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DETERMINING FUZZY DISTANCE THROUGH NON-SELF FUZZY CONTRACTIONS

Parbati SAHA
Department of Mathematics, Indian Institute of Engineering Science and Technology, Shibpur, Howrah - 711103, India
parbati_ saha@yahoo.co.in

Shantanu GURIA
Department of Mathematics, Indian Institute of Engineering Science and Technology, Shibpur, Howrah - 711103, India
sguria.math@gmail.com

Binayak S. CHOUDHURY
Department of Mathematics, Indian Institute of Engineering Science and Technology, Shibpur, Howrah - 711103, India
binayak12@yahoo.co.in

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Abstract: In the present work we solve the problem of finding the fuzzy distance between two subsets of a fuzzy metric space for which we use a non-self fuzzy contraction mapping from one set to the other. It is a fuzzy extension of the proximity point problem which is by its nature a problem of global optimization. The contraction is defined here by two control functions. We define a geometric property of the fuzzy metric space. The main result is illustrated with an example. Our result extends a fuzzy version of the Banach contraction mapping principle.

Keywords: Fuzzy Metric Spaces, Global Optimization, Pproximity Point, Non-Self $(\phi - \psi)$- Proximal Contraction, Optimal Approximate Solution, Fuzzy P-property.

MSC: $47H10, 54H25$.

1. INTRODUCTION

In this paper we establish a proximity point result in a fuzzy metric space so to find the fuzzy distance between two subsets. The problem originated from the work of Eldred et al [9] and has been well studied during the decade through works like [2, 5, 9, 15, 14, 16, 21, 22] . For our purpose we use a non-self contraction mapping which is defined by two control functions. The fuzzy metric space on which we deduce our results is as in George et al [10]. Due to its special features, it has become the platform of several extensions of metric related studies [1, 3, 4, 6, 7, 11, 12, 13, 19]. The problem sought to be considered here is essentially a global optimization problem which is solved by transforming it to a problem of finding the optimal approximate solution to a fixed point equation for a non-self contraction defined by use of two control functions. Control functions have been used in several fixed point problems in metric spaces [20]. Here, as the contraction function is non-self mapping, there is no exact solution of the fixed point equation. The following are two special features of the present work.

1. We define and use a non-self contraction with two control functions.
2. We define and use a geometric property in the fuzzy metric space.

2. MATHEMATICAL PRELIMINARIES

George and Veeramani in their paper [10] introduced the following definition of fuzzy metric space. Throughout this paper, we use this definition of fuzzy metric space.

Definition 1. *[10] The 3-tuple $(X, M, *)$ is called a fuzzy metric space if X is an arbitrary non-empty set, M is a fuzzy set on $X^2 \times (0, \infty)$ satisfying the following conditions for each $x, y, z \in X$ and $t, s > 0$:*

() $M(x, y, t) > 0$,

() $M(x, y, t) = 1$ *if and only if* $x = y$,

() $M(x, y, t) = M(y, x, t)$,

() $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$ *and*

() $M(x, y, .) : (0, \infty) \longrightarrow (0, 1]$ *is continuous,*

where $*$ is a continuous t-norm, that is, a continuous function $* : [0, 1]^2 \longrightarrow [0, 1]$ *such that*
(i) $a * b = b * a$ *for all* $a, b \in [0, 1]$,
(ii) $a * (b * c) = (a * b) * c$ *for all* $a, b, c \in [0, 1]$,
(iii) $a * 1 = a$ *for all* $a \in [0, 1]$,
(iv) $a * b \leq c * d$ *whenever* $a \leq c$ *and* $b \leq d$, *for each* $a, b, c, d \in [0, 1]$.

Let $(X, M, *)$ be a fuzzy metric space. For $t > 0$ and r with $0 < r < 1$, the open ball $B(x, t, r)$ with center $x \in X$ is defined by

$$B(x, t, r) = \{y \in X : M(x, y, t) > 1 - r\}.$$

A subset $A \subset X$ is called open if for each $x \in A$, there exist $t > 0$ and r with $0 < r < 1$ such that $B(x, t, r) \subset A$. Let τ denote the family of all open subsets of X. Then τ is a topology and is called the topology on X induced by the fuzzy metric M. The topology τ is a Hausdorff topology [10]. In fact, the definition 2.1 is a modification of the definition given in [17] for ensuring Hausdorff topology of the space.

Definition 2. *[10] Let $(X, M, *)$ be a fuzzy metric space. A sequence $\{x_n\}$ in X is said to be convergent to a point $x \in X$ if $\lim_{n\to\infty} M(x_n, x, t) = 1$ for all $t > 0$.*

Definition 3. *[10] Let $(X, M, *)$ be a fuzzy metric space. A sequence $\{x_n\}$ in X is called a Cauchy sequence if for each ε with $0 < \varepsilon < 1$ and $t > 0$, there exists a positive integer n_0 such that $M(x_n, x_m, t) > 1 - \varepsilon$ for each $n, m \geq n_0$.*

A fuzzy metric space is said to be complete if every Cauchy sequence is convergent in it.

The following lemma was proved by Grabiec [11] for fuzzy metric spaces defined by Kramosil et al [17]. The proof is also applicable to the fuzzy metric space given in definition 2.1.

Lemma 4. *[11] Let $(X, M, *)$ be a fuzzy metric space. Then $M(x, y, .)$ is non-decreasing for all $x, y \in X$.*

Lemma 5. *[18] M is a continuous function on $X^2 \times (0, \infty)$.*

We will require for use in our results the following two functions.

Definition 6. ***(Ψ-function)****[23] A function $\psi : [0, \infty) \to [0, \infty)$ is a Ψ- function if*

() ψ is nondecreasing and continuous,

() $\Sigma_{n=1}^{\infty} \psi^n(t) < \infty$ for all $t > 0$, where $\psi^{n+1}(t) = \psi(\psi^n(t)), n \geq 1$.

It is clear that $\psi(t) < t$ for all $t > 0$ whenever ψ is a Ψ-function.

The following function is an example of a ψ - function:

$$\psi(t) = \begin{cases} t - \frac{t^2}{2}, & \text{if } t \in [0, 1], \\ \frac{1}{2}, & t > 1. \end{cases}$$

Definition 7. *[20] A function $\phi : [0,\infty) \to [0,\infty)$ is a Φ- function if*
(i) ϕ is nondecreasing and continuous,
(ii) $\phi(0) = 0$.

Lemma 8. *[23] If $*$ is a continuous t-norm, and $\{\alpha_n\}$, $\{\beta_n\}$ and $\{\gamma_n\}$ are sequences such that $\alpha_n \to \alpha$, $\gamma_n \to \gamma$ as $n \to \infty$, then $\overline{\lim_{k\to\infty}}(\alpha_k * \beta_k * \gamma_k) = \alpha * \overline{\lim_{k\to\infty}} \beta_k * \gamma$ and*
$\underline{\lim_{k\to\infty}}(\alpha_k * \beta_k * \gamma_n) = \alpha * \underline{\lim_{k\to\infty}} \beta_k * \gamma$.

Lemma 9. *[23] Let $\{f(k,.) : [0,\infty) \to [0,1], k = 0,1,2,......\}$ be a sequence of functions such that $f(k,.)$ is continuous and monotone increasing for each $k \geq 0$. Then $\overline{\lim_{k\to\infty}} f(k,t)$ is a left continuous function in t and $\underline{\lim_{k\to\infty}} f(k,t)$ is a right continuous function in t.*

3. MAIN RESULTS

Definition 10. *[24] Let $(X, M, *)$ be a fuzzy metric space. The fuzzy distance of a point $x \in X$ from a nonempty subset A of X is*
$$M(x, A, t) = \sup_{a\in A} M(x, a, t) \text{ for all } t > 0$$
and the fuzzy distance between two nonempty subsets A and B of X is
$$M(A, B, t) = \sup\{M(a, b, t) : a \in A, b \in B\} \text{ for all } t > 0.$$

Let A and B be two nonempty disjoint subsets of a fuzzy metric space $(X, M, *)$. We write
$$A_0 = \{x \in A : \exists\ y \in B \text{ such that } M(x, y, t) = M(A, B, t) \text{ for all } t > 0\},$$
$$B_0 = \{y \in B : \exists\ x \in A \text{ such that } M(x, y, t) = M(A, B, t) \text{ for all } t > 0\}.$$

Definition 11. *Let $(X, M, *)$ be a fuzzy metric space and A, B are two non-empty subsets of X. An element $x^* \in A$ is defined as a fuzzy best proximity point of the mapping $f : A \to B$ if it satisfies the condition that for all $t > 0$*
$$M(x^*, fx^*, t) = M(A, B, t).$$

In the following we define a property of a pair of subsets in a fuzzy metric space. It is essentially a geometric property.

Definition 12. *Let (A, B) be a pair of nonempty disjoint subsets of a fuzzy metric space $(X, M, *)$. Then the pair (A, B) is said to satisfy the fuzzy P-property if for all $t > 0$ and $x_1, x_2 \in A$, $y_1, y_2 \in B$,*
$$M(x_1, y_1, t) = M(A, B, t) \text{ and } M(x_2, y_2, t) = M(A, B, t)$$
jointly implies that
$$M(x_1, x_2, t) = M(y_1, y_2, t).$$

The P-property is a geometric property which is automatically valid in Hilbert spaces for non- empty closed and convex pairs of sets [21], but does not hold in arbitrary Banach spaces. In metric spaces such property for pairs of subsets is separately assumed for specific purposes. The above definition is a fuzzy extension of that.

Definition 13. *Let $(X, M, *)$ be a fuzzy metric space and $f : A \to B$ be a mapping. The mapping f is non-self $(\phi - \psi)$- contraction mapping if there exist Ψ-function (definition 6) ψ, a Φ-function (definition 7) ϕ and $0 < c < 1$ such that for all $t > 0$ and $x, y \in A$ we have*

$$(\frac{1}{M(fx, fy, \phi(ct))} - 1) \leq \psi(\frac{1}{M(x, y, \phi(t))} - 1). \tag{3.1}$$

Note. The above contraction condition with some variations in the condition on ψ has already appeared in the context of fixed point studies in probabilistic metric spaces [8].

Theorem 14. *Let $(X, M, *)$ be a complete fuzzy metric space. Let A and B be two closed subsets of X and $f : A \to B$ be an $(\phi - \psi)-$ contractive mapping such that the following conditions are satisfied.*
(i) (A, B) satisfies the fuzzy P-property,
(ii) $f(A_0) \subseteq B_0$,
(iii) A_0 is nonempty,
Then there exists an element $x^* \in A$ which is a fuzzy best proximity point of f.
Proof. *By assumption (iii), A_0 is nonempty. Let $x_0 \in A_0$. Since $f(A_0) \subseteq B_0$, there exists $x_1 \in A_0$ such that*

$M(x_1, fx_0, t) = M(A, B, t)$ *for all* $t > 0$.

Again since $f(A_0) \subseteq B_0$, there exists $x_2 \in A_0$ such that

$M(x_2, fx_1, t) = M(A, B, t)$ *for all* $t > 0$.

Continuing this process, we construct a sequence $\{x_n\}$ in A_0 such that for all $n \geq 1$, for all $t > 0$,

$$M(x_n, fx_{n-1}, t) = M(A, B, t). \tag{3.2}$$

Also, we can write the above as

$$M(x_{n+1}, fx_n, t) = M(A, B, t) \text{ for all } n \geq 1, \text{for all } t > 0. \tag{3.3}$$

Since (A, B) satisfies the fuzzy P-property, we get from (3.2) and (3.3), for all $t > 0$

$$M(x_n, x_{n+1}, t) = M(fx_{n-1}, fx_n, t) \text{ for all } n > 1. \tag{3.4}$$

From the property of ϕ it is clear that for each $t > 0$ there exists $t_0 > 0$ such that $\phi(t_0) = t$.

Since f is $(\phi-\psi)-$ contraction and from the property of ϕ , we have for all $n \geq 1$, for all $t > 0$ there exist $t_0 > 0$ such that

$$\begin{aligned}(\frac{1}{M(fx_{n-1}, fx_n, t)} - 1) &= (\frac{1}{M(fx_{n-1}, fx_n, \phi(t_0))} - 1) \\ &\leq (\frac{1}{M(fx_{n-1}, fx_n, \phi(ct_0))} - 1) \\ &\leq \psi(\frac{1}{M(x_{n-1}, x_n, \phi(t_0))} - 1) \\ &= \psi(\frac{1}{M(x_{n-1}, x_n, t)} - 1)\end{aligned}$$

Therefore, we have for all $n \geq 1$, for all $t > 0$,

$$(\frac{1}{M(fx_{n-1}, fx_n, t)} - 1) \leq \psi(\frac{1}{M(x_{n-1}, x_n, t)} - 1). \tag{3.5}$$

Combining (3.4) *and* (3.5)*, we have for all $n \geq 1$, for all $t > 0$,*

$$(\frac{1}{M(x_n, x_{n+1}, t)} - 1) \leq \psi(\frac{1}{M(x_{n-1}, x_n, t)} - 1). \tag{3.6}$$

If for some $k > 0$, $x_k = x_{k+1}$, then x_k is a best proximity point of f.
Assuming $x_{n-1} \neq x_n$ for all $n \geq 1$, and making repeated applications of (3.6)*, we have for all $n \geq 1$, for all $t > 0$,*

$$(\frac{1}{M(x_n, x_{n+1}, t)} - 1) \leq \psi^n(\frac{1}{M(x_0, x_1, t)} - 1). \tag{3.7}$$

Taking $n \to \infty$ in the above inequality (3.7)*, for all $t > 0$, we obtain*
$\lim_{n\to\infty}(\frac{1}{M(x_n,x_{n+1},t)} - 1) \leq \lim_{n\to\infty}\psi^n(\frac{1}{M(x_0,x_1,t)} - 1) \to 0$ *as $n \to \infty$, (by a property of ψ).*
that is, $\lim_{n\to\infty}(\frac{1}{M(x_n,x_{n+1},t)} - 1) = 0$*, which implies that for all $t > 0$,*

$$\lim_{n\to\infty} M(x_n, x_{n+1}, t) = 1. \tag{3.8}$$

Next, we show that $\{x_n\}$ is a Cauchy sequence in A. We suppose, if possible, that $\{x_n\}$ is not a Cauchy sequence in A. Then definition 3 is not satisfied by the sequence $\{x_n\}$ and, therefore, there exist some $\epsilon > 0$ and some λ with $0 < \lambda < 1$, for which we can find two subsequences $\{x_{m(k)}\}$ and $\{x_{n(k)}\}$ of $\{x_n\}$ with
$n(k) > m(k) > k$ *such that*

$$M(x_{m(k)}, x_{n(k)}, \epsilon) \leq (1 - \lambda), \tag{3.9}$$

for all positive integer k.
We may choose the $n(k)$ as the smallest integer exceeding $m(k)$ for which (3.9) *holds. Then, for all positive integer k,*

$$M(x_{m(k)}, x_{n(k)-1}, \epsilon) > (1 - \lambda) \tag{3.10}$$

Then, for all $k \geq 1, 0 < s < \frac{\epsilon}{2}$*, we obtain,*

$$\begin{aligned}(1-\lambda) &\geq M(x_{m(k)}, x_{n(k)}, \epsilon) \\ &\geq M(x_{m(k)}, x_{m(k)-1}, s) * M(x_{m(k)-1}, x_{n(k)-1}, \epsilon - 2s) \\ &* M(x_{n(k)-1}, x_{n(k)}, s).\end{aligned} \tag{3.11}$$

For all $t > 0$*, we denote*

$$h_1(t) = \overline{\lim_{k\to\infty}} M(x_{m(k)-1}, x_{n(k)-1}, t). \tag{3.12}$$

Taking limit supremum on both sides of (3.11)*, using* (3.8)*, the properties of M and* $*$*, by lemma* (8)*, we obtain*

$$(1-\lambda) \geq 1 * \overline{\lim_{k\to\infty}} M(x_{m(k)-1}, x_{n(k)-1}, \epsilon - 2s) * 1 = h_1(\epsilon - 2s) \tag{3.13}$$

Since M is bounded within the range in [0,1], continuous and, by lemma 4, monotone increasing in the third variable t, it follows by an application of lemma 9 that h_1*, as given in* (3.12) *is continuous from the left. From the above, letting* $s \to 0$ *in* (3.13)*, it then follows that*

$$\overline{\lim_{k\to\infty}} M(x_{m(k)-1}, x_{n(k)-1}, \epsilon) \leq (1-\lambda). \tag{3.14}$$

Let,

$$h_2(t) = \underline{\lim}_{k\to\infty} M(x_{m(k)-1}, x_{n(k)-1}, t), t > 0. \tag{3.15}$$

Again, for all $k \geq 1, s > 0$,

$$\begin{aligned}M(x_{m(k)-1}, x_{n(k)-1}, \epsilon + s) &\geq M(x_{m(k)-1}, x_{m(k)}, s) * M(x_{m(k)}, x_{n(k)-1}, \epsilon) \\ &\geq M(x_{m(k)-1}, x_{m(k)}, s) * (1-\lambda), (by (3.10))\end{aligned} \tag{3.16}$$

Taking limit infimum as $k \to \infty$ *in* (3.16)*, by virtue of* (3.8)*, we obtain*

$$\begin{aligned}h_2(\epsilon + s) = \underline{\lim}_{k\to\infty} M(x_{m(k)-1}, x_{n(k)-1}, \epsilon + s) &\geq \underline{\lim}_{k\to\infty} M(x_{m(k)-1}, x_{m(k)}, s) * (1-\lambda) \\ &= 1 * (1-\lambda) = (1-\lambda).\end{aligned} \tag{3.17}$$

Since M is bounded within the range in [0,1], continuous and by lemma 4, it is monotone increasing in the third variable t, it follows by an application of lemma 9 that h_2*, as given in* (3.15) *is continuous from the right.*
From the above, letting $s \to 0$ *in* (3.17)*, it then follows that*

$$\underline{\lim}_{k\to\infty} M(x_{m(k)-1}, x_{n(k)-1}, \epsilon) \geq (1-\lambda). \tag{3.18}$$

The inequalities (3.14) *and* (3.18) *jointly imply that*

$$\lim_{k\to\infty} M(x_{m(k)-1}, x_{n(k)-1}, \epsilon) = (1-\lambda). \tag{3.19}$$

Again by (3.9),

$$\overline{\lim_{k\to\infty}} M(x_{m(k)}, x_{n(k)}, \epsilon) \leq (1-\lambda) \tag{3.20}$$

Also for all $k \geq 1, s > 0$, *we obtain*
$M(x_{m(k)}, x_{n(k)}, \epsilon+2s) \geq M(x_{m(k)}, x_{m(k)-1}, s) * M(x_{m(k)-1}, x_{n(k)-1}, \epsilon) * M(x_{n(k)-1}, x_{n(k)}, s)$
Taking limit infimum as $k \to \infty$ *in the above inequality, using* (3.8), (3.19) *and the properties of* M *and* $*$, *by lemma 8, we obtain*
$\underline{\lim}_{k\to\infty} M(x_{m(k)}, x_{n(k)}, \epsilon+2s) \geq 1 * \underline{\lim}_{k\to\infty} M(x_{m(k)-1}, x_{n(k)-1}, \epsilon) * 1 = 1-\lambda.$
Since M is bounded within the range in [0,1], is continuous and, by lemma 4, monotone increasing in the third variable t, it follows by an application of lemma 9 that $\underline{\lim}_{k\to\infty} M(x_{m(k)}, x_{n(k)}, t)$ *is continuous function of t from the right.*
Taking $s \to 0$ *in the above inequality, and using lemma 9, we obtain*

$$\underline{\lim}_{k\to\infty} M(x_{m(k)}, x_{n(k)}, \epsilon) \geq (1-\lambda), \tag{3.21}$$

Combining (3.20) *and* (3.21), *we obtain*

$$\lim_{k\to\infty} M(x_{m(k)}, x_{n(k)}, \epsilon) = (1-\lambda) \tag{3.22}$$

From (3.3), *we have*

$$M(x_{m(k)}, fx_{m(k)-1}, t) = M(A, B, t) \tag{3.23}$$

$$M(x_{n(k)}, fx_{n(k)-1}, t) = M(A, B, t) \tag{3.24}$$

Since (A, B) *satisfies the fuzzy P-property, we get from* (3.23) *and* (3.24), *for all* $t > 0$,

$$M(x_{m(k)}, x_{n(k)}, t) = M(fx_{m(k)-1}, fx_{n(k)-1}, t). \tag{3.25}$$

Now by the property of ϕ, *there exists* $\epsilon_0 > 0$ *such that* $\phi(\epsilon_0) = \epsilon$.
Therefore, from the above and by (3.25),

$$\begin{aligned}
(\frac{1}{M(x_{m(k)}, x_{n(k)},\ \epsilon)} - 1) &= (\frac{1}{M(fx_{m(k)-1}, fx_{n(k)-1},\ \epsilon)} - 1) \\
&= (\frac{1}{M(fx_{m(k)-1}, fx_{n(k)-1},\ \phi(\epsilon_0))} - 1) \\
&\leq (\frac{1}{M(fx_{m(k)-1}, fx_{n(k)-1},\ \phi(c\epsilon_0))} - 1) \\
&\leq \psi(\frac{1}{M(x_{m(k)-1}, x_{n(k)-1},\ \phi(\epsilon_0))} - 1) \\
&= \psi(\frac{1}{M(x_{m(k)-1}, x_{n(k)-1},\ \epsilon)} - 1)
\end{aligned}$$

Taking $k \to \infty$ *in the above inequality, we have*
$\left(\frac{1}{\lim\limits_{k\to\infty} M(x_{m(k)}, x_{n(k)}, \epsilon)} - 1\right) \leq \psi\left(\frac{1}{\lim\limits_{k\to\infty} M(x_{m(k)-1}, x_{n(k)-1}, \epsilon)} - 1\right)$. *(since* ψ *is continuous)*
Using (3.19) *and* (3.22)*, we have*

$$\left(\frac{1}{1-\lambda} - 1\right) \leq \psi\left(\frac{1}{1-\lambda} - 1\right) < \left(\frac{1}{1-\lambda} - 1\right),$$

which is a contradiction.
Thus, it is established that $\{x_n\}$ *is a Cauchy sequence. Since* $(X, M, *)$ *is complete, there exists* $x^* \in A$ *such that*

$$\lim_{n\to\infty} x_n = x^*.$$

Since f *is* $(\phi - \psi)$*- proximal contractive mapping, by using* (3.1)*, we have for all* $n \geq 0$, $t > 0$

$$\begin{aligned}
\left(\frac{1}{M(fx_n, fx^*, t)} - 1\right) &= \left(\frac{1}{M(fx_n, fx^*, \phi(t_0))} - 1\right) \\
&\leq \left(\frac{1}{M(fx_n, fx^*, \phi(ct_0))} - 1\right) \\
&\leq \psi\left(\frac{1}{M(x_n, x^*, \phi(t_0))} - 1\right) \\
&\leq \psi\left(\frac{1}{M(x_n, x^*, t)} - 1\right)
\end{aligned}$$

Taking limit $n \to \infty$ *on both sides of the above inequality, using the fact that* $\psi(0) = 0$*, we have*

$$fx_n \to fx^* \text{ as } n \to \infty.$$

From (3.3) *and the above limit, for all* $t > 0$

$$M(A, B, t) = M(x_{n+1}, fx_n, t) = M(x^*, fx^*, t) \text{ as } n \to \infty.$$

Therefore, for all $t > 0$, $M(x^*, fx^*, t) = M(A, B, t)$*. This completes the proof.*

4. ILLUSTRATION

Example 15. *Suppose that* $X = \mathbb{R}^2$ *with fuzzy metric space*

$$M((x, y), (x', y'), t) = \frac{t}{t + |x - x'| + |y - y'|} \text{ and minimum } t\text{-norm } *.$$

Consider the closed subsets A *and* B *in the topology induced by the fuzzy metric as*

$$A = \{(0, x) : x \in \mathbb{R}\},$$
$$B = \{(1, x) : x \in \mathbb{R}\}.$$

Let $\psi(t) = ct$ *and* $\phi(t) = t^2$*, where* $0 < c < 1$*. Let* $f : A \to B$ *be the mapping defined by*

$$f((0, x)) = (1, 1 - e^{-c^3 x}).$$

Here $M(A, B, t) = \frac{t}{1+t}$ *for all* $t > 0$.

Here $A_0 = A$ *and* $B_0 = B$ *and* $f(A_0) \subseteq B_0$.
Now we show that f *satisfies fuzzy* P*- property.*
Let $u_1 = (0, x_1)$, $u_2 = (0, x_2) \in A$ *and* $v_1 = (1, y_1)$, $v_2 = (1, y_2) \in B$ *with*

$$M(u_1, v_1, t) = M(A, B, t) \text{ for all } t > 0 \tag{4.1}$$

and

$$M(u_2, v_2, t) = M(A, B, t) \text{ for all } t > 0 \tag{4.2}$$

From (4.1), *we get for all* $t > 0$

$$\frac{t}{t+1+|x_1-y_1|} = \frac{t}{t+1},$$

which implies that $x_1 = y_1$.
Similarly from (4.2), *we get for all* $t > 0$

$$x_2 = y_2.$$

Now for all $t > 0$

$$\begin{aligned} M(u_1, u_2, t) &= \frac{t}{t+|x_1-x_2|} \\ &= \frac{t}{t+|y_1-y_2|} \\ &= M(v_1, v_2, t). \end{aligned}$$

Hence f *satisfies fuzzy* P*- property.*
Let $u = (0, x), v = (0, y) \in A$. *Without loss of generality, we may assume that* $x < y$.
Now for all $t > 0$,

$$\begin{aligned} \left(\frac{1}{M(fu, fv, \phi(ct))} - 1\right) &= \frac{|e^{-c^3x} - e^{-c^3y}|}{c^2t^2} \\ &= \frac{c^3e^{-c^3[x+\theta(y-x)]}|x-y|}{c^2t^2} \quad (\text{Using MVT, where } 0 < \theta < 1) \\ &\leq \frac{c|x-y|}{t^2} \\ &= c\left(\frac{1}{M(u, v, \phi(t))} - 1\right) \\ &= \psi\left(\frac{1}{M(u, v, \phi(t))} - 1\right). \end{aligned}$$

Hence f *satisfies* $(\phi - \psi)$*-proximal contraction.*
Here $(0, 0) \in A$ *is the best proximity point of* f.

Note: The above illustration indicates that our result is an effective generalization of the fuzzy Banach contraction mapping principle given by Gregori and Sapena [13] in complete fuzzy metric space since the latter is not applicable to the above example.

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