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APPLICATION OF QUEUING THEORY IN QUALITY CONTROL OF MULTI-STAGE FLEXIBLE FLOW SHOP

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Abstract: In this paper, we carry out queuing analysis to examine and integrate a quality control (inspection) process into a multi-stage flexible flow shop configured in an $M/M/1$ queue system. Our main concern is to improve the performance of multi-stage flexible flow shop production. Besides, this work aims to help managers in improving their efficiency and effectiveness, and in selecting a joint inspection, being the most suitable policy for minimizing inspection and queueing related costs. We adopt an analytical approach based on real-life data from a Kach company that produces disposable appliances. Queuing analysis reported in this work provides a basis for estimating and analyzing production systems by measures such as utilization, the percentage of the idle workstation, number of batches in the system, number of batches in the queue, expected time spent in the queue, and expected time spent in the system. The results indicate the performance of the company in relation to inspection and expected time.

Keywords: Queuing Theory, Multi-Stage Flexible Flow Shop, Quality Control.

MSC: 90B85, 90C26.

1. INTRODUCTION

Queuing theory is one of the oldest and most widely used quantitative analysis techniques. Basically, it is a mathematical approach used for the analysis of waiting lines. One of its applications is to analyze the congeries and delay of waiting in a line [5]. Queuing theory has a long history and has been used in applied problems of manufacturing, communication, inventory control, and etc. A queuing system is composed of three parts (1) the arrival or inputs to the system (sometimes referred to as the calling population), (2) the queue (or the waiting line itself), and (3) the service facility. In the queuing theory, it is usually supposed that the time between the two successively arrivals and the serving time follow a particular probability distribution [8].

One of the optimization problems that is increasingly dealt with is flexible flow shop management and automated multi-stage manufacturing systems. Due to high levels of flexibility and automation, an integrated quality control process is essential for an efficient total quality management system. Quality control must be integrated in an economic manner to ensure both proper systems functioning and high-quality outputs [11, 9]. Although steps have been taken to design such systems by queueing related techniques, no attempt has yet been made in rendering explicit quality control considerations [4, 3]. Several combined evaluation methodologies were developed in the literature. Pamucar and Ćirović [7] described an Adaptive Neuro Fuzzy Inference Guidance System (ANFIGS) that provides instructions to drivers based upon "optimum" route solutions for the problem of choice of road route under conditions of uncertainty. Vesković, et al. [12] considered a combination of the Delphi, SWARA (Step-Wise Weight Assessment Ratio Analysis) and MABAC (Multi-Attributive Border Approximation Area Comparison) methods as an evaluation and management models.

Applying evaluation methods in manufacturing systems have been extensively studied in the literature. Abdulshahed and Badi [1] applied an Adaptive Neuro-Fuzzy Inference System (ANFIS) approach for prediction of the workpiece surface roughness for the end milling process. A small number of fuzzy rules were used for building ANFIS models with the help of Subtractive clustering method (ANFIS-Subtractive clustering model). In many real scheduling situations, it is necessary to deal with the worker assignment and job scheduling together. However, in traditional scheduling problems, only the machine is assumed to be a constraint and there is not any constraint about workers. Asadi-Gangraj et al. [2] proposed a flexible flow shop scheduling problem with two simultaneous issues: finding the best worker assignment, and solving the corresponding scheduling problem. In single machine scheduling problems with availability constraints, machines are not available for one or more periods of time. Moslehi and Mashkani [6] considered a single machine scheduling problem with flexible and periodic availability constraints. In this problem, the maximum continuous working time for each machine increases in a stepwise manner with two different values allowed. Also, the duration of unavailability for each period depends on the maximum continuous working time of the machine in that same period, again with two different values allowed. The objective is to minimize the number of tardy jobs.

In this regard and according to the reviewed papers, this paper design a methodological and practical tool for handling the inspection function in controlling the quality of a flexible flow shop multi-stage production system. To do this, and for

simplicity, the quality control system is considered to be $M/M/1$ queue.
This paper discusses a case study about the application of Queuing Theory in Kach Company. The company produces Disposable Appliances. Because of the increase in its product demand, the company needs to examine the efficiency and other parameters of the production line to produce Disposable Appliances. To sum up, the objectives of this study are:

- To model a multi-stage flexible flow shop of the quality control process in Kach Company by using an appropriate analytical model of Queuing Theory.
- To examine the important performance of quality control (inspection) in multi-stage flexible flow shop production by using the developed queuing model, so that the performance improvement of the production can be achieved.
- To minimize inspection and queueing related costs.

The model, as outlined in Fig. 1, is then investigated to obtain greater insights regarding the mutual and joint effects of quality control and queueing design.

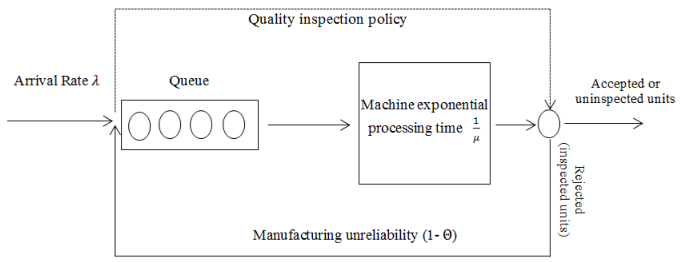

Fig 1: Quality control of the $M/M/1$ queue

The rest of this study is as follows:
In Section 2 the description of the problem is given. In the third Section, the $M/M/1$ queue quality control model is discussed. In the Section 4 a numerical example is illustrated to show the effectiveness of the proposed model. We conclude in Section 5.

2. DESCRIPTION OF THE PROBLEM

The problem is extracted from a real case study in a dairy disposable appliances production company. The production system consists of three main parts, which are located in three workstations; extruder, forming, and the print station. To produce each product, a three-stage process must be performed in three separate production workstations on raw materials. The first workstation is called extruder station consisting of 13 parallel processes. Each of these processes is responsible for producing different polystyrene sheets. The material used in this workstation is polystyrene which transforms to polystyrene sheets and rolls during melting and extruding. After the polystyrene sheets are produced in the extruder workstation, they are transferred to the forming workstation to produce disposable appliances of polyester sheets. The forming workstation consists of 24 parallel processes, each of which is responsible for the production of disposable glasses and containers; each molding machine can use different sheets and produce different glasses and disposable appliances. After the dishes were produced at the forming shop, they are sent to the print workstation for printing operations. The printing workstation also has 14 parallel processes, each of which can print several types of products. Generally, the structure of the production of disposable appliances is a parallel-series system as shown in Fig 2.

As we explained above, we show the relationship between queuing and quality control in Figure 1. Due to the output of the process, a quality control method is imposed with units that are re-supplied for reproduction. Consequently, if the production is quite reliable ($\Theta = 1$), or if the inspection is not performed, we basically consider the $M/M/1$ model; while for complete inspection and reliability of $0 \leq \Theta < 1$ production, we use the $M/M/1$ model with feedback. Other quality control methods (arbitrary) lead to different models of partial feedback from the $M/M/1$ queue. Specifically, the quality control methods defined by a barrier policy and intermittent inspections are formulated and studied in detail and with policy (minimizing costs) that are determined by analytical (and numerical) methods.

3. $M/M/1$ QUEUE WITH QUALITY CONTROL

According to the problem, each machine is considered as a service provider. So this model is one of the most common examples of queue models that is also known as the classic model. In this model, the jobs are according to the Poisson process and entered with parameter λ and the machining rate is μ. The job entry track is assumed to be independent of the number of jobs in the system. Because the system has a service provider, the exit rate is equal to the service rate of the system's unique servant [10].

The probability that the system has n jobs is then given in terms of four events.

1. A new job arrives (with probability λdt) when there are $(n-1)$ jobs in the system;
2. No job is inspected, and the processed job exits the system (with probability μdt) when there are $(n+1)$ jobs in the system;
3. A job is inspected ($W_n = 1$) and it is rejected, i.e. feedback to the queue and has to be reprocessed. This occurs with probability $\mu q dt$ when there are n jobs in the system; and
4. A job is inspected ($W_{n+1} = 1$) and it is accepted, i.e. there is the system with probability$(1-q)\mu dt$, when there are $(n+1)$ units in the system.

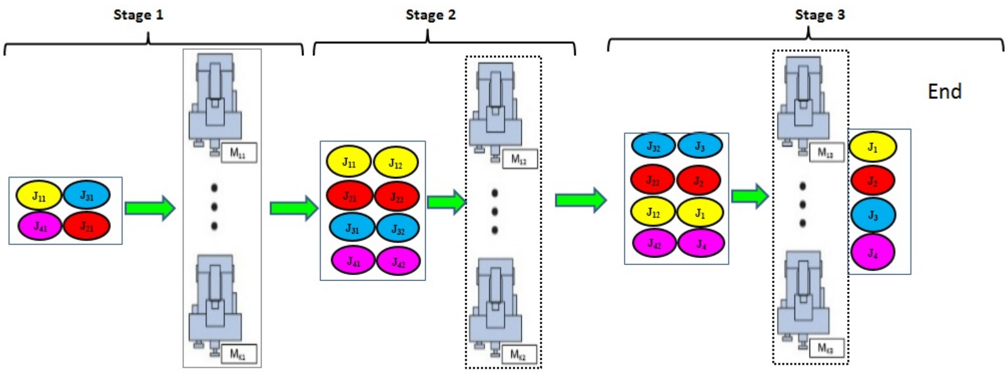

Fig 2: Structure of the production of disposable appliances

The last event consists of the complementary event of no new jobs coming in and no jobs processed (with probability, $(1-\lambda dt)$ $(1-\mu dt)$). The probability distribution of n jobs in the system is given by P_n,

$$P_n = p^n \beta_n / [1 + \sum_{j=1} p^j \beta_j] \tag{1}$$

$$\beta_n = \prod_{i=1} \frac{p - qw_i/\mu}{\sqrt{C - p + qw_i/\mu}} \tag{2}$$

$$C = \rho + \beta\sqrt{\rho} \tag{3}$$

$$q = \alpha \frac{\rho}{C - \rho} \tag{4}$$

$$\rho = C - p + q\frac{\Theta}{\mu},\ \forall 0 \leq \Theta < 1 \tag{5}$$

where,
λ: job arrival
μ: machining rate
$p = \frac{\lambda}{\mu}$,
r: the probability of inspecting a unit
$a = p/(l-q) < l$
The characteristics of the inspection-queueing systems defined in propositions 1-2 are studied next. Subsequently, these are derived to obtain optimal inspection procedures.
Let $N = E(n)$ be the average number of jobs in the system. We then have the results outlined in Eq. 6:
1. The random inspection:

$$N = \frac{r}{1-r},\ r = [\frac{\lambda}{\mu}]/[\alpha + (1-\alpha)(1-q)] \tag{6}$$

2. The C- alternating policy:

$$N = p[p^{2C} + (q - q^{C-2})p^C + q^C p^{C-1} + 1 - q]/(1-p)(1-p^C - q)(1-p^C) \quad (7)$$

For each of these, we then obtain

$$N_L = E(n-1) = N - (1-p_0)\, average\, queue\, length \quad (8)$$

$$W = N/\lambda\, average\, time\, spent\, in\, system \quad (9)$$

$$W_L = N_L/\lambda\, average\, waiting\, time \quad (10)$$

$$W_s = W - W_L = (1-P_0)/\lambda \geq 1/\mu\, finally,\, the\, average\, processing\, time \quad (11)$$

Economic analysis of the proposed model leads to strategically determine the role of decision making for organizations and its ability and capacity. It is required to optimize the queuing system. In other words, for an optimal system, it is needed to explore a system in which the total cost of quality control (inspection) is minimized.

$$Total\, Cost = C_s * C + C_h * N_L + C_L \lambda P_n + (C_W * N_L/\mu) + (C_I * (N - r + \frac{N_L}{\mu}))$$

where,
C_W: Cost of one job waiting in queue.
C_S: Cost per service per unit time.
C_h: holding cost per unit per unit time.
C_l: Cost of cancellation per job per unit time in queue.
C_I: Cost of keeping an idle server in a unit of time.
M: Number of the servers.

4. NUMERICAL EXAMPLE

In this section, the analytical results are summarized. The model is implemented in Kach production company. For intermittent C policies, numerical results were used to handle a sensitivity analysis on the decision variables. In Table 1, a sensitivity analysis for a given set of parameters has been shown to lead to an optimized policy. In other words, as long as there are q jobs or less in the system, a processed job is inspected.

Then, we can extract the following input data:
$\lambda = 8$, $\mu = 10$, $M = 7$, $C_S = 15$, $C_w = 13$, $C_h = 2$, $C_l = 10$, $C_I = 5$, $\rho = 0.5$

Table 1: A numerical example of evaluation the queuing in inspection policy

α	β	P_0	P_n	N	N_L	W	W_L	W_S	r	Total Cost
0.1	1.15	0.200	0.739	3.264	2.464	0.408	0.308	0.100	0.765	372.1768
0.2	1.30	0.184	0.677	3.384	2.568	0.423	0.321	0.102	0.771	371.6551
0.3	1.45	0.171	0.667	3.528	2.699	0.441	0.337	0.104	0.779	377.7354
0.4	1.00	0.162	0.656	3.696	2.858	0.462	0.357	0.105	0.787	385.3351
0.5	0.85	0.154	0.641	3.872	3.026	0.484	0.378	0.106	0.794	391.4890
0.6	0.75	0.147	0.633	4.056	3.203	0.507	0.400	0.107	0.802	398.5004
0.7	0.50	0.142	0.620	4.232	3.374	0.529	0.421	0.108	0.808	401.9314
0.8	0.30	0.138	0.615	4.408	3.546	0.551	0.443	0.108	0.815	410.3439
0.9	0.15	0.131	0.608	4.584	3.715	0.573	0.464	0.109	0.820	415.7687

As can be seen in Table 1, with the increase in α and decrease in β, the average of waiting times in the queue and queue length increases; as a result, the total cost has also increased proportionally. It shows that the reliability of the production process is lower by increasing inspections and increasing human resources in the organization. Another important aspect of the model is to integrate quality control, reliability, and cost analysis. This aspect is very important in decision making for top level managers. If a manager wants to have a high quality product, then the inspection cost increases and the reliability of the production system may decrease. On the other hand, if the reliability needs to be increased, then the quality may be influenced since the setting of machines may need to be changed. Therefore, it is a multi-dimensional decision model where the managers can decide based on the significance or priority of the decision variables.
This clearly indicates that the method used in flexible flow shop multi-stage production systems is effective and can be important in manager's decision making by combining knowledge of queuing and economic analysis in comparison with the standard quality inspection process.

5. CONCLUSION

This study examined the multi-stage flexible flow shop performance by using queuing theory to control quality at Kach Company, producing disposable appliances. The study has shown that by queuing theory we are able to analyze quality control in a production system such as utilization, the percentage of the idle workstation, number of the batch in the system, number of the batch in the queue, expected time spent in the queue, and expected time spent in the system. This study is based on independent queuing system $M/M/1$ having a flexible flow shop with 3 independent workstations. Our choice of such a model was thus motivated primarily by the need to demonstrate a methodological approach for designing jointly inspection and queueing systems. For practical purposes, more realistic assumptions regarding the processing rates, the arrival rates, the buffer capacity, the number of processing machines, and the multiplicity of workstations integrated into a network are required. The results obtained from the queuing model was compared to the standard data in the company to check the validity of queuing model. Accuracy between data used in the queuing theory and standard data in the company was 85.80%. As for future research directions, one can study the cubic impacts of cost, quality, and time in a queue system to investigate the quality control effects on production costs.

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