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BI- LEVEL PROGRAMMING DEA APPROACH FOR EFFICIENCY EVALUATION: A CASE STUDY OF INDIAN ELECTRONICS RETAIL STORES

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Abstract: Retail industry has witnessed enormous growth in the past decade in developing countries like India, China, and Brazil, owing to the upswing in globalization, growing trends in e-commerce, multi-format retailing, and increasing penetration of the internet. The growth of opportunities, on the other hand, have intensified the competition. It is important for retailers to gain a competitive edge in the market through innovative strategies and continuous improvement. Meticulous planning and efficiency in operations are the drivers for economic sustainability and profitability of the business. Important prerequisites to gain efficiency and planning for improvement is the evaluation of base level performance, defining benchmarks and evaluating effectiveness of the efforts taken in this direction. The studies in this domain existing in the literature have analysed efficiency of retail stores as a black box transforming input to outputs. This approach lacks transparency and overlooks the subprocesses, their characteristics and internal interaction and can be addressed considering the transformation process in a two stage system. Our study addresses the issue and proposes a Bi-level Programming DEA approach to evaluate the relative efficiency of multiple retail stores considering a network structure operating in a Stackelberg relation and defining benchmarks for inefficient stores. The approach enables computation of efficiency of each sub-stage as well as the overall efficiency of the stores. The proposed approach is validated through a case study of Indian electronic retail chain.

Keywords: Retail Stores, Electronics, Bi-level Programming, Data Envelopment Analysis, Benchmarking.

MSC: 90B85, 90C26.

1. INTRODUCTION

Retail is an important supply chain stage which creates an effective link between upstream and downstream supply chains (SC). Retailers are instrumental in controlling the circulation of material and information in supply chains as they create a balance between the customer order fulfilment and supply of materials from the upstream supply chain [43] Figure 1. Compared to the developed countries, there is greater retail trade potential in the developing countries like India, Brazil, China [39]. Indian retail market enrols for over 10% of the nation's GDP (Gross domestic Product and is expected to rise up to 60% near US$1.1 trillion by 2020 [24]. Indian retail industry is categorized into different sectors like food retailers, health and beauty products, clothing, footwear, and electronics goods. Among these sectors electronics retail (ER) has a remarkable impact on development of the industry due to rapid rate of urbanisation, rising income level of the middle class, lifestyle changes, digital penetration and support of the Goverments' policies. It is predicted that ER will boost up to 9 to 10 % nearby US $48.37 billion over next 4-5 years [25]. Growth opportunities in the ER sector have attracted new entrants, leading to increasing competition. There is a momentous opportunity for retailers to gain competitive positioning in the market. Effective competitive positioning requirement is coupled with the pressure to remain efficient in performance [31]. It is essential for retail chains to assess their stores' performance scientifically and then direct appropriate efforts to gain the benefit in the competitive environment [48]. In this direction this study proposes bi-level DEA [7] models for performance measurement of an Indian ER chain considering network structure with a Stackelberg game approach [53].

Charnes et al.[7] developed a DEA model for evaluating the relative efficiency of several Decision Making Units (DMUs) based on multiple inputs and outputs. Subsequently several scholars have contributed to theoretical developments and practical applications in diverse areas such as banking[8], education[41], transport[5], supply chains[52]and retail[29]. Conventional efficiency measurement models based on DEA considers DMUs as a black box wherein a set of inputs transforms to specific output(s)[12]. The existing literature of efficiency measurement related to retail stores also mostly follows the black box approach [29]. In retail stores the inputs such as inventory, human resource, store size, promotional expenses, operating expenses, and fixed assets are transformed into outputs such as product assortments, footfall, profit, sales and customer satisfaction [29, 65, 61]. The effectiveness of the several functions (that consumes these inputs) performed in retail stores such as store planning and design, display placement, inventory management, staff management, money and credit handling, and customer services, determines the extent of outputs and hence the efficiency of the store [29, 38, 31].

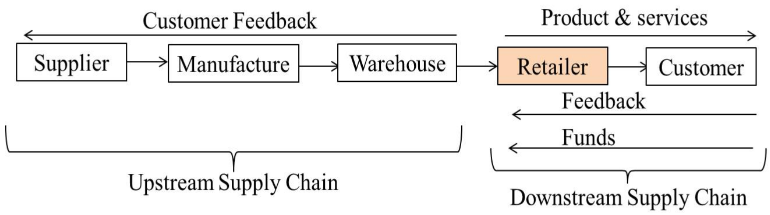

Figure 1: Traditional Supply Chain

Among these functions some including store planning and design, display placement, store level promotional activities are among the leading activities while others such as inventory management, selling and customer service and, money and credit handling are the following activities [16]. The decisions and inputs in the leading activities generate intermediate outputs (assortment and footfall) consumed as inputs in the following activities along with direct/shared inputs that yield the final retail outputs. In this scenario viewing the retail store as a black box for efficiency measurement lacks transparency and overlooks the subprocesses, their characteristics and internal interaction [61, 30].

There are studies in the literature with applications mainly in banking [46], hotel[23] and transportation[21]that discusses evaluation of efficiency in two stages (known as relational/network DEA model[28]). Few studies also discuss the supply chain efficiency measurement in a two stage network structure (TSNS) [52], while there is no significant research contribution dealing with the efficiency evaluation of retail stores considering network structures. In this direction our study proposes a bi-level network structure DEA model for retail chains[61] considering the two stages in the retail stores network structure operating according to the Stackelberg game relation. The bi-level DEA(BLDEA) model optimizes the weighted input cost [10] and determines the cost efficiency for each subsystem, the overall system and the optimal level of inputs for each subsystem within a retail store. The model validation and application is presented with a case study of an Indian ER chain.

The manuscript is organized as follows - Section 2 review the literature. Problem definition is discussed in section 3. Research methodology is described in section 4 along with the formulation of the BLDEA model and solution methodology. A case study is given in section 5. Section 6 presents the conclusion of the study.

2. LITERATURE REVIEW

Efficiency measurement of SC has gained the interest of several scholars [34, 18, 52]. Some studies have also investigate retail efficiency measurement of the retail echelon of the SC [9, 70, 31, 20]. The existing research related to retail efficiency evaluation has mostly considered the black box perspective ignoring the internal structures[66, 32, 63]. Our study addresses this issue and proposes a TSNS approach for evaluating efficiency of multiple retail stores. The following section discusses the literature and presents the literature gap and contributions of our study.

2.1. Performance Evaluation in Retail sector

Donthu and Yoo[12]applied basic (CCR) DEA model (Charnes et al.[7]) for efficiency measurement of fast food retail outlets considering inputs - carpet area, manager's experience, location and promotion expenses and two outputs - customer satisfaction and sales. In terms of the DMUs the studies have considered efficiency measurement of different departments within a store [38], multiple stores of a retail chain [31], efficiency with respect to competitive stores [1], multiple stores within a country and multiple countries [55]. From the application perspective the existing research in this domain mainly has applications in food and grocery retailers, apparel retail and home furnitur[19, 63, 55]. Table 1 presents the relevant literature in the last ten years.

Studies	**Highlights**
Yu and Ramanathan [66]	Evaluated the efficiency of Chinese retail firms using CCR DEA model with inputs (floor space and employees) and outputs (sales and profit) and investigated the efficiency score changes between 2000 and 2003 using MPI (Malmquist Productivity Index) and the effect of environmental variables employing bootstrapped Tobit regression model (TRM)
Gupta and Mittal[19]	Evaluated CCR DEA based efficiency grocery retail organization in NCR, India based on inputs (store size, SKUs, check points, no. of employees, employees cost and working hours) and outputs (sales and customer's conversion ratio).
Pande and Patel[38]	Scrutinized cost efficiency of pharmacy firm in NCR, India and derived the effect of footfalls, sales and operating expenses using TRM.
Gandhi and Shankar[17]	Demonstrated the measurement of economic efficiency of Indian retail firms between 2008 and 2010 following Yu and Ramanathan (2009).
Xavier et al.[63]	Presented evaluation of efficiency for Portuguese retail stores of a clothing retail company using CCR DEA.
Duman et al.[13]	Measured the retail performance of the US food industry integrating Fuzzy AHP, DEA and TOPSIS methodologies.
Ko et al.[31]	Measured the efficiency for Korean household retail chains and studied the influence of assortment and competitive environment on efficiency scores using TRM.
Gupta et al.[20]	Presented an approach for scientifically selecting key performance measures following a two stage methodology, efficiency measurement for retail stores with a case study of an Indian ER chain.

Table 1: Review of performance evaluation in the retail sector

It is evident from table 1 that the existing studies considered the single stage transformation of inputs to outputs for efficiency measurement of retails stores. This approach as discussed above overlooks the subsystems, lacks in exposition of internal interaction and hence limits the transparency in results (Wu[61]. These issues can be addressed considering a network (relational) structure (two-stage) for efficiency evaluation. The following section presents the review literature related to efficiency measurement considering a two stage network structure.

2.2. Efficiency Measurement in a Two-Stage Network (relational) Structure

TSNS DEA models have been discussed by several authors in different contexts[45, 8, 27]. The TSNS has mainly two approaches - independent and relational [28, 54]. In independent models, each stage is evaluated independently, while a relational model accesses the efficiency of two stages jointly considering intermediate measures and the overall efficiency of the DMU is calculated. Seiford and Zhu[44]) examined the efficiency of commercial banks of US based on two stage processes using basic DEA models. Profitability is computed in the first stage taking inputs - staff and assets, to produce profit and revenue. Using I stage output as II stage inputs (intermediate measures) to produce final output returns, market value and earning/share (defined as the marketability of the banks). Zhu[69] followed [44] to measure the performance of fortune 500 companies. Chen and Zhu[8] indicated [44]) approach is likely to compute a DMU as efficient overall while the individual stages are yet inefficient given the assumption that the two stages are operating independently. The efficiency of one stage is likely to be affected with the other stage due to the influence of intermediate measures. To address this issue, Chen and Zhu[8] proposed a linear model to optimize the efficiency measurment of two stages jointly. Kao and Hwang[27] argued that both of the above approaches don't provide the decomposition of the overall efficiency in terms of the sub-processes and proposed the multiplicative efficiency decomposition (MED) framework to decompose the total efficiency in a two-stage process. Kao[28] extended the concept of Kao and Hwang[27] and proposed a TSNS DEA model (RNDM) introducing sub-stages in series and parallel. Chen et al.[9] proposed overall efficiency decomposition through weighted sum of efficiency of the two stages as an alternative to [27] model. These studies considered only intermediate measures as input for the second stage in RNDM. Wu[61] extended the work considering direct and shared inputs along with intermediate measures in the second stage citing examples of banks where employees are the shared inputs and IT budget as the direct input for the second stage. The author proposed a bi-level RNDM with Stackelberg game relationship between stages to analyse the cost efficiency presenting applications from banking and manufacturing-distribution. Table 2 presents the relevant literature in the last ten years.

2.2.1. Efficiency Evaluation with Network Structure in Retail

Keh and Chu[29] evaluated efficiency of a US based grocery retail chain stores, in two-stages considering inputs - capital and labour; intermediate outputs assurance of product delivery, assortment, product information, accessibility and ambience; and output profit. The study ignores the interaction of the stages and doesn't analyse the overall system efficiency. Vaz et al.[56] presented two level efficiency measurement of stores of a Portuguese supermarket retail chain, considering store stections at I level and full store at the II level. The study uses the result of the first stage for setting the goals in the second stage to determine the potential improvement in sales through reallocation of shared resources used by sections, rather than considering leader-follower relation between the stages as considered in our study.

Studies	Objective	Application	Key highlights
Shahroudi et al.[47]	Efficiency measurement using RNDM	Iranian private insurance companies	Followed Kao and Hwang (2008) approach
Wanke[59]	Measured the efficiency for a two-stage system	Brazilian ports	Second stage inputs are described only by the intermediate output measure
Yu and Shi[67]	Efficiency measurement MED approach	Production process	Applies a heuristics method for solution of the proposed nonlinear model. Considers direct inputs in the II stage
Omrani and Keshavarz[35]	Efficiency measurement using RNDM	Iranian shipping company	Evaluated performance between the period 2008-2011
Yu et al.[64]	Efficiency measurement MED approach	Banking sector	Introduced the concept of cross efficiency in two stage system
Shafiee et al.[46]	Proposed a RNDM	Banking sector	Mixed integer linear programming (MILP) model for efficiency measurement
Toloo et al.[54]	Proposed a linear RNDM	University operations and banking industry	Measure the efficiency with shared inputs under CRS assumption
Zhou et al.[68]	Analysed multiple followers	Banking sector	Extend the concept of bi-level DEA [61]
Li et al.[33]	Efficiency measurement using RNDM	Fire protection system	Considered shared inputs and additive efficiency decomposition DEA mode
Chao et al.[6]	Developed a two-stage model	International container shipping companies	Analysed performance over multiple time periods
Wanke et al.[62]	Proposed a dynamic RNDM	Banking industry	Considered the interactions between the accounting and financial departments

Table 2: Review on efficiency measurement in TSNS

It is evident that while there are ample numbers of studies considering two stage relational network structures for efficiency measurement with applications in different industries; however the analysis of retail performance measurement in a network structure is discussed very limitedly. On this backdrop our study focuses on the performance evaluation of multiple stores of a retail chain considering a two-stage network structure operating in Stackelberg game relation.

Game theory has been gaining importance in many fields since 1928 since since the pioneering work of John von Neumann [57]. "Game theory provides a framework for modelling interactions between groups of decision-makers when individual actions jointly determine the outcome"[15]. Game theory models can be classified as cooperative and non-cooperative models[36]. The Stackelberg game (leader-follower game) approach was developed by Von Stackelberg[53] in (1952) also called as leader-follower game.Von Stackelberg introduced it as a static bilevel optimization model and has been applied in a lot of fields [46]. Esmaeili et al. [14] proposed models based on seller-buyer relation considering the stackelberg scenario. Sinha et al.[51] employed multi-leader-follower stackelberg model to determine optimal multi-period strategies for the aircraft manufacturing industry. Hjaila et al.[22] used stackelberg game approach to develop coordination between multi enterprise SC in an uncertain environment considering manufacturer as the leader. Rahmatiet al.[40] presented an approach based on stackelberg games to allocate optimal time for advertising in vehicular ad-hoc networks.

2.3. Research Gap and Contribution of the Study

From the ROL it is apparent that efficiency measurement of retail businesses employing DEA approaches has attracted the interest of several scholars. Important gaps have emerged from the review of the literature and in discussions with industry practitioners as follows

1. The existing research considers retail stores as a black box with single stage transformation of inputs into the desired outputs [66, 32, 17]. However discus-

sions with the industry experts highlighted the limitation for practical applications of this approach as internal structures and interactions are overlooked. The transformation process of retail inputs to outputs can be analysed as a two stage process considering the leader follower (Stackelberg game) relationship between the stages[68] to determine the relative efficiency of multiple retail stores. This approach considers the internal structures and interactions between the stages through intermediate inputs/outputs [28].

2. Existing studies have contributed to the theoretical development of the two stage network structure DEA models considering different theoretical[44, 8, 27] and practical aspects[69, 9, 59, 60]. Most of these studies limit the inputs of the II stage to I stage intermediate outputs. Mathematical formulation of the model becomes nonlinear considering direct inputs in II stage and/or shared inputs between stages. Few studies have also attempted to account shared/direct inputs as in [61, 68, 33] have also attempted to account shared/direct inputs. Our study is developed on the Wu[61]approach. Wu[61] model considers equal (unit) importance for all costs attached to inputs of both stages, while the relative importance may be different. Further while optimizing the distribution of shared resources, the model only considers the resource constraints and ignores the feasibility of solution in practice as it may provide insufficient/over allocation of shared resources.

Given the above research gaps the specific contributions of the study are as follows:

1. The study presents a method to evaluate the efficiency of retail stores of a chain considering TSNS following the Stackelberg game relation compared to the black box approach. A BLDEA efficiency measurement model is proposed considering shared as well as direct inputs in II stage along with intermediate measures. To the best of our information no existing work attempts to evaluate the measurement of efficiency of retail stores in a TSNS with shared resources among stages and direct resources in II stage.

2. Compared to the existing studies instead of assuming equal weight of input cost vectors, the proposed method integrates multi-criteria decision making (MCDM) method analytical hierarchical process (AHP) to compute the weights of cost vectors.

3. Our study further attempts to adapt the (Wu[61]) model including bounds on the allocation of shared resources between stages such that the optimal allocation of shared resources is also feasible in practice.

4. An illustration of the proposed approach is provided with a case study of an Indian electronics retail chain.

3. PROBLEM DEFINITION

Over the last one decade, the retail sector has observed sizable growth in developing countries and it is expected that the retail potential will continue to increase in the next decade[25]. The unit of analysis in consideration is an Indian electronic retail chain. Given the prevailing scenario of huge market potential with intense competition, the decision makers want to create maximum value for its customers, be efficient in terms of their performance and continuously improve their market base. In this direction the firm wants to establish a structured and scientific methodology for measuring the efficiency of their stores that can help them monitor their performance and guide the efforts directed towards improvement[31]. Discussions with the decision maker highlighted the limitations in adopting the existing approaches for performance evaluation as highlighted in section 2.3. This study attempts to address the following research questions.

1. What is an appropriate approach to examine the relative efficiency of several stores of a retail chain integrating the interaction of subprocesses and internal structures ?

2. What are the key inputs, intermediate and output measures to effectively evaluate the efficiency of a retail store?

3. What are the key inputs, intermediate and output measures to effectively evaluate the efficiency of a retail store?

4. How to distribute the shared resources within the internal structures to optimize the system efficiency?

The objectives of our the study includes

1. Formulate a framework for examining the relative efficiency of several stores of a retail chain integrating the effect of subprocesses and internal structures.

2. Identify the key measures in terms of inputs, intermediate measures and outputs.

3. Determine the relative importance of the input/output (I/O) measures.

4. Identify a suitable methodology for measuring efficiency of the sub-processes and overall store and formulate the mathematical model for the proposed framework including optimal distribution of shared resources.

5. Present a case study for model validation and demonstrate the solution methodology.

4. METHODOLOGY

For the development of conceptual framework and mathematical model for retail stores efficiency measurement taking into account the subprocesses, their characteristics and internal interaction [61, 30], first we need to understand the retail operations and the transformations processes. The functions (subprocesses) performed in a retail store can be viewed broadly at two levels - leading and following - as discussed in section 1. Each of the stages produces its output(s) based on the inputs received. Outputs of the following stage, in addition to its direct inputs are influenced by the leading stage decisions. The two stages also share common inputs like human resources.The specific nature of these I/O depends on the context and is determined through the review of literature and expert discussions. Considering this scenario the conceptual framework of the proposed model can be described by the network structure as shown in Figure 2 [67].

A DEA cost efficiency model formulated in bi-level programming structure enables simultaneous consideration of the objectives of the two levels and provides efficiency of the overall system as well as of the individual stages [61]. Efficiency

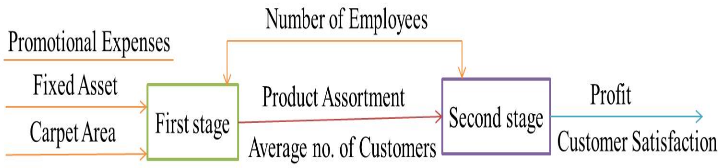

Figure 2: Network Structure of a Retail Store

measurement model for the network structure depicted in Figure 2 is hence formulated as a BLDEA model. In the following section we first discuss briefly the preliminaries of DEA cost efficiency model and bi-level programming followed by BLDEA model formulation.

4.1. Preliminaries

4.1.1. DEA

This section discusses the basic CCR DEA cost efficiency model[10]. Consider n DMUs under evaluation.

Notations j : index for number of DMUs; j=*1,2,..,n*. i : index for number of inputs; i=*1,2,..,m*. j : index for number of outputs; r=*1,2,..,s*. c_i : unit cost associated to the input x_i. x_{ij} : i^{th} input value of the j^{th} DMU. y_{rj} : r^{th} input value of the j^{th} DMU.

The linear programming (LP) model (M1) optimizes the cost for k^{th} DMU at the current level of outputs with the cost vector c= $\{c_i;i=1,...,m\}$ associated with the input vector x= $\{x_i;i=1,...,m\}$ for the k^{th} DMU.

$$\begin{aligned} cx^* &= \min \ \textstyle\sum_{i=1}^{m} c_i x_i \\ \text{s.t.} \quad & x_i \geq \textstyle\sum_{j=1}^{n} x_{ij}\lambda_j \ \ \forall i = 1,2,..,m \\ & y_{rk} \leq \textstyle\sum_{j=1}^{n} y_{rj}\lambda_j \ \ \forall r = 1,2,..,s \\ & \lambda_j \geq 0 \ \ \forall j \end{aligned} \tag{M1}$$

Solving model M1 we obtain the optimal value of the decision variables

$$(x_i, \lambda_j) = (x_i^*, \lambda_j^*)$$

and the optimal cost cx^*, the minimum cost for k^{th} DMU for the current level of outputs subject to the resource constraints. Based on the optimized input cost obtained on solving model M1 the cost efficiency of k^{th} DMU is calculated using

$$\theta_k = \frac{cx^*}{cx_k}$$

such that θ_k satisfies the following two properties

$$0 \leq \theta_k \leq 1, \quad \text{where } \theta_k = \begin{cases} 1 & k^{th}\ DMU\ is\ cost\ efficient \\ otherwise & k^{th}\ DMU\ is\ cost\ inefficient \end{cases}$$

4.1.2. Bi level Programming

Inspired from the Von Stackelberg's[53] game theory, bi-level programming (BLP) is a nested optimization problem applicable for optimizing the decisions at two levels connected in a hierarchical network structure. The two-level hierarchy under consideration is labelled as: upper (leader) and lower (follower) level. Wherein the two levels influence each other's decision making with their own independent objectives, decision variables and independent/shared restrictions. The follower decision is guided by apprehension of the leader's resolutions known to the follower partially, while the leader's level decision is optimized in the presence of complete information of the follower's decision, including its expected response to the leader's decision [61, 46, 68]. The standard mathematical formulation of a BLP problem is as follows

$$\begin{aligned} (PL1)\ & \min_{x\in X}\phi(x,y) = c_1x + d_1y \\ & \text{s.t. } A_1x + B_1y \leq b_1 \\ & \quad \text{where } y \text{ solves:} \\ (PF1)\ & \min\nolimits_{y\in Y}\ \gamma(x,y) = c_2x + d_2y \\ & \text{s.t. } A_2x + B_2y \leq b_2 \\ & \quad x \geq 0,\ y \geq 0 \end{aligned} \tag{M2}$$

Where, $x \in R^n$ and $y \in R^m$ represent upper and lower level decision variables, respectively. $c_1, c_2 \in R^n$ and $d_1, d_2 \in R^m$ are row coefficient vector; $A_1 \in R^{e\times n}$, $A_2 \in R^{f\times n}$ and $B_1 \in R^{e\times m}$, $B_2 \in R^{f\times m}$ are coefficient matrix; and $b_1 \in R^e$ and $b_2 \in R^f$ are associated constant column vector. $\phi(x, y)$ and $\gamma(x, y)$ are objective function of upper and lower levels, respectively.For further details of Bi level Programming please refer to[4].

Even though a BLP model (M2) is characterised by linear functions the nature of problem is non-convex and NP-hard as the follower's level problem is nested within the leader's problem[2]. Shi et al.[49] proposed a single level linear optimization model transformation for deriving the solution of the model (M2). The transformation is based on Karush–Kuhn–Tucker (KKT) optimality conditions, wherein the leaders' problem is redefined by including the follower's KKT conditions as constraints. The authors later proposed an extended branch and bound algorithm to solve the single level problem[50].

4.2. Bi-level programming DEA cost efficiency model

4.2.1. Cost efficiency optimization model

Consider n DMUs structured into two level decision hierarchies operating in Stackelberg conditions. The leading level problem is to optimize the level of its direct inputs along with the inputs it shares with the following level to yield the desired outputs. The following stage consumes intermediate measures and guided by the leader's decision the follower's level optimizes inputs for producing the final outputs. The mathematical formulation of the model is given as follows

Notations: c^1 is unit cost associated with shared inputs for the leader/follower; c^2 is unit cost associated with direct inputs for the leader; c^3 is unit cost associated with direct inputs for the follower; c^4 is unit cost associated with intermediate I/O for the follower;

For the j^{th} DMU; j=1,2,..,n. X^s_{lj} is shared input vector of the leader; X^d_{lj} is direct input vector of the leader; X^s_{fj} is shared input vector of the follower; X^d_{fj} is direct input vector of the follower; Y^I_j is intermediate I/O vector of the leader and follower; Y_{lj} is direct output vector of the leader; Y_{fj} is direct output vector of the follower.

Decision variables: $\bar{X^s_{lj}}$ is optimal shared input vector of the leader; $\bar{X^d_{lj}}$ is optimal direct input vector of the leader; $\bar{X^s_{fj}}$ is optimal shared input vector of the follower; $\bar{Y^I_j}$ is optimal intermediate I/O vector of the leader and follower;

k^{th} DMU's Leader's Problem

$$
\begin{aligned}
(PL2) \quad & \min_{\bar{X^s_{lk}},\bar{X^d_{lk}},\lambda} (c^1\bar{X^s_{lk}} + c^2\bar{X^d_{lk}}) + (c^1\bar{X^s_{fk}} + c^3\bar{X^d_{fk}} + c^4\bar{Y^I_k}) \\
& \text{s.t. } \bar{X^s_{lk}} \geq \textstyle\sum_{j=1}^n \lambda_j X^s_{lj} \\
& \bar{X^d_{lk}} \geq \textstyle\sum_{j=1}^n \lambda_j X^d_{lj} \\
& Y^I_k \leq \textstyle\sum_{j=1}^n \lambda_j Y^I_j \\
& Y_{lk} \leq \textstyle\sum_{j=1}^n \lambda_j Y_{lj} \\
& \bar{X^s_{lk}} + \bar{X^s_{fk}} \leq M \qquad (1)
\end{aligned}
$$

k^{th} DMU's Follower's Problem

$$
\begin{aligned}
(PF2) \quad & \min_{\bar{X^s_{fk}},\bar{X^d_{fk}},\bar{Y^I_k},\pi} (c^1\bar{X^s_{fk}} + c^3\bar{X^d_{fk}} + c^4\bar{Y^I_k}) \\
& \text{s.t. } \bar{X^s_{fk}} \geq \textstyle\sum_{j=1}^n \pi_j X^s_{fj} \\
& \bar{X^d_{fk}} \geq \textstyle\sum_{j=1}^n \pi_j X^d_{fj} \\
& \bar{Y^I_k} \geq \textstyle\sum_{j=1}^n \pi_j Y^I_j \\
& Y_{fk} \leq \textstyle\sum_{j=1}^n \pi_j Y_{fj} \\
& \bar{X^s_{lk}}, \bar{X^d_{lk}}, \bar{X^s_{fk}}, \bar{X^d_{fk}}, \lambda, \pi \geq 0
\end{aligned}
\qquad \text{(M3)}
$$

The last constraint in the leader problem is the resource constraint with M (a known and constant value) as upper bound on the shared resource consumption.

The model (M3) determines the optimal level of inputs for leader and follower. The optimal solution obtained based on the solving model (M3) distributes the shared resources between two stages and can result in insufficient/over allocation to a stage limiting the practical applicability of the solution. For example, the proportion of shared resources allocated any stage can range from 0% to 100% according to model (M3). If the shared resource is human resource), there is a minimum requirement of human resource at both the stages and the follower stage is more human resource intensive than the leader stage. To resolve this issue in the next section we propose an extension of the model (M3) introducing bounds on the shared resource allocation. Value of bounds can be obtained from the decision maker or based on industry standards.

4.2.2. Cost efficiency optimization model with Bounds on Shared Resource Distribution

k^{th} DMU's Leader's Problem

$$(PL3) \quad \min_{\bar{X^s_{lk}},\bar{X^d_{lk}},\lambda} (c^1\bar{X^s_{lk}} + c^2\bar{X^d_{lk}}) + (c^1\bar{X^s_{fk}} + c^3\bar{X^d_{fk}} + c^4\bar{Y^I_k})$$

$$\alpha M \leq \bar{X^s_{lk}} \leq \beta M \quad (2)$$

k^{th} DMU's Follower's Problem

$$(PF3) \quad \min_{\bar{X^s_{fk}},\bar{X^d_{fk}},\bar{Y^I_k},\pi} (c^1\bar{X^s_{fk}} + c^3\bar{X^d_{fk}} + c^4\bar{Y^I_k})$$

$$\bar{X^s_{fk}} \geq \beta_1 M \quad (3)$$

$$\bar{X^s_{lk}}, \bar{X^d_{lk}}, \bar{X^s_{fk}}, \bar{X^d_{fk}}, \lambda, \pi, \alpha, \beta, \beta_1 \geq 0$$

(M4)

The model (M4) is redefined with bounds on shared resources along with all other constraints as in (M3). Here the constraint (2) and (3) defines the feasible range of allocation of shared resources for the leader and follower levels respectively

4.3. Solution Method

Solving the model (M4) we obtain the optimal solution $\{\bar{X}, \lambda, \pi : \bar{X}_k, \lambda_k, \pi_k, k = 1, 2, .., n\}$. The model (M4) is a nested non-convex and NP hard problem. The model (M4) can be solved by transforming the BLDEA model into a single linear optimization model. The following theorem based on KKT conditions is applied for obtaining the transformation[49].

Theorem1 : if u,v and w are dual variables associated with the constraints of leaders and followers problem in model (M4), then a necessary and sufficient condition that $(\bar{x}, \bar{y})$ is an optimal solution to the BLDEA problem (M4) is that there exist the row vectors $\bar{u}$,$\bar{v}$ and $\bar{w}$ such that $(\bar{x}, \bar{y}, \bar{u}, \bar{v}, \bar{w})$ solves the model

$$\begin{aligned} &\min F(x, y) = c_1x + d_1y \\ &\text{s.t. } A_1x + B_1y \leq b_1 \\ &\quad A_2x + B_2y \leq b_2 \\ &\quad uB_1 + vB_2 - w = -d_2 \\ &\quad u(b_1 - A_1x - B_1y) + v(b_2 - A_2x - B_2y) + wy = 0 \\ &\quad x \geq 0,\ y \geq 0,\ u \geq 0,\ v \geq 0,\ w \geq 0 \end{aligned} \quad \text{(M5)}$$

where $x = \begin{pmatrix} \bar{X}^s_{lk} \\ X^d_{lk} \\ \lambda \end{pmatrix}, \quad y = \begin{pmatrix} \bar{X}^s_{fk} \\ X^d_{fk} \\ Y^I_k \\ \pi \end{pmatrix}$

$c_1 = \begin{pmatrix} c^1 & c^2 & 0 \end{pmatrix}, \quad d_1 = \begin{pmatrix} c^1 & c^3 & c^4 & 0 \end{pmatrix}, \quad c_2 = 0, \quad d_2 = \begin{pmatrix} c^1 & c^3 & c^4 & 0 \end{pmatrix},$
$X^s_l = \begin{pmatrix} X^s_{l1} & X^s_{l2} & \dots & X^s_{ln} \end{pmatrix}, \quad X^d_l = \begin{pmatrix} X^d_{l1} & X^d_{l2} & \dots & X^d_{ln} \end{pmatrix}, \quad Y_l = \begin{pmatrix} Y_{l1} & Y_{l2} & \dots & Y_{ln} \end{pmatrix},$
$Y^I = \begin{pmatrix} Y^I_1 & Y^I_2 & \dots & Y^I_n \end{pmatrix}, \quad X^s_f = \begin{pmatrix} X^s_{f1} & X^s_{f2} & \dots & X^f_{ln} \end{pmatrix}, \quad X^d_f = \begin{pmatrix} X^d_{f1} & X^d_{f2} & \dots & X^d_{fn} \end{pmatrix},$
$Y_f = \begin{pmatrix} Y_{f1} & Y_{f2} & \dots & Y_{fn} \end{pmatrix}, \quad 0 = \begin{pmatrix} 0 & 0 & \dots & 0 \end{pmatrix}$

$$A_1 = \begin{pmatrix} -1 & 0 & X^s_l \\ 0 & -1 & X^d_l \\ 0 & 0 & -Y^I \\ 0 & 0 & -Y_l \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}, \quad B_1 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad b_1 = \begin{pmatrix} 0 \\ 0 \\ -Y^I_k \\ -Y_{lk} \\ M \\ \beta M \\ -\alpha M \end{pmatrix},$$

$$B_2 = \begin{pmatrix} -1 & 0 & 0 & X^s_f \\ 0 & -1 & 0 & X^d_f \\ 0 & 0 & -1 & Y^I \\ 0 & 0 & 0 & -Y_f \\ -1 & 0 & 0 & 0 \end{pmatrix}, \quad b_2 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ -Y_{fk} \\ M \\ -\beta_1 M \end{pmatrix}, \quad A_2 = 0$$

The single level transformed model (M5) is non-linear in nature. Softwares such as LINGO, Mathematica, and R are available to solve the robust optimization problems. In this paper we have coded the model (M5) on optimization software LINGO to obtain the solution. LINGO software provides a variety of solvers including global, integer, linear, non-linear and general to solve the robust optimization problems. The model is solved once with respect to each DMU to obtain the optimal level of inputs for both levels of the DMU using Global Solver. The solution for model (M3) can be obtained as a special case of model (M4). The cost efficiency for bi-level system under the model (M5) can be obtained from the following proposition.

Proposition 1: Cost efficiency of the k^{th} leader (follower) is the ratio of optimal and observed cost of the leader (follower).

Optimal cost of the k^{th} leader is $c^1\bar{X}^s_{lk} + c^2\bar{X}^d_{lk}$ (4)

Thus the cost efficiency of the k^{th} leader is defined as

$$\theta^l_k = \frac{c^1\bar{X}^s_{lk} + c^2\bar{X}^d_{lk}}{c^1X^s_{lk} + c^2X^d_{lk}}, \quad \text{where } \theta^l_k = \begin{cases} 1 & iff\ k^{th}\ leader\ is\ cost\ efficient \\ else & iff\ k^{th}\ DMU\ is\ cost\ inefficient \end{cases} \tag{5}$$

Optimal cost of the k^{th} follower is $\quad c^1\bar{X}^s_{fk}+c^3\bar{X}^d_{fk}+c^4\bar{Y}^I_k$ (6)

$$\theta^f_k = \frac{c^1\bar{X}^s_{fk} + c^3\bar{X}^d_{fk} + c^4\bar{Y}^I_k}{c^1X^s_{fk} + c^3X^d_{fk} + c^4Y^I_k}, \text{where } \theta^l_k = \begin{cases} 1 & iff\ k^{th}\ leader\ is\ cost\ efficient \\ else & iff\ k^{th}\ DMU\ is\ cost\ inefficient \end{cases} \tag{7}$$

and the cost efficiency of the k^{th} DMU is defined as

$$\theta_k^s = \frac{c^1 \bar{X}_{lk}^s + c^2 \bar{X}_{lk}^d + c^1 \bar{X}_{fk}^s + c^3 \bar{X}_{fk}^d + c^4 \bar{Y}_k^I}{c^1 X_{lk}^s + c^2 X_{lk}^d + c^1 X_{fk}^s + c^3 X_{fk}^d + c^4 Y_k^I}, \text{where } \theta_k^s = \begin{cases} 1 & iff\ k^{th}\ leader\ is\ cost\ efficient \\ else & iff\ k^{th}\ DMU\ is\ cost\ inefficient \end{cases} \quad (8)$$

Proposition 2: $\theta_k^s = 1$ *iff* $\theta_k^l = 1$ and $\theta_k^f = 1$. This implies k^{th} system is efficient when it's both stages are efficient respectively.

4.3.1. Benchmarking for Inefficient Units

BLDEA model can also be used to determine the benchmarks for inefficient DMUs as follows[61].

Benchmarking unit of Leader (follower): if the optimal value of $\lambda_j \geq 0 (\pi_j \geq 0)$ in the solution of model (M5) solved with respect to DMU *k*, then the DMU *j* is a benchmark for the DMU *k*.

Benchmarking unit of system: if the optimal value of $\lambda_j \geq 0$ *and* $\pi_j \geq 0$ in the solution of model (M5) solved with respect to DMU *k*, then DMU *j* is a benchmark for the DMU *k*.

5. APPLICATION

The proposed framework is validated through a case study of an Indian electronics retail chain. The details of the retail chain are not shared here due to commercial confidentiality of information. The firm sells a broad assortment of home electronics products of several brands. The case company wants to focus on improvement of its store's performance basis the current performance. The objective is to identify the important dimensions for measuring the relative efficiency of its stores, their respective efficiencies and benchmarks for inefficient stores to target improvement efforts. The decision makers found the traditional black box approach for efficiency measurement of retail stores limiting in providing the appropriate measure of efficiency as it ignores the internal structures and processes. Following the proposed two stage network structure based approach (Figure 2) and the BLDEA models (M3 and M4) efficiency of stores is measured.

5.1. Data

The data is collected for 24 stores of the retail chain (numbered from R1- R24). Through the detailed review of literature inputs, intermediate I/O, and outputs are identified[20]. The identified key measures for both stages are identified and described in Table 3.

For the output customer satisfaction average value of customer satisfaction is provided based on the past data collected in store in routine basis measured on 5 point likert scale[12] for other I/O the firm provided modified data from the stores.

	Key Measures	References
1^{st} stage inputs	No. of employess	Izadikhah et al.[26]; Zhou et al.[68]; Ko et al.[31]
1^{st} stage inputs	Carpet area	Duman et al.[13]; Ko et al.[31]
1^{st} stage inputs	Fixed assets	Yu et al.[64];Wu[61];De Jorge Moreno[11]
1^{st} stage inputs	Promotional expenses	Donthu and Yoo[12]
2^{nd} stage inputs (direct)	Operating expenses	Balios et al.[1]; Xavier et al.[63]
2^{nd} stage inputs (direct)	Average inventory cost	De Jorge Moreno[11];Pestana Barros[37]
Intermediate measures	No. of customers	Lau[32]; Ko et al.[31]
Intermediate measures	Product assortment	Keh and Chu[29]; Betancourt et al.[3]
2^{nd} stage outputs	Profit	Lau[32]; Gandhi and Shankar[17] (2014)
Intermediate measures	Customer Satisfaction	Donthu and Yoo[12]
Shared Input	No. of employees	Wu[61]; Zhou et al.[68] ; Ko et al.[31]

Table 3: Description of key I/O measures

5.2. Results Analysis and Managerial Implications

5.2.1. Results

For solving the mathematical models to measure the cost efficiency of the retail stores the Model M3 and M4 are coded on lingo 11.0 software. First model (M3) is solved to obtain the optimal system cost and optimized value of all inputs. Cost weights $c^1 = c^{11}, c^{12}; c^2 = c^{21}, c^{22}, c^{23}; c^3 = c^{31}, c^{32}; c^4 = c^{41}, c^{42}$ calculated through paired comparison of inputs employing AHP[42, 58]methodology taking expert responses. The calculated values of the cost weights are listed in Table 4 and model (M3) is solved substituting the values of the other parameters (I/O) for measuring the bi-level efficiency of 24 retail stores. The modified data of I/O for the 24 stores is collected from the stores. This data is not shared in the manuscript due to commercial confidentiality. Solving the model (M3) applying KKT conditions as in model (M5) we obtained the total optimized cost and optimal level of inputs for both stages. Using equations (4-7) optimized cost and efficiency of both stages are computed and the cost efficiency of the retail stores is determined using equation (8). The results of cost efficiency are tabulated in Table 5 (column 2-5).

Cost Input	Weight	Cost Input	Weight	Cost Input	Weight
c^{11}	0.55	c^{12}	0.45	c^{21}	0.25
c^{22}	0.12	c^{23}	0.08	c^{31}	0.13
c^{32}	0.09	c^{41}	0.27	c^{42}	0.0

Table 4: Input Cost Weights

From the results (Table 5) we can see retail stores R3, R4 and R17 are efficient store (given $\theta_k^s = 1;\ k = 3, 4\,\text{and}\,17$) and can act as benchmark for inefficient stores. Store R21 is efficient ($\theta_k^f = 1$) in the following stage but inefficient in the leading stage ($\theta_k^l \leq 0.99$) and hence the store is not efficient overall ($\theta_k^s \leq 1; k = 21$). The results justify the application of bi-level efficiency measurement, as it clearly indicates the improvement is required in the performance of the leader than the overall system. Using the results of the model (M3) we also obtain the benchmarking reference sets (based on Section 4.3.1) for the inefficient stores as listed in column 5 and 6 of Table 5. For the store R21 the store (R4 and R17) form the benchmarking reference set for the leading stage. Similarly benchmark reference sets for all inefficient stores are defined. The results obtained on solving

Retail stores	Efficiency			Reference Sets		Shared Input allocation (in %)	
	System	Stage I	Stage II	Stage I	Stage II	Stage I	Stage II
R1	0.92	0.88	0.98	R4,R17	R17	24.6	75.4
R2	0.83	0.85	0.81	R4,R17	R3,R21	19.5	51.5
R3	1.00	1.00	1.00	-	-	15.7	84.3
R4	1.00	1.00	1.00	-	-	23.9	76.1
R5	0.84	0.78	0.93	R4,R17	R17,R21	21.9	57.0
R6	0.87	0.79	0.98	R4,R17	R17,R21	22.1	69.9
R7	0.87	0.81	0.96	R4,R17	R17	22.4	67.4
R8	0.89	0.83	0.96	R4,R17	R17,R21	20.7	52.4
R9	0.91	0.90	0.91	R4,R17	R3,R21	24.1	74.9
R10	0.88	0.89	0.86	R4,R17	R3,R23	24.8	66.6
R11	0.82	0.81	0.83	R4,R17	R17,R21	23.4	63.7
R12	0.88	0.81	0.97	R4,R17	R17,R21	23.6	76.4
R13	0.96	0.98	0.95	R4,R17	R3,R21	21.9	78.1
R14	0.96	0.94	0.98	R4,R17	R17,R21	25.0	75.0
R15	0.98	0.99	0.98	R4,R17	R17,R21	20.3	70.5
R16	0.89	0.96	0.82	R4,R17	R3,R21	17.5	66.9
R17	1.00	1.00	1.00	-	-	21.0	79.0
R18	0.93	0.88	0.99	R4,R17	R17	25.3	72.9
R19	0.94	0.95	0.92	R4,R17	R17,R21	24.7	64.5
R20	0.99	0.99	0.97	R4,R17	R17,R21	17.3	66.0
R21	0.99	0.99	1.00	R4,R17	-	21.4	77.4
R22	0.85	0.84	0.87	R4,R17	R3,R21	22.6	58.8
R23	0.83	0.76	0.93	R4,R17	R17,R21	21.2	64.4
R24	0.84	0.80	0.90	R4,R17	R17,R21	23.4	63.5

Table 5: Results of Cost efficiency for model (M3) and Benchmarking Reference Set

Retail stores	Efficiency			Reference Sets		Shared Input allocation (in %)	
	System	Stage I	Stage II	Stage I	Stage II	Stage I	Stage II
R1	0.96	0.94	0.98	R4,R17	R17	24.6	75.1
R2	0.83	0.85	0.81	R4,R17	R3,R21	28.0	72.0
R3	1.00	1.00	1.00	-	-	20.0	80.0
R4	1.00	1.00	1.00	-	-	23.9	76.1
R5	0.84	0.78	0.93	R4,R17	R17,R21	27.8	72.2
R6	0.87	0.82	0.94	R4,R17	R17,R21	24.01	76.0
R7	0.87	0.81	0.96	R4,R17	R17	24.2	75.8
R8	0.89	0.88	0.92	R4,R17	R17,R21	28.4	71.6
R9	0.91	0.87	0.961	R4,R17	R3,R21	24.1	75.9
R10	0.88	0.91	0.84	R4,R17	R3,R23	24.8	75.2
R11	0.82	0.81	0.83	R4,R17	R17,R21	26.9	73.1
R12	0.88	0.84	0.92	R4,R17	R17,R21	23.6	76.4
R13	0.97	0.99	0.95	R4,R17	R3,R21	21.9	78.1
R14	0.96	0.94	0.98	R4,R17	R17,R21	24.5	75.5
R15	0.99	0.99	0.99	R4,R17	R17,R21	22.3	77.7
R16	0.91	0.88	0.95	R4,R17	R3,R21	20.6	79.4
R17	1.00	1.00	1.00	-	-	21.0	79.0
R18	0.93	0.96	0.90	R4,R17	R17	25.8	74.2
R19	0.94	0.95	0.92	R4,R17	R17,R21	27.9	72.1
R20	0.97	0.98	0.95	R4,R17	R17,R21	23.0	77.0
R21	0.97	0.96	1.00	R4,R17	-	21.6	78.4
R22	0.85	0.91	0.78	R4,R17	R3,R21	27.5	72.5
R23	0.83	0.77	0.91	R4,R17	R17,R21	24.8	75.2
R24	0.84	0.80	0.91	R4,R17	R17,R21	27.1	72.9

Table 6: Bi-level efficiency of electronics retail stores using model (M4)

the model (M3) are presented to the decision maker. Decision maker indicated the infeasibility of solution with respect to the optimal distribution of the shared input (number of employees, see column 7 and 8, Table 5). For example the optimal allocation of shared input in the leading stage of Store R3 and R21 is less than 20% (refer Table 3). It was discussed that certain minimum (maximum) proportion of human resources should be allocated to each stage to ensure proper operations. To resolve this issue we proposed an extension of the model (M3) as described in (M4) incorporating bounds on the distribution of shared resources. The result of efficiency computation, benchmark reference sets and distribution of shared resources obtained from model (M4) are demonstrated in Table 6. The results of model (M4) ensure allocation of employees between 20-40% for the leading stage and above 40% for the following stage as prescribed by the decision maker (refer Table 6) and also optimize the overall cost efficiency.

5.2.2. Managerial Implication

The central idea of the study presented in this paper is to develop a TSNS approach for measuring the relative performance of multiple retail stores defined in terms of efficiency[29, 56, 61]. In the past few years increasing potential in the retail sector in the emerging economies have attracted interest of global as well as local businesses towards investment in retailing, which on the other hand has intensified the competition in the market. To gain the competitive edge in the market it is essential for the retail firms to focus on innovation and continuous efforts towards improvement[31]. Any improvement effort is required to be based on the current performance level and aspired targets. That makes it imperative for firms to adopt a suitable approach for the measurement of performance such that appropriate and target oriented improvements can be undertaken. There is a wide literature on efficiency measurement approaches [38]. DEA has remained a popular methodology in these studies with focus on theoretical and/or application perspective. Some of these studies have also discussed the efficiency measurement approaches applicable to retail stores considering different scenarios[38, 31]. An important issue overlooked specifically in relation to retail stores is the consideration of internal structures and interactions for efficiency measurement that limits the application of the existing studies[61]. Given this literature gap the study presented here develops a relative efficiency measurement approach considering subprocesses, their characteristics and internal interactions. Following are the specific theoretical and managerial implications this study

1.Effectiveness of any effort undertaken by a firm towards improvement and even the decision to support the deployment of resources in a project directed towards improvement is to be backed by the appropriate measure of the base level performance. Further the literature[66, 13] supports the fact that a relative measure of performance provides better insights than an absolute measure. In order to support this prerequisite and measure the performance relatively for the units under evaluation, this paper presents a scientific approach for measurement of relative performance in terms of efficiency for multiple retail stores.

2.Exploration of research provided us studies that addresses the efficiency measurement frameworks for retail stores. Discussion with experts and the decision maker[32, 38] highlighted the limitation of the existing studies in practice as they consider a DMU (retail stores in our case) as a black box[61]. Problem with the black approach is that it overlooks the subsystems and their characteristics and hence lacks in exposition of internal interactions and bring ambiguity in results [61]. The existing research support defining the retail store's efficiency measurement in a two stage network structure such that the two stages operate in a Stackelberg relation wherein the leading stage influence the decision of the follower and vica versa[53]. Our study present a cost efficiency measurement approach for retail stores in a TSNS framework under Stackelberg relation.

3. Following the proposed approach one can compute the relative cost efficiency of each of the 2 stages as well as the overall system. As discussed in the result section, the proposed approach brings more transparency in result. For a DMU to be efficiency it is necessary that both stages are efficient. The results of the case

study presented here clearly depicts that, it possible that one of the stage (leading or following) may be efficient and the other may be inefficient. Such transparency helps directing the improvement targeted in the right direction.

4.The input cost optimization models (M3 and M4) not only find implications in computing the relative cost efficiency of stores but also provides the optimal level of inputs at the optimum cost. The modified model M4 fine-tunes the results for application in practise by further optimizing the distribution of shared resources. For the results of the models we can also infer the benchmarking reference sets of the inefficient stores at the system as well as subprocess level.

6. CONCLUSION

This paper develops a bi-level programming DEA approach for relative efficiency measurement in context to electronics retail stores considering a TSNS with stages operating in Stackelberg game relation. The efficiency of any retail chain is defined by the efficiency of the functions performed in the stores such as store planning and design, display placement, inventory and supply chain management, premises maintenance, labor management, money and credit handling, and customer services. Some are functions such as store planning and design, display placement, store level promotional activities leads in decision making to the other. The leading decisions influence the following and vica versa. In this situation, measuring the efficiency of the retail outlets as a black box lacks internal interaction, transparency and overlooks the subprocesses. The existing literature has overlooked this issue. Our study addresses this concern by considering a TSNS for measuring the relative efficiencies of retail stores. To demonstrate the validity of the proposed framework a case study of Indian electronics retail chain is presented. Results of the study provides important insights to decision makers and guides them for planning improvement strategies for the inefficient stores and benchmarking. The scope of the study is limited to the efficiency measurement at one point of time and can be further extended through studies that analyse the efficiency measurement over a period of time and draw comparative results. Researchers can also study the effect of environmental factors in efficiency measurement.

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