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DECISION MAKINGS IN DISCOUNT PRICING POLICY FOR IMPERFECT PRODUCTION SYSTEM

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Abstract: In this paper, we discussed the effects of discount price on demand and profit in a diminishing market. A production plan has been suggested for an imperfect production system. Here, demand is considered to be price sensitive and negative power function of the selling price. This problem is solved by optimization, using the Hessian matrix of order three. The main objective is to find the optimal expected average profit, optimal selling price, discount rate, backorder level, and lot-size. The recommendations are provided to offer a price discount for limited sale season on different occasions. A numerical example is presented to validate the model and is graphically illustrated accordingly.

Keywords: Inventory, Dynamic pricing, Price-discount Dependent Demand, Optimal Price Settings, Imperfect Item, Rework, Shortage, Partial Backlogging.

MSC: 90B05, 90B30, 90B50.

1. INTRODUCTION

Inventory management plays a significant role as it ensures product quality to be maintained and effectively tackles transactions related to consumer goods. Inventory management is an essential requirement that facilitates smooth operation of business affairs in retail stores, warehouses and production systems. Pricing policy plays a vital role to maintain inventory and it also influences demand of the product.

Production planning is another aspect that researchers are attracted to recover overages and shortages of any products, the company should carefully develop the production system to enrich the business. The production system does not guarantee hundred percent perfection, in real situations it might produce some imperfect items too. For imperfect production, there could be several reasons such as poor quality raw materials, unskilled labour, and machinery malfunction. Imperfect items can be bifurcated into two types which would include; rework able and scrap.

Our goal is to develop an imperfect production inventory model considering the partial backlogging situation and selling price-sensitive demand pattern, with discount in selling price. First, we review the prevalent work related to the study.

1.1. Review of Litetature

Traditionally, the basic inventory control model was first developed by Harris [8]. He introduced an EOQ (Economic Order Quantity) model which informs a company about how much should be ordered and when orders should bear place so that the total costs will be minimized. Many researchers like Wilson [30]. Arrow *et al.* [13] and Whitin *et al.* [36] analyzed and reviewed Harris model. Abad [25] incorporated a joint price and lot size determination problem model in which the supplier provided incremental quantity discounts to the retailer on purchase of items. Abad [26] studied an inventory model for deteriorating items, in which shortage was allowed and partially backlogged. Wee [10] determined the pricing and replenishment model for deteriorating items which considered exponential decreasing demand with time. Wee and Yu [11] derived an inventory model for deteriorating product by providing a temporary price discount within a short time period.

Abad [27] developed an inventory model with backlogging and considering price sensitive demand. Viswanath and Wang [35] studied the effectiveness of quantity discount and volume discount coordination mechanisms. Yang [24] also introduced an optimal replenishment policy for price sensitive demand. Salameh and Jaber [19] suggested an economic quantity model for imperfect quantity items. In this model, the screening process was chaired to detect the imperfect items. They suggested that the imperfect products were sold at a discounted price.

Konstantaras *et al.* [12] presented a production inventory model using an inspection process. Perfect items were sent to a working inventory warehouse in equal batches and imperfect items were either sold to another secondary store at a lower price or were reworked so that they may be kept at a new shop. In their model, they considered the number of batches as a decision variable. Jaber *et al.* [21] designed an economic production quantity model in which they were considering the time horizon as finite and infinite with learning effect. In that model, they also presented two types of learning curves logistic form and powder form due to the learning effect, imperfect products witnessed a gradual reduction. Roy and Chaudhuri [37] explored a model in which the production rate also depended on selling price per unit. They considered constant deterioration and extended the proposed demand function quadratic price, dependent or stochastically fluctuating demand pattern. The two-stage supply chain consisted of one vendor and one buyer. Joglekar *et al.* [28] presented an inventory model in which he projected that increasing price strategy is better for the e-tailer as compared to constant price strategy. This particular model is applicable to products that are more price-sensitive. The model is illustrated with a numerical example and compares with price and time. Sajadieh *et al.* [20] proposed a model to find the relevant profit maximizing decision variable values. This model is based on the joint total profit of both the vendor and the buyer. If buyers and vendors cooperate with each other and demand is more price-sensitive then the model is more beneficial for any business regimentation.

Banerjee and Sharma [32] considered an inventory model for seasonal products. When the products had seasonal demand rate which depended on time and price both, they took price as a decision variable and also profit function was the concave function of time and conditionally joint concave function of selling price and time. Tripathi *et al.* [5] investigated an economic quantity model in which they have considered demand rate as a function of the selling price, and holding cost as time-dependent. This model is a deterministic inventory model for deteriorating items. In this model they used two cases, one with shortage and the second was without shortage. As per their findings, it was observed that the optimum average profit in without shortage was more than that of shortage.

Sana [33]conceptualized an economic order quantity (EOQ) model in which they assumed demand function as price dependent and they also assumed deterioration rate of the defective item as time proportional. They discussed the shortage followed by an inventory of replenishment. They developed this model over an infinite time horizon for perishable products. Sana [34] suggested an inventory model in which they have considered demand function as quadratic function and the selling price is augmented in each cycle, but demand decreases quadratically with the selling price. They studied many changes in the rate of demand. In case the demand function is taken as a negative function of price, it cannot be done so in practical scenarios. Shah *et al.* [14] reconsidered the model presented by Sony and Shah by using selling price as decision variable and ending inventory to be positive or zero for finite time horizon. They also assumed limited floor space, maximum profit which kept deteriorating as a constant. On the basis of this assumption, they developed an algorithm to find the optimal decision policy. Yang [23]outlined a piecewise production inventory model for imperfect products of price-sensitive demand. They indicated that multiple production cycles were better than a single production cycle. His model was successful as it was a good opportunity to raise product prices if there was an increase in demand parameters.

Preservation technology is very useful for the present scenario and as well as for perishable type commodities. This idea provides more effectiveness in business. Khedlekar *et al.* [38] designed an EOQ, in this model the demand for products is price sensitive and linearly decreasing rate. They considered the profit as the concave function of the optimal selling price, they also stated that the optimal selling price, the length of the replenishment cycle and the optimal preservation concept investment simultaneously. Mishra [39] proposed a model for single-manufacturer single-retailer by incorporating preservation technology cost for deteriorating item and determined optimal retail price, replenishment cycle and the cost of preservation technology.

Taleizadeh and Noori-daryan [1] presented a production inventory model with a three-level decentralized supply chain with price-sensitive demand. Haider *et al.* [18] presented an economic production quantity (EPQ) model in which they studied that if a discount is provided for a defective item and a rework process is applied then it was possible to derive maximum profit. Teksan and Geunes [41] studied an economic order quantity model for finished goods. In this model, they assumed that the demand rate was more price-sensitive for both, suppliers and customers. Taleizadeh *et al.* [2] outlined an imperfect production inventory model without shortages. Most recently, Pal and Adhikari [3] developed an imperfect production inventory model with exponential partial backlogging with rework. In this model, they assumed that all imperfect quality products are reworked after regular production process and demand rate was price-sensitive and it was monotonically decreasing function selling price.

1.2. Review on Discount Policy

An efficient and balanced discount schedule will reflect economical costs at both buyers and seller in the business. There are two general types of quantity discount schedule offered by supplier: all unit discounts and the incremental discount. Purchasing big quantities in all-units discount schedule results in small unit price of the whole lot; whereas, in incremental discount schedule, the small unit facility is available only to units purchased above a specified quantity. Bastain [16] described a dynamic lot-size problem under discounting which allows a speculative motion for holding commodity. He derived a method that determined the first lot-size decision in a rolling horizon environment, using forecast data of the minimum possible number of future periods. Martin [9] Martin gave an alternative perspective on the quantity discount - pricing problem. He generalised the multiple price break excluding the buyers operating parameter from consideration, with the exception of price dependent demand.

The active area of research in inventory models is a model with temporary price discount. Carlson *et al.* [17] derived an EOQ and optimal quantity model under both all-units and incremental quantity discount when, ordering cost, holding cost and purchase cost are incurred on date-terms supplier credit. Payment dates for the three cost components need not to be same. Bakar [29] described an inventory model in which he developed discount scenario for placing special order at discounted price when the company's regular ordering cycle coincides with the end of the discount period. Wee and Yu [40] emphasized the fact that some items may deteriorate during shortage. Models for exponentially deteriorating items with temporary price discount were considered under regular and non-regular replenishment time. The main aim was to maximise total savings during the temporary price discount period. Aucamp and Kuzdrall [6]introduced an inventory model in which they formulated the order quantity which minimised discounted cash flows for a one time sale. When the sale was consummated, the current inventory may be at or might exceed the used reorder point. In the later case, the company may decide to buy nothing, if large minimum order quantity is required in order to obtain price discount.

Fear vengeance from stronger competitors to temporary price changes designed to stimulate demand often leads vendors to search for alternate financial incentive to be used as substitutes or in conjunction with price discounts. Arcelus *et al.* [7] studied the retailers pricing, credit and inventory policies for deteriorating items in response to temporary price incentive model. In this model they discussed the retailer's profit-maximizing retail promotion strategy, when confronted with a vendor's trade promotion offer of credit and price discount on the purchase of regular or perishable items. In the same vein, Shah *et al.* [4] introduced an EOQ model for time-dependent deterioration rate with temporary price discount. In this model he considered a temporary price discount when items in an inventory system are subject to deterioration with respect to time. Abad [22] presented an inventory model,in which they characterized the buyer's response to temporary price reduction. They outlined a search procedure for determining the optimal purchase lot size for the buyer in response to the temporary price discount offered by the supplier.

The offer of price discount by the supplier increases the demand and attracts more retailers, as well as increase the cash-flow. Several researchers have studied temporary price discount and proposed various inventory models gain deeper insight into the relationship between price discounts and order policy. Shah [15] suggested an inventory model in which demand depends on price and discount policy. Tripathi and Tomar [31] established an inventory model with optimal order policy for deteriorating items with time-dependent demand in response to a temporary price discount linked to order quantity. In this study, they discussed the possible effects of a temporary price discount offered by the suppliers' replenishment policy for defective items.

We have considered an imperfect production inventory model with a discount for selling price. We assume that every defective product is reworked and no scrap product is produced during production as well as reworking runtime. We allowed shortage which is partially backlogged at the beginning and considering backlogging rate as variable and impatient behaviour of the customer. The price of goods is definitely shown to the customer at the beginning of the time cycle in many situations. So it is very difficult to take different price within the same inventory cycle. In this paper we deal with four issues: first, what will be the selling price of items, second how much the discount is in selling price, third, how much inventory should be produced and fourth, at what time period shortage would be allowed in order to maximize the expected average total profit.

2. ASSUMPTIONS & NOTATIONS

We have designed the proposed model by using the following assumptions and notations:

Assumption

1. This model is designed for an infinite time horizon,
2. This model is developed for a single item,
3. Production rate if perfect item p is constant and production rate of imperfect quality items is $p_d = xp$, where x is continuous random variable,
4. In this model shortages occur at the beginning of the cycle and during the shortage time interval, a fraction of the demand varying with waiting time is backlogged for the clients, who have the patience to wait, assume that customer's impatient function is
$B(\tau) = e^{-\alpha\tau}$, $\alpha > 0$,
5. After the regular production process all imperfect quality items are reworked,
6. The holding cost for both perfect and imperfect items are same,
7. Every constant cost like inspection cost and purchasing cost are included within the production cost of the items,
8. The demand function of the model is $D(s) = \varphi(\xi s)^{-\eta}$.

Notation

$D(s)$ – Demand function for good products,
$I(t)$ – On-hand inventory of product at time t in j^{th} cycle,
p – Production rate for perfect item unit per unit time,
p_d – Production rate for imperfect quantity items unit per unit time,
x – Percentage of produced imperfect quality items which is random variable,
$f(x)$ – Probability density function of x,
r – Rework rate of imperfect quality item per units per unit time,
ω – Backorder level,
$B(\tau)$ – Customers impatient function, where τ is the waiting time of a customer,
c_h – Holding cost per item per unit time,
c_{h_1} – Holding cost of reworked item per item per unit time,
c_p – Production cost per unit of item,
c_b – Backorder cost per item,
c_k – Per production set-up cost,
c_l – Lost sale cost per item,
s – Selling price per item,
ξ – Discount rate,
η – Price parameter of demand function,
φ – Stock dependent parameter,
Π – The total profit,
Π_{ATP} – Average total profit,
Π_{EATP} – Excepted average total profit.

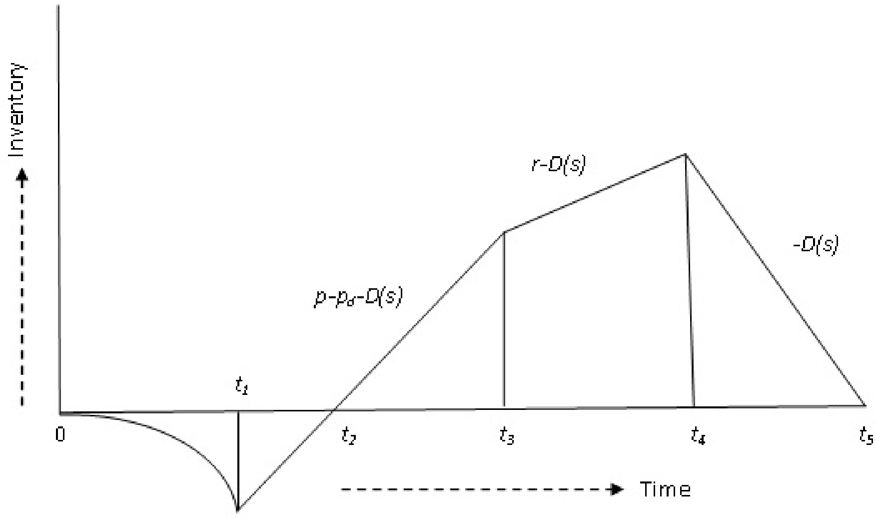

Figure 1: Logistic diagram of EPQ model with shortage of non defective items

3. THE MATHEMATICAL MODEL

Suppose a business starts with the shortage of products which are partially backlogged. The backlogging rate depends on waiting time of customer, that is $B(\tau) = e^{-a\tau}$, $a > 0$, where τ is waiting time and $\tau = t_1 - t$. Suppose the production starts at time t_1 and it continues up to time t_3. Due to production run, all the products which are backlogged, during time period $[0, t_2]$ are delivered at time t_2. The production process is not hundred percent perfect, however the production rate is considered constant. The qx amount of defective item is produced by the total production. The rework rate of defective products is r, and these are reworked after the regular production process. $\frac{qx}{r}$ is the amount of time required for reworking of defective products, where qx is total items produced and r is rework rate. There is the same price for good products and reworked product as well as the demand rate depends on selling price and defined as,

$$D(s) = \varphi(\xi s)^{-\eta} \tag{1}$$

We take $T_i = t_i - t_{i-1}$.
From Figure 1,
for the time period $0 \leq t \leq t_1$, the differential equation governing the inventory level is

$$\frac{dI}{dt} = -D(s)B(\tau) \tag{2}$$

with the boundary condition$I(0) = 0, I(t_1) = -\omega$ and $\tau = t_1 - t$.
The solution of above differential equation by using the boundary condition is

$$I(t) = \frac{D(s)e^{-at} - e^{a(t_1-t)}}{a} \tag{3}$$

and using the boundary condition $I(t_1) = -\omega$, we get

$$\omega = \frac{D(s)(1-e^{-at_1})}{a} \tag{4}$$

The backorder cost during $0 \leq t \leq t_1$ is

$$c_b \int_0^{t_1} (I(t))dt = \frac{c_b D(s)\left\{1 - at_1 e^{-at_1} - e^{-at_1}\right\}}{a^2} \tag{5}$$

The demand rate is D(s), out of this only $D(s)e^{-a(t_1-t)}$ is fullfilled during $[0, t_1]$ and $D(s) - D(s)e^{-a(t_1-t)}$ which not fullfilled. Then the cost of lost sale is given by

$$c_l \int_0^{t_1} D(s)\big(1 - e^{-a(t_1-t)}\big)dt = \frac{c_l D(s)\left\{at_1 - 1 + e^{-at_1}\right\}}{a} \tag{6}$$

For the time interval $t_1 \leq t \leq t_2$, the governing differential equation of inventory level is

$$\frac{dI}{dt} = p - p_d - D(s) \tag{7}$$

with boundary condition $I(t_1) = -\omega$ and $I(t_2) = 0$
Then the solution of above differential equation is

$$I(t) = \left\{(1-x)p - D(s)\right\}(t - t_2) \tag{8}$$

using the condition $I(t) = -\omega$, we have

$$\omega = \left\{(1-x)p - D(s)\right\}T_2, \tag{9}$$

where $T_2 = t_2 - t_1$
The cost of backorder in time interval $t_1 \leq t \leq t_2$ is

$$c_b \int_0^{t_1} I(t)dt = \frac{c_b \omega T_2}{2} \tag{10}$$

Eq. (9) & Eq. (10) leads the back order cost during $t_1 \leq t \leq t_2$

$$= \frac{c_b \omega^2}{2\left\{(1-x)p - D(s)\right\}} \tag{11}$$

For the time interval $t_2 \leq t \leq t_3$, the governing differential equation of inventory level is

$$\frac{dI}{dt} = p - p_d - D(s) \tag{12}$$

with boundary condition $I(t_2) = 0$, $I(t_3) = z_3$, where z_3 is inventory level of good product.
Then the solution of above differential equation is

$$I(t) = \Big\{(1-x)p - D(s)\Big\}(t - t_2) \tag{13}$$

using $I(t_3) = z_3$, we get

$$z_3 = \Big\{(1-x)p - D(s)\Big\}T_3 \tag{14}$$

The holding cost of good item for the time period $t_2 \leq t \leq t_3$ is

$$c_h \int_{t_2}^{t_3} I(t)dt = \frac{c_h z_3 T_3}{2} \tag{15}$$

Now $T_2 + T_3 = \frac{q}{p}$, using the Eq. (9) & Eq. (14) the holding cost is

$$= \frac{c_h}{2}\Big\{(1-x)p - D(s)\Big\}\frac{q^2}{p^2} - \frac{c_h q\omega}{p} + \frac{c_h\omega^2}{2\Big\{(1-x)p - D(s)\Big\}} \tag{16}$$

The differential equation for time period $t_3 \leq t \leq t_4$

$$\frac{dI}{dt} = r - D(s) \tag{17}$$

with boundary condition $I(t_3) = z_3$, $I(t_4) = z_4$, where z_4 is the highest inventory level of good items

$$I(t) = z_3 + \{r - D(s)\}(t - t_3) \tag{18}$$

by using the condition $I(t_4) = z_4$

$$z_4 - z_3 = \{r - D(s)\}T_4 \tag{19}$$

After some simplification and putting $T_4 = \frac{qx}{r}$, we get

$$z_4 = q\Big\{1 - \frac{D(s)(r+x)}{pr}\Big\} - \omega \tag{20}$$

Holding cost for good poducts for the time interval $t_3 \leq t \leq t_4$ is given by

$$c_h \int_{t_2}^{t_3} I(t)dt = \frac{c_h}{2}(z_3 + z_4)T_4 \tag{21}$$

Putting the value from Eq. (19) then holding cost

$$\begin{aligned}
&= \frac{c_h T_4}{2}\Big\{z_3 + z_3 + \big(r - D(s)\big)T_4\Big\} \\
&= c_h T_4 z_3 + \frac{c_h\{r - D(s)\}{T_4}^2}{2} \\
&= c_h\Big\{(1-x)p - D(s)\Big\}T_3 T_4 \quad \text{by Eq.(14)} \\
&= c_h\Big\{(1-x)p - D(s)\Big\}\Big\{\frac{q}{p} - \frac{\omega}{\big((1-x)p - D(s)\big)}\Big\}\frac{qx}{r} + \Big\{r - D(s)\Big\}\frac{q^2x^2}{r^2} \\
&= c_h\Big\{(1-x)p - D(s)\Big\}\frac{q^2x}{pr} - \frac{c_h\omega qx}{r} + \frac{c_h}{2}\Big\{r - D(s)\Big\}\frac{q^2x^2}{r^2}
\end{aligned} \tag{22}$$

Now from Figure 2, it can be seen that the defective products are produced during the time interval $t_1 \leq t \leq t_3$ at rate p_d. The defective products are reworked perfectly during the time interval $[t_3, t_4]$ by the rework rate r. In this system there are no defective items after time $t = t_4$.

The differential equation for time period $t_4 \leq t \leq t_5$, that shows inventory level is

$$\frac{dI}{dt} = -D(s) \tag{23}$$

with boundary conditions $I(t_4) = z_4$, $I(t_5) = 0$
Then the solution of this differential equation will be,

$$I(t) = D(s)(t_5 - t) \tag{24}$$

$$\text{By using } \; I(t) = z_4, \qquad z_4 = D(s)T_5 \tag{25}$$

Holding cost for the time interval $t_4 \leq t \leq t_5$ is given by

$$\begin{aligned}
c_h \int_{t_4}^{t_4} I(t)dt &= \frac{c_h}{2} z_4 T_5 \\
&= \frac{c_h {z_4}^2}{2D(s)} \\
&= \frac{c_h}{2D(s)}\left[q\Big\{1 - \frac{\beta(r+x)}{pr}\Big\} - \omega\right]^2
\end{aligned} \tag{26}$$

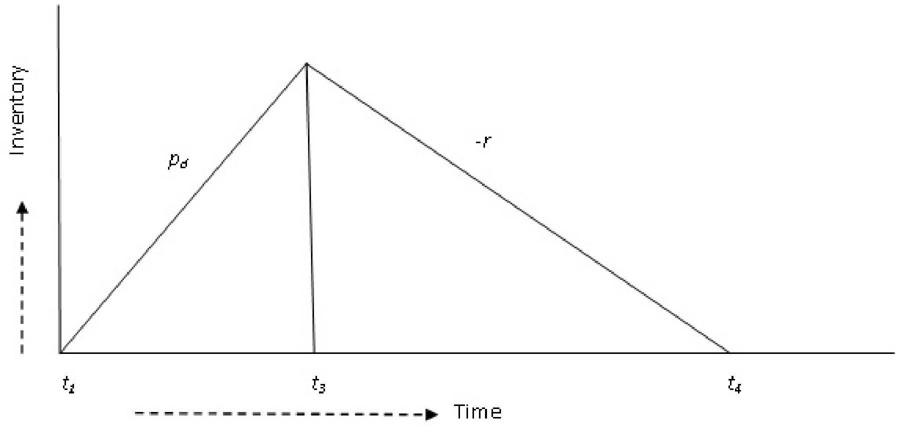

Figure 2: Logistic diagram of EPQ model of defective items

The inventory of defective products is given in Figure(2) then the differential equation for time period $t_1 \leq t \leq t_3$

$$\frac{dI_d}{dt} = p_d, \text{ with boudary condition } I_d(t_1) = 0, \ \ I_d(t_3) = qx \tag{27}$$

Then the solution is

$$I_d(t) = p_d(t - t_1) \tag{28}$$

Holding cost for the defective products is

$$c_h \int_{t_1}^{t_3} I_d(t)dt = \frac{c_h q^2 x}{2p} \tag{29}$$

For time interval $t_3 \leq t \leq t_4$ the governing differential equation inventory level of the defective item, is given by

$$\frac{dI_d}{dt} = -r, \text{ with boundary condition } I_d(t_3) = qx, \ \ I_d(t_4) = 0 \tag{30}$$

Then the solution is

$$I_d(t) = r(t_4 - t) \tag{31}$$

The holding cost of reworked items

$$c_{h_r} \int_{t_3}^{t_4} I_d(t)dt = \frac{c_{h_1} q^2 x^2}{2r} \tag{32}$$

The total profit = Revenue - total cost

The total profit = Revenue - (backorder cost + cost of lost sale + holding cost for good and defective products + holding cost for reworked items + purchase cost +repairing cost for defective items + set-up cost)

$$\begin{aligned}\Pi(q,t_1,s) =& sq - \frac{c_b D(s)\Big\{1-at_1e^{-at_1}-e^{-at_1}\Big\}}{a^2} - \frac{c_l D(s)\Big\{at_1-1+e^{-at_1}\Big\}}{a} \\ &- \frac{c_b\omega^2}{2\Big\{(1-x)p-D(s)\Big\}} - \frac{c_h}{2}(1-x)\frac{q^2}{p} + \frac{c_h}{2}\frac{D(s)q^2}{p^2} + \frac{c_h q\omega}{p} \\ &- \frac{c_h\omega^2}{2\Big\{(1-x)p-D(s)\Big\}} - c_h\Big\{(1-x)p-D(s)\Big\}\frac{q^2x}{pr} + \frac{c_h\omega qx}{r} \\ &- \frac{c_h}{2}\Big\{r-D(s)\Big\}\frac{q^2x^2}{r^2} - \frac{c_h}{2D(s)}\left[q\Big\{1-\frac{\beta(r+x)}{pr}\Big\}-\omega\right]^2 \\ &- \frac{c_hq^2x}{2p} - \frac{c_{h_1}q^2x^2}{2r} - c_pq - c_rqx - k\end{aligned} \tag{33}$$

The total average profit of the model,

$$\begin{aligned}\Pi_{ATP} =& \frac{D(s)}{q}\Pi(q,t_1,s) \\ =& \frac{D(s)}{q}\Bigg[sq - \frac{c_b D(s)\Big\{1-at_1e^{-at_1}-e^{-at_1}\Big\}}{a^2} - \frac{c_l D(s)\Big\{at_1-1+e^{-at_1}\Big\}}{a} \\ &- \frac{c_b\omega^2}{2\Big\{(1-x)p-D(s)\Big\}} - \frac{c_h}{2}(1-x)\frac{q^2}{p} + \frac{c_h}{2}\frac{D(s)q^2}{p^2} + \frac{c_h q\omega}{p} \\ &- \frac{c_h\omega^2}{2\Big\{(1-x)p-D(s)\Big\}} - c_h\Big\{(1-x)p-D(s)\Big\}\frac{q^2x}{pr} + \frac{c_h\omega qx}{r} \\ &- \frac{c_h}{2}\Big\{r-D(s)\Big\}\frac{q^2x^2}{r^2} - \frac{c_h}{2D(s)}\left[q\Big\{1-\frac{\beta(r+x)}{pr}\Big\}-\omega\right]^2 \\ &- \frac{c_hq^2x}{2p} - \frac{c_{h_1}q^2x^2}{2r} - c_pq - c_rqx - k\Bigg]\end{aligned} \tag{34}$$

The total expected average profit of the model,

$$\begin{aligned}\Pi_{EATP} =& \frac{D(s)}{q}\Bigg[sq - \frac{c_b D(s)\Big\{1 - at_1 e^{-at_1} - e^{-at_1}\Big\}}{a^2} - \frac{c_l D(s)\Big\{at_1 - 1 + e^{-at_1}\Big\}}{a} \\ &- \frac{c_b \omega^2}{2\Big\{(1-m)p - D(s)\Big\}} - \frac{c_h}{2}(1-m)\frac{q^2}{p} + \frac{c_h}{2}\frac{D(s)q^2}{p^2} + \frac{c_h q\omega}{p} \\ &- \frac{c_h \omega^2}{2\Big\{(1-m)p - D(s)\Big\}} - c_h\Big\{(1-m)p - D(s)\Big\}\frac{q^2 m}{pr} + \frac{c_h \omega q m}{r} \\ &- \frac{c_h}{2}\Big\{r - D(s)\Big\}\frac{q^2(m^2+\sigma^2)}{r^2} - \frac{c_h}{2D(s)}\left[q\Big\{1 - \frac{\beta(r+m)}{pr}\Big\} - \omega\right]^2 \\ &- \frac{c_h q^2 m}{2p} - \frac{c_{h_1} q^2(m^2+\sigma^2)}{2r} - c_p q - c_r q m - k\Bigg]\end{aligned} \tag{35}$$

From Eq. (4) & Eq. (35)

$$\Pi_{EATP} = f_1(q, s, t_1) = u_0(s) + u_1(s, t_1) + \frac{u_2(s, t_1)}{\Psi(s)q} \tag{36}$$

Where

$$\begin{aligned}u_0(s) =& x_{00} + x_{01}D(s) + x_{02}D(s)^2 \\ u_1(s, t_1) =& w_1(s) + w_2(s)e^{-at_1} \\ u_2(s, t_1) =& v_1(s)e^{-2at_1} + \big\{v_2(s) + t_1 v_3(s)\big\}e^{-at_1} + v_4(s)t_1 + v_5(s) \\ \Psi(s) =& 2a^2\big\{(1-m)p - D(s)\big\} \\ v_1(s) =& \lambda_{11}D(s)^2 + \lambda_{12}D(s)^3 \\ v_2(s) =& \lambda_{21}D(s) + \lambda_{22}D(s)^2 + \lambda_{22}D(s)^3 \\ v_3(s) =& \lambda_{31}D(s)^2 + \lambda_{32}D(s)^3 \\ v_4(s) =& \lambda_{41}D(s)^2 + \lambda_{42}D(s)^3 \\ v_5(s) =& \lambda_{51}D(s) + \lambda_{52}D(s)^2 + \lambda_{53}D(s)^3 \\ w_1(s) =& x_{11}D(s) + x_{12}D(s)^2 \\ w_2(s) =& x_{21}D(s) + x_{22}D(s)^2\end{aligned}$$

$$\lambda_{11} =-c_h p(1-m); \lambda_{12} = -c_b; \lambda_{21} = 0$$
$$\lambda_{22} =2c_b p(1-m) + 2\{c_h p(1-m) - c_l p(1-m)a\}; \lambda_{23} = 2c_l a$$
$$\lambda_{31} =2c_b p(1-m)a; \lambda_{32} = -2c_b a; \lambda_{41} = -2c_l p(1-m)a^2$$
$$\lambda_{51} =2kp(1-m)a^2; \lambda_{52} = 2c_b p(1-m) + \{-c_h p(1-m) + c_l p(1-m)a\}$$
$$\lambda_{53} =c_b - 2c_l a; x_{00} = \frac{-c_h}{2};$$
$$x_{01} =-\frac{c_h(1-m)}{2p} - \frac{c_h m}{2p} - \frac{c_h(1-m)m}{r} - \frac{c_h(m^2+\sigma^2)}{2r} - \frac{c_{h_1}(m^2+\sigma^2)}{2r} + \frac{c_h(m+r)}{pr}$$
$$x_{02} =\frac{c_h}{2p^2} + \frac{c_h m}{pr} + \frac{c_h(m^2+\sigma^2)}{2r^2} - \frac{c_h(m+r)^2}{2p^2r^2}; x_{11} = -c_p + s - c_r m + \frac{c_h}{a}$$
$$x_{12} =\frac{c_h m}{ra} - \frac{c_h m}{pra}; x_{21} = -\frac{c_h}{a}; x_{22} = -\frac{c_h m}{ra} + \frac{c_h m}{pra}$$

Proposition 1. *The profit function $f_1(q, s, t_1)$ is concave if the corresponding Hessian matrix H of expected profit function is negative definite.*
Where

$$H=\begin{pmatrix} \frac{\partial^2 f_1}{\partial q^2} & \frac{\partial^2 f_1}{\partial s\partial q} & \frac{\partial^2 f_1}{\partial q\partial t_1} \\ \frac{\partial^2 f_1}{\partial s\partial q} & \frac{\partial^2 f_1}{\partial s^2} & \frac{\partial^2 f_1}{\partial t_1\partial s} \\ \frac{\partial^2 f_1}{\partial q\partial t_1} & \frac{\partial^2 f_1}{\partial t_1\partial s} & \frac{\partial^2 f_1}{\partial {t_1}^2} \end{pmatrix}$$

Proof : We have

$$\Pi_{EATP} = f_1(q, s, t_1) = u_0(s) + u_1(s, t_1) + \frac{u_2(s, t_1)}{\Psi(s)q}$$

$$\frac{\partial f_1}{\partial q} =x_{00} + x_{01}D(s) + x_{02}D(s)^2 - \frac{v_1(s)e^{-2at_1} + \{v_2(s) + t_1v_3(s)\}e^{-at_1} + v_4(s)t_1 + v_5(s)}{q^2\Psi(s)}$$

$$\frac{\partial f_1}{\partial s} =w_1'(s) + w_2'(s)e^{-at_1} + q\{x_{01}D'(s) + 2x_{02}D'(s)D(s)\}$$
$$- \frac{\Big\{v_1(s)e^{-2at_1} + \{v_2(s) + t_1v_3(s)\}e^{-at_1} + v_4(s)t_1 + v_5(s)\Big\}\Psi'(s)}{q\Psi(s)^2}$$
$$+ \frac{v_1'(s)e^{-2at_1} + \{v_2'(s) + t_1v_3'(s)\}e^{-at_1} + v_4'(s)t_1 + v_5'(s)}{q\Psi(s)}$$

$$\frac{\partial f_1}{\partial t_1} =-aw_2(s)e^{-at_1} + \frac{-2av_1(s)e^{-2at_1} + e^{-at_1}v_3(s) - \{v_2(s) + t_1v_3(s)\}ae^{-at_1} + v_4(s)}{q\Psi(s)}$$

Solve above equations by putting

$$\frac{\partial f_1}{\partial q} = 0, \frac{\partial f_1}{\partial s} = 0, \frac{\partial f_1}{\partial t_1} = 0$$

and get the values of variable q, s, t_1

$$x_{00} + x_{01}D(s) + x_{02}D(s)^2 - \frac{v_1(s)e^{-2at_1} + \{v_2(s) + t_1v_3(s)\}e^{-at_1} + v_4(s)t_1 + v_5(s)}{q^2\Psi(s)} = 0$$

Then

$$q = \sqrt{\frac{v_1(s)e^{-2at_1} + \big\{v_2(s) + t_1 v_3(s)\big\}e^{-at_1} + v_4(s)t_1 + v_5(s)}{\big\{x_{00} + x_{01}D(s) + x_{02}D(s)^2\big\}\Psi(s)}} \tag{37}$$

Substituting the value of q in Eq. $\frac{\partial f_1}{\partial s} = 0$ & $\frac{\partial f_1}{\partial t_1} = 0$ and solving them, we get the solution of decision variable q, s, t_1 of the model.
If the second order condition of optimization method satisfies then above solution will be optimal.
Now the second order derivatives

$$\frac{\partial^2 f_1}{\partial q^2} = \frac{2\big[v_1(s)e^{-2at_1} + \big\{v_2(s) + t_1 v_3(s)\big\}e^{-at_1} + v_4(s)t_1 + v_5(s)\big]}{q^2\Psi(s)} \tag{38}$$

$$\frac{\partial^2 f_1}{\partial q \partial t_1} = -\frac{-2v_1(s)e^{-2at_1} + v_3(s)e^{-at_1} + v_4(s) - \big\{v_2(s) + t_1 v_3(s)\big\}ae^{-at_1}}{q^2\Psi(s)} \tag{39}$$

$$\begin{aligned}\frac{\partial^2 f_1}{\partial s^2} = &- \frac{2\Big\{v_1'(s)e^{-2at_1} + \big\{v_2'(s) + t_1 v_3'(s)\big\}e^{-at_1} + v_4'(s)t_1 + v_5'(s)\Big\}\Psi'(s)}{q\Psi(s)^2} \\ &+ \frac{2\Big\{v_1(s)e^{-2at_1} + \big\{v_2(s) + t_1 v_3(s)\big\}e^{-at_1} + v_4(s)t_1 + v_5(s)\Big\}\{\Psi'(s)\}^2}{q\Psi(s)^3} \\ &+ \frac{v_1''(s)e^{-2at_1} + \big\{v_2''(s) + t_1 v_3''(s)\big\}e^{-at_1} + v_4''(s)t_1 + v_5''(s)}{q\Psi(s)} \\ &+ w_1''(s) + w_2''(s)e^{-2at_1} + q\big[x_{01}D''(s) + 2x_{02}D''(s)D(s) + 2x_{02}\{D'(s)\}^2\big]\end{aligned} \tag{40}$$

$$\begin{aligned}\frac{\partial^2 f_1}{\partial s \partial q} = &- \frac{\Big\{v_1'(s)e^{-2at_1} + \big\{v_2'(s) + t_1 v_3'(s)\big\}e^{-at_1} + v_4'(s)t_1 + v_5'(s)\Big\}}{q^2\Psi(s)} \\ &+ \frac{\Big\{v_1(s)e^{-2at_1} + \big\{v_2(s) + t_1 v_3(s)\big\}e^{-at_1} + v_4(s)t_1 + v_5(s)\Big\}\{\Psi'(s)\}}{q\Psi(s)^2} \\ &+ 2x_{01}x_{02}D'(s) + D'(s)D(s)\end{aligned} \tag{41}$$

$$\frac{\partial^2 f_1}{\partial {t_1}^2} = a^2 w_2(s)e^{-at_1} + \frac{4a^2 v_1(s)e^{-2at_1} - 2e^{-at_1}v_3(s) + \big\{v_2(s) + t_1 v_3(s)\big\}a^2 e^{-at_1}}{q\Psi(s)} \tag{42}$$

$$\frac{\partial^2 f_1}{\partial s \partial t_1} = \frac{-2av_1'(s)e^{-2at_1} + v_3'(s)e^{-at_1} + v_4'(s) - \big\{v_2'(s) + t_1 v_3'(s)\big\}ae^{-at_1}}{q\Psi(s)} - aw_2'(s)e^{-at_1}$$
$$- \frac{\big[-2av_1(s)e^{-2at_1} + e^{-at_1}v_3(s) - \big\{v_2(s) + t_1 v_3(s)\big\}ae^{-at_1} + v_4(s)\big]\Psi'(s)}{q\Psi(s)^2} \tag{43}$$

putting all values of second derivatives in Hessian matrix

$$\text{H}= \begin{pmatrix} \frac{\partial^2 f_1}{\partial q^2} & \frac{\partial^2 f_1}{\partial s \partial q} & \frac{\partial^2 f_1}{\partial q \partial t_1} \\ \frac{\partial^2 f_1}{\partial s \partial q} & \frac{\partial^2 f_1}{\partial s^2} & \frac{\partial^2 f_1}{\partial t_1 \partial s} \\ \frac{\partial^2 f_1}{\partial q \partial t_1} & \frac{\partial^2 f_1}{\partial t_1 \partial s} & \frac{\partial^2 f_1}{\partial {t_1}^2} \end{pmatrix}$$

and solve, if all eigen values are negative i.e Hessian matrix of expected profit function is negative definite, then the profit function is concave.

4. NUMERICAL EXAMPLE & SENSITIVITY ANALYSIS

Consider two numerical example taking the demand function as given in Eq. (1). First Ex. is based on No discount in selling price and another one is based on discount on selling price.

Example 1 We consider the demand function D(s) as $D(s) = \varphi(\xi s)^{-\eta}$ and the value of the parameter in appropriate units are $\eta = 1.2$, $c_l = 1.5$ unit per unit time, $c_b = 0.5$ unit per unit time, $k = 600$, $c_h = 0.2$ unit per unit time, $c_{h_1} = .9$ unit per unit time, $c_r = 1.5$ per unit, $c_p = 4$ per unit, $\varphi = 1400$, $r = 1100$ units per unit time, $\alpha = .8$, $\xi = 1$, $m = 0.05$, $\sigma^2 = \frac{1}{1100}$, $p = 800$ units per unit time, and randam variable which follows uniform distribution in the interval $(0, 0.1)$. Then the optimal values for the model are $f_1^* = 540.81$, $s^* = 34.42$, $q^* = 361$ and $t_1^* = 1.31$. These values are optimal as the eigen value of the Hessian matrix H are negative. i.e -1.510, -0.12, -0.00042. So the profit function is concave.

Example 2 By using above data of Ex.1 and giving discount 20% (i.e $\xi = .2$) on selling price, then the optimal values for the model are $f_1^* = 4038.38$, $s^* = 27.87$, $q^* = 1378.73$ and $t_1^* = 0.65$. These values are optimal as the eigen value of the Hessian matrix H are negative. i.e -37.87, -1.44, -0.00009. So the profit function is concave.
Clearly, the profit function by giving discount in price is $f_1^* = 4038.38$, is more than the profit function $f_1^* = 540.81$ without giving discount in price. Hence the discount pricing policy performs in diclining market.

4.1. Sensitivity Analysis

We observe the sensitivity of the key parameters that help decision makers to take appropriate decision on their marketing strategy.

On increasing the power parameter of demand function in discount pricing policy, the profit function decreases continuously and shortage period also reduces (table 1). This ravels that the product having less value of power parameter gives better result in discount price policy. The similar results obtained in without discount policy.

If the demand function parameter φ increases, the expected average profit, and lot size increase, while the selling price and shortage period decreases in both discount and without discount policy (table 2). Also, demand follows the same pattern. However the profit, by giving 20% discount on selling price, is more than without discount.

From Table 3, we noticed that in discount and without discount situation both, the optimal lot size, shortage period and selling price are increasing with increasing set-up cost, and we also find that expected profit decreases with increasing set-up cost. On increasing the holding cost of lot size, shortages decreases for both discount and without discount policy (table 4). This analysis that less holding cost permit to store more items in a warehouse or in a shop. The same result followed for perfect and imperfect items.

Besides this on increasing holding cost the selling price of items increases accordingly. This reveals that an increase in the cost of item provides high selling price and this shows the robustness of the proposed model. The graphical presentation of the model shows that profit function is concave with respect to the required quantity and price. Also the profit function is concave with respect to required time and price. There is a minor decreasing change in expected average profit. It is clear that higher holding cost provides less lot size. So smaller commodity causing increases the shortage period. In this situation, the expected average total profit is in decreasing order.

Now we have followed the graphical analysis method in three-dimensional (3D) plots for the profit function Π_{EATP}. Figure 3-4 presents the piecewise 3D plots for the profit function, versus the two corresponding variables. The profit function Π_{EATP} is concave function in terms of s and q (Fig 3). Also the profit function Π_{EATP} is concave function in terms of s and t_1 (Fig 4).

Table 1: Sensitive analysis for parameter η

η		s	q	t_1	f_1
1.2	No discount	34.42	361	1.31	540.81
	Discount	27.86	1378	0.65	4038.37
1.3	No discount	24.56	378	1.27	376.68
	Discount	19.84	1738	0.61	3448.61
1.4	No discount	19.91	373	1.28	267.18
	Discount	15.90	2060	0.59	3026.97
1.5	No discount	17.28	356	1.32	190.90
	Discount	13.56	2362	0.59	2705.72
1.6	No discount	15.65	332	1.37	136.37
	Discount	12.02	2650	0.58	2452.74

Table 2: Sensitive analysis for parameter φ

φ		s	q	t_1	f_1
1400	No discount	34.42	361	1.31	540.81
	Discount	27.86	1378	0.65	4038.37
1600	No discount	33.64	393	1.24	623.13
	Discount	27.64	1552	0.62	4629.86
1800	No discount	33.02	425.01	1.18	705.78
	Discount	27.47	1736	0.61	5222.18
2000	No discount	32.50	454	1.13	788.73
	Discount	27.33	1933	0.60	5815.18
2200	discount	32.07	483	1.09	871.92
	Discount	27.22	2147	0.59	6408.73

Table 3: Sensitive analysis for setup cost k

k		s	q	t_1	f_1
600	No discount	34.42	361	1.31	540.81
	Discount	27.86	1378	0.65	4038.37
700	No discount	35.42	382	1.39	535.37
	Discount	28.16	1471	0.69	4022.18
800	No discount	36.38	402	1.47	530.40
	Discount	28.44	1556	0.73	4007.18
900	No discount	37.31	419	1.54	525.79
	Discount	28.71	1634	0.77	3993.17

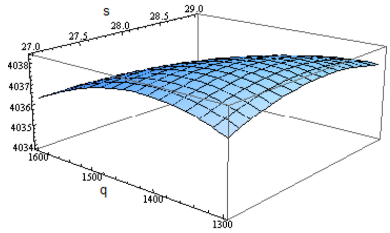

Figure 3: Total profit versus quantity and price

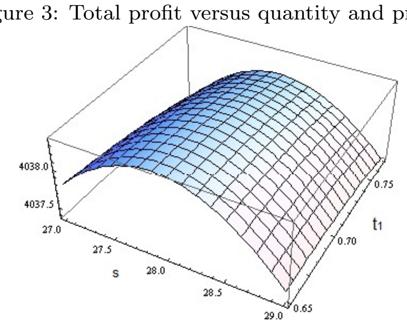

Figure 4: Total profit versus time and price

Table 4: Sensitive analysis for holding cost c_h & c_{h_1}

c_h		s	q	t_1	f_1
0.2	No discount	34.42	361	1.31	540.81
	Discount	27.86	1378	0.65	4038.37
0.3	No discount	36.98	283	1.52	526.83
	Discount	28.55	1114	0.76	3998.59
0.4	No discount	39.22	236	1.69	515.74
	Discount	29.10	958	0.84	3967.04
0.5	No discount	41.25	205	1.82	506.64
	Discount	29.56	853	0.91	3940.7

5. CONCLUSION AND RECOMMENDATIONS

We have developed a price-discount policy for declining market by considering demand as a negative power function of the selling price. The sensitive analysis provided for discount price policy has less time period than without discount price policy. This reveals the discount price policy might offer a shorter period for clearance of stock or at a festival time. The shortage occurs at the beginning because of more increased cost, but it helps to protect the product and optimize the selling price. The optimal profit for discount pricing policy is more than without discount pricing policy. The manufacturer has to take decision for giving a discount at different occasions.

Sensitivity on holding cost reveals that less holding cost permits to store more items in a warehouse or in a shop. On increasing holding cost the selling price of items increases. This reveals that high increasing cost of the item provides high selling price and this shows the robustness of the proposed model. One can formulate the proposed model in a fuzzy environment. It also could be considered as stochastic demand and variable holding cost.

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