

$$a_{th}\le (C_{t+1}-C_t)z_{th} \qquad h\in\mathbb{H}, t=|\mathbb{T}|-1,$$

$$\sum_{s\in\mathbb{S}}(y_{sh}+r_sD_{sh})=\sum_{t=0}^{|\mathbb{T}|-1}a_{th} \qquad h\in\mathbb{H},$$

$$C_0=0, a_{0h}=0 \qquad h\in\mathbb{H},$$

Lower level constraints:

$$\sum_{h\in HC} y_{sh}+r_s\sum_{h\in HC} D_{sh}-q=\sum_{h\in HC} D_{sh}, \qquad s\in\mathbb{S}$$

$$\sum_{h\in HP} y_{sh}+r_s\sum_{h\in HP} D_{sh}+q=\sum_{h\in HP} D_{sh}, \qquad s\in\mathbb{S}$$

$$y_{sh}+r_sD_{sh}\le\bar{C}_{sh} \qquad s\in\mathbb{S}, h\in\mathbb{H};$$

$$r_s\le 1 \qquad s\in\mathbb{S};$$

Linearized complementary slackness conditions:

$$\bar{C}_{sh}-y_{sh}-r_sD_{sh}\le(1-\alpha_{sh}^1)BN^1 \qquad s\in\mathbb{S}, h\in\mathbb{H};$$

$$\delta_{sh}\le\alpha_{sh}^1 BN^2 \qquad s\in\mathbb{S}, h\in\mathbb{H};$$

$$1 - r_s \leq (1 - \alpha_s^2) BN^3 \qquad s \in \mathbb{S};$$
$$\lambda_s \leq \alpha_s^2 BN^4 \qquad s \in \mathbb{S};$$
$$\mu_s^{HC} - \delta_{sh} - p_h \geq (\alpha_{sh}^3 - 1) BN^5 \qquad s \in \mathbb{S}, h \in HC;$$
$$y_{sh} \leq \alpha_{sh}^3 BN^6 \qquad s \in \mathbb{S}, h \in HC;$$
$$\mu_s^{HP} - \delta_{sh} - p_h \geq (\alpha_{sh}^4 - 1) BN^7 \qquad s \in \mathbb{S}, h \in HP;$$
$$y_{sh} \leq \alpha_{sh}^4 BN^8 \qquad s \in \mathbb{S}, h \in HP;$$
$$\mu_s^{HC} \sum_{h \in HC} D_{sh} + \mu_s^{HP} \sum_{h \in HP} D_{sh} - \delta_{sh} \sum_{h \in \mathbb{H}} D_{sh} - \lambda_s - \sum_{h \in \mathbb{H}} D_{sh} P_h \geq (\alpha_s^5 - 1) BN^9 \qquad s \in \mathbb{S};$$
$$r_s \leq \alpha_s^5 BN^{10} \qquad s \in \mathbb{S};$$
$$\sum_{s \in \mathbb{S}} \mu_s^{HP} - \sum_{s \in \mathbb{S}} \mu_s^{HC} - W + B \geq (\alpha^6 - 1) BN^{11};$$
$$q \leq \alpha^6 BN^{12};$$

Primary variables:

$$z_{th} \in \{0, 1\}, a_{th} \geq 0, p_h \geq 0, y_{sh} \geq 0, r_s \geq 0, q \geq 0 \qquad t \in \mathbb{T}, s \in \mathbb{S}, h \in \mathbb{H}.$$

Dual variables:

$$\delta_{sh} \geq 0, \lambda_s \geq 0, \mu_s^{HP}, \mu_s^{HC} \qquad s \in \mathbb{S}, h \in \mathbb{H};$$

Auxiliary variables:

$$\alpha_{sh}^1, \ldots, \alpha^6 \in \{0, 1\} \qquad s \in \mathbb{S}, h \in \mathbb{H};$$

where $BN^{10}, \ldots, BN^{12}$ are the large numbers.

5. NUMERICAL RESULTS

In this section, we study the sensitivity of the model to parameter changes. First, we study the interactions of the model parameters on the small-size instances, and then, we launch the model on the data provided by EDF. We consider one type of tariff: Off-Peak / Peak as it is one of the most subscribed tariffs. The price per kWh of HC hour is cheaper than one of HP hour.

The computational experiments were performed on a PC Intel Xeon X5675, 3 GHz, RAM 96 GB running under the Windows Server 2008 operating system. We use ILOG CPLEX 11.0 as an optimization mixed integer programming solver.

$\mathbb{H} = \{1, \dots, 4\}$	4-hour planning horizon
$HC = \{1, 2\}$ $HP = \{3, 4\}$	off-peak and peak hours
$(P_h) = (10, 10, 15, 15)$ euro per kWh	current prices
$\mathbb{S} = \{1, 2\}$	2 consumer segments
$(D_{sh}) = (10,\ 5,\ 15,\ 17\ ;\ 2,\ 12,\ 35,\ 45\ )$ kW	electricity demand
$\mathbb{T} = \{1, 2, 3\}$	3 power technologies
$(C_t) = (20, 56, 80)$ kW	power capacity for each technology
$(F_t) = (0, 2, 7)$ euro per kW	production costs for each technology
$\bar{C}_{sh} = 141^*$ kW	upper bound on hourly consumption
*a large number equal to the total demand.	

Table 1: Test instance. Input data.

5.1. Test instance

The input data for the test instance are shown in Table 1. In this instance, a time horizon of 4 hours is divided into 2 peak and 2 off-peak hours. The total initial consumption during the off-peak hours is less than during the peak hours: 29kW and 112kW, respectively. To cover this demand, the company involves all technologies: first technology covers the first 20kWs of demands, and if it is not enough, the second technology is used to cover the next 36 kWs, and then the remaining technology is involved to cover the rest demanded kWs.

Table 2, lines 2–3 present the values of $Profit$, $Sales$, and $Costs$ under the existing pricing policy (column $Prices$). Analysing the initial demand profile, it can be seen that the first technology is not used with its full capacity, as 11kW (8kW and 3kW during the first and the second HC hours, respectively) are under loaded. Thus, the company could reduce the production costs if these 11kW were shifted from some HP to these HC hours. The question to be answered is "Which prices and bonuses should be assign to interest the consumers to change their way of consumption?".

Table 2, lines 5–11 show the results obtained by the model with the different parameters, such as bonus B and unwillingness of customers W to pass to the new pricing policy.

First, we fix the bonus B to zero and change W: 1, 3.3, and 3.5 (lines 5–7). We can see that given these input data, the model finds the new prices such that HC hours are less expensive than HP hours (column $Prices$ in Table 2), and results to the less production costs (column $Costs$ in Table 2) with a new way of electricity consumption. The solutions with q = 11kW (lines 5, 6), most likely indicate that 1 and 3.3 are too small values for W in comparison with other input parameters. Under these parameter values, the model is not sensitive to W, since 11kW is the maximum amount of energy that might be shifted from HP to HC hours to involve the less expensive production technology entirely. If we increase the unwillingness of consumers W up to 3.5, then it starts playing its role in the model resulting in only 6.4kW of consumption shifted from HP to HC hours. The reluctance of customers is a quite difficult parameter to be estimated. Additional statistical or data mining methods should be involved to appropriately measure its values. Our goal is to show its impact on the model solutions.

Table 2, lines 8–11 present the impact of the bonus value B under the fixed parameter W. The results show that B equaled to 0.3 is not enough to encourage consumers to shift their loads from HP to HC hours although around 70% of consumers of the second segment ($r_2 = 0.33$) are choosing the new pricing policy. When B is equal to 0.7, 0.75, and 0.8, we observe the shifts from HP to HC hours. Remember, the bonuses represent the additional costs for the company, whereas they make HC hours less expensive for the consumers.

Fig. 3–5 show graphically the hourly electricity consumption profiles when $B = 0$. Similar consumption profiles have been observed when $B > 0$, so for brevity we do not present them graphically. The columns along a horizontal axis depict the demand profile for two segments together at each hour; a left vertical axis shows the amount of consumed energy; the right vertical axis shows graphically the capacities of each technology. The dashed (solid) line connects the points that represent the total amount of demand at each hour under the existing (new) prices. We can see that the consumption with the new pricing policy is always covered by two less expensive production technologies.

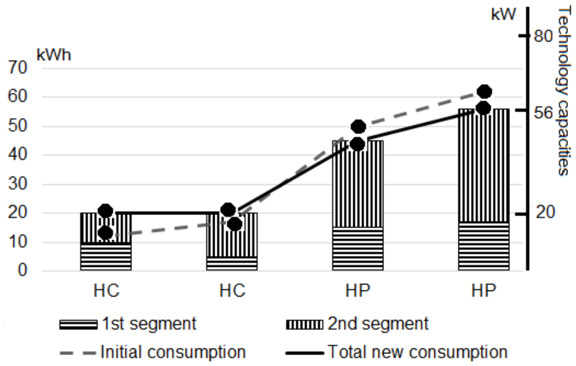

Figure 3: Impact of consumers' reluctance on the consumption shifts, provided $B = 0, W = 1$.

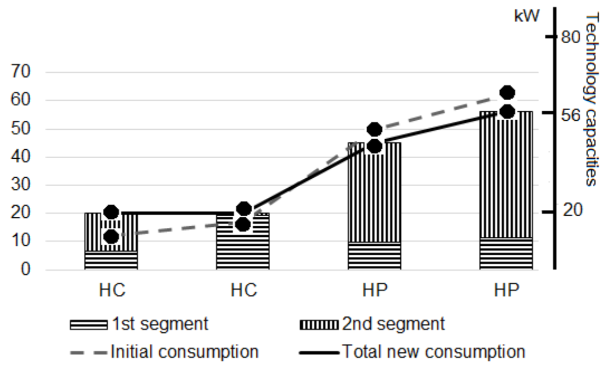

Figure 4: Impact of consumers' reluctance on the consumption shifts, provided $B = 0, W = 3.3$

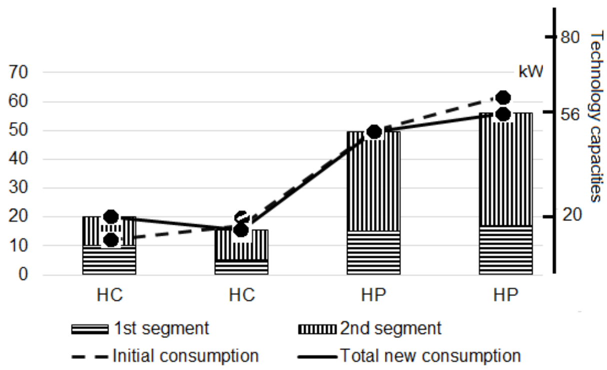

Figure 5: Impact of consumers' reluctance on the consumption shifts, provided $B = 0, W = 3.5$

Table 3 shows the impact of production costs on the behaviour of the model. We change the production costs F_t for each technology, keeping the previously tested values of parameters B and W. We observe that as soon as the production costs for the first technology is increased (F_1 from 0 to 1 euro per kW), there is no shift from HP to HC hours in the optimal solutions, Table 3 lines 6–7. However, if we increase the production costs for the second technology (F_2 from 2 to 20 euro per kW), then the optimal solution involves the less expensive technology entirely (the shifts are 11kW), Table 3 line 12. The higher price for the third technology does not have the same impact as it has on the first technology: the second technology is 6kW under loaded for the 3rd HP hour in the initial consumption profile, so it is more profitable for the consumers to shift these 6kW from the 4th to the 3rd HP hour than to change their habits shifting from HP to HC hours, Table 3 line 17.

1	B	W	Prices HC hour;HP hour	Profit	Sales	Costs	q	$(r_1; r_2)$
2			Initial values					
3			10;15	1796	1970	174		
4			Results					
5	0	1	13.7 ; 14.4	1840.3	1962.3	122	11	$(1; 0)$
6	0	3.3	12.5 ; 14.5	1830.5	1957.2	122	11	$(0.65; 0.05)$
7	0	3.5	12.5 ; 14.5	1826	1957.2	131.2	6.4	$(1; 0.87)$
8	0.3	3.5	12.8 ; 14.5	1826	1970	144	0	$(1; 0.33)$
9	0.7	3.5	13.1 ; 14.5	1826	1961.7	131.2	6.4	$(1; 0.86)$
10	0.75	3.5	13.2 ; 14.44	1826	1956.3	122	11	$(1; 0.86)$
11	0.8	3.5	13.2 ; 14.43	1826	1956.8	122	11	$(1; 0.85)$

Table 2: Impact of reluctance and bonus parameters on prices, profit, sales and costs

1	B	W	Prices HC hour; HP hour	Profit	Sales	Costs	q	$(r_1; r_2)$
2			Initial values					
3			10;15	1727	1970	243		
4	Prod. costs: $(F_t) = (\mathbf{1}, 2, 7)$							
5			Results					
6	0	3.5	12.6 ; 14.6	1757	1970	213	0	$(1; 0.87)$
7	0.7	3.5	13.15 ; 14.45	1757	1970	213	0	$(1; 0.87)$
8			Initial values					
9			10 ; 15	539	1970	1431		
10	Prod. costs : $(F_t) = (\mathbf{1}, \mathbf{20}, 7)$							
11			Results					
12	0.7	3.5	13.15 ; 14.45	960	1955.7	988	11	$(1; 0.17)$
13			Initial values					
14			10 ; 15	1349	1970	621		
15	Prod. costs: $(F_t) = (\mathbf{1}, 2, \mathbf{70})$							
16			Results					
17	0.7	3.5	13.15;14.45	1757	1970	213	0	$(1; 0.86)$

Table 3: Impact of production costs

5.2. Instance with realistic consumption profiles

We had at our disposal the real hourly consumption profiles and prices, Fig. 6 shows the initial hourly demand. There are two power generation technologies: the first on limited to total production of 3000 MW, the second one operating from 3000 MW up to 9000 MW. We see that the current consumption is not smooth and for some hours (1, 10, 13,. . ., 19 and 24) the second power technology is mandatory. Table 4 contains the real input data. Notice the given production costs per kW (1 cent per kW for the first technology, and 3.5 cents for the second one) are significantly lower than hourly prices per kW (104.4 cents per kWh for

HC hour, and 151 cents for HP hour). Our goal is to find new hourly prices, to scale bonuses and customers' reluctance in accordance with other data to smooth electricity consumption, and to reduce production costs.

$\mathbb{S} = \{1\}$	one consumer segment
$\mathbb{H} = \{1, \ldots, 24\}$	24-hour planning horizon
$HC = \{24, 1, \ldots, 7\}$	off-peak hours
$HP = \{8, \ldots, 23\}$	peak hours
$\mathbb{T} = \{1, 2\}$	2 power technologies
$(C_t) = (3000, 9000)$ MW	power capacities for each technology
$(F_t) = (1, 3.5)$ cents per kW	production costs for each technology
$P_h = 104.4$ cents per kWh $h \in HC$	current price for HC hour
$P_h = 151$ cents per kWh $h \in HP$	current price for HP hour
$\bar{C}_{sh} = 9000$ MW	upper bound on hourly consumption

Table 4: Real input data

1	B (euros per MW)	W	Prices HC hour ; HP hour	Profit (in %)	Sales (in %)	Costs (in %)	q (in MW)
2	0	0.1	141.5 ; 141.5	+1.3	-0.0	-80	1008.2
3	0	10	139 ; 142.1	+1.27	-0.03	-80	1008.2
4	0	100	117.6 ; 147.6	+1.2	0.0	-74	0
5	20	100	129.6 ; 144.52	+1.2	0	-74	0

$+(-)$ means increase (decrease) in % in comparison with initial values

Table 5: Results on realistic consumption profiles

We have tested the model with different values of bonuses and reluctance. First, we consider W equaled to 0.1, 10 and 100. Table 5 presents the obtained results. We observe that when $W = 0.1$ or 10 (lines 2 and 3), consumers shift 1008.2MW from HP to HC hours without any bonus ($B = 0$). It means that these values of W are not scaled properly with respect to other data. The induced shifts allow the company to decrease generation costs of 80%. Because of the significant difference between the production costs and hourly prices per kW, a reduction of 80% in $Costs$ induces an increase of only 1.3% in $Profit$. In spite of the slight decrease in $Sales$ the company's $Profit$ increases by 1.3% thanks to the reduction in production costs. The new prices for HP hours are lower so the optimal consumption profiles in optimistic case are smooth during HP hours. According to the new consumption profiles, the first power production technology operates almost at full limit hour by hour, and the company can drastically reduce the use of the second technology (only three times at HP hours 13, 16, and 17), Fig. 7.

If W is increased up to 100, then the customers' reluctance to consumption changes is so high that there is no shift from HP to HC hours, i.e. $q = 0$. However, the new consumption profiles are smoother and the number of peaks is reduced, resulting in a reduction of costs of 74%, Fig. 8. If bonus B is increased up to 20 euros per MW, then it does not induce a shift from HP to HC hours, line 5. Increasing bonus do not make sense for these input data because the production costs become too high to maintain profit.

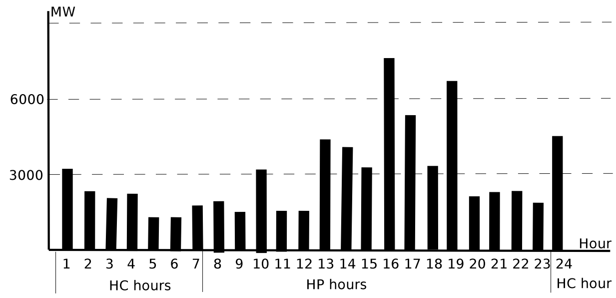

Figure 6: Real electricity consumption profiles hour by hour.

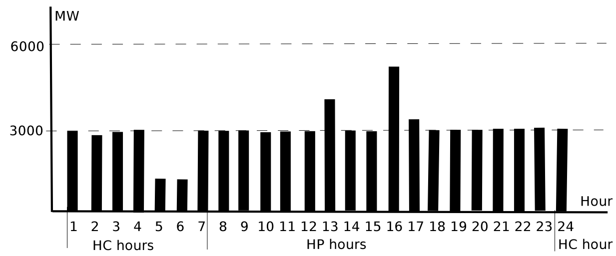

Figure 7: Impact of parameter changes. $B = 0$, $W = 0.1$.

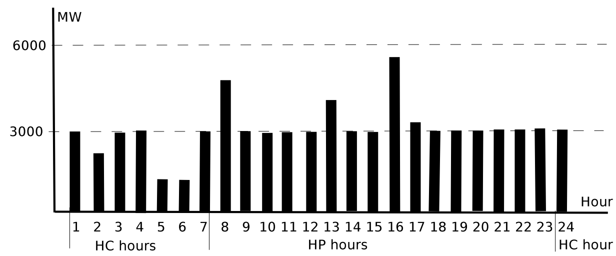

Figure 8: Impact of parameter changes. $B = 20$ euros per MW, $W = 100$.

The largest size instances presented here have about 500 variables and constraints. All of them have been solved optimally in less than one minute. However, since the total number of variables and constraints in the single level formulation is $\mathcal{O}(\mathbb{S} * \mathbb{H} + \mathbb{T} * \mathbb{H})$, the bigger size input data might increase computational time. In this case, some preprocessing steps (to fix the values of some variables) might be done to speed-up the solution process and the bounds on the large numbers $BN^{10}, \ldots, BN^{12}$ have to be tightened.

6. CONCLUSION

In this work we have proposed a demand side management model to determine the new electricity prices, optimising the existing ones. The new pricing policy aims to modify consumers' behaviour, shifting the consumption from peak to off-peak hours, resulting in a decrease of electricity production costs.

We have used bilevel optimization that allows us to model the situation in which a decision of a load serving entity depends on the optimal consumers' decision. As a matter of fact, the particularity of the model is to anticipate the consumers' behaviour and to introduce bonuses to encourage consumers to shift their loads. We have considered the following driven parameters: consumers' reluctance, bonuses, and production costs. Our computational experiments have shown their impact on prices and the new consumption profiles.

This model can be used by a load serving entity as a tool to test macro-level decisions that aim to encourage the residential consumers to be more rational regarding hourly electricity consumption.

The model has been applied to the test instances and real life demand profiles were provided by the Electricity of France company. The test instances allowed analysing the impact of the driven parameters on the consumers' behaviour and the involved power generation. The optimal solutions to the realistic case are characterized by the reduction in consumption peaks. This also results in a decrease in power generation costs thanks to the limited usage of the more expensive technology, required in case of consumption peaks.

Consumers' reluctance and bonuses might be different for each customer's segment. Thus, a further perspective of this work is to introduce several different segments in the model and test it on given and appropriately scaled data. Also, this model could be adapted to account for uncertainty aspects related to consumers' behaviour or the electricity provider strategic behaviour.

The solution approach used in this work is based on the replacement of the lower level problem by its Karush-Kuhn-Tucker conditions, resulting in one level programming problem. It estimates the optimistic profit for the company. In perspective, this approach might be modified to estimate the company's profit in a pessimistic case.

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