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LINEAR PROGRAMMING PROBLEMS WITH SOME MULTI-CHOICE FUZZY PARAMETERS

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Abstract: In this paper, we consider some Multi-choice linear programming (MCLP) problems where the alternative values of the multi-choice parameters are fuzzy numbers. There are some real-life situations where we need to choose a value for a parameter from a set of different choices to optimize our objective, and those values of the parameters can be imprecise or fuzzy. We formulate these situations as a mathematical model by using some fuzzy numbers for the alternatives. A defuzzification method based on incentre point of a triangle has been used to find the defuzzified values of the fuzzy numbers. We determine an equivalent crisp multi-choice linear programming model. To tackle the multi-choice parameters, we use Lagranges interpolating polynomials. Then, we establish a transformed mixed integer nonlinear programming problem. By solving the transformed non-linear programming model, we obtain the optimal solution for the original problem. Finally, two numerical examples are presented to demonstrate the proposed model and methodology.

Keywords: Linear Programming, Triangular Fuzzy Number, Trapezoidal Fuzzy Number, Multi-choice Programming, Fuzzy Programming.

MSC: 90C05, 90C11, 90C70.

1. INTRODUCTION

In real life decision making situations, we face several types of optimization problems. The articles related to both optimization methods and models can be found very frequently in the literature of optimization. Every new real-life decision-making situation always needs a new type of optimization model and methodology to be developed in order to resolve the situation. In general, these optimization models are known as mathematical programming models.

However, when modeling this kind of decision-making situation into a mathematical programming problem, we have to provide values of the parameters as the input. To do so, we ask the experts to give us the information for the parameters of the model. From the practical point of view, he/she may provide us with some vague values. e.g., for the cost coefficient included in the objective function, we are told that the first cost coefficient is about 10$, the second one is more than 15$, etc. Also, the decision maker may have several alternative values for any parameter of the problem. Then, this kind of parameter is known as the multi-choice parameter, and the linear programming (LP) problem with this type of parameter is known as multi-choice linear programming problem [10]. In these problems, a decision maker needs to find the suitable value for each parameter from the corresponding set of alternative values so that the objective function attains its optimal value. The methods to solve this kind of problem are proposed in [10, 11, 12].

In the previous paragraph, we have described that in many practical/real-life situations, knowledge about the data (i.e., values of the parameters of a model) is not purely deterministic but rather vagu. So, we may have a situation where multi-choiceness and fuzziness are present at the same time in the model. For example, suppose a production company produces different types of products and sells them in the market directly. The company wants to maximize the profit subject to the demand for each product in the market. Now, for each product, there are several markets where it can be sold but the company has to choose one of the markets to sell the particular product. So, multi-choiceness is in the process of selecting a market for the specific product. Also, the selling prices and the demands for these products in different markets are different. Now depending upon the market modes, the company can set the selling prices and demands as crisp numbers or as fuzzy numbers. These types of optimization problems are related to multi-choiceness and fuzziness.

Zadeh [17] was the first researcher to introduce the fuzzy set theoretic concept, which Zimmermann [15] applied to solve linear programming with several objective functions by considering the fuzziness in the constraints. Buckley [14] used possibility distribution for solving the fuzzy linear programming problem. Tanaka et al. [18] dealt with an LP problem with fuzzy parameters in the constraints. Delgado et al. [19] studied a general model for fuzzy LP problems in which the technological and resource parameters of the constraints are fuzzy numbers. Afterward, several authors considered various types of fuzzy LP problems and proposed several approaches to solve them. In the field of fuzzy linear programming, research is going on in two categories:
(*i*) Fuzziness in decision parameters
(*ii*) Fuzziness in decision variables,
However, in most cases, to solve the fuzzy LP, the model is reduced to a crisp programming problem. Defuzzification methods are very popular to establish these

crisp models for fuzzy LP. The most commonly used defuzzification method is ranking fuzzy numbers method. Various types of ranking functions have been introduced in the literature [2, 3, 13, 20], and some of them have been used for solving LP problems with fuzzy parameters.

Aggarwal and Sharma [22] have considered a fully fuzzy multi-objective programming problem where the parameters and decision variables are fuzzy variables. The resource parameters of the constraints are multi-choice with only two alternatives, represented by fuzzy numbers. They use ranking function to get the crisp value of the fuzzy number. In this paper, we have developed a methodology to solve a general multi-choice linear programming problem where each alternative value of the multi-choice parameters are considered as trapezoidal fuzzy numbers and/or triangular fuzzy numbers. A defuzzification method based on the incentre point of a triangle is used to find the crisp value of a fuzzy number.

2. PRELIMINARIES

In this Section, we present some basic concepts of fuzzy sets and fuzzy numbers so as some operations on fuzzy numbers. (**Fuzzy Set:**) Let X be the universe which is a classical set of objects, and its generic elements are denoted by x. A fuzzy set $\widetilde{A}$ of X can be represented as $\tilde{A} = \{(x, \mu_{\tilde{A}}(x))|x \in X\}$, where $\mu_{\tilde{A}} : X \to [0,1]$. Also $\mu_{\tilde{A}}(x)$ is called membership function. A fuzzy set $\tilde{A}$ is called normal if and only if $\sup_x \mu_{\tilde{A}}(x)$=1. (**Convex Fuzzy Set:**) A fuzzy set $\tilde{A}$ in X is called a convex set if and only if for every pair point x_1 and x_2 in X, the membership function of $\tilde{A}$ satisfies the inequality:

$$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)), \quad \textit{where} \quad \lambda \in [0,1].$$

(α**-level set:**) For a given $\alpha \in [0,1]$, the α-level set of a fuzzy set $\tilde{A}$ is defined as an ordinary set A_α of elements x such that the membership function value $\mu_{\tilde{A}}(x)$ of x exceeds α, i.e.,

$$A_\alpha = \{x \in X|\mu_{\tilde{A}}(x) \geq \alpha\}. \tag{1}$$

(**Fuzzy Number:**)[7] Let $\tilde{A}$ be a fuzzy set and its membership function is given by $\mu_{\tilde{A}} : X \to [0,1]$ if
(*i*) $\tilde{A}$ is upper semi-continuous.
(*ii*) $\tilde{A}$ is normal.
(*iii*) $\tilde{A}$ is convex.
(*iv*) The closed convex hull of $\tilde{A}$, $\tilde{A}_0 = cl[co\{x \in X, \mu_{\tilde{A}}(x) > 0\}]$ is cored.
then A is a fuzzy number. (**LR Fuzzy Number:**)[7] A function denoted by L or R is called a reference function of fuzzy numbers iff
(*i*) $L(x) = L(-x)$;
(*ii*) $L(0) = 1$;
(*iii*) $L(x)$ is non increasing on $[0, +\infty)$.

Let $L(x)$ and $R(x)$ be the reference functions of fuzzy number $\tilde{A}$, then it is said to be an LR type fuzzy number iff

$$\mu_{\tilde{A}_1}(x) = \begin{cases} L(\frac{m-x}{\alpha}), & x \leq m, \alpha > 0; \\ R(\frac{x-m}{\beta}), & m \leq x, \beta > 0; \end{cases} \tag{2}$$

where m is called the *mean* of the fuzzy number $\tilde{A}$ and α, β are the left and right *spread*, respectively. LR fuzzy numbers are denoted by (m, α, β).

(**Triangular Fuzzy Number**)[7] A fuzzy number denoted by the triplet $\tilde{A}=(a, b, c)$ is a triangular fuzzy number with a piecewise linear membership function $\mu_{\tilde{A}}(x)$, defined by:

$$\mu_{\tilde{A}}(x)=\begin{cases} \frac{x-a}{b-a}, & a \leq x \leq b; \\ \frac{c-x}{c-b}, & b \leq x \leq c; \\ 0, & \text{otherwise.} \end{cases}$$

(**Trapezoidal Fuzzy Number)**[7] A fuzzy number denoted by the quartet $\tilde{A}=(a, b, c, d)$ is a trapezoidal fuzzy number with a piecewise linear membership function $\mu_{\tilde{A}}(x)$, defined by:

$$\mu_{\tilde{A}}(x)=\begin{cases} \frac{x-a}{b-a}, & a \leq x \leq b; \\ 1, & b \leq x \leq c; \\ \frac{d-x}{d-c}, & c \leq x \leq d; \\ 0, & \text{otherwise.} \end{cases}$$

2.1. Operations of Fuzzy Number

Based on the extension principle[17], arithmetic operations on the fuzzy numbers can be defined as: If $\tilde{A}_1$ and $\tilde{A}_2$ are two fuzzy numbers and $'*'$ is a binary operation $(+, -, \times, /)$, then membership of $\tilde{A}_1(*)\tilde{A}_2$ is defined as:

$$\mu_{\tilde{A}_1(*)\tilde{A}_2}(z) = \sup_{\substack{z=x*y \\ x\in\tilde{A}_1, y\in\tilde{A}_2}} \min\{\mu_{\tilde{A}_1}(x), \mu_{\tilde{A}_2}(y)\} \tag{3}$$

Some important properties of operations on triangular (trapezoidal) fuzzy number are as follows:
(*i*) The results from addition or subtraction between triangular (trapezoidal) fuzzy numbers are also triangular (trapezoidal) fuzzy numbers.
(*ii*) The results from multiplication or division are not triangular (trapezoidal) fuzzy numbers.
(*iii*) Max or min operation does not give triangular (trapezoidal) fuzzy number.
If we consider two triangular fuzzy numbers $\tilde{A}_1 = (a_1, b_1, c_1)$ and $\tilde{A}_2 = (a_2, b_2, c_2)$ then,

$$\tilde{A}_1(+)\tilde{A}_2 = (a_1 + a_2, b_1 + b_2, c_1 + c_2) \tag{4}$$

$$\tilde{A}_1(-)\tilde{A}_2 = (a_1 - a_2, b_1 - b_2, c_1 - c_2) \tag{5}$$

and $\tilde{A}_1(\times)\tilde{A}_2 = \tilde{B}$, where membership function of $\tilde{B}$ is given by [8]:

$$\mu_{\tilde{B}}(y) = \begin{cases} \frac{-(a_1b_2+a_2b_1-2a_1b_2)+\sqrt{(a_1b_2-a_2b_1)^2+4(b1-a_1)(b_2-a_2)y}}{2(b1-a_1)(b_2-a_2)}, & a_1a_2 \le y \le b_1b_2; \\ \frac{-(c_1b_2+c_2b_1-2c_1c_2)+\sqrt{(c_1b_2-b_1c_2)^2+4(c_1-b_1)(c_2-b_2)y}}{2(c_1-b_1)(c_2-b_2)}, & b_1b_2 \le y \le c_1c_2; \\ 0, & \text{otherwise}. \end{cases} \quad (6)$$

If we consider two trapezoidal fuzzy numbers $\tilde{A}_1 = (a_1, a_2, a_3, a_4)$ and $\tilde{A}_2 = (b_1, b_2, b_3, b_4)$ then,

$$\tilde{A}_1(+)\tilde{A}_2 = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4) \quad (7)$$

$$\tilde{A}_1(-)\tilde{A}_2 = (a_1 - b_2, a_2 - b_1, a_3 + b_3, a_4 + b_4) \quad (8)$$

and $\tilde{A}_1(\times)\tilde{A}_2 = \tilde{B}$, where membership function of $\tilde{B}$ is given by [9]:

$$\mu_{\tilde{B}}(y) = \begin{cases} \frac{-(a_1b_2+a_2b_1-2a_1b_1)+\sqrt{(a_1b_2-a_2b_1)^2+4(a_2-a_1)(b_2-b_1)y}}{2(a_2-a_1)(b_2-b_1)}, & a_1b_1 \le y \le a_2b_2; \\ 1, & a_2b_2 \le y \le a_3b_3; \\ \frac{-(a_4b_3+a_3b_4-2a_4b_4)+\sqrt{(a_4b_3-a_3b_4)^2+4(a_3-a_4)(b_3-b_4)y}}{2(a_3-a_4)(b_3-b_4)}, & a_3b_3 \le y \le a_4b_4; \\ 0, & \text{otherwise}. \end{cases} \quad (9)$$

3. MATHEMATICAL MODEL OF MULTI-CHOICE FUZZY LINEAR PROGRAMMING PROBLEM

(Multi-choice Fuzzy Parameter): If a parameter requires to choose a value from a set of different fuzzy numbers, then that parameter is called multi-choice fuzzy parameter. A multi-choice fuzzy linear programming problem is a modified linear programming problem where the objective function and the constraints contain some multi-choice fuzzy parameters. i.e., each alternative value of a multi-choice parameter is a fuzzy number. The mathematical model of the multi-choice fuzzy linear programming problem can be formulated as:

$$\max / \min : z = \sum_{j=1}^{n} \{\tilde{c}_j^{(1)}, \tilde{c}_j^{(2)}, \ldots, \tilde{c}_j^{(k_j)}\} x_j \quad (10)$$

subject to

$$\sum_{j=1}^{n} \{\tilde{a}_{ij}^{(1)}, \tilde{a}_{ij}^{(2)}, \ldots, \tilde{a}_{ij}^{(p_{ij})}\} x_j \le \{\tilde{b}_i^{(1)}, \tilde{b}_i^{(2)}, \ldots, \tilde{b}_i^{(r_i)}\}, \qquad i = 1, 2, \ldots, m \quad (11)$$

$$x_j \ge 0, \quad j = 1, 2, 3, ..., n \quad (12)$$

In the formulated model, alternative value $\tilde{c}_j^{(l)}$, $l = 1, 2, \ldots, k_j$ of multi-choice cost coefficient c_j, $j = 1, 2, \ldots, n$, $\tilde{a}_{ij}^{(s)}$, $s = 1, 2, \ldots, p_{ij}$ of multi-choice technological coefficient $a_{ij}, i = 1, 2, \ldots, m; j = 1, 2, \ldots, n$, and $\tilde{b}_i^{(t)}$, $t = 1, 2, \ldots, r_i$ of multi-choice resource variable $b_i, i = 1, 2, \ldots, m$ are considered as either triangular or trapezoidal fuzzy number (for the purpose of discussion). The decision variables x_j ($j = 1, 2, \ldots, n$) are deterministic in the problem.

4. PROPOSED METHODOLOGY

The parametric space of the multi-choice fuzzy linear programming problem (10)-(12) contains multi-choice fuzzy parameters. Hence, we are not able to apply any direct method to solve this problem. So, we developed a methodology to tackle the multi-choice fuzzy parameters. To find the crisp values of the fuzzy numbers, we use the concept of defuzzification based on incentre point of a triangle [24]. Then, to tackle the multi-choice parameters with crisp value, we use Lagrange's interpolating polynomial approach. The methodology is discussed in the following subsections.

4.1. Defuzzification of the Fuzzy Number Based incentre point

Incentre of a triangle is the intersecting point of the three angle bisector of the triangle. It is a unique point of a triangle that is also the center of the inscribed circle of the triangle. To ranking the fuzzy numbers, the concept of incentre point of a triangle is used by Rouhparvar et al. [24]. We use this concept of defuzzification based on incentre point of a triangle to defuzzify the fuzzy numbers and to establish the crisp model of the problem. According to the definition, the incentre ($I = (I_x, I_y)$) of a triangle ABC is calculated as:

$$I_x = \frac{A_x|BC| + B_x|AC| + C_x|AB|}{|AB| + |BC| + |CA|} \qquad I_y = \frac{A_y|BC| + B_y|AC| + C_y|AB|}{|AB| + |BC| + |CA|} \tag{13}$$

where A_x, B_x, C_x are the x-coordinates, and A_y, B_y, C_y are the y-coordinates of the vertices of the triangle.

Now, we define the defuzzified values for the different kinds of fuzzy numbers.

4.1.1. Defuzzifier of the Triangular Fuzzy Number

Let $\tilde{A}$=(a, b, c) be triangular fuzzy number. The incentre of the triangular fuzzy number $\tilde{A}$ is shown in Fig 1.

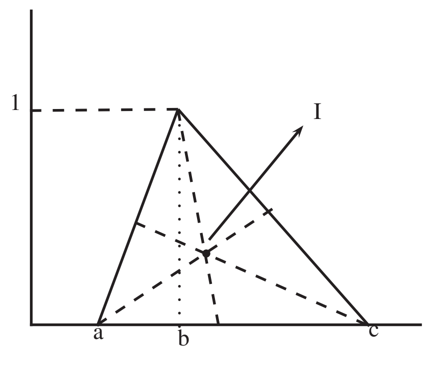

Figure 1: Incentre of the triangular fuzzy number $\tilde{A}$

The defuzzifier of the fuzzy number $\tilde{A}$ is calculated as:

$$A_{defuzz} = \frac{a\sqrt{1+(c-b)^2} + b(c-a) + c\sqrt{1+(b-a)^2}}{\sqrt{1+(c-b)^2} + (c-a) + \sqrt{1+(b-a)^2}} \tag{14}$$

If $\tilde{A}$=(a, b, c) is the generalized triangular LR-fuzzy number, then the defuzzifier of $\tilde{A}$ is given by:

$$A_{defuzz} = \frac{a\alpha + b\beta + c\gamma}{\alpha + \beta + \gamma} \tag{15}$$

where $\alpha = \int_b^c \sqrt{1+(A_R(x))'}dx$, $\beta = (c-a)$ and $\gamma = \int_a^b \sqrt{1+(A_L(x))'}dx$

4.1.2. Defuzzifier of the Trapezoidal Fuzzy Number

Let us consider a trapezoidal fuzzy number $\tilde{A}$=(a, b, c, d), shown by Fig 2. To find the defuzzifier of the trapezoidal fuzzy number, first we divide the trapezoid into two disjoint triangles $\triangle apd$ and $\triangle pqd$. Then, the defuzzifier of the trapezoidal fuzzy number can be defined as the average of the defuzzifier of these two triangles. Hence, the defuzzifier of the trapezoidal fuzzy number $\tilde{A}$=(a, b, c, d) is given by:

$$A_{defuzz} = \frac{1}{2}\Big(\frac{a\sqrt{1+(d-b)^2} + b(d-a) + d\sqrt{1+(b-a)^2}}{\sqrt{1+(d-b)^2} + (d-a) + \sqrt{1+(b-a)^2}} + \frac{b\sqrt{1+(d-c)^2} + d(c-b) + c\sqrt{1+(d-b)^2}}{\sqrt{1+(d-c)^2} + (c-b) + \sqrt{1+(d-b)^2}}\Big) \tag{16}$$

If we divide the trapezoid $apqd$ into two disjoint triangles $\triangle apq$ and $\triangle aqd$, then the defuzzifier is given by:

$$A'_{defuzz} = \frac{1}{2}\Big(\frac{c\sqrt{1+(a-b)^2} + a(c-b) + b\sqrt{1+(c-a)^2}}{\sqrt{1+(a-b)^2} + (c-b) + \sqrt{1+(c-a)^2}} + \frac{(a\sqrt{1+(d-c)^2} + c(d-a) + d\sqrt{1+(c-a)^2}}{\sqrt{1+(d-c)^2} + (d-a) + \sqrt{1+(c-a)^2}}\Big) \tag{17}$$

Note that, $A_{defuzz} - A'_{defuzz} = 0 \Rightarrow A_{defuzz} = A'_{defuzz}$, i.e., the defuzzified values of the trapezoidal fuzzy number for the two considered cases are the same. Hence, the defuzzified value of a trapezoidal fuzzy number does not depend on the choice of a triangles set. In rest of the paper, we use the formula (17) to find the defuzzified value of a trapezoidal fuzzy number.

If $\tilde{A}$=(a, b, c, d) is the generalized trapezoidal LR-fuzzy number, then the defuzzifier of $\tilde{A}$ is given by:

$$A_{defuzz} = \frac{1}{2}\Big(\frac{a\alpha_1 + b\beta_1 + d\gamma_1}{\alpha_1 + \beta_1 + \gamma_1} + \frac{b\alpha_2 + d\beta_2 + c\gamma_2}{\alpha_2 + \beta_2 + \gamma_2}\Big) \tag{18}$$

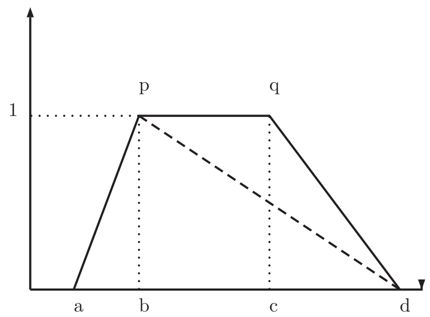

Figure 2: The trapezoidal fuzzy number $\tilde{A}$

where $\alpha_1 = \sqrt{1+(d-b)^2}$, $\beta_1 = (d-a)$ and $\gamma_1 = \int_a^b \sqrt{1+(A_L(x))'}dx$;
$\alpha_2 = \int_b^c \sqrt{1+(A_R(x))'}dx$, $\beta_2 = (c-b)$ and $\gamma_2 = \sqrt{1+(d-b)^2}$

4.2. Multi-choice Fuzzy Linear Programming Problem and Its Crisp Model

Let us consider the alternative values of a multi-choice fuzzy parameters of the model (10)-(12) as triangular fuzzy numbers. They are given by $\tilde{c}_j^{(l)} = (c_{j1}^{(l)}, c_{j2}^{(l)}, c_{j3}^{(l)})$ $(l = 1,2,\dots,k_j;\ j = 1,2,\dots,n)$, $\tilde{a}_{ij}^{(s)} = (a_{ij1}^{(s)}, a_{ij2}^{(s)}, a_{ij3}^{(s)})$ $(s = 1,2,\dots,p_{ij};\ i = 1,2,\dots,m;\ j = 1,2,\dots,n)$, and $\tilde{b}_i^{(t)} = (b_{i1}^{(t)}, b_{i2}^{(t)}, b_{i3}^{(t)})$, $(t = 1,2,\dots,r_i;\ i = 1,2,\dots,m)$. The crisp values of these fuzzy numbers are obtained by using the method described in Sub-section 4.1. The crisp values of the fuzzy numbers $\hat{c}_j^{(l)}$, $\hat{a}_{ij}^{(s)}$ and $\hat{b}_i^{(t)}$ are given by:

$$\hat{c}_j^{(l)} = \frac{c_{j1}^{(l)}\sqrt{1+(c_{j3}^{(l)}-c_{j2}^{(l)})^2}+c_{j2}^{(l)}(c_{j3}^{(l)}-c_{j1}^{(l)})+c_{j3}^{(l)}\sqrt{1+(c_{j2}^{(l)}-c_{j1}^{(l)})^2}}{\sqrt{1+(c_{j3}^{(l)}-c_{j2}^{(l)})^2}+(c_{j3}^{(l)}-c_{j1}^{(l)})+\sqrt{1+(c_{j2}^{(l)}-c_{j1}^{(l)})^2}},\ l = 1,2,\dots,k_j;\ j = 1,2,\dots,n$$

$$\hat{a}_{ij}^{(s)} = \frac{a_{ij1}^{(s)}\sqrt{1+(a_{ij3}^{(s)}-a_{ij2}^{(s)})^2}+a_{ij2}^{(s)}(a_{ij3}^{(s)}-a_{ij1}^{(s)})+a_{ij3}^{(s)}\sqrt{1+(a_{ij2}^{(s)}-a_{ij1}^{(s)})^2}}{\sqrt{1+(a_{ij3}^{(s)}-a_{ij2}^{(s)})^2}+(a_{ij3}^{(s)}-a_{ij1}^{(s)})+\sqrt{1+(a_{ij2}^{(s)}-a_{ij1}^{(s)})^2}},\ s = 1,2,\dots,p_{ij};\ i = 1,2,\dots,m;$$

$j = 1,2,\dots,n$

$$\hat{b}_i^{(t)} = \frac{b_{i1}^{(t)}\sqrt{1+(b_{i3}^{(t)}-b_{i2}^{(t)})^2}+b_{i2}^{(t)}(b_{i3}^{(t)}-b_{i1}^{(t)})+b_{i3}^{(t)}\sqrt{1+(b_{i2}^{(t)}-b_{i1}^{(t)})^2}}{\sqrt{1+(b_{i3}^{(t)}-b_{i2}^{(t)})^2}+(b_{i3}^{(t)}-b_{i1}^{(t)})+\sqrt{1+(b_{i2}^{(t)}-b_{i1}^{(t)})^2}},\ t = 1,2,\dots,r_i;\ i = 1,2,\dots,m$$

respectively.

Similarly, if we consider the trapezoidal fuzzy numbers, then by using the definition of the defuzzifier for the trapezoidal fuzzy number given by (17), we can obtain the crisp values of the fuzzy numbers.

Substituting these crisp values for the respective fuzzy number, we obtain a crisp multi-choice linear programming problem of the model as:

$$\max / \min : z = \sum_{j=1}^{n} \{\hat{c}_j^{(1)}, \hat{c}_j^{(2)}, \dots, \hat{c}_j^{(k_j)}\} x_j \tag{19}$$

subject to

$$\sum_{j=1}^{n}\{\hat{a}_{ij}^{(1)},\hat{a}_{ij}^{(2)},\ldots,\hat{a}_{ij}^{(p_{ij})}\}x_j \le \{\hat{b}_i^{(1)},\hat{b}_i^{(2)},\ldots,\hat{b}_i^{(r_i)}\},\ i=1,2,\ldots,m \tag{20}$$

$$x_j \ge 0,\ j=1,2,\ldots,n \tag{21}$$

4.3. Multi-choice Fuzzy Linear Programming Problem and Its Transformed Model

In the previous sub-section (4.2), we have obtained a crisp model of the problem. There is no fuzziness in the parametric space of the problem. Still, we are not able to apply any optimization method due to the presence of multi-choice parameters. To tackle the multi-choice parameters of the problem (19)-(21), we use Lagrange's interpolating polynomial. For the multi-choice parameter $\hat{c}_j$ $j = 1, 2, ..., n$, we introduce an integer variable u_j that takes k_j number of values. We formulate a Lagrange interpolating polynomial $f_{\hat{c}_j}(u_j)$, which passes through all the k_j number of points given by Table-1.

Following the Lagrange's formula [4], we obtain the interpolating polynomial

Table 1: Data table for multi-choice parameter $\hat{c}_j$

u_j	0	1	2	$\cdots$	k_j-1
$f_{\hat{c}_j}(u_j)$	$\hat{c}_j^{(1)}$	$\hat{c}_j^{(2)}$	$\hat{c}_j^{(3)}$	$\cdots$	$\hat{c}_j^{(k_j)}$

for the multi-choice parameter $\hat{c}_j$ ($j = 1, 2, ..., n$) as:

$$\begin{aligned} f_{\hat{c}_j}(u_j) = \quad & \frac{(u_j-1)(u_j-2)\cdots(u_j-k_j+1)}{(-1)^{(k_j-1)}(k_j-1)!}\hat{c}_j^{(1)} + \frac{u_j(u_j-2)\cdots(u_j-k_j+1)}{(-1)^{(k_j-2)}(k_j-2)!}\hat{c}_j^{(2)} \\ & + \frac{u_j(u_j-1)(u_j-3)\cdots(u_j-k_j+1)}{(-1)^{(k_j-3)}2!(k_j-3)!}\hat{c}_j^{(3)} + \cdots \\ & + \frac{u_j(u_j-1)(u_j-2)\cdots(u_j-k_j+2)}{(k_j-1)!}\hat{c}_j^{(k_j)}. \end{aligned} \tag{22}$$

Similarly, we introduce an integer variable w_{ij} to tackle the multi-choice parameter $\hat{a}_{ij}$. The integer variable w_{ij} takes p_{ij} number of different values. Following the Lagrange's formula, we construct an interpolating polynomial $f_{\hat{a}_{ij}}(w_{ij})$. The interpolating polynomial $f_{\hat{a}_{ij}}(w_{ij})$ passes through all the p_{ij} number of points which are given by Table-2. The interpolating polynomial can be written as:

Table 2: Data table for multi-choice parameter $\hat{a}_{ij}$

w_{ij}	0	1	2	$\cdots$	$p_{ij}-1$
$f_{\hat{a}_{ij}}(w_{ij})$	$\hat{a}_{ij}^{(1)}$	$\hat{a}_{ij}^{(2)}$	$\hat{a}_{ij}^{(3)}$	$\cdots$	$\hat{a}_{ij}^{(p_{ij})}$

$$\begin{aligned} f_{\hat{a}_{ij}}(w_{ij}) = \quad & \frac{(w_{ij}-1)(w_{ij}-2)\cdots(w_{ij}-p_{ij}+1)}{(-1)^{(p_{ij}-1)}(p_{ij}-1)!}\hat{a}_{ij}^{(1)} + \frac{w_{ij}(w_{ij}-2)\cdots(w_{ij}-p_{ij}+1)}{(-1)^{(p_{ij}-2)}(p_{ij}-2)!}\hat{a}_{ij}^{(2)} \\ & + \frac{w_{ij}(w_{ij}-1)(w_{ij}-3)\cdots(w_{ij}-p_{ij}+1)}{(-1)^{(p_{ij}-3)}2!(p_{ij}-3)!}\hat{a}_{ij}^{(3)} + \cdots \qquad (23) \\ & + \frac{w_{ij}(w_{ij}-1)(w_{ij}-2)\cdots(w_{ij}-p_{ij}+2)}{(p_{ij}-1)!}\hat{a}_{ij}^{(p_{ij})}, \quad i = 1,2,...,m; j = 1,2,...,n. \end{aligned}$$

Similarly, we introduce a new integer variable v_i for the multi-choice parameter $\hat{b}_i$. We formulate the corresponding interpolating polynomial $f_{\hat{b}_i}(v_i)$, which passes through the data points given by Table-3. The interpolating polynomial can be

Table 3: Data table for multi-choice parameter $\hat{b}_i$

v_i	0	1	2	$\cdots$	$r_i - 1$
$f_{\hat{b}_i}(v_i)$	$\hat{b}_i^{(1)}$	$\hat{b}_i^{(2)}$	$\hat{b}_i^{(3)}$	$\cdots$	$\hat{b}_i^{(r_i)}$

formulated as:

$$\begin{aligned} f_{\hat{b}_i}(v_i) = \quad & \frac{(v_i-1)(v_i-2)\cdots(v_i-r_i+1)}{(-1)^{(r_i-1)}(r_i-1)!}\hat{b}_i^{(1)} + \frac{v_i(v_i-2)\cdots(v_i-r_i+1)}{(-1)^{(r_i-2)}(r_i-2)!}\hat{b}_i^{(2)} \\ & + \frac{v_i(v_i-1)(v_i-3)\cdots(v_i-r_i+1)}{(-1)^{(r_i-3)}2!(r_i-3)!}\hat{b}_i^{(3)} + \cdots \qquad (24) \\ & + \frac{v_i(v_i-1)(v_i-2)\cdots(v_i-r_i+2)}{(r_i-1)!}\hat{b}_i^{(r_i)}, \quad i = 1,2,...,m. \end{aligned}$$

Replacing the multi-choice parameters c_{kj}, a_{ij} and b_i ($i = 1,2,...,m$; $j = 1,2,...,n$) present in the problem (19-21) by interpolating polynomials $f_{\hat{c}_j}(u_j)$, $f_{\hat{a}_{ij}}(w_{ij})$ and $f_{\hat{b}_i}(v_i)$, respectively, we obtain a mixed integer nonlinear programming problem as:

$$\max / \min : z = \sum_{j=1}^{n} f_{\hat{c}_j}(u_j)x_j \qquad (25)$$

subject to

$$\sum_{j=1}^{n} f_{\hat{a}_{ij}}(w_{ij})x_j \le f_{\hat{b}_i}(v_i), \qquad i = 1,2,\ldots,m \qquad (26)$$

$$x_j \ge 0 \qquad j = 1,2,...,n \qquad (27)$$

$$0 \le u_j \le k_j - 1 \qquad (28)$$

$$0 \le w_{ij} \le p_{ij} - 1 \qquad (29)$$

$$0 \le v_i \le r_i - 1 \qquad (30)$$

$$u_j, w_{ij}, v_i \in \mathbb{N}_0 \quad i = 1,2,3,...,m; \; j = 1,2,...,n. \qquad (31)$$

4.4. Steps to solve Multi-choice Fuzzy Linear Programming Problem

Using the following steps, one can solve the multi-choice fuzzy linear programming problem:

Step 1: With the defuzzifier, defined in Subsection 4.1, find the defuzzified values of the fuzzy numbers present in the problem.

Step 2: Replace the fuzzy numbers of the problem by their corresponding defuzzified values and establish the corresponding crisp multi-choice linear programming model.

Step 3: Establish the Lagrange's interpolating polynomial for each and every multi-choice parameter of the crisp model.

Step 4: Replace the multi-choice parameters by the corresponding Lagrange's interpolating polynomial to establish a mixed integer non linear programming model.

Step 5: To obtain the optimal solution of the considered model, solve the corresponding transformed mixed integer non linear programming model by using any non linear programming solver software package, namely LINGO 11.0 [21], GAMS, MATHEMATICA 9 [25].

5. NUMERICAL EXAMPLES

In this Section, we discuss two numerical examples to illustrate the described methodology to solve multi-choice fuzzy linear programming problems.

Example-1: Let us consider the following multi-choice fuzzy linear programming problem.

$$\min : Z = \{\widetilde{95}, \widetilde{96}, \widetilde{99}\}x_1 + \{\widetilde{30.5}, \widetilde{31}, \widetilde{33}, \widetilde{33.5}\}x_2 + \{\widetilde{24}, \widetilde{25}, \widetilde{26}\}x_3 \tag{32}$$

subject to

$$2x_1 + 3x_2 + x_3 \geq \{\widetilde{990}, \widetilde{1100}, \widetilde{1160}, \widetilde{1170}\} \tag{33}$$

$$\widetilde{6}x_1 + \{\widetilde{0.18}, \widetilde{0.28}, \widetilde{0.31}, \widetilde{0.38}\}x_2 + \{\widetilde{0.15}, \widetilde{0.16}, \widetilde{0.16}\}x_3 \geq \widetilde{510} \tag{34}$$

$$\widetilde{50}x_1 + \{\widetilde{10.6}, \widetilde{11.3}, \widetilde{12.3}, \widetilde{12.5}\}x_2 + \{\widetilde{5}, \widetilde{5}, \widetilde{6}\}x_3 \geq \{\widetilde{2100}, \widetilde{2200}, \widetilde{2250}, \widetilde{2310}\} \tag{35}$$

$$x_j \geq 0, \quad j = 1, 2, 3. \tag{36}$$

The alternative values of the cost coefficient of the problem are defined as trapezoidal fuzzy numbers and the other fuzzy parameters present in the problem are all triangular fuzzy numbers. The trapezoidal fuzzy numbers are given by $\widetilde{95} = (92, 95, 96, 100)$, $\widetilde{96} = (91, 95, 96, 98)$, $\widetilde{99} = (85, 96, 99, 103)$; $\widetilde{30.5} = (30, 30.5, 33.5, 36)$, $\widetilde{31} = (30, 31, 33, 35)$, $\widetilde{33} = (31, 33, 34, 36)$, $\widetilde{33.5} = (31.5, 33.5, 34.5, 36)$ and $\widetilde{24} = (22, 24, 26, 27)$, $\widetilde{25} = (22, 24, 25, 27)$, $\widetilde{26} = (23, 25, 26, 28)$. The triangular fuzzy numbers present in the problem are given by $\widetilde{1100} = (1000, 1100, 1200)$, $\widetilde{1170} = (990, 1170, 1260)$, $\widetilde{1160} = (950, 1160, 1210)$, $\widetilde{990} = (935, 990, 1320)$; $\widetilde{50} = (45, 50, 60)$; $\widetilde{11.3} = (11, 11.3, 12)$, $\widetilde{12.5} = (11, 12.5, 12.8)$, $\widetilde{10.6} = (10, 10.6, 11.5)$, $\widetilde{12.3} = (11.5, 12.3, 13.5)$; $\widetilde{5} = (4, 5, 7)$, $\widetilde{5} = (4.2, 5, 6.5)$, $\widetilde{6} = (4.5, 6, 6.8)$; $\widetilde{2200} = (2000, 2200, 2500)$, $\widetilde{2250} =$

(1980, 2250, 2700), $\widetilde{2100} = (2000, 2100, 2400)$, $\widetilde{2310} = (1980, 2310, 2530)$; $\widetilde{6} = (4, 6, 7)$; $\widetilde{0.28} = (0.25, 0.28, 0.3)$, $\widetilde{0.18} = (0.14, 0.18, 0.29)$, $\widetilde{0.38} = (0.3, 0.38, 0.4)$, $\widetilde{0.31} = (0.3, 0.31, 0.35)$; $\widetilde{0.15} = (0.1, 0.15, 0.18)$, $\widetilde{0.16} = (0.12, 0.16, 0.2)$, $\widetilde{0.16} = (0.12, 0.16, 0.18)$ and $\widetilde{510} = (450, 510, 600)$.

The defuzzified values of the triangular fuzzy numbers and trapezoidal fuzzy numbers are calculated by using eq.-14 and eq.-17, respectively. After substituting the triangular and trapezoidal fuzzy numbers by their crisp values, we obtain a multi-choice linear programming problem. The crisp model is given by:

$$\min: Z = \{95.5098, 95.4718, 97.4806\}x_1 + \{32.1064, 32.0445, 33.5, 33.9833\}x_2 + \{24.9555, 24.5, 25.5\}x_3$$

subject to

$$2x_1 + 3x_2 + x_3 \geq \{990.0038, 1100, 1169.9986, 1159.9962\}$$
$$5.9109x_1 + \{0.2751, 0.2124, 0.3515, 0.3246\}x_2 + \{0.1404, 0.16, 0.1503\}x_3 \geq 510.0014$$
$$50.0246x_1 + \{11.4117, 12.2794, 10.6604, 12.3593\}x_2 + \{5.089, 5.089, 5.911\}x_3 \geq \{2200, 2250.0004, 2100.0017, 2309.9996\}$$
$$x_j \geq 0, \quad j = 1, 2, 3.$$

Using the interpolating polynomial to tackle the multi-choice parameter present in the crisp problem, we obtain a mixed integer non-linear programming problem. The transformed problem is given by:

$$\min: Z' = (95.5098 - 1.0614u_1 + 1.0234u_1^2)x_1 + (-0.41493u_2^3 + 2.0035u_2^2 - 1.65047u_2 + 32.1064)x_2 + (24.9555 - 1.18325u_3 + 0.72775u_3^2)x_3$$

subject to

$$2x_1 + 3x_2 + x_3 \geq 990.0038 + 116.66u_4 0.0029u_4^2 - 6.66723u_4^3$$
$$5.9109x_1 + (-0.0613u_5^3 + 0.2824u_5^2 - 0.2862u_5 + 0.2751)x_2 + (-0.01465u_6^2 + 0.03425u_6 + 0.1404)x_3 \geq 510.0014$$
$$50.0246x_1 + (0.96743u_7^3 - 4.14565u_7^2 + 4.045917u_7 + 11.4117)x_2 + (0.411u_8^2 - 0.411u_8 + 5.089)x_3 \geq 93.332617u_9^3 - 379.9974u_9^2 + 336.665183u_9 + 2200$$

$$x_j \geq 0, \quad j = 1, 2, 3.$$
$$0 \leq u_i \leq 2, \quad i = 1, 3, 6, 8.$$
$$0 \leq u_i \leq 3, \quad i = 2, 4, 5, 7, 9.$$

The above problem is a non-linear programming problem, and by using any non-linear programming solver software, we can obtain optimal solution to the problem.
The solutions of the problem obtained by using different solvers are presented in the Table 4.

Table 4: Obtained Solution For Different Solver

Solver	Objective Value	Optimal Point (Continuous Variable)	Optimal Point (Integer Variable)
Lingo 11.0	15754.08	(69.889,283.409,0)	(1,1,0,0,2,0,1,1,1)
Gams 23.5	16019.226	(73.194,281.205,0)	(0,0,0,0,0,0,3,0,0)
Mathematica 9	15754.1	(69.889,283.409,0)	(1,1,2,0,2,2,1,2,0)
MATLAB 2017a (using GA)	15756.855	(69.889,283.407,0)	(0,1,0,0,2,0,2,1,2)

From the solution table, we observe that LINGO and MATHEMATICA solvers give the same solution, whereas the solution obtained by the GAMS solver is very high. We use a genetic algorithm to solve the problem and obtain a solution that is very close to the solution obtained by LINGO and MATHEMATICA. Here, the solution obtained by the GAMS solver is very much inefficient.

Example-2: Let us consider the following maximization type multi-choice fuzzy linear programming problem.

$$\max : F = \{\widetilde{18}, \widetilde{20}, \widetilde{21}\}x_1 + \{\widetilde{10}, \widetilde{12}, \widetilde{14}, \widetilde{15}, \widetilde{16}\}x_2 + \{\widetilde{15}, \widetilde{18}, \widetilde{19}\}x_3 + \{\widetilde{21}, \widetilde{23}, \widetilde{24}, \widetilde{25}\}x_4 \quad (37)$$

subject to

$$2x_1 + 4x_2 + 3x_3 + 3x_3 \leq \{\widetilde{210}, \widetilde{220}, \widetilde{225}, \widetilde{232}\} \quad (38)$$

$$\{\widetilde{3}, \widetilde{4}, \widetilde{5}\}x_1 + \{\widetilde{1}, \widetilde{2}\}x_2 + \widetilde{6}x_3 + \{\widetilde{2}, \widetilde{2.5}, \widetilde{3}\}x_4 \leq \widetilde{180} \quad (39)$$

$$\{\widetilde{2}, \widetilde{4}\}x_1 + \{\widetilde{1}, \widetilde{2}, \widetilde{3}\}x_2 + \{\widetilde{1}, \widetilde{2}, \widetilde{3}, \widetilde{4}\}x_3 + x_4 \geq \{\widetilde{220}, \widetilde{230}, \widetilde{245}\} \quad (40)$$

$$x_j \geq 0, \quad j = 1, 2, 3, 4. \quad (41)$$

We consider that all the fuzzy parameters present in the problem are triangular fuzzy numbers. The fuzzy numbers are given by $\widetilde{18} = (15, 18, 20)$, $\widetilde{20} = (19, 20, 22)$, $\widetilde{21} = (18, 21, 23)$; $\widetilde{10} = (9, 10, 12)$, $\widetilde{12} = (10, 12, 13)$, $\widetilde{14} = (11, 14, 16)$, $\widetilde{15} = (12, 15, 17)$, $\widetilde{16} = (14, 16, 18)$; $\widetilde{15} = (14, 15, 17)$, $\widetilde{18} = (16, 18, 20)$, $\widetilde{19} = (17, 19, 20)$; $\widetilde{21} = (19, 21, 22)$, $\widetilde{23} = (20, 23, 24)$, $\widetilde{24} = (22, 24, 28)$, $\widetilde{25} = (21, 25, 26)$; $\widetilde{210} = (205, 210, 220)$, $\widetilde{220} = (210, 220, 225)$, $\widetilde{225} = (215, 225, 240)$, $\widetilde{232} = (210, 232, 235)$; $\widetilde{3} = (2, 3, 4)$, $\widetilde{4} = (2, 4, 5)$, $\widetilde{5} = (3, 5, 7)$; $\widetilde{1} = (0, 1, 2)$, $\widetilde{2} = (0, 2, 3)$; $\widetilde{6} = (4, 6, 7)$; $\widetilde{2} = (1, 2, 5)$, $\widetilde{2.5} = (1, 2.5, 3)$, $\widetilde{3} = (1, 3, 4)$; $\widetilde{180} = (170, 180, 187)$; $\widetilde{2} = (0, 2, 3)$, $\widetilde{4} = (3, 4, 6)$; $\widetilde{1} = (0, 1, 3)$, $\widetilde{2} = (1, 2, 3)$, $\widetilde{3} = (1, 3, 5)$; $\widetilde{1} = (0.5, 1, 2)$, $\widetilde{2} = (0.5, 2, 3.5)$, $\widetilde{3} = (0, 3, 5)$, $\widetilde{4} = (2, 4, 6)$; $\widetilde{220} = (200, 220, 230)$, $\widetilde{230} = (220, 230, 245)$, $\widetilde{245} = (215, 245, 250)$.
Based on Subsection 4.1, we calculate the crisp values of triangular fuzzy numbers. Substituting the fuzzy numbers by their crisp values in the problem, we

obtain a multi-choice linear programming problem. Replacing each multi-choice parameter by the corresponding interpolating polynomial, we obtain a mixed integer programming problem, given by:

$$\begin{aligned}
&\max : F' = (17.9631 + 2.752v_1 - 0.626v_1^2)x_1 \\
&+(0.09882v_2^4 - 0.8067v_2^3 + 1.84353v_2^2 + 0.68614v_2 + 10.0891)x_2 \\
&+(15.0891 + 3.9109v_3 - v_3^2)x_3 \\
&+(20.9109 + 2.4854v_4 - 0.5883v_4^2 + 0.066v_4^3)x_4
\end{aligned}$$

subject to

$$\begin{aligned}
&2x_1 + 4x_2 + 3x_3 + 3x_4 \leq (1.134483v_5^3 - 5.8624v_5^2 + 14.67872v_5 + 210.0246) \\
&(3 + 0.8218v_6 + 0.0891v_6^2)x_1 + (1 + 0.9109v_7)x_2 \\
&+5.9109x_3 + (0.17605v_8^2 + 0.04035v_8 + 2.126)x_4 \leq 179.9894 \\
&(1.9109 + 2.1782v_9)x_1 + (0.04455v_{10}^2 + 0.86635v_{10} + 1.0891)x_2 \\
&+(0.001467v_{11}^3 + 0.0281v_{11}^2 + 0.86853v_{11} + 1.1019)x_3 \\
&+x4 \geq 219.9876 + 7.5558v_{12} + 2.4649v_{12}^2
\end{aligned}$$

$$\begin{aligned}
&x_j \geq 0, \quad j = 1, 2, 3, 4. \\
&0 \leq v_i \leq 1, \quad i = 7, 9. \\
&0 \leq v_i \leq 2, \quad i = 1, 3, 6, 8, 10, 12. \\
&0 \leq v_i \leq 3, \quad i = 4, 5, 11. \\
&0 \leq v_2 \leq 4.
\end{aligned}$$

The solution to the problem obtained by using different solvers are presented in Table 5.

Table 5: Obtained Solution For Different Solver

Solver	Objective Value	Optimal Point (Continuous Variable)	Optimal Point (Integer Variable)
Lingo 11.0	1732.078	(32.278,19.3258,0,30.024)	(2,4,1,3,3,0,0,0,1,2,2,0)
Gams 23.5	1732.078	(32.278,19.3258,0,30.024)	(2,4,0,3,3,0,0,0,1,2,0,0)
Mathematica 9	1732.078	(32.278,19.3258,0,30.024)	(2,4,0,3,3,0,0,0,1,2,0,0)
MATLAB 2017a (using GA)	1704.851	(32.7913, 20.862, 0.5297, 27.1039)	(2,4,2,3,3,0,0,0,1,2,1,0)

From the solution table, we notice that most of the solvers attain the same solution. Using Genetic Algorithm, we obtain a different solution. We observe that, the number of integer variable is increased for the second problem so as the error in solution.

6. CONCLUSION

The present paper deals with a particular type of linear programming problem where all the involved parameters are assumed as multi-choice fuzzy parameters. A detailed solution methodology has been provided: first, defuzzification of fuzzy numbers is based on the incentre point of a triangle; then, interpolating polynomials are formulated for the defuzzified multi-choice parameters; finally, the obtained crisp non-linear programming problem is solved by using different non-linear programming solvers. Two numerical examples are presented to demonstrate the detailed solution procedure. Many Operations Research problems have some multi-choice parameters where complete information is not provided, and such parameters are given as multi-choice fuzzy parameters. This type of situation also occurs in some portfolio selection, project management problem, and supply chain problems. In this paper, only the model with single objective function is included. This concept can be extended for multi-objective and multi-level models. The decision variables involved in the problem can be either discrete or continuous depending on the model. From the computational experiments, we observe that the presence of more multi-choice fuzzy variables increases the computational complexity of the problem.

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