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THREE DIMENSIONAL BOUNDED TRANSPORTATION PROBLEM

Kavita GUPTA
Department of Mathematics, Kirori Mal College, University of Delhi, Delhi-110007, India
gupta_kavita31@yahoo.com

Ritu ARORA
Department of Mathematics,Keshav Mahavidyalaya, University of Delhi, Delhi, India
ritunarora@yahoo.in

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Abstract: This article presents a methodology to solve a three dimensional bounded transportation problem. Bounded transportation problem is expounded by defining a parallel bounded transportation problem. Equivalence between the two problems is established. The proposed model is applied to public distribution system of North Delhi. A computing software LINGO 17.0 is used to solve the application.

Keywords: Transportation, Bounded Transportation Problem, Rim Conditions, Optimal Solution, LINGO 17.0.

MSC: 90C08, 90B06.

1. INTRODUCTION

A classical transportation problem [6] is a special class of linear programming problem. In this problem, a homogenous commodity is available in known quantity at each of the m origins which needs to be transported to each of n destinations. The cost of transporting a unit of the commodity from any origin to any destination is known. The objective is to determine a transportation schedule which minimizes the total transportation cost . This classical transportation model is based on the assumption that the total quantity required at the destinations is equal to the total quantity available at the origins. But in real life, there are situations when more than one commodity is to be transported. In such a situation, classical transportation problem is extended to multi-dimensions. Sometimes, the supply and demand of a firm may also vary. Due to the varying nature of demand, supply, and type of commodity, there is a need to fix bounds on demand,supply, and commodity constraints. This gives rise to a bounded transportation problem.

Dahiya and Verma [4] discussed bounded transportation problem in two dimensions. Zhang et al. [12] presented three different uncertain solid transportation models in which both variable cost and fixed cost are taken into consideration. They designed an algorithm on the basis of tabu search algorithm and theory of uncertainty. Gupta et.al. [5] formulated a capacitated transportation problem with linear and fractional objective functions which are conflicting in nature. Ozdemir et al. [10]presented a multi-location transshipment problem in which the decision variables are bounded.

In 1981, Misra et al. [9] gave a note on solid transportation problem with bounds on rim conditions. In 1988, Bandopadhyaya et al. [2] discussed transportation problem in multi- dimensions subjected to mixed type of axial constraints. Malhotra et al. [8] studied time-minimization transportation problem in three dimensions. Bandopadhyaya et al.[3] discussed impaired flow in a solid transportation problem. Jalil et al.[7] proposed a multi-level decision making model for an uncertain multi-index transportation problem. Tzeng et al.[11] explained the planning of annual coal purchase by formulating a fuzzy solid transportation problem. They provided an allocation schedule of Taipower, the official authority of Taiwan. In 2004, Arora and Khurana [1] formulated indefinite quadratic transportation problem in three dimensions. In this paper, they perceived cost-time trade off pairs but the problem that they have studied is not bounded. This motivated the authors to study three dimensional bounded transportation problem.

This paper is organized as follows : In section 2, a mathematical model of a three dimensional bounded transportation problem is presented. In section 3, a related transportation problem is presented. In section 4, equivalence between the two problems is ratified.In section 5, the devised model solves the problem of public distribution system of North Delhi.

Notations

The following notations are used throughout the paper:
$I = \{1, 2, ..., m\}$ represents m origins.
$J = \{1, 2, ..., n\}$ represents n destinations.
$K = \{1, 2, ..., p\}$ represents p types of products to be transported.
x_{ijk} = amount of k^{th} type of product shipped from the i^{th} origin to the j^{th} destination.
c_{ijk} = per unit cost of shipping the k^{th} type of product from the i^{th} origin to the j^{th} destination irrespective of the amount of the product shipped, so long as $x_{ijk} > 0$.
a_{jk} and A_{jk} are respectively the minimum and maximum amount of the k^{th} type of product demanded by the j^{th} destination from all the origins.
b_{ki} and B_{ki} are respectively the minimum and maximum amount of the k^{th} type

of product available at the i^{th} origin to be shipped to all the destinations.
e_{ki} and E_{ki} are respectively the minimum and maximum amount of units of all types of products to be supplied from i^{th} origin to the j^{th} destination.
l_{ijk} and u_{ijk} are respectively the lower bounds and upper bounds on the amount of k^{th} type of product shipped from the i^{th} origin to the j^{th} destination.

2. MATHEMATICAL MODEL OF A THREE DIMENSIONAL BOUNDED TRANSPORTATION PROBLEM (P_1)

Mathematically, the three dimensional bounded transportation problem is defined as follows

$$\min\left\{\sum_{i\in I}\sum_{j\in J}\sum_{k\in K} c_{ijk}x_{ijk}\right\} \qquad (P_1)$$

subject to

$$b_{ki} \le \sum_{j\in J} x_{ijk} \le B_{ki}, \forall i \in I; \forall k \in K \tag{1}$$

$$a_{jk} \le \sum_{i\in I} x_{ijk} \le A_{jk}, \forall j \in J; \forall k \in K \tag{2}$$

$$e_{ij} \le \sum_{k\in K} x_{ijk} \le E_{ij}, \forall i \in I; \forall j \in J \tag{3}$$

$$l_{ijk} \le x_{ijk} \le u_{ijk} \text{ and integers } \forall i \in I; \forall j \in J; \forall k \in K \tag{4}$$

The problem (P_1) is unbalanced as the quantity of distinct products received by all the destinations,the quantity of products supplied from all the origins to all destinations, and the quantity of distinct types of products supplied from all the origins are not equal. The problem (P_1) will possess a feasible solution when it is balanced. We introduce a dummy row , a dummy column and a dummy product to balance problem (P_1). This leads to the formulation of a related transportation problem which is balanced and hence, possess a feasible solution.

3. RELATED TRANSPORTATION PROBLEM (P_2)

Let $I^{'} = \{1, 2, ..., m + 1\}$ represents $m + 1$ origins.
$J^{'} = \{1, 2, ..., n + 1\}$ represents $n + 1$ destinations.
$K^{'} = \{1, 2, ..., p + 1\}$ represents $p + 1$ types of products.

$$\min\left\{\sum_{i\in I'}\sum_{j\in J'}\sum_{k\in K'} c^{'}_{ijk}y_{ijk}\right\} \qquad (P_2)$$

subject to

$$\sum_{j\in J'} y_{ijk} = B^{'}_{ki}, \forall i \in I^{'}; \forall k \in K^{'} \tag{5}$$

$$\sum_{i\in I'} y_{ijk} = A^{'}_{jk}, \forall j \in J^{'}; \forall k \in K^{'} \tag{6}$$

$$\sum_{k\in K'} y_{ijk} = E^{'}_{ij}, \forall i \in I^{'}; \forall j \in J^{'} \tag{7}$$

$$l_{ijk} \leq y_{ijk} \leq u_{ijk} \text{ and integers } \forall i \in I; \forall j \in J; \forall k \in K \tag{8}$$

$$0 \leq y_{i,n+1,k} \leq B_{ki} - b_{ki} \ \ , \forall i \in I; \forall k \in K \tag{9}$$

$$0 \leq y_{m+1,j,k} \leq A_{jk} - a_{jk} \ \ , \forall j \in j; \forall k \in K \tag{10}$$

$$0 \leq y_{i,j,p+1} \leq E_{ij} - e_{ij} \ \ , \forall i \in I; \forall j \in J \tag{11}$$

$$y_{i,n+1,p+1} \geq 0 \ \ , \forall i \in I \tag{12}$$

$$y_{m+1,j,p+1} \geq 0 \ \ , \forall j \in J \tag{13}$$

$$y_{m+1,n+1,k} \geq 0 \ \ , \forall k \in K \tag{14}$$

$$y_{m+1,n+1,p+1} \geq 0 \tag{15}$$

$$A^{'}_{jk} = A_{jk} \ \ , \forall j \in J; \forall k \in K \tag{16}$$

$$A^{'}_{n+1,k} = \sum_{i\in I} B_{ki} \ \ , \forall k \in K \tag{17}$$

$$B^{'}_{ki} = B_{ki} \ \ \forall i \in I, \forall k \in K \tag{18}$$

$$B^{'}_{k,m+1} = \sum_{j\in J} A_{jk} \ \ , \forall k \in K \tag{19}$$

$$E^{'}_{ij} = E_{ij} \ \ \forall i \in I, \forall j \in J \tag{20}$$

$$E'_{i,n+1} = \sum_{k \in K} B_{ki} \quad ,\forall i \in I \tag{21}$$

$$B'_{p+1,i} = \sum_{j \in J} E_{ij} \quad ,\forall i \in I \tag{22}$$

$$A_{n+1,p+1} = B_{p+1,m+1} = E_{m+1,n+1} = \sum_{i \in I} \sum_{j \in J} E_{ij} \quad ,\forall i \in I; \forall j \in J \tag{23}$$

$$A'_{j,p+1} = \sum_{i \in I} E_{ij} \quad ,\forall j \in J \tag{24}$$

$$E'_{m+1,j} = \sum_{k \in K} A_{jk} \quad ,\forall j \in J \tag{25}$$

$$c'_{ijk} = c_{ijk} \quad \forall i \in I, \forall j \in J; \forall k \in K \tag{26}$$

$$c'_{m+1,j,k} = c'_{i,n+1,k} = c'_{i,j,p+1} \quad ,\forall i \in I; \forall j \in J; \forall k \in K \tag{27}$$

$$c'_{m+1,n+1,p+1} = 0 \tag{28}$$

4. THEORETICAL DEVELOPMENT

4.1. Existence of feasible solution of problem (P_2)

A feasible solution to problem (P_2) will exist if the quantity of distinct products received by all the destinations,the quantity of products supplied from all the origins to all destinations, and the quantity of distinct types of products supplied from all the origins are equal [1]. Mathematically,

$$\sum_{i \in I'} \sum_{k \in K'} B'_{ki} = \sum_{i \in I'} \sum_{j \in J'} E'_{ij} = \sum_{j \in J'} \sum_{k \in K'} A'_{jk}$$

For this, we will prove a series of theorems given below.

Theorem 1.

$$\sum_{i \in I'} \sum_{k \in K'} B'_{ki} = \sum_{i \in I'} \sum_{j \in J'} E'_{ij} \tag{29}$$

Proof. We first show that distinct products supplied by the i^{th} origin is equal to the quantity of products received by all destinations from the i^{th} origin.
That is, $\sum\limits_{k\in K'} B'_{ki} = \sum\limits_{j\in J'} E'_{ij}$.
Now, $\sum\limits_{k\in K'} B'_{ki} = \sum\limits_{k\in K} B'_{ki} + B'_{p+1,i}$
$= E'_{i,n+1} + \sum\limits_{j\in J} E_{ij}, \forall i \in I$ by (21) and (22)
$= E'_{i,n+1} + \sum\limits_{j\in J} E'_{ij}$ by (20)
$= \sum\limits_{j\in J'} E'_{ij}$
$\Rightarrow \sum\limits_{k\in K'} B'_{ki} = \sum\limits_{j\in J'} E'_{ij}$
Summing over $i \in I'$, we get
$\sum\limits_{i\in I'} \sum\limits_{k\in K'} B'_{ki} = \sum\limits_{i\in I'} \sum\limits_{j\in J'} E'_{ij}$ □

Theorem 2.

$$\sum_{k\in K'} \sum_{j\in J'} A'_{jk} = \sum_{k\in K'} \sum_{i\in I'} B'_{ki} \tag{30}$$

Proof. We first show that k^{th} type of product received by all destinations is equal to the k^{th} type of product supplied from all origins.
That is, $\sum\limits_{j\in J'} A'_{jk} = \sum\limits_{i\in I'} B'_{ki}$.
Now, $\sum\limits_{j\in J'} A'_{jk} = \sum\limits_{j\in J} A'_{jk} + A'_{n+1,k}$
$= B'_{k,m+1} + \sum\limits_{i\in I} B_{ki}; \forall k \in K$ by (16), (17) and (19)
$= B'_{k,m+1} + \sum\limits_{i\in I} B'_{ki}$ by (18)
$= \sum\limits_{i\in I'} B'_{ki}$
$\Rightarrow \sum\limits_{j\in J'} A'_{jk} = \sum\limits_{i\in I'} B'_{ki}$
Summing over $k \in K'$, we get
$\sum\limits_{k\in K'} \sum\limits_{j\in J'} A'_{jk} = \sum\limits_{k\in K'} \sum\limits_{i\in I'} B'_{ki}$ □

Theorem 3.

$$\sum_{j\in J'} \sum_{i\in I'} E'_{ij} = \sum_{j\in J'} \sum_{k\in K'} A'_{jk} \tag{31}$$

Proof. We first show that quantity of products supplied from all origins to j^{th} destination is equal to distinct products received by j^{th} destination.

That is, $\sum_{i\in I'} E'_{ij} = \sum_{k\in K'} A'_{jk}$.
Now, $\sum_{i\in I'} E'_{ij} = \sum_{i\in I} E'_{ij} + E'_{m+1,j}$
$= A'_{j,p+1} + \sum_{k\in K} A_{jk}, \forall j \in J$ by (20), (24) and (25)
$= A'_{j,p+1} + \sum_{k\in K} A'_{jk}, \forall j \in J$ by (16)
$= \sum_{k\in K'} A'_{jk}$
$\Rightarrow \sum_{i\in I'} E'_{ij} = \sum_{k\in K'} A'_{jk}$
Summing over $j \in J'$, we get
$\sum_{j\in J'} \sum_{i\in I'} E'_{ij} = \sum_{j\in J'} \sum_{k\in K'} A'_{jk}$ □

Theorem 4. *To every feasible solution of problem (P_1), there exists a feasible solution to problem (P_2) and conversely.*

Proof. Let $\{y_{ijk}\}$ be a feasible solution to problem (P_2).
Define $\{x_{ijk}\}, i \in I, j \in J, k \in K$ by the following transformation.

$$x_{ijk} = y_{ijk} \ \ \forall i \in I; \forall j \in J; \forall k \in K \tag{32}$$

As $l_{ijk} \le y_{ijk} \le u_{ijk} \ \ \forall i \in I; \forall j \in J; \forall k \in K$ by (8),
therefore, $l_{ijk} \le x_{ijk} \le u_{ijk} \ \ \forall i \in I; \forall j \in J; \forall k \in K$ by (32).
Also, $\sum_{j\in J'} y_{ijk} = B'_{ki}, \forall i \in I'; \forall k \in K'$ by (5).
$\Rightarrow \sum_{j\in J} y_{ijk} + y_{i,n+1,k} = B_{ki}$, *because* $B'_{ki} = B_{ki} \ \ \forall i \in I; \forall k \in K$ by (18).
$\Rightarrow \sum_{j\in J} y_{ijk} = B_{ki} - y_{i,n+1,k}, \ \ \forall i \in I; \forall k \in K$
$\Rightarrow \sum_{j\in J} y_{ijk} \le B_{ki}$, *because* $y_{i,n+1,k} \ge 0, \ \ \forall i \in I; \forall k \in K$
Also, $0 \le y_{i,n+1,k} \le B_{ki} - b_{ki}, \ \ \forall i \in I; \forall k \in K$ by (9)
$\Rightarrow 0 \ge -y_{i,n+1,k} \ge b_{ki} - B_{ki}$
$\Rightarrow B_{ki} \ge B_{ki} - t_{i,n+1,k} \ge b_{ki}$
$\Rightarrow b_{ki} \le \sum_{j\in J} y_{ijk} \le B_{ki}, \ \ \forall i \in I; \forall k \in K$
$\Rightarrow b_{ki} \le \sum_{j\in J} x_{ijk} \le B_{ki}, \ \ \forall i \in I; \forall k \in K$ by (32)
Now, $\sum_{i\in I'} y_{ijk} = A'_{jk}, \forall j \in J'; \forall k \in K'$ by (6)
$\Rightarrow \sum_{i\in I} y_{ijk} + y_{m+1,j,k} = A_{jk}, \forall j \in J; \forall k \in K$ by (16)
$\Rightarrow \sum_{i\in I} y_{ijk} = A_{jk} - y_{m+1,j,k}, \forall j \in J; \forall k \in K$
$\Rightarrow \sum_{i\in I} y_{ijk} \le A_{jk}$, *because* $y_{m+1,j,k} \ge 0, \ \ \forall j \in J; \forall k \in K$
Also, $0 \le y_{m+1,j,k} \le A_{jk} - a_{jk}, \ \ \forall j \in J; \forall k \in K$ by (10)

$\Rightarrow a_{jk} \leq \sum_{i \in I} y_{ijk} \leq A_{jk} \ \ \forall j \in J; \forall k \in K$
$\Rightarrow a_{jk} \leq \sum_{i \in I} x_{ijk} \leq A_{jk} \ \ \forall j \in J; \forall k \in K$ by (32).
Similarly, we can show that $e_{ij} \leq \sum_{k \in K} x_{ijk} \leq E_{ij}, \forall i \in I; \forall j \in J$
So,$\{x_{ijk}\}, i \in I, j \in J, k \in K$ defined by (32) is a feasible solution to problem (P_1).
Conversely, let $\{x_{ijk}\}, i \in I, j \in J, k \in K$ be a feasible solution to problem (P_1).

$$\text{Define } \ y_{ijk} = x_{ijk} \ \ \forall i \in I; \forall j \in J; \forall k \in K \tag{33}$$

$$y_{m+1,j,k} = A_{jk} - \sum_{i \in I} x_{ijk} \ \ \forall j \in J; \forall k \in K \tag{34}$$

$$y_{i,n+1,k} = B_{ki} - \sum_{j \in J} x_{ijk} \ \ \forall i \in I; \forall k \in K \tag{35}$$

$$y_{i,j,p+1} = E_{ij} - \sum_{k \in K} x_{ijk} \ \ \forall i \in I; \forall j \in J \tag{36}$$

$$y_{m+1,n+1,p+1} = \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} x_{ijk} \tag{37}$$

$$y_{i,n+1,p+1} = \sum_{j \in J} \sum_{k \in K} x_{ijk} \ \ \forall i \in I \tag{38}$$

$$y_{m+1,j,p+1} = \sum_{i \in I} \sum_{k \in K} x_{ijk} \ \ \forall j \in J \tag{39}$$

$$y_{m+1,n+1,k} = \sum_{i \in I} \sum_{j \in J} x_{ijk} \ \ \forall k \in K \tag{40}$$

Since, $l_{ijk} \leq x_{ijk} \leq u_{ijk} \ \ \forall i \in I; \forall j \in J; \forall k \in K$ by (4),
$\Rightarrow l_{ijk} \leq y_{ijk} \leq u_{ijk} \ \ \forall i \in I; \forall j \in J; \forall k \in K$ by (33).
Also, $\sum_{j \in J'} y_{ijk} = \sum_{j \in J} y_{ijk} + y_{i,n+1,k}$
$= \sum_{j \in J} x_{ijk} + B_{ki} - \sum_{j \in J} x_{ijk}$ by (33) and (35).
$= B_{ki}, \ \ i \in I, k \in K.$
$= B'_{ki}, \ \ i \in I, k \in K$ by (18).
Also, $\sum_{j \in J'} y_{i,j,p+1} = \sum_{j \in J} y_{i,j,p+1} + y_{i,n+1,p+1}$
$= \sum_{j \in J} E_{ij} - \sum_{j \in J} \sum_{k \in K} x_{ijk} + y_{i,n+1,p+1}$ by (36).
$= \sum_{j \in J} E_{ij} - \sum_{j \in J} \sum_{k \in K} x_{ijk} + \sum_{j \in J} \sum_{k \in K} x_{ijk}$ by (38).

$= \sum_{j \in J} E_{ij}$
$= B'_{p+1,i}$ by (22).
Also, $\sum_{j \in J'} y_{m+1,j,k} = \sum_{j \in J} y_{m+1,j,k} + y_{m+1,n+1,k}$
$= \sum_{j \in J} A_{jk} - \sum_{j \in J} \sum_{i \in I} x_{ijk} + \sum_{i \in I} \sum_{j \in J} x_{ijk}$ by (34)and (40).
$= \sum_{j \in J} A_{jk}$
$= B'_{k,m+1}$ by (19).
Now, $\sum_{i \in I'} y_{ijk} = \sum_{i \in I} y_{ijk} + y_{m+1,j,k}$
$= \sum_{i \in I} x_{ijk} + A_{jk} - \sum_{i \in I} x_{ijk}$ $, \forall j \in J; \forall k \in K$ by (33)and (34).
$= A_{jk}, \;\; j \in J, k \in K.$
$= A'_{jk}, \;\; j \in J, k \in K$ by (16).
Also, $\sum_{i \in I'} y_{i,n+1,k} = \sum_{i \in I} y_{i,n+1,k} + y_{m+1,n+1,k}$
$= \sum_{i \in I} (B_{ki} - \sum_{j \in J} x_{ijk}) + \sum_{i \in I} \sum_{j \in J} x_{ijk}$ by (35)and (40).
$= \sum_{i \in I} B_{ki}$
$= A'_{n+1,k}$ $, \forall k \in K$ by (17).
Also, $\sum_{i \in I'} y_{i,j,p+1} = \sum_{i \in I} y_{i,j,p+1} + y_{m+1,j,p+1}$
$= \sum_{i \in I} (E_{ij} - \sum_{k \in K} x_{ijk}) + \sum_{i \in I} \sum_{k \in K} x_{ijk}$ by (36)and (39).
$= \sum_{i \in I} E_{ij}$
$= A'_{j,p+1}$ $, \forall j \in J$ by (24).
Now, $\sum_{k \in K'} y_{ijk} = \sum_{k \in K} y_{ijk} + y_{i,j,p+1}$
$= \sum_{k \in K} x_{ijk} + E_{ij} - \sum_{k \in K} x_{ijk}$ by (36)and (38).
$= E_{ij}, \;\; i \in I, j \in J.$
$= E'_{ij}, \;\; i \in I, j \in J$ by (20).
Also, $\sum_{k \in K'} y_{i,n+1,k} = \sum_{k \in K} y_{i,n+1,k} + y_{i,n+1,p+1}$
$= \sum_{k \in K} (B_{ki} - \sum_{j \in J} x_{ijk}) + \sum_{k \in K} \sum_{j \in J} x_{ijk}$ by (35)and (38).
$= \sum_{k \in K} B_{ki}$
$= E'_{i,n+1}$ $, \forall i \in I$ by (21).
Also, $\sum_{k \in K'} y_{m+1,j,k} = \sum_{k \in K} y_{m+1,j,k} + y_{m+1,j,p+1}$
$= \sum_{k \in K} (A_{jk} - \sum_{i \in I} x_{ijk}) + \sum_{k \in K} \sum_{i \in I} x_{ijk}$ by (34)and (39).
$= \sum_{k \in K} A_{jk}$
$= E'_{m+1,j}$ $, \forall j \in J$ by (25).

Now, $a_{jk} \leq \sum_{i \in I} x_{ijk} \leq A_{jk}, \forall j \in J; \forall k \in K$ by (2),
$\Rightarrow -a_{jk} \geq -\sum_{i \in I} x_{ijk} \geq -A_{jk}, \forall j \in J; \forall k \in K$
$\Rightarrow A_{jk} - a_{jk} \geq A_{jk} - \sum_{i \in I} x_{ijk} \geq A_{jk} - A_{jk}$
$\Rightarrow 0 \leq y_{m+1,j,k} \leq A_{jk} - a_{jk}, \quad \forall j \in J; \forall k \in K$ by (34) which proves (10)
Now, $e_{ij} \leq \sum_{k \in K} x_{ijk} \leq E_{ij}, \forall i \in I; \forall j \in J$ by (3),
$\Rightarrow -e_{ij} \geq -\sum_{k \in K} x_{ijk} \geq -E_{ij}, \forall j \in J; \forall i \in I$
$\Rightarrow E_{ij} - e_{ij} \geq E_{ij} - \sum_{k \in K} x_{ijk} \geq 0$
$\Rightarrow 0 \leq y_{i,j,p+1} \leq E_{ij} - e_{ij}, \quad \forall i \in I; \forall j \in J$ by (36) which proves (11)
Now, $b_{ki} \leq \sum_{j \in J} x_{ijk} \leq B_{ki}, \forall i \in I; \forall k \in K$ by (1),
$\Rightarrow -b_{ki} \geq -\sum_{j \in J} x_{ijk} \geq -B_{ki}, \forall i \in I; \forall k \in K$
$\Rightarrow B_{ki} - b_{ki} \geq B_{ki} - \sum_{j \in J} x_{ijk} \geq 0$
$\Rightarrow 0 \leq y_{i,n+1,k} \leq B_{ki} - b_{ki}, \quad \forall i \in I; \forall k \in K$ by (35) which proves (9)
Clearly, $y_{m+1,n+1,p+1} = \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} x_{ijk} \geq 0$
All the above arguments show that $\{y_{ijk}\}$ is a feasible solution to problem (P_2). □

Theorem 5. *The value of the objective function of problem (P_1) at a feasible solution is equal to the value of the objective function of problem (P_2) at its corresponding feasible solution and conversely.*

Proof. Let $\{y_{ijk} : i \in I', j \in J', k \in K'\}$ be a feasible solution to problem (P_2) and $\{x_{ijk} : i \in I, j \in J, k \in K\}$ be the corresponding feasible solution to problem (P_1).
Let z = objective function value of problem (P_2) at $\{y_{ijk} : i \in I', j \in J', k \in K'\}$
$\Rightarrow z = \sum_{i \in I'} \sum_{j \in J'} \sum_{k \in K'} c'_{ijk} y_{ijk}$
$= \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} c'_{ijk} y_{ijk} + \sum_{i \in I} \sum_{j \in J} c'_{i,j,p+1} y_{i,j,p+1} + \sum_{i \in I} \sum_{k \in K} c'_{i,n+1,k} y_{i,n+1,k}$
$+ \sum_{j \in J} \sum_{k \in K} c'_{m+1,j,k} y_{m+1,j,k} + c'_{m+1,n+1,p+1} y_{m+1,n+1,p+1}$
$= \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} c_{ijk} x_{ijk}$ by(26),(27)(28)and (33).
= value of the objective function of problem (P_1) at $\{x_{ijk} : i \in I, j \in J, k \in K\}$.
Converse can be proved similarly. □

Theorem 6. *There is one to one correspondence between optimal solution of problem (P_1) and optimal solution of problem (P_2).*

Proof. Let $\{x^0_{ijk} : i \in I, j \in J, k \in K\}$ be the optimal solution to problem (P_1). Let the objective function value be z^0 at this optimal solution. Since $\{x^0_{ijk} : i \in I, j \in J, k \in K\}$ is an optimal solution to problem (P_1), so it is a feasible solution also. Therefore, by theorem (4), there exists a corresponding feasible solution of problem (P_2). Let $\{t^0_{ijk} : i \in I^{'}, j \in J^{'}, k \in K^{'}\}$ be the corresponding feasible solution of problem (P_2). Then by theorem (5), the objective function value yielded by $\{t^0_{ijk} : i \in I^{'}, j \in J^{'}, k \in K^{'}\}$ is same as the objective function value yielded by $\{x^0_{ijk} : i \in I, j \in J, k \in K\}$. Therefore, the objective function value yielded by $\{t^0_{ijk} : i \in I^{'}, j \in J^{'}, k \in K^{'}\}$ is also z^0. Let, if possible,$\{t^0_{ijk} : i \in I^{'}, j \in J^{'}, k \in K^{'}\}$ is not an optimal solution to problem (P_2). Then, there exists a feasible solution $\{t^{'}_{ijk} : i \in I^{'}, j \in J^{'}, k \in K^{'}\}$, say, to problem ($P_2$) with the objective function value $z^{'} < z^0$. Let $\{x^{'}_{ijk} : i \in I, j \in J, k \in K\}$ be the corresponding feasible solution to problem (P_1). Then by theorem (5), $\sum\limits_{i \in I} \sum\limits_{j \in J} \sum\limits_{k \in K} c_{ijk} x^{'}_{ijk} = z^{'}$, which is a contradiction to the assumption that $\{x^0_{ijk} : i \in I, j \in J, k \in K\}$is an optimal solution of problem (P_1). Similarly, an optimal solution to problem (P_2) will give an optimal solution to problem (P_1). □

5. APPLICATION OF THREE DIMENSIONAL BOUNDED TRANSPORTATION PROBLEM TO PUBLIC DISTRIBUTION SYSTEM OF NORTH DELHI

The government stores wheat, rice, and sugar in three godowns at Civil Lines, Sadar Bazar, and Nai Sarak. The products are transported from these godowns to fair price shops at Ashok Vihar, Daryaganj, Karol Bagh, and Rohini. The cost (in thousand rupees per quintal) of transporting these products from the godowns to fair price shops are given in Table 1. The minimum and maximum availability of these products (in quintals) at three godowns are mentioned in column 7 of Table 1. Moreover, the minimum and maximum demand of these products (in quintals) at fair price shops are mentioned in row 5. The minimum and maximum quantity (in quintals) of products received by each fair price shop from each godown are given in the last row of each cell. Lower bound (LB) and upper bound (UB) on the quantity (in quintals) of wheat, rice, and sugar supplied from each godown to each fair price shop are given in the respective cell in brackets under row heading (LB,UB) in Table 1. The objective is to determine the quantity of each type of product to be shipped from each godown to each fair price shop such that the overall transportation cost is minimum and the rim conditions are satisfied.

5.1. Formulation of the problem of public distribution system as a three dimensional capacitated transportation problem

Let I={1,2,3} be three godowns at Civil Lines, Sadar Bazar, and Nai Sarak respectively.
J={1,2,3,4} be fair price shops at Ashok Vihar, Daryaganj, Karol Bagh, and Rohini respectively.
K={1,2,3} be the set of three products such as wheat, rice, and sugar respectively.
x_{ijk} = quantity of k^{th} type of product shipped from the i^{th} godown to the j^{th} fair price shop.
Mathematically,

Godowns↓	Fair Price Shop → Commodity↓	Ashok Vihar	Darya-Ganj	Karol Bagh	Rohini	Supply (Min , Max)
Civil Lines	Wheat→	2(0,30)	4(1,30)	3(2,50)	4(0,40)	(5,30)
	Rice→	3(1,60)	2(2,70)	1(0,70)	4(0,50)	(3,40)
	Sugar→	1(0,70)	2(0,50)	3(0,40)	1(0,50)	(2,70)
	(LB,UB)→	(0,60)	(2,50)	(2,30)	(0,40)	–
Sadar Bazar	Wheat→	3(1,40)	2(0,30)	5(0,50)	3(0,70)	(3,50)
	Rice→	1(1,70)	4(1,60)	2(2,80)	4(0,50)	(5,40)
	Sugar→	2(0,50)	3(0,60)	3(2,70)	2(0,80)	(2,60)
	(LB,UB)→	(0,70)	(3,50)	(2,60)	(2,50)	–
Nai Sarak	Wheat→	1(1,40)	4(0,30)	2(0,50)	1(0,70)	(2,50)
	Rice→	2(1,70)	3(1,60)	3(1,50)	2(0,60)	(3,50)
	Sugar→	3(2,80)	2(2,70)	4(2,50)	3(0,60)	(5,50)
	(LB,UB)→	(0,60)	(0,60)	(0,60)	(0,60)	–
Demand (Min, Max)	Wheat→	(3,50)	(5,30)	(3,50)	(2,70)	–
	Rice→	(5,50)	(3,40)	(5,40)	(0,60)	–
	Sugar→	(2,50)	(2,70)	(2,60)	(0,40)	–

Table 1: cost and capacity bounds

$$\min z = (2x_{111} + 3x_{112} + x_{113}) + (4x_{121} + 2x_{122} + 2x_{123})$$
$$+ (3x_{131} + x_{132} + 3x_{133}) + (4x_{141} + 4x_{142} + x_{143}) + (3x_{211} + x_{212} + 2x_{213})$$
$$+ (2x_{221} + 4x_{222} + 3x_{223}) + (5x_{231} + 2x_{232} + 3x_{233}) + (3x_{241} + 4x_{242} + 2x_{243})$$
$$+ (x_{311} + 2x_{312} + 3x_{313}) + (4x_{321} + 3x_{322} + 2x_{323}) + (2x_{331} + 3x_{332} + 4x_{333})$$
$$+ (x_{341} + 2x_{342} + 3x_{343}) \qquad (P_3)$$

subject to the constraints

$5 \leq x_{111} + x_{121} + x_{131} + x_{141} \leq 30$
$3 \leq x_{112} + x_{122} + x_{132} + x_{142} \leq 40$
$2 \leq x_{113} + x_{123} + x_{133} + x_{143} \leq 70$
$3 \leq x_{211} + x_{221} + x_{231} + x_{241} \leq 50$
$5 \leq x_{212} + x_{222} + x_{232} + x_{242} \leq 40$
$2 \leq x_{213} + x_{223} + x_{233} + x_{243} \leq 60$
$2 \leq x_{311} + x_{321} + x_{331} + x_{341} \leq 50$
$3 \leq x_{312} + x_{322} + x_{332} + x_{342} \leq 50$
$5 \leq x_{313} + x_{323} + x_{333} + x_{343} \leq 50$
$3 \leq x_{111} + x_{211} + x_{311} \leq 50$
$5 \leq x_{112} + x_{212} + x_{312} \leq 50$
$2 \leq x_{113} + x_{213} + x_{313} \leq 50$
$5 \leq x_{121} + x_{221} + x_{321} \leq 30$
$3 \leq x_{122} + x_{222} + x_{322} \leq 40$
$2 \leq x_{123} + x_{223} + x_{323} \leq 70$
$3 \leq x_{131} + x_{231} + x_{331} \leq 50$
$5 \leq x_{132} + x_{232} + x_{332} \leq 40$
$2 \leq x_{133} + x_{233} + x_{333} \leq 60$
$2 \leq x_{141} + x_{241} + x_{341} \leq 70$
$0 \leq x_{142} + x_{242} + x_{342} \leq 60$

$0 \leq x_{143} + x_{243} + x_{343} \leq 40$
$0 \leq x_{111} + x_{112} + x_{113} \leq 60$
$2 \leq x_{121} + x_{122} + x_{123} \leq 50$
$2 \leq x_{131} + x_{132} + x_{133} \leq 30$
$0 \leq x_{141} + x_{142} + x_{143} \leq 40$
$0 \leq x_{211} + x_{212} + x_{213} \leq 70$
$3 \leq x_{221} + x_{222} + x_{223} \leq 50$
$2 \leq x_{231} + x_{232} + x_{233} \leq 60$
$2 \leq x_{241} + x_{242} + x_{243} \leq 50$
$0 \leq x_{311} + x_{312} + x_{313} \leq 60$
$0 \leq x_{321} + x_{322} + x_{323} \leq 60$
$0 \leq x_{331} + x_{332} + x_{333} \leq 60$
$0 \leq x_{341} + x_{342} + x_{343} \leq 60$
$0 \leq x_{111} \leq 30, 1 \leq x_{112} \leq 60, 0 \leq x_{113} \leq 70$
$1 \leq x_{121} \leq 30, 2 \leq x_{122} \leq 70, 0 \leq x_{123} \leq 50$
$2 \leq x_{131} \leq 50, 0 \leq x_{132} \leq 70, 0 \leq x_{133} \leq 40$
$0 \leq x_{141} \leq 40, 0 \leq x_{142} \leq 50, 0 \leq x_{143} \leq 50$
$1 \leq x_{211} \leq 40, 1 \leq x_{212} \leq 70, 0 \leq x_{213} \leq 50$
$0 \leq x_{221} \leq 30, 1 \leq x_{222} \leq 60, 0 \leq x_{223} \leq 60$
$0 \leq x_{231} \leq 50, 2 \leq x_{232} \leq 80, 2 \leq x_{233} \leq 70$
$0 \leq x_{241} \leq 70, 0 \leq x_{242} \leq 50, 0 \leq x_{243} \leq 80$
$1 \leq x_{311} \leq 40, 1 \leq x_{312} \leq 70, 2 \leq x_{313} \leq 80$
$0 \leq x_{321} \leq 30, 1 \leq x_{322} \leq 60, 2 \leq x_{323} \leq 70$
$0 \leq x_{331} \leq 50, 1 \leq x_{332} \leq 50, 2 \leq x_{333} \leq 50$
$0 \leq x_{341} \leq 70, 0 \leq x_{342} \leq 60, 0 \leq x_{343} \leq 60$

Introduce a dummy origin, a dummy destination, and a dummy product in problem (P_3)). The related transportation problem is defined as follows.

$\min z = (2y_{111} + 3y_{112} + y_{113} + 0y_{114}) + (4y_{121} + 2y_{122} + 2y_{123} + 0y_{124}) + (3y_{131} + y_{132} + 3y_{133} + 0y_{134}) + (4y_{141} + 4y_{142} + y_{143} + 0y_{144})$
$+ (0y_{151} + 0y_{152} + 0y_{153} + 0y_{154}) + (3y_{211} + y_{212} + 2y_{213} + 0y_{214}) + (2y_{221} + 4y_{222} + 3y_{223} + 0y_{224}) + (5y_{231} + 2y_{232} + 3y_{233} + 0y_{234})$
$+ (3y_{241} + 4y_{242} + 2y_{243} + 0y_{244}) + (0y_{251} + 0y_{252} + 0y_{253} + 0y_{254}) + (y_{311} + 2y_{312} + 3y_{313} + 0y_{314}) + (4y_{321} + 3y_{322} + 2y_{323} + 0y_{324})$
$+ (2y_{331} + 3y_{332} + 4y_{333} + 0y_{334}) + (y_{341} + 2y_{342} + 3y_{343} + 0y_{344}) + (0y_{351} + 0y_{352} + 0y_{353} + 0y_{354}) + (0y_{411} + 0y_{412} + 0y_{413} + 0y_{414})$
$+ (0y_{421} + 0y_{422} + 0y_{423} + 0y_{424}) + (0y_{431} + 0y_{432} + 0y_{433} + 0y_{434}) + (0y_{441} + 0y_{442} + 0y_{443} + 0y_{444}) + (0y_{451} + 0y_{452} + 0y_{453} + 0y_{454}) \quad (P_4)$

subject to the constraints:

$y_{111} + y_{121} + y_{131} + y_{141} + y_{151} = 30$
$y_{112} + y_{122} + y_{132} + y_{142} + y_{152} = 40$
$y_{113} + y_{123} + y_{133} + y_{143} + y_{153} = 70$
$y_{114} + y_{124} + y_{134} + y_{144} + y_{154} = 180$
$y_{211} + y_{221} + y_{231} + y_{241} + y_{251} = 50$
$y_{212} + y_{222} + y_{232} + y_{242} + y_{252} = 40$
$y_{213} + y_{223} + y_{233} + y_{243} + y_{253} = 60$

$y_{214} + y_{224} + y_{234} + y_{244} + y_{254} = 230$
$y_{311} + y_{321} + y_{331} + y_{341} + y_{351} = 50$
$y_{312} + y_{322} + y_{332} + y_{342} + y_{352} = 50$
$y_{313} + y_{323} + y_{333} + y_{343} + y_{353} = 50$
$y_{314} + y_{324} + y_{334} + y_{344} + y_{354} = 240$
$y_{411} + y_{421} + y_{431} + y_{441} + y_{451} = 200$
$y_{412} + y_{422} + y_{432} + y_{442} + y_{452} = 190$
$y_{413} + y_{423} + y_{433} + y_{443} + y_{453} = 220$
$y_{414} + y_{424} + y_{434} + y_{444} + y_{454} = 650$
$y_{111} + y_{211} + y_{311} + y_{411} = 50$
$y_{112} + y_{212} + y_{312} + y_{412} = 50$
$y_{113} + y_{213} + y_{313} + y_{413} = 50$
$y_{114} + y_{214} + y_{314} + y_{414} = 190$
$y_{121} + y_{221} + y_{321} + y_{421} = 30$
$y_{122} + y_{222} + y_{322} + y_{422} = 40$
$y_{123} + y_{223} + y_{323} + y_{423} = 70$
$y_{124} + y_{224} + y_{324} + y_{424} = 160$
$y_{131} + y_{231} + y_{331} + y_{431} = 50$
$y_{132} + y_{232} + y_{332} + y_{432} = 40$
$y_{133} + y_{233} + y_{333} + y_{433} = 60$
$y_{134} + y_{234} + y_{334} + y_{434} = 150$
$y_{151} + y_{251} + y_{351} + y_{451} = 130$
$y_{152} + y_{252} + y_{352} + y_{452} = 130$
$y_{153} + y_{253} + y_{353} + y_{453} = 180$
$y_{154} + y_{254} + y_{354} + y_{454} = 650$
$y_{141} + y_{241} + y_{341} + y_{441} = 70$
$y_{142} + y_{242} + y_{342} + y_{442} = 60$
$y_{143} + y_{243} + y_{343} + y_{443} = 40$
$y_{144} + y_{244} + y_{344} + y_{444} = 150$
$y_{111} + y_{112} + y_{113} + y_{114} = 60$
$y_{121} + y_{122} + y_{123} + y_{124} = 50$
$y_{131} + y_{132} + y_{133} + y_{134} = 30$
$y_{141} + y_{142} + y_{143} + y_{144} = 40$
$y_{151} + y_{152} + y_{153} + y_{154} = 140$
$y_{211} + y_{212} + y_{213} + y_{214} = 70$
$y_{221} + y_{222} + y_{223} + y_{224} = 50$
$y_{231} + y_{232} + y_{233} + y_{234} = 60$
$y_{241} + y_{242} + y_{243} + y_{244} = 50$
$y_{251} + y_{252} + y_{253} + y_{254} = 150$
$y_{311} + y_{312} + y_{313} + y_{314} = 60$
$y_{321} + y_{322} + y_{323} + y_{324} = 60$
$y_{331} + y_{332} + y_{333} + y_{334} = 60$
$y_{341} + y_{342} + y_{343} + y_{344} = 60$
$y_{351} + y_{352} + y_{353} + y_{354} = 150$
$y_{411} + y_{412} + y_{413} + y_{414} = 150$

$y_{421} + y_{422} + y_{423} + y_{424} = 140$
$y_{431} + y_{432} + y_{433} + y_{434} = 150$
$y_{441} + y_{442} + y_{443} + y_{444} = 170$
$y_{451} + y_{452} + y_{453} + y_{454} = 650$
$0 \leq y_{111} \leq 30, 1 \leq y_{112} \leq 60, 0 \leq y_{113} \leq 70, 0 \leq y_{114} \leq 60, 0 \leq y_{151} \leq 25$
$1 \leq y_{121} \leq 30, 2 \leq y_{122} \leq 70, 0 \leq y_{123} \leq 50, 0 \leq y_{124} \leq 48, 0 \leq y_{152} \leq 37$
$2 \leq y_{131} \leq 50, 0 \leq y_{132} \leq 70, 0 \leq y_{133} \leq 40, 0 \leq y_{134} \leq 28, 0 \leq y_{153} \leq 68$
$0 \leq y_{141} \leq 40, 0 \leq y_{142} \leq 50, 0 \leq y_{143} \leq 50, 0 \leq y_{144} \leq 40, y_{154} \geq 0$
$1 \leq y_{211} \leq 40, 1 \leq y_{212} \leq 70, 0 \leq y_{213} \leq 50, 0 \leq y_{214} \leq 70, 0 \leq y_{221} \leq 30$
$1 \leq y_{222} \leq 60, 0 \leq y_{223} \leq 60, 0 \leq y_{224} \leq 47, 0 \leq y_{231} \leq 50, 2 \leq y_{232} \leq 80$
$2 \leq y_{233} \leq 70, 0 \leq y_{234} \leq 58, 0 \leq y_{241} \leq 70, 0 \leq y_{242} \leq 50, 0 \leq y_{243} \leq 80$
$0 \leq y_{244} \leq 48, 0 \leq y_{251} \leq 47, 0 \leq y_{252} \leq 35, 0 \leq y_{253} \leq 58, y_{254} \geq 0$
$1 \leq y_{311} \leq 40, 1 \leq y_{312} \leq 70, 2 \leq y_{313} \leq 80, 0 \leq y_{314} \leq 60, 0 \leq y_{321} \leq 30$
$1 \leq y_{322} \leq 60, 2 \leq y_{323} \leq 70, 0 \leq y_{324} \leq 60, 0 \leq y_{331} \leq 50, 1 \leq y_{332} \leq 50$
$2 \leq y_{333} \leq 50, 0 \leq y_{334} \leq 60, 0 \leq y_{341} \leq 70, 0 \leq y_{342} \leq 60, 0 \leq y_{343} \leq 60$
$0 \leq y_{344} \leq 60, 0 \leq y_{351} \leq 48, 0 \leq y_{352} \leq 47, 0 \leq y_{353} \leq 45, y_{354} \geq 0$
$0 \leq y_{411} \leq 47, 0 \leq y_{412} \leq 45, 0 \leq y_{413} \leq 48, y_{414} \geq 0, 0 \leq y_{421} \leq 25$
$0 \leq y_{422} \leq 37, 0 \leq y_{423} \leq 68, y_{424} \geq 0, 0 \leq y_{431} \leq 47, 0 \leq y_{432} \leq 35$
$0 \leq y_{433} \leq 58, y_{434} \geq 0, 0 \leq y_{441} \leq 68, 0 \leq y_{442} \leq 60, 0 \leq y_{443} \leq 40$
$y_{444} \geq 0, y_{451} \geq 0, y_{452} \geq 0, y_{453} \geq 0, y_{454} \geq 0$

5.2. Solution

We solved problem (P_4) on LINGO 17.0 and obtained the solution given in Table 2 and Table 3. From the solution of problem (P_4), we obtained the solution of problem (P_3), which is shown in Table 4.

Interpretation: $x_{111} = 1.00$ means that 1 quintal of wheat is supplied from the godown at Civil lines to fair price shop at Ashok Vihar. Similarly, value of other decision variables can be interpreted. The overall minimum transportation cost is eighty five thousand rupees when this transportation schedule is followed.

Objective Function Value	$z = 85.000$
Infeasibilities	0.000
Total solver iterations	35
Elapsed runtime seconds	0.05
Model Class	LP
Total variables	80
Nonlinear variables	0
Integer variables	0
Total constraints	57
Nonlinear constraints	0
Total non-zeros	276
Nonlinear non-zeros	0

Table 2: Global optimal solution obtained on LINGO 17.0

$y_{111} = 1.00$	$y_{121} = 1.00$	$y_{131} = 3.00$	$y_{141} = 0.00$	$y_{151} = 25.00$
$y_{112} = 1.00$	$y_{122} = 2.00$	$y_{132} = 2.00$	$y_{142} = 0.00$	$y_{152} = 35.00$
$y_{113} = 0.00$	$y_{123} = 0.00$	$y_{133} = 0.00$	$y_{143} = 2.00$	$y_{153} = 68.00$
$y_{114} = 58.00$	$y_{124} = 47.00$	$y_{134} = 25.00$	$y_{144} = 38.00$	$y_{154} = 12.00$
$y_{211} = 1.00$	$y_{221} = 4.00$	$y_{231} = 0.00$	$y_{241} = 0.00$	$y_{251} = 45.00$
$y_{212} = 3.00$	$y_{222} = 1.00$	$y_{232} = 2.00$	$y_{242} = 0.00$	$y_{252} = 34.00$
$y_{213} = 0.00$	$y_{223} = 0.00$	$y_{233} = 2.00$	$y_{243} = 2.00$	$y_{253} = 56.00$
$y_{214} = 66.00$	$y_{224} = 45.00$	$y_{234} = 56.00$	$y_{244} = 48.00$	$y_{254} = 15.00$
$y_{311} = 1.00$	$y_{321} = 0.00$	$y_{331} = 0.00$	$y_{341} = 2.00$	$y_{351} = 47.00$
$y_{312} = 1.00$	$y_{322} = 1.00$	$y_{332} = 1.00$	$y_{342} = 0.00$	$y_{352} = 47.00$
$y_{313} = 2.00$	$y_{323} = 2.00$	$y_{333} = 2.00$	$y_{343} = 0.00$	$y_{353} = 44.00$
$y_{314} = 56.00$	$y_{324} = 57.00$	$y_{334} = 57.00$	$y_{344} = 58.00$	$y_{354} = 12.00$
$y_{411} = 47.00$	$y_{421} = 25.00$	$y_{431} = 47.00$	$y_{441} = 68.00$	$y_{451} = 13.00$
$y_{412} = 45.00$	$y_{422} = 36.00$	$y_{432} = 35.00$	$y_{442} = 60.00$	$y_{452} = 14.00$
$y_{413} = 48.00$	$y_{423} = 68.00$	$y_{433} = 56.00$	$y_{443} = 36.00$	$y_{453} = 12.00$
$y_{414} = 10.00$	$y_{424} = 11.00$	$y_{434} = 12.00$	$y_{444} = 6.00$	$y_{454} = 611.00$

Table 3: Solution of problem (P_4)

$x_{111} = 1.00$	$x_{121} = 1.00$	$x_{131} = 3.00$	$x_{141} = 0.00$
$x_{112} = 1.00$	$x_{122} = 2.00$	$x_{132} = 2.00$	$x_{142} = 0.00$
$x_{113} = 0.00$	$x_{123} = 0.00$	$x_{133} = 0.00$	$x_{143} = 2.00$
$x_{211} = 1.00$	$x_{221} = 4.00$	$x_{231} = 0.00$	$x_{241} = 0.00$
$x_{212} = 3.00$	$x_{222} = 1.00$	$x_{232} = 2.00$	$x_{242} = 0.00$
$x_{213} = 0.00$	$x_{223} = 0.00$	$x_{233} = 2.00$	$x_{243} = 2.00$
$x_{311} = 1.00$	$x_{321} = 0.00$	$x_{331} = 0.00$	$x_{341} = 2.00$
$x_{312} = 1.00$	$x_{322} = 1.00$	$x_{332} = 1.00$	$x_{342} = 0.00$
$x_{313} = 2.00$	$x_{323} = 2.00$	$x_{333} = 2.00$	$x_{343} = 0.00$

Table 4: Solution of problem (P_3)

6. CONCLUSIONS AND SUGGESTIONS

In this study, we formulated a transportation problem in three dimensions. In this problem, the demand,supply, and commodity constraints are bounded . Moreover, the decision variables are also bounded. We first convert the original problem into a related transportation problem by introducing a dummy row, a dummy column, and a dummy product. The equivalence between the original problem and the related transportation problem is shown.It is shown that there is one to one correspondence between optimal solution of the original problem and optimal solution of related transportation problem. The developed model is then applied to the problem of Public Distribution System of North Delhi. A computing software LINGO 17.0 is used to solve the problem. This study has managerial implications. It can assist the managers of small scale and large scale industries in the selection of distributors, customers, quantity of variety of products, etc.The prospective research work of this paper is to apply the developed model to multi-index bounded transportation problem with specified flow.

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