Yugoslav Journal of Operations Research
28 (2018), Number 1, 93–105
DOI: 10.2298/YJOR161113005M

CRITICALITY ASSESSMENT RISK; CONTRIBUTION OF FUZZY LOGIC

S. MASMOUDI
Faculty of Economics and Management Sciences, University of Sfax, Tunisia Road Aéroport Km 4, BP 1088, Sfax 3018, Tunisia
soniamasmoudi@yahoo.fr

M. M. DHIAF
Emirates College of Technology, P.O. Box 41009, Abu Dhabi, UAE
mohamedmahjoub.dhiaf@fsegs.rnu.tn

Received: November 2016 / Accepted: March 2017

Abstract: In order to determine the criticality of a risk, an assessment of the probability of occurrence (notion of frequency) and of the impact (notion of severity) are to be estimated. The criticality is the product of the probability of its occurrence and the impact that the risk has on the project, hence on the whole company. So, the practice of matrix or the criticality grid considering these two dimensions is necessary. However, the criticality grid involves the insufficiencies inherent to the subjective behavior of expert judgments and to the imprecise information engaged in the assessment of the risk. Taking into account the problems of the imperfection implied in the Conventional Criticality Matrix (CCM), the objective of this work is to develop a Fuzzy Criticality Matrix (FCM) to overcome these difficulties. The proposed model aims at improving the system of fuzzy inference. The proposed approach is applied to a test system which is the company SAROST S.A.

Keywords: Criticality, Fuzzy Inference System, (Conventional/Fuzzy) Criticality Matrix (CCM) / (FCM).

MSC: 90B06, 03B52, 93C42.

1. INTRODUCTION

Although the criticality grid is mostly used in current applications, it has deficiencies inseparable from the subjective behavior of assessors judgments and imprecise information used in risk assessment.

The information used may be contaminated with imperfection and ambiguity. In addition, they may be in insufficient quantity, thus not providing quantitative handling. Indeed, as the probability and impact are defined by linguistic qualifiers to set out the concrete condition, they cause different meanings of risk parameters as the interpretation of the linguistic terms such as ”rare”, ”possible”, ”minor injuries”, etc. So, they may differ from one grid to another, from one expert to another, or from one sector to another as the results of the assessment, and according to the method or approach used to determine one of the dimensions, severity and probability.

To remedy this problem of subjectivity and partial assessment of the grid, it seems essential to know how to treat regular and logical subjective data, and their qualitative and quantitative analysis.

Fuzzy logic seems the most adequate method to act in a concrete case where nothing can be regulated in advance or even definitively. In other words, the uncertainty is the keyword of the recourse to the fuzzy thinking.

To prove the importance of the approach proposed in the criticality assessment risk based on a fuzzy inference system in the presence of uncertain information, we devote this work to an experimental study in SAROST Company.

To do this, we present the criticality grid used by the company. Based on the data obtained from this grid, we develop the fuzzy scales as the basis of fuzzy rules.

2. PRESENTATION OF THE CONVENTIONAL CRITICALITY MATRIX

To assess the criticality and risk acceptability, SAROST Company uses a qualitative approach based on the following grid:

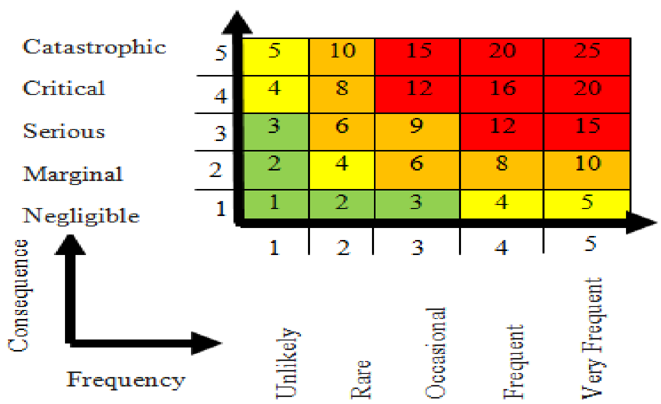

Figure 1: Criticality Matrix used by the SAROST Company

The description of the frequency, severity and criticality scales will be presented as follows:

-Frequency Scale The probability of contingency is examined through five levels and estimated by using a listed frequency 10^{-n}/ year. The occurrence probability classes on scales are defined qualitatively (with a language designation for each class) and quantitatively (with numerical ordinal ranking from 1 to 5).

Level	Designations	Qualitatives Descriptions
1	Unlikely	Extremely unlikely over time of life of the installation
2	Rare	Very unlikely over time of life of the installation
3	Occasional	unlikely over time of life of the installation
4	Frequent	Possible over time of life of the installation
5	Very Frequent	likely over time of life of the installation

Table 1: Frequency Scale

-Severity Scale Estimation of severity is based on the consequences of a proposed scenario from an ordinal scale with five levels (1 to 5).

Level	Consequences	Sigifications
1	Negligible	Some minor injuries Damage weak and reversible
2	Marginal	Numerous minor injuries and some injuries with disabilities significant and reversible damage or low and irreversible
3	Severe	Severe injuries Unit shut
4	Critical	Many injured with disabilities Significant and irreversible damage
5	Catastrophic	Death of occupants Plant shutdown

Table 2: Severity Scale

-Criticality Scale The multiplication of severity levels by those frequencies completes the risk assessment. The criticality risk, characterized by a number corresponding to each grid cell, varies from 1 (minimum acceptable risk) to 25 (maximum acceptable risk).

Level	Consequences	Sigifications	Score
1	Acceptable (Green area)	Reduced risk, no necessary dispositions : Preventive measures already in place seem sufficient	1, 2, 3
2	Tolerable (Yellow area)	medium risk , verify the need for risk reduction : prevention measures are desirable and a review of the position must be achieved	4, 5
3	Critical (Orange area)	High risk, provisions for reducing: preventive measures are required with emphasis on collective protection	6, 8, 9, 10
4	Unacceptable (Red area)	unacceptable risk, emergency provisions for the reduction: prevention measures must be applied immediately	12, 15, 16, 20 , 25

Table 3: Criticality Scale

These values will be used by the Company to define the fuzzy input data.

3. FUZZY INFERENCE SYSTEMS

The theory of fuzzy sets due to Zadeh [9] offers the one of most adequate method for assessing criticality in the presence of uncertainty and imprecision. Namely, uncertainty and imprecision are the keywords in the use of fuzzy thinking in which quantitative and symbolic revelation of information, by qualifying natural language, can be processed, Bouchon [1].

According to Freska [3], to allow the analyst to handle natural and informative knowledge of expert judgments, fuzzy criticality assessment of risks accesses the direct use of measures in the form of fuzzy sets. Although qualitative and imprecise, corrective actions accepting risk control may be initiated, Hicks and Fayek [4] and Xu *e*t al. [8].

The current work aims to improve conventional criticality matrices by describing an inference system according to Mamdani and Assilian [5]. to better manage industrial risks. A fuzzy rule base is created from a fuzzy representation taking into account linguistic values of frequency scales, severity and risk. To determine a criticality risk index, data frequency and severity are applied to Fuzzy Inference System (FIS) using fuzzy logic operations.

The general methodology of Fuzzy Inference Systems is given in Figure 2.

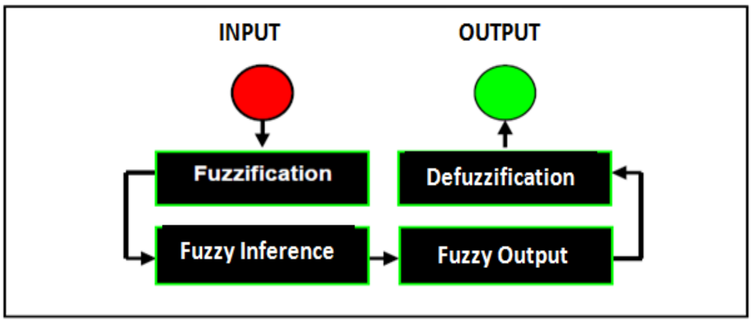

Figure 2: Flowchart Fuzzy Inference System (FIS)

Source: Mamdani and Assilian (1975)

In what follows, we will briefly present the details specific to each stage of this method as follows:

- ¢ The fuzzification operation allows to pass from the real domain to the fuzzy domain by the fuzzy quantification of the real values of a variable. It consists in converting ordinary input data f_0 and g_0 in their symbolic representation by calculating the degrees of membership $\mu_B(g_0)$ and $\mu_A(f_0)$.
- ¢ ¢ The fuzzy inference transforms the input fuzzy sets, created in the first part of fuzzification, into the output fuzzy sets using linguistic rules and fuzzy implication operations. A rule must be in the form if condition, then conclusion. During this second stage, we generate a lot of controls under

the form of linguistic variables (control by rule). According to Roger Jang and Gulley [7], the fuzzy output is obtained using the max-min method of inference according to the following sub-steps:

(i) **Locate the activation level of each rule:** truth value of antecedent (the premise) of each R_i rule is calculated and then applied to the part of "conclusion" of this rule. Formally, the activation level of the rule is defined by:

$$\alpha = min\left(\mu_A\left(f_0\right), \mu_B\left(g_0\right)\right)$$

(ii) **Inferencing:** In the deduction step, the output of each rule is calculated from the conjunction operator. The calculation is as follows:

$$\mu_{c'}(r) = min\left(\alpha, \mu_c(r)\right)$$

(iii) **Aggregation:** The overall output of the system is obtained by combining specific outputs for each rule using the max union operator:

$$\mu_{c'^{agr}}(r) = \max_{i=1,\ldots,n} \mu_{c'}(r)$$

- ¢ Defuzzifier determines the quantitative values in terms of the functions of memberships of the linguistic variables. To calculate the characteristic value of a set of the output, the Center Of Gravity (COG) method is used:

$$r_0 = \frac{\int_{reW} \mu_{c'^{agr}}(r).r.dr}{\int_{reW} \mu_{c'^{agr}}(r).dr}$$

4. FUZZY CRITICALITY MATRIX (FCM) PROPOSED

During a risk assessment, it is mandatory to consider thoroughly all relevant scenarios indicating each probability of occurrence and potential severity of the consequences. The integration of these two dimensions to criticality matrix is necessary to achieve so.

The information concerning the event occurrence probability and severity of their consequences are generally imprecise and / or uncertain. These facts have impact on conventional criticality matrix as well. Hence, the present work is introduced in this context and aims in improving conventional criticality matrix for better control of industrial risks. Until now, and towards a revision of practices in the evaluation and risk analysis, many reforms and improvements are needed. Therefore, it is essential to bring the theory of fuzzy sets in the criticality assessment of risks in the presence of imperfect data. The coherence of these proofs justifies the requirement to make appeal a fuzzy criticality assessment of risks. This will be subject of the following section.

4.1. Structure of the Fuzzy Criticality Matrix (FCM)

The following figure shows the overall structure of the process model of fuzzy criticality matrix proposed.

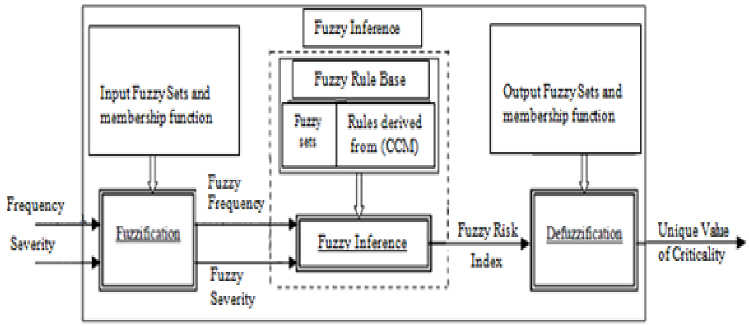

Figure 3: Global Procedure of Criticality Assessment based on Fuzzy Rules: Global Model Structure of the (FCM)

Towards a description of the risk parameters and risk levels, fuzzy partitions are used by this model. According to Zadeh [10], the first operation consists of fuzzification corresponding data to parameters of the calibrated risk graph. The purpose of this process is determining the membership functions. Thus, fuzzy intervals replace crisp intervals with trapezoidal membership functions.

The fixation of the borders of an ordinary interval as a mean value of a fuzzy number under the form of upper and lower expectation remains the fundamental notion of this transformation, according to Dubois and Prade [2].

The following sections will elaborate in more details on the steps of the proposed approach.

4.2. Input and Output Variables

The fuzzy rules-based system associated with conventional criticality matrix considers a both parameters G and F as input variables, and criticality as the unique output variable.

4.3. Development of the Fuzzy Scales

The robust development of fuzzy scales relies heavily on the passing of ordinary scale intervals to fuzzy scales. This transformation seems to offer the average value of a fuzzy interval as an inverse problem. In front of this situation, we have taken as reference the recent work of Nait-Said *e*t al. [6]. In this work, the authors propose below equations derived by considering the transformation of an ordinary interval of borders E^* and E_* in a fuzzy interval "Q" likewise the inverse problem determining the average value a fuzzy interval. This interval is closed and bounded by the expected values calculated by mean of the upper and lower distribution functions.

$$E(Q) = [E*(Q), E^{*}(Q)]$$

Where,

$$E_{*}(Q) = \inf E(Q) = \int_{-\infty}^{+\infty} udF^{*}(u)$$

$$E^{*}(Q) = \sup E(Q) = \int_{-\infty}^{+\infty} udF_{*}(u)$$

F^{*} and F_{*} represent the upper and lower distribution functions of P. P belongs to the set of probability measures $P(Q)$, defined on the support of Q. Referring to the expressions described above, the following equations can be demonstrated:

$$E_{*}(Q) = q_{-} - \frac{\alpha}{2}$$

$$E^{*}(Q) = q_{+} + \frac{\beta}{2}$$

Where α represents the left spreading and β represents the right spreading.

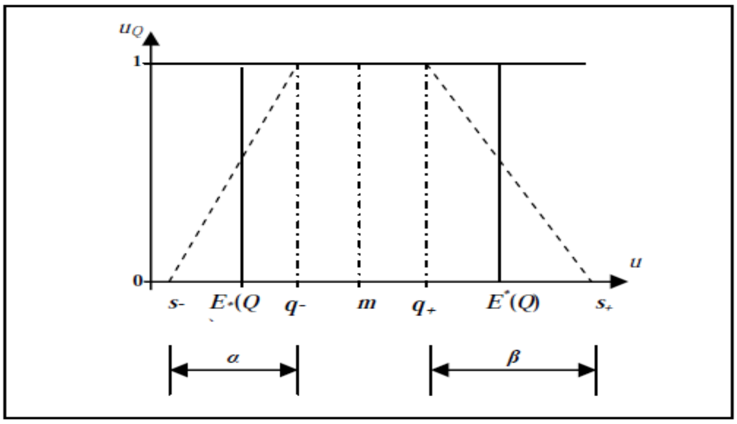

Figure 4: Transformation of an Ordinary Interval in a Fuzzy Interval

These results influence the fact that the width of the average value is a linear function of α and β, Dubois and Prade [2]. Referring respectively to these last two expressions, the determination of these distributions will be the topic of the following sections.

The calculation of α and β requires the calculation of the average value m in the interval $[E_{*}, E^{*}]$, first, and then $q-$ and $q+$ borders of the core by using respectively the average value of subdivisions $[E_*, m]$ and $[m, E^*]$. To determine $m, q - andq+$, the arithmetic mean as well as the geometric mean are used at the same time, regardless of the scale of the universe used (either linear or nonlinear).

The passage of an ordinary interval in a fuzzy interval on a linear scale is explained by the Figure 4. Take for example the α distribution and the lower border $s-$ of Q:

$$\begin{aligned}
\alpha &= 2(q_- - E_*) = 2\left(\frac{E_* + m}{2} - E_*\right) \\
&= m - E_* = \frac{E_* + E^*}{2} - E_* \\
&= \frac{E^* + E_*}{2} \\
s_- &= q_- - \alpha
\end{aligned}$$

5. FUZZY ASSESSMENT OF CRITICALITY RISK

The set of the terms used in the conventional criticality matrix: universe of discourse, fuzzy partitions as well as rules If / Then of the conventional criticality matrix will be the subject of the following section.

5.1. Establishment of the Fuzzy Partitions (Fuzzification)

The severity scale, frequency and criticality of which we reproduce in the following are exposed in Table 4. Regularly partitioned, the severity and criticality scales are presented as continuous ordinal scales. The orders of magnitude of the frequency scale (as the predominant parameter) are estimated based on expert judgment present on site by making recourse to their own knowledge as well as on the return to experiences. The number of linguistic values given to every input and output variable defines the addition of this variable universe of discourse. The fuzzy partitions are given in Table 4. The data are displayed in Figure 5 (a, b, c).

- *Definition of the Fuzzy Scales of the Parameters G, F, and R*

- Severity Scale: Fuzzy scale of severity includes five fuzzy sets: ″Negligible″, ″Marginal″, ″Serious″, ″Critical″ and ″Catastrophic″. These sets are described on a space of severity within 1 to 5. We wish to present an example of a scale for the measure of the severity parameter in Figure 5(a).

- Frequency Scale: As shown in Figure 5 (b), the frequency is defined by five fuzzy sets, namely ″Unlikely ″, ″Rare″, ″Occasional″, ″Frequent″ and ″Very Frequent″ (almost sure). The varying values from 10^{-9}/year to 10/year are indicated on a logarithmic scale. The discrete interval $[10^{-9}, 10^{-7}]$, indicating a rare event,

Risk Parameters	Linguistic Value	Qualitative Description	Quantitative Description /Rating
	Negligible	Some minor injuries Damage weak and reversible	1=G<2
Severity (G)	Marginal	Numerous minor injuries and some injuries with disabilities significant and reversible damage or low and irreversible	1<G<3
Severity (G)	Serious	Severe injuries Unit shut	2<G<4
	Critical	Many injured with disabilities Significant and irreversible damage	3<G<5
	Catastrophic	Death of occupants Plant shutdown	4<G$\leq$ 5
	Unlikely	Extremely unlikely over time of life of the installation	10^{-9}<F1$\leq 10^{-7}$
	Rare	Very unlikely over time of life of the installation	10^{-3}<F2$\leq 10^{-2}$
Frequency (F)	Occasional	unlikely over time of life of the installation	10^{-2}<F3$\leq 10^{-1}$
	Frequent	Possible over time of life of the installation	10^{-1}<F4$\leq$ 1
	Very Frequent (almost sure)	likelyover time of life of the installation	F5>1
	Acceptable	Reduced risk, no necessary dispositions : Preventive measures already in place seem sufficient	$0\leq R \leq 2$
Criticality (R)	Tolerable (to monitor)	medium risk , verify the need for risk reduction : prevention measures are desirable and a review of the position must be achieved	$1\leq R \leq 3$
	Critical	High risk, provisions for reducing: preventive measures are required with emphasis on collective protection	$2\leq R \leq 4$
	Unacceptable (necessary actions)	unacceptable risk, emergency provisions for the reduction: prevention measures must be applied immediately	$3\leq R \leq 5$

Table 4: Qualitative and Quantitative Definition of Risk Parameters

is attributed to the linguistic value ”Unlikely” defined in the criticality matrix as ”Number of times a year where the dangerous event occurs”. This interval has transformed into a fuzzy interval with absence of negative part.

The terminal interval $[1, 10]$] indicates the mean value of the fuzzy set ”Very Frequent” which upper border may be modified by giving greater importance to the dangerous situation. The adjustment of the increasing part of (almost sure) is determined by taking of the interval which precedes the upper border of the core of the fuzzy set ”Frequent” as a starting point. The goal of such adjustment is twofold. First, it removes the negative part of the aggregated fuzzy interval to the term ”Very Frequent” (almost sure) which has no sense from the point of view 'Number of hazards'. Second, it neglects the crossing between more than two fuzzy sets, which prejudge diverse values giving no meaning to the category ”Very Frequent” (almost sure).

- Criticality Scale: The criticality of risk as unique output variable is defined on a universe of discourse values between 0 and 5. Four fuzzy sets are included in the scale, namely ”Acceptable”, ”Tolerable”, ”Critical” and ”Unacceptable” (Figure 5(c)).

Symbols Frequency	E*	E*	M	q-	q+	s-	S*-	s+	S*+
unlikely	1.00E-09	1.00E-07	1.00E-08	3.16E-09	3.16E-08	-1.16E-09	1.00E-09	1.68E-07	/
Rare	0.001	0.01	3.16E-03	1.78E-03	5.62E-03	2.22E-04	/	1.44E-02	/
Occasional	0.01	0.1	3.16E-02	1.78E-02	5.62E-02	2.22E-03	/	1.44E-01	/
Frequent	0.1	1	3.16E-01	1.78E-01	5.62E-01	2.22E-02	/	1.44E-00	/
Very Frequent (Almost Sure)	1	10	3.16E-00	1.78E-00	5.62E-00	2.22E-00	/	1.44E+01	1.00E+01

Table 5: Transformation of crisp intervals into fuzzy intervals:Numerical Results of Partition Fuzzy Frequency Parameter Intervals

Table 5 summarizes the numerical results of the transformation for the parameter Frequency.

To determine the degrees of membership to the involved fuzzy sets, the input data of various parameters of the considered scenario are fuzzified by placing them on the corresponding fuzzy scales by means of the simulator Matlab Toolbox, as in Figure 5 (a, b, c).

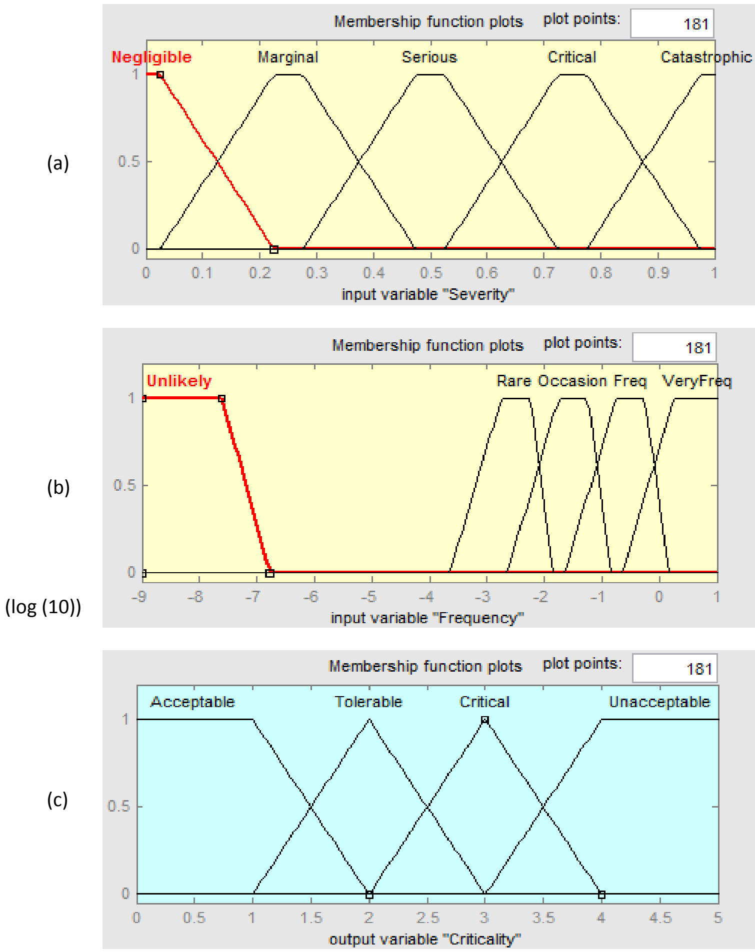

Figure 5: Membership functions generated for: (a) Severity (b) Frequency and (c) Criticality

5.2. *Establishment of the Fuzzy Rules: Fuzzy Conclusion Assessment (Fuzzy Inference)*

By reference to the criticality grid in Figure 1 and to build the base of fuzzy rules, the association of fuzzy sets corresponding to the premises and to the conclusions of the rules will be necessary after establishment of fuzzy partitions.

Therefore, the fuzzy inference algorithm Mamdani (max-min model) is chosen to transform qualitative rules in an interpretable quantitative result. By using the ”min” operator, the conjunction and the implication of premises of the rule are transformed into the rule truth value. The ”max” operator is used for the aggregation of the resulting fuzzy outputs.

Table 6 regroups an example of the combination rules of the parameters F and G. The rule 1, for example, should be read as follows:

"If the Frequency is Unlikely and the Severity is Negligible, then the risk is Acceptable".

The critical surface derived from all rules is given in figure 6.

Rules	Frequency (F)	Severity (G)	Criticality (R)
1	Unlikely	Negligible	Acceptable
2	Unlikely	Marginal	Acceptable
3	Unlikely	Serious	Acceptable
4	Unlikely	Critical	Tolerable (to monitor)
5	Unlikely	Catastrophic	Tolerable (surveiller)
6	Rare	Negligible	Acceptable
7	Rare	Marginal	Tolerable
8	Rare	Serious	Critical
9	Rare	Critical	Critical
10	Rare	Catastrophic	Critical
.	.	.	..
.	.	.	..
25	Very Frequent	Catastrophic	Unacceptable

Table 6: Rules of combination of risk parameters

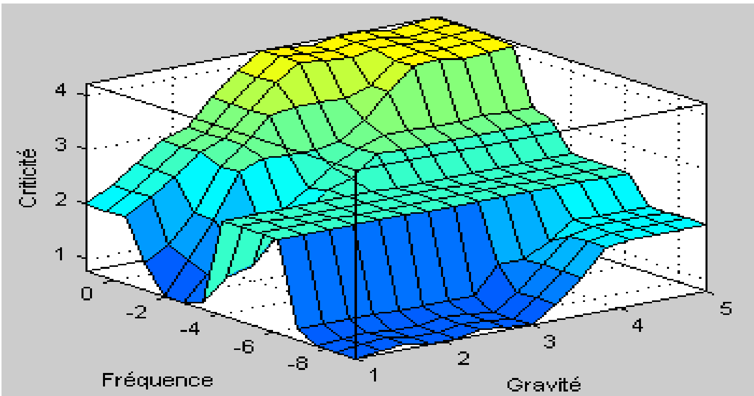

Figure 6: Fuzzy Criticality Surface

6. CONCLUSION

In the term of this work, we can conclude that the imperative analysis of criticality risk must take into account the uncertainties and imprecise data concerning risk parameters. In this regard, this paper proposes a fuzzy risk assessment approach. The proposed approach aims to improve the Conventional Criticality Matrix (CCM) by using fuzzy criticality matrix instead.

To validate the proposed approach, we examined two main properties of a fuzzy rule base, the coherence and the consistency. Our base of fuzzy rules is developed by relying on a persistent risk graph model while noting the absence of abstract rules between the established redundant fuzzy rules. The validation methodology allows varying the values associated with risk parameters of knowing the severity and that of the frequency. These two parameters are preserved at the level of conventional criticality matrix and can be easily adjusted to improved criticality matrices.

The final results given by this new matrix can be directly compared with those obtained by semi-quantitative and quantitative methods such as Fault Tree Analysis (FTA), the Layer Of Protection Analysis (LOPA) and Quantitative Risk Assessment (QRA).

REFERENCES

[1] Bouchon-Meunier, B., *Fuzzy logic and its applications - Artificial life*, Addison - Wesley France, Paris, 1995.

[2] Dubois, D., and Prade, H., "The mean value of a fuzzy number", *Fuzzy Sets and Syst.*, 24 (1987) 279–300.

[3] Freska, C., "Linguistic description of human judgments in experts systems and in the "Soft" sciences", *in: M. M. Gupta and E. Sanchez (eds.), Approximate reasoning in decision analysis, North-Holland, Publishing Company*, 1982, 297–305.

[4] Hicks, F., and Fayek, A., "Forecasting ice jam risk at fort mcmurray, AB, using fuzzy logic", in: *Proceedings of the 16th IAHR International Symposium on Ice Dunedin*, New Zealand, 2002, 112–118.

[5] Mamdani, E., and Assilian, S., "An experiment in linguistic synthesis with a fuzzy logic controller " *International J. of Man-Machine Studies*, 7 (1975) 1–13.

[6] Nait-Said, R., and al., "Modified risk graph method using fuzzy rule-based approach" *Journal of Hazardous Materials*, 164 (1) (2009) 651–658.

[7] Roger Jang, J. S., and Gulley, N., "MATLAB, Fuzzy Logic Toolbox", April (1997).

[8] Xu, K., and al., "Fuzzy assessment of FMEA for engine systems" *Reliability Engineering System Safety*, 75 (1) (2002) 17–29.

[9] Zadeh, L. A., "Fuzzy sets", *Information and Control*, 8 (1965) 338–353.

[10] Zadeh, L. A., "A theory of approximate reasoning", in: *Machine, Intelligence, New York*, 1979, 149–194.