# UNDERSTANDING MULTI-STAGE DIFFUSION PROCESS IN PRESENCE OF ATTRITION OF POTENTIAL MARKET AND RELATED PRICING POLICY

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**Abstract:** In this paper, a mathematical model using a three-stage structure is developed to describe the diffusion process of high-technology products. This article investigates the significance of the informed and disinterested potential adopters on the growth pattern of innovation. The suitability of the proposed methodology for real-life scenarios is validated by fitting the model to the actual sales and price data sets from the electronics and semiconductor industries. The present study also incorporates the influence of dynamic price on the decision of the adoption of new products. An optimization problem is also formulated as an optimal control problem with the objective of profit maximization to determine the optimal price of the new technology.

**Keywords:** Awareness, Adoption, Attrition, Disinterested potential customers, Dynamic price, Innovation diffusion modeling, Motivation, Optimal control theory.

**MSC:** 90B85, 90C26.

## 1. INTRODUCTION

The demand for product innovation is rising in every market sector and, therefore, it is necessary for a company to continue to innovate to have an advantage in the competitive environment. Technological breakthroughs (innovations) are regarded as a process of creating an improved product (or process, service, technology, etc.) with enhanced characteristics that have additional economic and social values. Worldwide, innovation serves as a leading technology solution, which results from the skilled workforce and business activities [14]. Successful innovation, thus helps enterprises to attain economic growth, but the risks associated with introducing a new product in the high technology market are inescapable, and the rate of an innovation failure is formidable that leads to enormous financial difficulty and predicament to the organization [13, 1]. Therefore, the business firms should analyze the probable cause of product failure to reduce the risk related to the decision-making process. Researchers and academicians have suggested that for commercial success, a company needs to balance mainly two functions, namely, innovation and marketing [50]. By innovating, a firm can add value to the consumers; through optimizing marketing strategies, a company can increase the market appeal of their innovative products to take adequate benefits of the new product.

Moreover, the study of new product adoption and diffusion behavior becomes essential for analyzing the growth pattern of a new product. Diffusion of innovation is a process to depict, model, and replicate the sales behavior phenomenon of the new technological products. Diffusion of innovation is accomplished when potential buyers accept the technology and use it for a certain period [46]. It primarily attempts to elucidate the potential buyers intention towards the innovation and their adoption behavior. S-shaped diffusion models have been used widely by the researchers to study the changing flow of cumulative buyers over time [51]. The chief pragmatic diffusion model in marketing literature is Bass [4] innovation diffusion model that exhibits the new products growth rate by taking into consideration the positive word-of-mouth effect of the adopters. The Bass model explicitly classifies the adopters into innovators (adopts the product due to the effect of mass-communication) and imitators (adopts the product due to the effect of interpersonal communication), who with time increasingly purchase an innovation. The Bass model and its various modified structures have effectively estimated the sales pattern of various new products (and services) and have provided very good fit to the actual sales data. Although, the Bass model has been broadly used for modeling of the adoption process of a wide range of products, it suffers from some limitations as well. The major limitation of the Bass model is that it does not incorporate the effect of marketing-mix variables such as price, and advertising intensity in predicting the sales of a new product. The marketing variables of an innovation play an important part in the diffusion process. The short-term, as well as long-term growth of any innovation depends only on how an organization controls and manages these variables for an interminable period [50].

In 1983, Rogers [38] claimed that the potential adopters decision about innova-

tion did not occur immediately; instead, it takes place with time and is composed of a sequence of steps. An individual goes through five stages of the adoption process before confirming their purchasing decision. The five stages are Knowledge, Persuasion, Decision, Implementation, and Confirmation [38]. The outcome of every stage may influence the diffusion dynamics of innovation. Therefore, to model a more plausible diffusion process, the new product adoption modeling should be based on the stage structure. Recently, authors [2, 3, 10] have studied the adoption behavior using the three-stage process: awareness, motivation, and eventual adoption. The first stage encompasses cognizance of the products information among the potential market. In this stage, potential adopters learn about the existence of the innovation. Now, if the well-aware potential adopters are interested in the innovation, they will evaluate the innovations merits and demerits along with its marketing-mix variables in the second stage, i.e., in the motivation stage. In the final stage, the positively motivated prospective buyers will confirm their purchase and become the ultimate adopters of the innovation.

In most of the diffusion modeling approaches, the market potential is assumed constant that will ultimately purchase the innovation. However, not every individual will be convinced to buy innovation. The potential adopters are exposed to the product and gain knowledge in favor or against its adoption. Therefore, in this study, the focus is on developing a new diffusion modeling for technology innovation using the stage structure by taking into account the rate of disinterested prospective adopters. The proposed study seeks to model the diffusion process of technological innovations that have competitive substitutes in the marketplace. Additionally, the proposed model meticulously evaluates the impact of dynamic price on the consumer buying behavior. Pricing is possibly the most complex marketing-mix variable. Companies try to control the prices to obtain the maximum profit and at the same time satisfy consumers varying needs. Therefore, to find the optimal pricing policy, firstly, a dynamic three-stage diffusion model is structured using the ordinary differential equations. Then, an optimization problem is constructed whose objective function measures the maximum discounted profit value for the firm. Furthermore, to solve the optimization model, optimal control theory is employed to obtain an appropriate control variable, i.e., the price of the product, which maximizes the profit function subject to motivation rate, sales rate, and control variable constraint. The problem also assimilates the cost-learning phenomenon.

The remaining paper is summarized in the following sections. In section 2, a detailed review of past literature is presented. The proposed innovation diffusion model using three-stage adoption process is developed in section 3. Following that, in section 4, the parameter estimation and forecasting performance results of the proposed model are produced. After that, section 5 discusses the control model for the optimal pricing policy. The applicability of the proposed innovation diffusion model is corroborated in Section 6 through the numerical example. In section 7, some vital managerial insights are mentioned, and finally, in section 8, the paper conclusion is conferred.

## 2. LITERATURE REVIEW

For the past several decades, an innovation is regarded as one of the prime factors of economic increase. As a result, researchers have developed various mathematical models in the literature to examine the growth pattern of innovation. Since 1960, analysts have conducted different researches on studying the applicability of the innovation diffusion model [4, 12, 15, 16, 33]. Out of all the early studies, Bass [4] published the most influential and triumphant new product diffusion model for investigating the sales behavior of new consumer durables. The critical feature of the Bass model is that it includes the contagion processes in the diffusion of innovation using word-of-mouth and interpersonal communications [32]. The diffusion process in the Bass model is based on the conditional probability that a potential customer will buy the new product given that he/she has not yet adopted it, is a linear function of the previous buyers. The model is conceptually thorough and empirically vigorous with many previous sales datasets. The sales parameters of the model can be estimated from the actual adoption data. The parameter estimation method and issues are explained meticulously in [22, 28, 42, 43, 48].

Also, practitioners have suggested an innumerable extensiones of the Bass model that modify the adoption process over time by introducing marketing-mix variables in the diffusion model. Robinson and Lakhani [37] and Jain and Rao [19] have shown the significance of price on the diffusion rate of innovation. Mahajan and Peterson [31], Kalish [25] and Josrgensen [23] have measured the potential market size as a function of price. Meanwhile, a few researchers have analyzed the impact of other marketing variables such as advertising, production, and quality on the products growth. Horsky and Simon [18] have explored the positive outcomes of advertising on the sales growth of infrequently purchased innovations. Bass et al. [5] have proposed a generalized Bass model that reflects the effect of both the pricing and advertising variables on the innovation growth structure. In addition, Teng and Thompson [44] have proposed the interrelationship between optimal pricing and quality policies for the new product diffusion by considering cost learning curve. Soon After, Mesak [34] modeled the diffusion process to optimize the pricing policy and the optimal warranty length, which are dependent on the pricing function. Later, Sethi and Bass [41] have pointed out that price has an inverse relation with the adoption rate irrespective of the discount rate and the time duration. More recently, Lin [29] simultaneously analyzed the marketing and production problem for a monopolist firm in a dynamic setting using the price, quality, and cumulative sales dependent demand function. Further, Jha at al. [21] have analyzed the impact of price and promotional efforts on the evolution of sales intensity in the segmented market.

Moreover, some authors have attempted to model the diffusion process in the multi-stage framework. Kalish [24] formulated the adoption assessment based on two-stage diffusion process: awareness, and adoption. Later, Jain et al. [20] and Ho et al. [17] have assumed that the buyers go through three stages of adoption; from being the potential market to waiting adopters and waiting applicants, they are either lost-customer or eventual adopters. Furthermore, Anand et al. [2] have

also proposed the innovation diffusion model with the stage structure using 3-stage Erlang distribution. Afterward, Chanda and Das [10] and Anand *et* al. [3] have formulated the multi-stage diffusion model in a dynamic environment. All the above-mentioned multistage models (except [10]) assumed that every aware individual would eventually get motivated to make the purchase.

Nevertheless, in the realistic framework, this assumption does not hold, as there is a probability that a fraction of individuals who are aware of the innovation may find the information unfavorable, and thus leave the system altogether. Moreover, the major drawback of [10] is that they considered the rate of adoption as constant. However, in marketing literature, it has been established that the growth of a technology innovation follows an S-shaped structure [11, 36]. The purpose of the current study is to extend the former diffusion models with stage structures by introducing the impact of potential customers' attrition behavior. The paper combines the multi-stage diffusion dynamics and the effects of disinterested potential buyers in a form of attrition in the adoption process. In the past, Libai *et* al. [27] made the initial study of the influence of attrition on the growth of an innovation (services).

The present diffusion model is developed using three stages of the adoption process, namely, product awareness (or product cognizance) stage, motivation (or persuasion) stage, and eventual adoption (or confirmation) stage. It is assumed that the knowledge about the innovation is widened using mass-media advertisements and word-of-mouth communication. Moreover, the persuasion for adoption by the prospective customers is considered to be influenced by the price of the innovation, which is taken as a control variable that helps in optimizing the total sales of the product. Further, the logistic time-dependent distribution function given by Kapur *et* al. [26] is used for measuring the adoption rate of the innovation. Evaluation of the proposed methodology was performed on the actual sales and price data set of high-technology products to understand the estimation and forecasting capability of the proposed model. This paper also presents the numerical analysis to illustrate the practical application of the current research, which was missing in the extant literature [2, 10, 25].

## 3. MODELING FRAMEWORK

The proposed model exhibits the diffusion process of a newly launched product with a fixed potential market of size Here denotes the number of the target populace who are unaware of the innovations existence and they go through various stages before actually purchasing the product. In the current modeling framework, adoption of an innovation has been reflected through three stages as categorized in [2, 3].

### 3.1. Assumptions

The fundamental assumptions taken into consideration while formulating the model are:

(i) The diffusion process is specified using the stage structure.

(ii) In the present study, the adoption of innovation consists of three steps: awareness, motivation and eventual adoption.

(iii) The aware population is classified into two categories, innovators and imitators, based the mode through which they have received the products information.

(iv) After seeking the knowledge about the innovation, an informed individual will become favorable towards the product only if he/she is content with the received information. A fixed rate of rejection is considered that indicates the percent of people who will leave the system due to dissatisfaction with the product information.

(v) The well-informed potential market will evaluate the innovation based on its price.

(vi) The utility of the product has an inverse relation with the amount it has to be payed for.

(vii) The selling price of the product is assumed dynamic, it modifies with time.

(viii) The rate at which adoption occurs in the final stage is modeled using the logistic function to incorporate the learning phenomenon.

(ix) Each adoption has an equal probability and occurs independently.

(x) At any point in time, there is a correlation between the sales rate and the number of remaining potentially motivated adopters.

*Stage 1: Diffusion of product cognizance or product awareness*

In the first step, individuals are exposed to innovation and learn about its existence. During this stage, unaware individuals are transferred from the unacquainted group into the prospective buyers. This stage is also known as information pursuing and knowledge processing stage as potential customers gather information about advantages and disadvantages of the product. It is presumed that there are two prime factors that facilitates the diffusion of product information: mass-media advertisements, and interpersonal communications. It is believed that a person gains knowledge about the existence of an innovation and exchange this awareness with other people. This is how an individual spread the information successively in the social system [9].

Moreover, an aware potential buyer will adopt the innovation only if he/she is convinced and satisfied with the information they have received. The interested target market is called positively aware population, and others who are not satisfied with the innovation characteristics and features will quit the system, and they are known as the disinterested or dissatisfied customers. This behavior of prospective

buyers who are leaving in the initial step of the adoption process can be termed as customer attrition.

The mathematical interpretation of the spread of information can be expressed using following differential equation [27]:

$$\frac{dN_1(t)}{dt} = \dot{N_1}(t) = \phi(Z - N_1(t)) + \varphi(1-\delta)\frac{N_1(t)}{Z}(Z - N_1(t)) - \delta N_1(t) \tag{1}$$

where $N_1(t)$ represents the total number of potential customers aware of the product but not yet purchased the innovation by the time t.

$\delta$ represents the rate of attrition or the proportion of the disinterested adopters, i.e., the percentage of cognizant individuals who are not dissatisfied with the product information and therefore, left the market.

$\phi$ denotes the coefficient of innovation that determines the rate with which customers adopt an innovation after getting influenced by the external factors, such as mass-media advertisements, and $\varphi$ is the coefficient of imitation that denotes the rate of adoption by the potential market only after getting positive feedback from the previous adopters.

On solving (1) using the initial condition, $N_1(0) = 0$, the following closed-form result is obtained:

$$N_1(t) = \bar{Z}\left(\frac{1 - e^{-(\bar{\phi}+\bar{\varphi})t}}{1 + \frac{\bar{\varphi}}{\bar{\phi}}e^{-(\bar{\phi}+\bar{\varphi})t}}\right) \tag{2}$$

The equation (2) represents the cumulative number of informed customers in timet,

where

$$\bar{Z} = Z\left(\frac{\Delta + \beta}{2\varphi(1-\delta)}\right) \tag{3}$$

$$\bar{\phi} = \frac{\Delta - \beta}{2}; \bar{\varphi} = \frac{\Delta + \beta}{2} \tag{4}$$

$$\beta = \varphi(1-\delta) - \phi - \delta \tag{5}$$

$$\Delta = \sqrt{\beta^2 + 4\varphi(1-\delta)\phi} \tag{6}$$

*Stage 2: Motivation for the purchase of the product*

The selling price of an innovation is the essential marketing-mix variable that helps in forming the persons opinion about the product. Therefore, after acquiring the knowledge about the innovation, the potential customer is persuaded to buy it after evaluating its price. It may be noted that there is always an objective difference between the customers attitude towards the price and the manufacturers perception of the price. The manufacturer sets prices with the aim to maximize the overall profit of the firm, while at the same time, the customers are considered to be coherent in the attempts to make a purchase when the price is lower [30]. Therefore, there should be a tradeoff between the customers utility and the sellers aim to maximize the profit.

Therefore, the price acts as a motivating factor for customers to make the purchases. Provided, that $\psi(t)$ is the pricing function at any time $t$, the total number of potentially motivated adopters is given as:

$$N_2(t) = N_1(t)\psi(t) \tag{7}$$

where $\psi(t)$ is the pricing function that takes the subsequent negative exponential structure:

$$\psi(t) = e^{-\alpha P(t)} \tag{8}$$

where $\alpha$ denotes the positive parameter expressing the price elasticity.

The cumulative motivated customers are obtained by using the functional form of equation (8) in equation (7):

$$N_1(t) = \bar{Z}e^{-\alpha P(t)}\left(\frac{1 - e^{-(\bar{\phi}+\bar{\varphi})t}}{1 + \frac{\bar{\varphi}}{\bar{\phi}}e^{-(\bar{\phi}+\bar{\varphi})t}}\right) \tag{9}$$

*Stage 3: Actual adoption of the product*

Once the target market is definite about the price of the innovation, the potentially motivated people are persuaded to buy it. Thus, the present buyers who have made the purchase become the resulting adopters. The differential equation for the sales rate at any time t is given as:

$$\frac{dN_3(t)}{dt} = \lambda(t)(N_2(t) - N_3(t)) \tag{10}$$

where $\lambda(t)$ denotes the rate with which the current potential market is adopting an innovation, and it is assumed to take the following logistic functional form:

$$\lambda(t) = \frac{\lambda}{1 + \gamma e^{-\lambda t}} \tag{11}$$

On substituting (9) and (11) in (10), the actual adoption rate can be written as:

$$\dot{N_3}(t) = \frac{\lambda}{1 + \gamma e^{-\lambda t}} \left[ \bar{Z} e^{-\alpha P(t)} \left( \frac{1 - e^{-(\bar{\phi} + \bar{\varphi})t}}{1 + \frac{\bar{\varphi}}{\phi} e^{-(\bar{\phi} + \bar{\varphi})t}} \right) - N_3(t) \right] \quad (12)$$

where equation (12) denotes the adopters at any time t with dynamic saturation level that depends upon the customers attrition behavior and price of the product.

## 4. PARAMETER ESTIMATION AND MODEL VALIDATION

The formulated mathematical model can be fitted to several past sales data to predict the pace of the diffusion process. The predicted values of diffusion rate assist in evaluating the accuracy of the developed model, and is helpful for comprehending market growth and planning for future levels of consumer demand [47]. The past sales data sets are acquired from electronic and semiconductor industries. The data from electronic industry are of Room air conditioner (*DSI*) and Color Television (*DSII*), and these were examined by Bass *et al.* [5]. The second industry from which the data have been collected are of semiconductors. The dataset from the semiconductor industry include data of different generations of Dynamic Random Access Memory (DRAM). In this paper, two generations, namely, 256 K (*DSIII*) and 4 M (*DSIV*) are considered for analysis purpose; these datasets are obtained from Victor and Ausubel [49].

The Nonlinear Least Square (NLLS) method for nonlinear regression is applied to estimate the parameters of the formulated model and to verify the fit between the proposed model and the actual data [42]. The data analysis is performed using the statistical software package known as SAS [40]. The predicted values of diffusion parameters for the given model on all the four datasets are listed in Table 1. Moreover, the results of the goodness-of-fit criteria such as coefficient of determination ($R^2$) Mean Square Error (MSE) and Adjusted ($R^2$), which measure the performance and fit precision of the proposed model are summarized in Table 2. From Tables 1 and 2, it can be inferred that the proposed model produces highly significant parameter estimates provides good fit to the four data sets. The results of the performance measures also indicate that the proposed model is well suitable for predicting the sales pattern of high-technology products. Pictorial representations of the predicted sales and the actual sales for different datasets are shown in Figure 1.

## 5. OPTIMAL CONTROL METHOD FOR PRICING POLICY

### 5.1. Control Model

In this paper, an optimal dynamic pricing policy of a new technology product is modeled to maximize the profit for the firm. Moreover, cumulative adoption is a powerful index that nearly all companies have the concern. Therefore, the proposed three-step diffusion process is incorporated to describe the cumulative sales and determine the pricing strategies. The current work assumes that a firm

| Parameters | Datasets | | | |
|---|---|---|---|---|
| | DSI | DSII | DSIII | DSIV |
| | Estimates | Estimates | Estimates | Estimates |
| $\bar{Z}$ | 47391.06 | 95620.59 | 4894.676 | 7888.303 |
| $\bar{\phi}$ | 0.13577 | 0.034855 | 0.426075 | 0.252885 |
| $\bar{\varphi}$ | 0.351128 | 0.707697 | 0.590538 | 0.616585 |
| $\delta$ | 0.01 | 0.00101 | 0.008 | 0.01 |
| $\alpha$ | 0.003246 | 0.001514 | 0.003 | 0.003 |
| $\lambda$ | 0.306448 | 0.52043 | 0.913781 | 0.962708 |
| $\gamma$ | 11 | 36.642 | 574.7113 | 2259.178 |

Table 1: Parameter Estimates

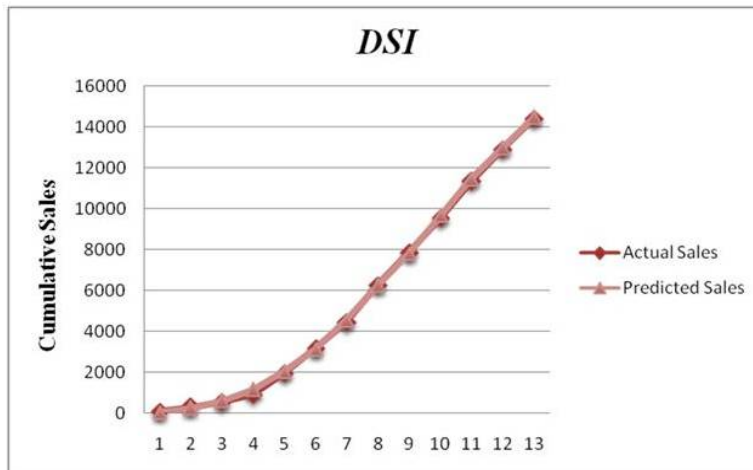| Datasets | $R^2$ | Adjusted $R^2$ | MSE |
|---|---|---|---|
| DSI | 0.9696 | 0.9595 | 17256.0 |
| DSII | 0.9880 | 0.9785 | 128971.0 |
| DSIII | 0.9754 | 0.9649 | 4475.8 |
| DSIV | 0.9917 | 0.9896 | 3873.6 |

Table 2: Performance Measures



Figure 1: Goodness of fit curve for DSI

aims to control the price of its product over a fixed planning horizon T. The following optimization problem is developed to analyze the pricing policy for an innovation:

$$Maximize\ \Pi(N_3(t), P(t)) = \int_0^T (P(t) - C(t))\dot{N}_3(t)e^{-rt}dt \tag{13}$$

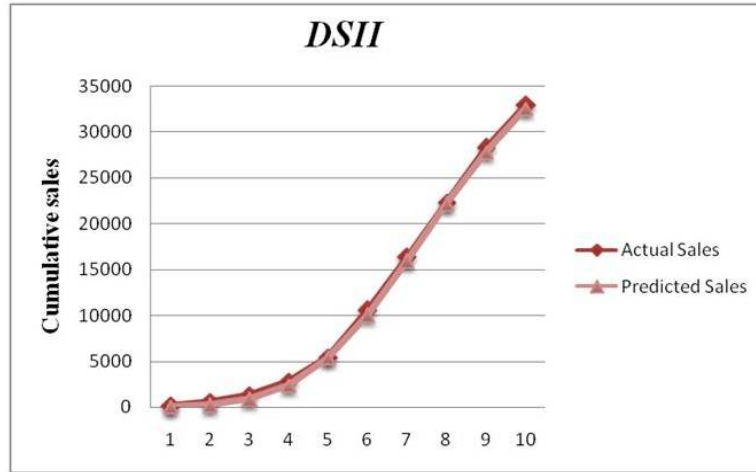where $\Pi$ is the present value of the total profit discounted by a rate r over a

Figure 2: Goodness of fit curve for DSII
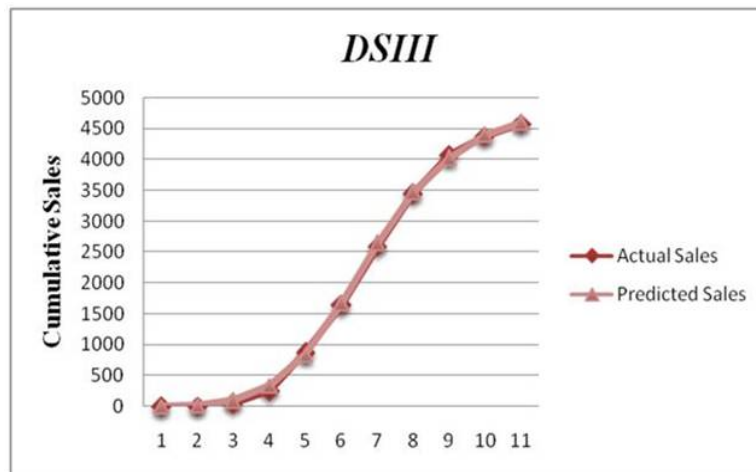


Figure 3: Goodness of fit curve for DSIII

fixed period T. In the developed control model, the problem is about maximizing the companys total discounted worth over a fixed period between $t = 0$ to $t = T$ at a constant discount rate r by optimizing its product selling price using the sales rate of innovation.

Here, the problem also includes the cost learning effect [35], and the marginal cost $C(t)$ is assumed as a function of total sales and the price of the product, i.e.

$$C(t) = C(N_3(t), P(t)) \tag{14}$$

The present optimization problem is solved using the optimal control approach
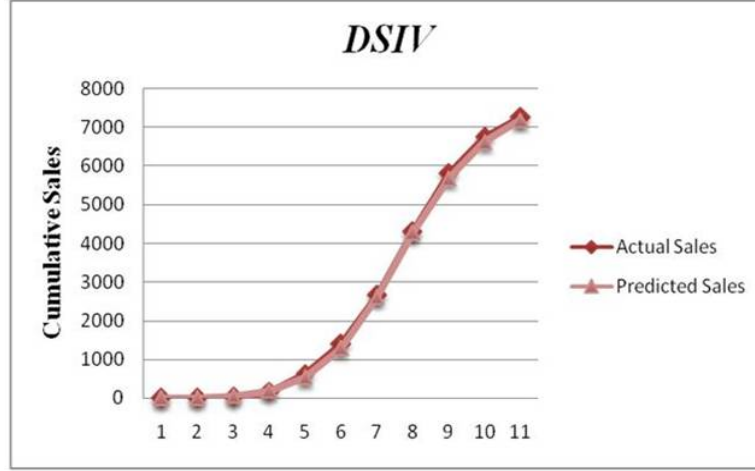
Figure 4: Goodness of fit curve for DSIV

for which $N_3(t)$ acts as a state variable while $P(t)$ is the control variable. Therefore, the sales rate equations for the given optimal control problem are:

$$x_1(t) = \dot{N}_2(t) = e^{-\alpha P(t)}\dot{N}_1(t) - \alpha \dot{P}(t)e^{-\alpha P(t)}N_1(t) \tag{15}$$

$$x_2(t) = \dot{N_3}(t) = \frac{\lambda}{1 + \gamma e^{-\lambda t}}\left[\bar{Z}e^{-\alpha P(t)}\left(\frac{1 - e^{-(\bar{\phi}+\bar{\varphi})t}}{1 + \frac{\bar{\varphi}}{\bar{\phi}}e^{-(\bar{\phi}+\bar{\varphi})t}}\right) - N_3(t)\right] \tag{16}$$

The functions $x_1(t)$ and $x_2(t)$ are twice differentiable and have an inverse relation to price, i.e. $\frac{\partial x_1(t)}{\partial P(t)} < 0$ and $\frac{\partial x_2(t)}{\partial P(t)} < 0$.

### 5.2. Optimality Necessary Conditions

Solving the above-mentioned control problem is in correspondence with optimizing the *current-value Hamiltonian function* in a given time frame [6, 18, 45], and it is defined as:

$$H_c(t) = \big\{(P(t) - C(t))\big\}x_2(t) + \lambda(t)x_1(t) + \mu(t)x_2(t) \tag{17}$$

where $\lambda(t)$ and $\mu(t)$ represent the *current-value multipliers* or *costate variables* associated with, that exhibit small fluctuation in the objective value for a slight deviation in the state variables, respectively.

The Hamiltonian function $H_c(t)$ objectively represents the overall revenue generated by incorporating both current $\big\{(P(t) - C(t))x_2(t)\big\}$ as well as future $\big\{\lambda(t)x_1(t) + \mu(t)x_2(t)\big\}$ advantages, taken into consideration. The Hamiltonian

function under dynamic market environment is explained by employing the necessary and sufficient optimality conditions of *Pontryagins Maximum Principle* [45]. Therefore, by using the *Pontryagin Maximum Principal*, the multiplier equations along with the *transversality conditions* for the control problem are given as:

$$\frac{d}{dt}\lambda(t) = \dot{\lambda}(t) = r\lambda(t) - \frac{\partial H_c(t)}{\partial N_2(t)}; \qquad \lambda(T) = 0 \tag{18}$$

$$\frac{d}{dt}\mu(t) = \dot{\mu}(t) = r\mu(t) - \frac{\partial H_c(t)}{\partial N_3(t)}; \qquad \mu(T) = 0 \tag{19}$$

The transversality condition depicts what should be satisfied at the end of the planning horizon. Now, by further solving equations (18) and (19), the following optimal results are obtained:

$$\dot{\lambda}(t) = r\lambda(t) - \lambda(t)\frac{\partial x_1(t)}{\partial N_2(t)} \tag{20}$$

$$\dot{\mu}(t) = r\mu(t) - \left\{ P(t) - C(t) + \mu(t) \right\}\frac{\partial x_2(t)}{\partial N_3(t)} + \frac{\partial C(t)}{\partial N_3(t)}x_2(t) \tag{21}$$

On integrating the equations (20) and (21) and by utilizing the transversality condition, we get the optimal values of *costate* variables:

$$\lambda(t) = e^{rt}\int_t^T \left\{ \lambda(t)\frac{\partial x_1(t)}{\partial N_2(t)} \right\}e^{-rs}ds \tag{22}$$

$$\mu(t) = e^{rt}\int_t^T \left\{ \left( P(t) - C(t) + \mu(t) \right)\frac{\partial x_2(t)}{\partial N_3(t)} + \frac{\partial C(t)}{\partial N_3(t)}x_2(t) \right\}e^{-rs}ds \tag{23}$$

### 5.3. Optimal Pricing Policy

The necessary condition for obtaining the optimal pricing function is characterized as:

$$\frac{\partial H_c(t)}{\partial P(t)} = 0 \tag{24}$$

$$\implies \left( 1 - \frac{\partial C(t)}{\partial P(t)} \right)x_2(t) + \left\{ P(t) - C(t) + \mu(t) \right\}\frac{\partial x_2(t)}{\partial P(t)} + \lambda(t)\frac{\partial x_1(t)}{\partial P(t)} = 0 \tag{25}$$

and the second-order optimal condition is

$$\frac{\partial^2 H_c(t)}{\partial P^2(t)} < 0 \tag{26}$$

By applying Managasarian Sufficiency Theorem [7] to the control problem, the second-order condition of optimality indicates that the Hamiltonian $H_c$ is concave in both the control variable and the state variables. Therefore, using the above necessary conditions, the optimal price can be derived and it has the following expression:

$$P^*(t) = \big\{C(t) - \mu(t)\big\} - \lambda(t)\frac{x_{1P}(t)}{x_{2P}(t)} - \frac{(1 - C_P)x_2(t)}{x_{2P}(t)} \tag{27}$$

where $x_{1P} = \frac{\partial x_1(t)}{\partial P(t)}$; $x_{2P} = \frac{\partial x_2(t)}{\partial P(t)}$ and $C_P = \frac{\partial C(t)}{\partial P(t)}$

The control theory problem in the continuous framework is changed into its discrete counterpart by a method known as discretization, given by [39]. Discrete models preserve the characteristics of continuous-time models and can be represented by the following difference equations:

$$\Delta N_2(k) = \frac{N_2(k+1) - N_2(k)}{\Delta} \tag{28}$$

$$\Delta N_3(k) = \frac{N_3(k+1) - N_3(k)}{\Delta} \tag{29}$$

where $\Delta$ is a constant time interval and $t = k \cdot \Delta$ and $\lim_{x \to \infty}(1 + x)^1/x = e$. Therefore, the corresponding control problem in discrete-time is given as:

$$Maximize \ \Pi(N_3(k), P(k)) = \sum_{k=0}^{T} \big(P(k) - C(N_3(k), P(k))\big)\Delta N_3(k) \cdot \frac{1}{(1+r)^k}$$

$$...(30a)$$

Subject to:

$$N_2(k+1) = N_2(k) + \Delta\left(\left\{e^{-\alpha P(k)}\left[\left(\phi + \varphi(1-\delta)\frac{N_1(k)}{Z}\right)(Z - N_1(k)) - \delta N_1(k)\right]\right\}\right.$$

$$\left. -\alpha\left[\frac{P(k+1) - P(k)}{\Delta}\right]e^{-\alpha P(k)}N_1(k)\right); \qquad k = 1, ..., T \qquad ...(30b)$$

$$N_3(k+1) = N_3(k) + \Delta\left[\frac{\lambda}{1+\gamma(1-\Delta\lambda)^{k+1}}\right.$$

$$\left.\left(\bar{Z}e^{-\alpha P(k)}\left(\frac{1-(1-\Delta(\bar{\phi}+\bar{\varphi}))^k}{1+\frac{\varphi}{\phi}(1-\Delta(\bar{\phi}+\bar{\varphi}))^k}\right) - N_3(k)\right)\right]; \quad k = 1,...T \qquad (30c)$$

## 6. NUMERICAL ILLUSTRATION

In this section, a numerical example is presented to analyze the influence of dynamic price on the adoption of an innovation. The proposed optimal control problem (P1) is solved numerically to derive the results of optimal price trajectory using the values of sales parameters, estimated by the empirical analysis. For solving purpose, the cost parameter is assumed constant. The objective of the above nonlinear optimization problem is to maximize the total discounted revenue generated by selling the innovation. To solve the discrete problem, optimization tool known as LINGO 15.0 has been utilized. The complete time horizon is divided into 12 equal discrete-time points.

### 6.1. Example from semiconductor industry

By positioning an innovation into a marketplace, business firms have to deal with a significant challenge: the enterprises must make sure that the price of their innovation must be in line with the consumers utility while maintaining the profit. Therefore, from the producers viewpoint, setting up an appropriate pricing strategy is imperative to stimulate customers adoption rate and for achieving ultimate profits. In this section, the optimal pricing strategy for the DRAM (Dynamic Random Access Memory) semiconductors is evaluated by using the historical global sales data set of the DRAM chip set to understand the real-life application of the proposed optimization problem. DRAMs are the largest volume commodity semiconductors that constitute around 11% of the entire semiconductor market. They are primarily used to store information of the CPU (Central Processing Units). DRAMs found their application in electronic products such as computer systems, hard-drives, printers, mobile handsets, as well as in telecommunication systems: GPS (Global Positioning Systems) or PDAs (Personal Digital Assistants). They are also extensively implemented in digital consumer products, e.g. video recorders, set-top boxes, and gaming console.

The parameter values of 4 Mb DRAM (DSIV) acquired from nonlinear regression have been used for the optimization model and the values of rest of the parameters are listed in Table 3. On solving the constrained maximization problem by using the parameter values, the optimal profit of $ 12,20,989 is attained. The optimal results for control variable $P(k)$ and state variables, $N_2(k)$ and $N_3(k)$, are given in Table 4. From the results, it can be inferred that the optimal price of the new technology decreases over time, which accelerates the adoption process.

Thus, selling price acts as a crucial deciding factor for adoption. Moreover, at any instant of time, the number of motivated individuals, $N_2(k)$, is higher as compared to eventual adopters, $N3(k)$. Figure 5 depicts the relationship between price and motivated potential customers, and eventual adopters.

| Parameters | Values | Parameters | Values |
|:---:|:---:|:---:|:---:|
| $\bar{Z}$ | 7888.303 | $\lambda$ | 0.962708 |
| $Z$ | 7979.33 | $\gamma$ | 2259.178 |
| $\bar{\phi}$ | 0.252885 | $C$ | 15 |
| $\bar{\varphi}$ | 0.616585 | $\Delta$ | 0.911 |
| $\phi$ | 0.25 | $r$ | 0.05 |
| $\varphi$ | 0.63 | $N_2(0)$ | 10 |
| $\delta$ | 0.01 | $N_3(0)$ | 10 |
| $\alpha$ | 0.003 | | |

Table 3: Parameter Values

| K | $N_2(k)$ | $N_3(k)$ | $P(k)$ |
|:---:|:---:|:---:|:---:|
| 1 | 10 | 10 | 1249.008 |
| 2 | 76.44574 | 12.18988 | 1180.670 |
| 3 | 187.8308 | 43.46960 | 1029.968 |
| 4 | 423.7791 | 219.6351 | 812.0937 |
| 5 | 799.3760 | 603.9681 | 631.1996 |
| 6 | 1264.810 | 1110.325 | 500.7205 |
| 7 | 1778.030 | 1675.907 | 403.3621 |
| 8 | 2316.870 | 2268.729 | 327.0089 |
| 9 | 2872.786 | 2872.786 | 264.3560 |
| 10 | 3483.462 | 3483.462 | 207.3228 |
| 11 | 4142.688 | 4142.688 | 155.4373 |
| 12 | 4880.249 | 4849.340 | 105.7541 |

Table 4: Optimal Results

## 6.2. Sensitivity Analysis

In this section, a detailed sensitivity analysis is conducted to analyze the effect of critical parameters such as the rate of attrition, price elasticity, and the rate of imitation, on the optimization problem and the pricing strategy. Table 5 provides the respective ranges of these parameters used for the analysis purpose. On solving the maximization problem by using values of the parameters, following conclusions can be established:

a. *Effect of the rate of attrition*

The parameter $\delta$ captures a percentage of those individuals who are acquainted with the product existence but are dissatisfied with the information they have
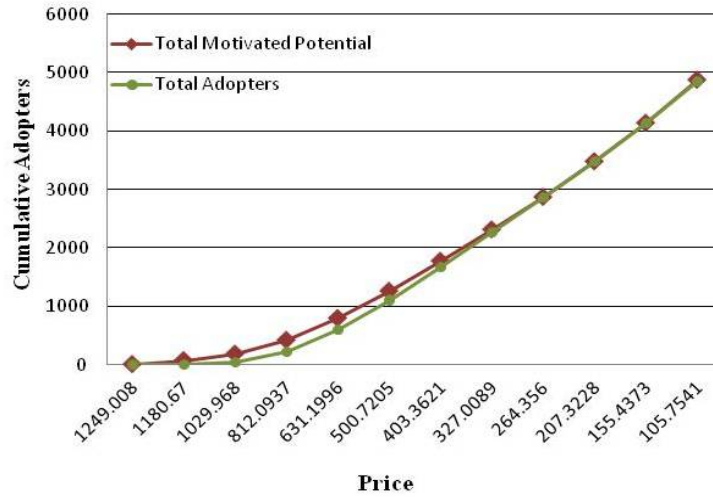
Figure 5: Relationship between Price and Motivated Potential and Eventual Adopters

| Parameters | Min. Value | Max. Value |
|:---:|:---:|:---:|
| $\delta$ | 0.005 | 0.02 |
| $\alpha$ | 0.0015 | 0.006 |
| $\varphi$ | 0.378 | 0.882 |

Table 5: Parameter ranges for sensitivity analysis

received, and thus decide not to adopt the innovation. Notice that when the value of $\delta$ increases, the total profit and the number of motivated potential customers decrease. Moreover, the company set the price somewhat high to maintain the profit level. For instance, when increases by 50%, i.e., its value changes from 0.01 to 0.015, then the profit decreases to $ 12,220,081, and total motivated prospective buyers by the end of the planning horizon reduce to 4,792 units. Also, the company raises the base price of innovation to $ 1,316.

b. *Effect of price elasticity*

The price elasticity $\alpha$ denotes the marginal effect of price change on the growth of the motivated potential market for Color TV. Now when price elasticity increases, the profit function and the total adopters decrease. The increase in the value of $\alpha$ also leads to price reduction. Thus, the optimal pricing strategy the company follows for the high value of $\alpha$ is to slash the prices to encourage sales. For instance, when $\alpha$ is increased from 0.003 to 0.0045, the firm reduces the base price of their product by 30% on average, i.e., the price decreases from $1,249 to $862. Moreover, profit function also substantially decreases. In such a scenario,

the company only achieves a maximum profit of $ 6,93,910.

   c. *Effect of word-of-mouth (or imitation rate)*

   The most important catalyst for product awareness and information diffusion is interpersonal communication. Potential customers seek knowledge from their associates and receive feedback from them that shapes their buying decision. The parameter $\varphi$ is the rate of imitation that represents the word-of-mouth effect that built interest in potential customers to gain more knowledge about the innovation. The rate of imitation is positively correlated with the profit function. As $\varphi$ increases, the number of positively aware as well as motivated individuals will increase, which in turn increases the total adoption. Subsequently, a considerable increase in profit function is observed. Moreover, the optimal pricing strategy when referral power is stronger, is to increase the base price. For instance, when $\varphi$ changes from 0.63 to 0.756, total adoption rises from 4,849 units to 4,867 units. The maximum profit of $ 12,26,748 is attained. Also, the company quotes $ 1,284 as the base price of the DRAM semiconductor.

## 7. MANAGERIAL IMPLICATIONS

   In the marketing literature, the interest of researchers for modeling and forecasting the diffusion of innovations is longstanding. The study of the diffusion of innovation is also essential for the managerial insights to understand and predict the dissemination of innovation among potential buyers. The present research and the results of the numerical illustration provide various avenues for decision makers and render certain managerial recommendations. The proposed three-stage diffusion model, incorporating the fraction of a disinterested prospective audience, is helpful in understating the diffusion curve of a new product more efficaciously. Moreover, the majority of purchasers are likely to be price sensitive, ie, do not want to pay premium prices for a new product. Thus, the customers purchasing decision is a function of the products price. Therefore, it is crucial for business firms to evaluate the optimal pricing policy of their new offerings. The proposed analysis assists the company in planning an effective course of action for building an optimal pricing policy. Furthermore, an optimization model is beneficial in fabricating a better business strategy for profit maximization.

## 8. CONCLUDING REMARKS

   The past research allows to assert that there is a definite time delay between the diffusion of information and the final adoption of a new technological product. Also, not all well-informed individuals will become eventual adopters because an informed customer will make a purchase decision only if he/she is content with the received information.
   In this research, a three-stage innovation diffusion model is investigated that explore the significance of dissatisfaction on the diffusion process. The first stage

of the diffusion process contributes to spreading the awareness of an innovation among the intended market. The proposed model also encompasses the aftermath of unfavorable information and of the positive word-of-mouth effect in the acknowledgment of an innovation. The developed model presents highly reliable parameter estimates and felicitous goodness of fit measures.

The paper also constructs the maximization model and yields an approach for optimizing the pricing decisions of the innovation. A non-linear optimization problem has been further solved through optimal control approach, where the concern of the objective function is to maximize the total discounted profit function. The discount rate is considered to cater customers time preference for money. For the control problem, the potentially motivated population and cumulative sales represent the system or the state variables, which are optimized by controlling the price variable. In addition, a discrete version of the optimal control problem has been solved numerically using the optimization tool LINGO 15.0 to obtain the maximum profit value. Moreover, the numerical example also depicts the impact of price variation in the growth of the innovation. Sensitivity analysis is also performed to analyze the influence of the critical parameters on the maximization problem.

This study also forms a basis for future scholars to explore other control variables such as advertising spending, warranty length, etc. to shape different adoption stages of innovation. Moreover, the proposed research can be further studied in a multi-generational framework wherein subsequent generations of innovation exist simultaneously in the marketplace. Thus, the optimal pricing policies for different generations can be inferred in the future.

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