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REPLENISHMENT POLICIES FOR IMPERFECT INVENTORY SYSTEM UNDER NATURAL IDLE TIME AND SHORTAGES

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Abstract: Most of the offline businesses do not run continuously for the entire day, therefore, the concept of idle time is inevitable. Natural idle time is referred to as the closing time when no demand is being fulfilled by the seller, the unproductive time when labor is not getting utilized but could be. In view of this, this paper investigates replenishment policies for the retailer who runs his business for only a part of a day and closes his shop for the remaining time. The demand gets fulfilled only during the opening time period while no customer is entertained during the closing part of the day. The retailer also faces issues of imperfect quality items in the lot received from his supplier. Thus, he carries out a rigorous inspection process of the entire lot so as to fulfill the demand with perfect quality items only. Under the above-stated circumstances, it is difficult to avoid shortages in the model. Thus, the model assumes fully backlogged shortages and is solved under a profit maximizing framework. The model is exemplified

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to understand its behavior. Further, to gain some managerial insights and to check the robustness of the model parameters, detailed sensitivity analysis is performed.

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MSC: 90B05, 90B25, 13P25.

1. INTRODUCTION AND LITERATURE OVERVIEW

The following section describes the motivation behind the developed framework. It also presents an overview of the existing literature relevant to the present study. This helps in sketching differences between our study and the previously done work.

1.1. Motivation

Traditional models assume that firms continuously work without any break in their systems. However, it is unviable to avoid natural pause time in which the labor is put to rest by the decision maker so as to keep up with their respective efficiencies. Therefore, the firms generally operate for a particular part of a day as is seen in most of the retail shops which are open for only a fixed number of hours (say 10 a.m. to 10 p.m.). Rest of the day is referred to as idle time, i.e., 10 p.m. to 10 a.m. The demand therefore gets fulfilled only in the non-idle time and so a discontinuous demand pattern is observed instead of the continuous one. When such a scenario is investigated under the imperfect quality environment, the outcome gets closer to reality. Due to the presence of defectives in the batch received by the supplier, a careful inspection becomes a necessity so as the prevention of the defectives reaching end customers. This proportion of defectives is analyzed over a period of time and estimated using the previous data. In the research so far, a discontinuous demand pattern due to the incorporation of idle time in the imperfect quality environment and fully-backlogged shortages have not been explored by the researchers. This has motivated us to study such a combination of practical problems, which are helpful for the firms.

1.2. Literature Review

Majority of the research since the development of classical EOQ model tacitly assumed a continuous demand throughout the cycle, which is not the case in practice. Most of the retail businesses go through a leisure time where no demand is being fulfilled. The concept of natural idle time was first introduced by De [3] under the assumption of wrongly measured demand rate. Later, Das *et al.* [2] presented a back-ordered EOQ model under the assumption of natural idle time where demand during shortage decreases exponentially with time. Further, Karmakar *et al.* [11] investigated optimum ordering policy for a product which is deteriorating in nature under the consideration of imprecise demand and natural idle time.

All the aforementioned papers have the assumption that all the procured units do not have any defect, which is unrealistic. Hence, a lot of research has been done in last few decades eliminating the unrealistic assumption of perfect items production in classical EOQ model by Harris [4]. The presence of imperfect items in the supply chain has the potential to make the product flow unreliable. Thus, screening of items before selling gets crucial, especially for non-manufacturing firms. Some of the initial research addressing the production of imperfect items includes Rosenblatt and Lee [17], Schwaller [20], Jaber *et al.* [8], Lee and Rosenblatt [13], Salameh and Jaber [18], Huang [7], Zhang and Gerchak [24], Jaggi and Mittal [9]. Later, Lin [5] presented an inventory model considering defectives as random variables, where the demand is distribution-free and lead-time is controllable. Further, Alamri *et al.* [1] explored an ordering model in which number of defectives decrease according to a learning curve. Soon after, Manna *et al.* [14] gave an EPQ model where the production rate affected the number of defectives produced. Jauhari *et al.* [10] studied a supply chain model with defectives and screening errors. Recently, Kazemi *et al.* [12] investigated an imperfect supply process to analyze the effect of carbon emission on optimal replenishment policies.

Referring to a supply chain with imperfect items, there is a considerable probability that shortages may ocurre. In the light of this, Hsu and Hsu [6] gave an integrated model where the production process is faulty and backorders are planned. Then, Moussawi-Haidar *et al.* [15] allowed shortages, thus extending the work of Jaggi and Mittal [9]. Further, Taleizadeh *et al.* [22] inspected an ordering model where shortages are partially back-ordered, incorporating reparation of imperfect products. Sanjai and Periyasamy [19] studied two different cases under imperfect production and rework, first without shortages and the other with shortages. Pal and Mahapatra [16] further explored a model with stochastic demand. Taleizadeh [21] studied a constrained integrated imperfect system under the assumptions of preventive maintenance and partial backordering.

Our Contribution: Inspired by the aforementioned studies, the current study focuses on the idle time opted by the firms to facilitate smooth functioning of their labor for a longer period of time. Though, this idle time ends up raising the total costs to the firm but prevents it from long term breaks in its performance. Idle time cost refers to the salary of employees even when they are off-duty, the security protection of the shop/firm which includes cost of guards, cameras, and also the cost of maintaining/ preserving the inventory from damage of external factors viz. temperature, humidity, rodents, etc. When this scenario is explored under the presence of defectives, it gives more pragmatic results useful for the firm. To protect goodwill loss, it is essential for firms to screen their lots before allowing them to enter the mainstream. Thus, the defectives are first inspected and then only the screened perfect items are sold to the end customers in the proposed model. Just like demand is satisfied during the opening time of the firm, shortages are built and eliminated during its non-idle time only. Shortages are sometimes beneficial for the seller especially when they can be completely backlogged owing to the loyal nature of the customers. Thus, the present model is applicable to a variety of retail industries that target ease for their resources, customer satisfaction, and high standards of quality. Hence, the proposed model can be observed as a closer representation of today's retail industries like pharmaceuticals, plastics, electronics, furniture, etc.

Rest of the paper takes the following structure: Section 2 specifies the nomenclature and assumptions used. Section 3 presents the model formulation. Section 4 delivers an algorithm to get the optimal values, also proving the concavity of the profit function. Section 5 validates the theory with the help of a numerical experiment. Section 6 checks the robustness of the model using sensitivity analysis. Section 7 summarizes the paper by highlighting key points. Section 8 provides insights into future work directions and further discusses the model limitations.

2. MODEL DEVELOPMENT

In this section, we give the nomenclature and assumptions, followed to build up the proposed mathematical framework.

2.1. Parametric notation and description

Parameters

D	Rate of demand in units/ unit time
α	Defective proportion ($0 < \alpha < 1$)
x	Rate of screening in units/ unit time; $x > D$
A	Proportion of screening rate with which backorders are satisfied
t_1	Opening time duration per day
t_2	Closing time duration per day ($t_1 + t_2 = 1$)
T	Cycle length
K	Ordering cost per cycle in \$
c	Purchase cost in \$/item
s	Selling price in \$/item
d	Inspection cost in \$/item
v	Salvage price of each defective item ($< s$) in \$/item
b	Cost of backordering in \$/unit/unit time
h	Cost of storage in \$/unit/unit time
i	Cost of idle time in \$/unit time
$*$	In superscript defines the optimum value

Dependent variables

θ_1	Number of days taken to fulfill shortages
$\theta_1 + \theta_2$	Number of days to finish inspection of the entire lot

Decision variables

m	Number of days inventory should be kept -1
n	Number of days shortages should be allowed -1

Dependent decision variables

Q	Order Size
B	Backorder Level

2.2. Assumptions

The mathematical model is proposed under the following assumptions.

(i) There is presence of defectives in the received lot whose proportion is estimated from the past data.
(ii) Rate of demand is known with certainty, constant and occurs at a uniform rate. Further, demand is fulfilled using perfect items only.
(iii) The rate of screening is more than the demand rate.
(iv) Single product type is under consideration.
(v) Natural idle time begins from the closing time of the shop up till its next opening time. The sum of the duration of opening time and closing time is one day.
(vi) Demand starts to get fulfilled with the opening time of the shop and stops at the beginning of the closing time.
(vii) Time horizon is infinite (days).
(viii) Backlogging is permitted and is fully backlogged in nature.
(ix) Lead time is insignificant.

3. MATHEMATICAL MODELING

The following section gives a detailed description of the problem under consideration and a formulation of the mathematical model befitting the above illustrated assumptions.

3.1. Problem Definition

The proposed model discusses about few problems faced by the retail industries. Generally, the retail industries do not operate endlessly for all day and night and thus go through a natural leisure time, also called idle time. The active hours of the day are characterized by the demand fulfillment leading to inventory depletion, process of shortage building and elimination, inspection of the lot and similar processes which aid in smooth functioning of the inventory systems. The idle time counts on the conservation of the stored inventory in terms of power consumption, working of security for protection, installation of cameras for safety, telephone for emergency communication, etc. All the idle time processes increase the total cost for the retailer but aid in providing longer and smoother operation. So, the aim is to effectively control the idle time costs without hampering the performance of the firm. Another common challenge faced by retailers is that of compromised quality of items delivered to them by their suppliers. Imperfect quality items may be present in the received lot due to mishandling at supplier's end, wear and tear during transit, damage/ breakage at retailer's end, etc. So, in order to sustain with the competency of marketplace, the retailer needs to sort these defectives before selling them to the end customers. Thus, a rigorous and error-free inspection process is inevitable at his end to match-up with the supreme standards of quality prevailing in the global markets. Therefore, the demand gets fulfilled by perfect items only. Finally, fully-backlogged shortages are also allowed in the model to bring it closer to reality. Hence, the retailer's objective is to jointly optimize the optimal ordering quantity along with the optimal backorder level. Net profit is attained by subtracting the total of all cost components viz. cost of ordering, purchase, inspection, holding, and shortage from the revenue, which is obtained by the sales of perfect items along with imperfect quality items.

3.2. Mathematical Formulation

The following section develops a mathematical formulation of the proposed model which fits the above-stated problem description and the assumptions of our model. Figure 1 demonstrates the nature of inventory curve graphically. The cycle starts as soon as lot size Q is received, which is inspected to isolate imperfect items.

Total perfect items sorted after performing inspection process on the lot size Q are $(1-\alpha)Q$. Further, the rate at which perfect items will be attained after inspection is $(1-\alpha)x$. These items are required to satisfy usual demand along with backorders at the rate 'D' and 'Ax' respectively. Hence, the overall depletion occurs at rate '$Ax+D$' such that $(1-\alpha)x = Ax + D$ which implies

$$A = (1-\alpha) - \frac{D}{x} \tag{1}$$

Accordingly, cycle time is given as

$$T = (1-\alpha)\frac{Q}{Dt_1} \tag{2}$$

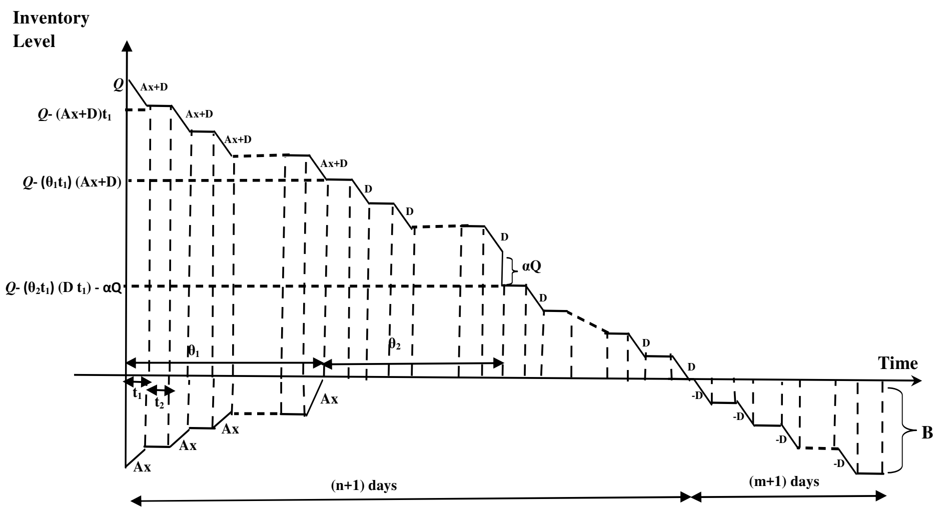

Figure 1: Inventory behavior representation

Let the entire inventory depletion time be denoted as $n+1$ days. It has been assumed that the level of inventory vanishes when the working duration (t_1) ends on the last day. So, we have

$$n+1=\frac{(1-\alpha)Q-B}{Dt_1} \tag{3}$$

Let the shortage building time be denoted as $m+1$ days. So, we have

$$m+1=\frac{B}{Dt_1} \tag{4}$$

Here $m,n \in Z^+$; $n>m$.

Also, the cycle length can be summed up as

$$T=n+m+2 \text{ days.} \tag{5}$$

From (2) and (5),

$$Q=\frac{(n+m+2)Dt_1}{(1-\alpha)} \tag{6}$$

From the commencement of cycle time up to θ_1 days, demand along with shortages are satisfied simultaneously from perfect items, tagged so by the inspection process, with rate $(1-\alpha)x$ and $(1-\alpha)x-D$ respectively.

Let all the shortages get satisfied in θ_1 $(\theta_1 \geq 1)$ number of days, so we get

$$\theta_1 t_1=\frac{B}{(1-\alpha)x-D} \tag{7}$$

$$B=\theta_1 t_1\left[(1-\alpha)x-D\right] \tag{8}$$

From (4) and (7),

$$\theta_1=\frac{(m+1)D}{(1-\alpha)x-D} \tag{9}$$

Let the number of days required for completion of inspection process be $(\theta_1+\theta_2)$ where $(\theta_1,\theta_2 \geq 1)$.

So, the total inspection time is calculated as

$$(\theta_1+\theta_2)t_1=\frac{Q}{x} \tag{10}$$

From (6), (9) and (10),

$$\theta_2=\frac{(n+m+2)D}{(1-\alpha)x}-\frac{(m+1)D}{(1-\alpha)x-D} \tag{11}$$

3.3. Components of Sales Revenue

The components of revenue are evaluated as follows

$$R_1\text{: Sales from only good quality items} = s(1-\alpha)Q \tag{12}$$

$$R_2\text{: Sales from only imperfect quality items} = v\alpha Q \tag{13}$$

Hence, the total revenue of the retailer is obtained as follows

$$T.R. = R_1 + R_2 = s(1-\alpha)Q + v\alpha Q \tag{14}$$

3.4. Components of Inventory System Costs

Various cost components applicable are as follows

(i) Cost of ordering per cycle, i.e.,

$$OC = A \tag{15}$$

(ii) Cost of purchase per cycle, i.e.,

$$PC = cQ \tag{16}$$

(iii) Cost of inspection per cycle, i.e.,

$$IC = dQ \tag{17}$$

(iv) Cost of storage per cycle, i.e., SC is the cost of holding all items during operational time as well as non-operational time, i.e.,

$$\begin{aligned}
HC = \frac{h}{2}&\{\{2Q - [(1-\alpha)xt_1]\}t_1 + \{2Q - 3[(1-\alpha)xt_1]\}t_1 \\
&+ \{2Q - 5[(1-\alpha)xt_1]\}t_1 + \ldots \theta_1 \text{ times}\} \\
&+ \frac{h}{2}\{\{2[Q - \theta_1(1-\alpha)xt_1] - Dt_1\}t_1 \\
&+ \{2[Q - \theta_1(1-\alpha)xt_1] - 3Dt_1\} \\
&+ \{2[Q - \theta_1(1-\alpha)xt_1] - 5Dt_1\} + \ldots \theta_2 \text{ times}\} \\
&+ \frac{h}{2}\{\{2\{[Q - \theta_1(1-\alpha)xt_1-] - \theta_2 Dt_1 - \alpha Q\} - Dt_1\}t_1 \\
&+ \{2\{[Q - \theta_1(1-\alpha)xt_1-] - \theta_2 Dt_1 - \alpha Q\} - 3Dt_1\}t_1 \\
&+ \{2\{[Q - \theta_1(1-\alpha)xt_1] - \theta_2 Dt_1 - \alpha Q\} - 5Dt_1\}t_1 \\
&+ \ldots (n - \theta_1 - \theta_2) \text{ times}\} + \frac{h}{2}Dt_1^2 \\
&+ h\{\{Q - [(1-\alpha)xt_1]\}t_2 + \{Q - 2[(1-\alpha)xt_1]\}t_2 \\
&+ \{Q - 3[(1-\alpha)xt_1]\}t_2 + \ldots \theta_1 \text{ times}\} \\
&+ h\{\{[Q - \theta_1(1-\alpha)xt_1] - Dt_1\}t_2
\end{aligned}$$

$$+\{[Q-\theta_1(1-\alpha)xt_1]-2Dt_1\}t_2$$
$$+\{[Q-\theta_1(1-\alpha)xt_1]-3Dt_1\}t_2+\ldots(\theta_2-1)\,\text{times}$$
$$\{[Q-\theta_1(1-\alpha)xt_1-]-\theta_2Dt_1-\alpha Q\}t_2\}$$
$$+h\{\{\{[Q-\theta_1(1-\alpha)xt_1-]-\theta_2Dt_1-\alpha Q\}-Dt_1\}t_2$$
$$+\{\{[Q-\theta_1(1-\alpha)xt_1-]-\theta_2Dt_1-\alpha Q\}-2Dt_1\}t_2$$
$$+\{\{[Q-\theta_1(1-\alpha)xt_1-]-\theta_2Dt_1-\alpha Q\}-3Dt_1\}t_2$$
$$+\ldots(n-\theta_1-\theta_2)\,\text{times}\} \quad (18)$$

(v) Idle Time Cost is the cost incurred when no labor/machinery is in-process, i.e.,

$$ITC=i(n+m+2)t_2 \quad (19)$$

(vi) Backordering cost is the cost of the total shortages occurred, i.e.,

$$BC=b\left\{\frac{1}{2}Dt_1^2+\frac{1}{2}(Dt_1+2Dt_1)t_1+\frac{1}{2}(2Dt_1+3Dt_1)t_1+\ldots(m+1)\,\text{times}\right\}$$
$$=\frac{1}{2}bDt_1^2(m+1)^2 \quad (20)$$

3.5. Retailer's Total Cost

Total cost (TC), which is the sum of all costs, for one cycle is as follows

$$TC=OC+PC+HC+IC+ITC+BC$$

Using equations (15)-(20), we get

$$TC=A+cQ+dQ+\frac{h}{2}\{\{2Q-[(1-\alpha)xt_1]\}t_1+\{2Q-3[(1-\alpha)xt_1]\}t_1$$
$$+\{2Q-5[(1-\alpha)xt_1]\}t_1+\ldots\theta_1\,\text{times}\}$$
$$+\frac{h}{2}\{\{2[Q-\theta_1(1-\alpha)xt_1]-Dt_1\}t_1+\{2[Q-\theta_1(1-\alpha)xt_1]-3Dt_1\}$$
$$+\{2[Q-\theta_1(1-\alpha)xt_1]-5Dt_1\}+\ldots\theta_2\,\text{times}\}$$
$$+\frac{h}{2}\{\{2\{[Q-\theta_1(1-\alpha)xt_1-]-\theta_2Dt_1-\alpha Q\}-Dt_1\}t_1$$
$$+\{2\{[Q-\theta_1(1-\alpha)xt_1-]-\theta_2Dt_1-\alpha Q\}-3Dt_1\}t_1$$
$$+\{2\{[Q-\theta_1(1-\alpha)xt_1-]-\theta_2Dt_1-\alpha Q\}-5Dt_1\}t_1$$
$$+\ldots(n-\theta_1-\theta_2)\,\text{times}\}+\frac{h}{2}Dt_1^2+h\{\{Q-[(1-\alpha)xt_1]\}t_2$$
$$+\{Q-2[(1-\alpha)xt_1]\}t_2+\{Q-3[(1-\alpha)xt_1]\}t_2+\ldots\theta_1\,\text{times}\}$$
$$+h\{\{[Q-\theta_1(1-\alpha)xt_1]-Dt_1\}t_2+\{[Q-\theta_1(1-\alpha)xt_1]-2Dt_1\}t_2$$
$$+\{[Q-\theta_1(1-\alpha)xt_1]-3Dt_1\}t_2+\ldots(\theta_2-1)\,\text{times}$$

$$\begin{aligned}
&\{[Q-\theta_1(1-\alpha)xt_1-]-\theta_2 Dt_1-\alpha Q\}t_2\} \\
&+h\{\{\{[Q-\theta_1(1-\alpha)xt_1-]-\theta_2 Dt_1-\alpha Q\}-Dt_1\}t_2 \\
&+\{\{[Q-\theta_1(1-\alpha)xt_1-]-\theta_2 Dt_1-\alpha Q\}-2Dt_1\}t_2 \\
&+\{\{[Q-\theta_1(1-\alpha)xt_1-]-\theta_2 Dt_1-\alpha Q\}-3Dt_1\}t_2 \\
&+\ldots(n-\theta_1-\theta_2)\,\text{times}\} \\
&+i(n+m+2)t_2+\frac{1}{2}bDt_1^2(m+1)^2 \qquad (21)
\end{aligned}$$

$$\begin{aligned}
TC = {} & A+cQ+dQ+hQt_1\theta_1-\frac{h}{2}(1-\alpha)xt_1^2\theta_1^2 \\
&+h[Q-\theta_1(1-\alpha)xt_1]t_1\theta_2-\frac{h}{2}Dt_1^2\theta_2^2 \\
&+h\left[Q-\theta_1(1-\alpha)xt_1-\theta_2 Dt_1-\alpha Q\right]t_1(n-\theta_1-\theta_2) \\
&-\frac{h}{2}Dt_1^2(n-\theta_1-\theta_2)^2+\frac{h}{2}Dt_1^2 \\
&+hQt_2\theta_1-h(1-\alpha)xt_1t_2\frac{\theta_1(\theta_1+1)}{2}+h\left[Q-\theta_1(1-\alpha)xt_1\right]t_2(\theta_2-1) \\
&-hDt_1t_2\frac{\theta_2(\theta_2-1)}{2}+h\left[Q-\theta_1(1-\alpha)xt_1-\theta_2 Dt_1-\alpha Q\right]t_2 \\
&+h\left(Q-\theta_1(1-\alpha)xt_1-\theta_2 Dt_1-\alpha Q\right)t_2(n-\theta_1-\theta_2) \\
&-hDt_1t_2\frac{(n-\theta_1-\theta_2)(n-\theta_1-\theta_2+1)}{2}+i(n+m+2)t_2 \\
&+\frac{b}{2}Dt_1^2(m+1)^2 \qquad (22)
\end{aligned}$$

3.6. Retailer's Total Profit

Total Profit (TP) = Total revenue (TR) − Total Cost (TC)

Total profit after substituting the values from equations (14) and (22) is given by

$$\begin{aligned}
TP = {} & s(1-\alpha)Q+v\alpha Q-A-cQ-dQ-hQt_1\theta_1 \\
&+\frac{h}{2}(1-\alpha)xt_1^2\theta_1^2-h[Q-\theta_1(1-\alpha)xt_1]t_1\theta_2 \\
&+\frac{h}{2}Dt_1^2\theta_2^2-h[Q-\theta_1(1-\alpha)xt_1-\theta_2 Dt_1-\alpha Q]t_1(n-\theta_1-\theta_2) \\
&+\frac{h}{2}Dt_1^2(n-\theta_1-\theta_2)^2-\frac{h}{2}Dt_1^2 \\
&-hQt_2\theta_1+h(1-\alpha)xt_1t_2\frac{\theta_1(\theta_1+1)}{2}-h[Q-\theta_1(1-\alpha)xt_1]t_2(\theta_2-1) \\
&+hDt_1t_2\frac{\theta_2(\theta_2-1)}{2}-h[Q-\theta_1(1-\alpha)xt_1-\theta_2 Dt_1-\alpha Q]t_2 \\
&-h[Q-\theta_1(1-\alpha)xt_1-\theta_2 Dt_1-\alpha Q]t_2(n-\theta_1-\theta_2) \\
&+hDt_1t_2\frac{(n-\theta_1-\theta_2)(n-\theta_1-\theta_2+1)}{2}-i(n+m+2)t_2
\end{aligned}$$

$$- \frac{b}{2} D t_1^2 (m+1)^2 \tag{23}$$

Substituting the values of θ_1, θ_2 and Q from equations (7), (11) and (6) respectively,

$$\begin{aligned}
TP(m,n) = {} & s(1-\alpha)\frac{(n+m+2)Dt_1}{(1-\alpha)} + v\alpha\frac{(n+m+2)Dt_1}{(1-\alpha)} \\
& - A - (c+d)\frac{(n+m+2)Dt_1}{(1-\alpha)} - h\frac{(n+m+2)Dt_1}{(1-\alpha)} \\
& t_1\frac{(m+1)D}{(1-\alpha)x-D} + \frac{h}{2}(1-\alpha)xt_1^2\left[\frac{(m+1)D}{(1-\alpha)x-D}\right]^2 \\
& - ht_1\left[\frac{(n+m+2)Dt_1}{(1-\alpha)} - \frac{(m+1)D}{(1-\alpha)x-D}(1-\alpha)xt_1\right] \\
& \left[\frac{(n+m+2)D}{(1-\alpha)x} - \frac{(m+1)D}{(1-\alpha)x-D}\right] \\
& + \frac{h}{2}Dt_1^2\left[\frac{(n+m+2)D}{(1-\alpha)x} - \frac{(m+1)D}{(1-\alpha)x-D}\right]^2 \\
& - ht_1\left[\frac{(n+m+2)Dt_1}{(1-\alpha)} - \frac{(m+1)D}{(1-\alpha)x-D}(1-\alpha)xt_1\right. \\
& - \left[\frac{(n+m+2)D}{(1-\alpha)x} - \frac{(m+1)D}{(1-\alpha)x-D}\right]Dt_1 \\
& \left.-\alpha\frac{(n+m+2)Dt_1}{(1-\alpha)}\right]\left[n - \frac{(n+m+2)D}{(1-\alpha)x}\right] \\
& + \frac{h}{2}Dt_1^2\left[n - \frac{(n+m+2)D}{(1-\alpha)x}\right]^2 - \frac{h}{2}Dt_1^2 \\
& - h\frac{(n+m+2)Dt_1}{(1-\alpha)}t_2\frac{(m+1)D}{(1-\alpha)x-D} \\
& + \frac{h}{2}(1-\alpha)xt_1t_2\frac{(m+1)D}{(1-\alpha)x-D}\left[\frac{(m+1)D}{(1-\alpha)x-D} + 1\right] \\
& - ht_2\left[\frac{(n+m+2)Dt_1}{(1-\alpha)} - \frac{(m+1)D}{(1-\alpha)x-D}(1-\alpha)xt_1\right] \\
& \cdot\left[\frac{(n+m+2)D}{(1-\alpha)x} - \frac{(m+1)D}{(1-\alpha)x-D} - 1\right] \\
& + \frac{h}{2}Dt_1t_2\left[\frac{(n+m+2)D}{(1-\alpha)x} - \frac{(m+1)D}{(1-\alpha)x-D}\right] \\
& \cdot\left[\frac{(n+m+2)D}{(1-\alpha)x} - \frac{(m+1)D}{(1-\alpha)x-D} - 1\right] \\
& - ht_2\left[\frac{(n+m+2)Dt_1}{(1-\alpha)} - \frac{(m+1)D}{(1-\alpha)x-D}(1-\alpha)xt_1\right.
\end{aligned}$$

$$
\begin{aligned}
&-\left[\frac{(n+m+2)D}{(1-\alpha)x}-\frac{(m+1)D}{(1-\alpha)x-D}\right]Dt_1-\alpha\frac{(n+m+2)Dt_1}{(1-\alpha)}\Bigg]\\
&-ht_2\left[\frac{(n+m+2)Dt_1}{(1-\alpha)}-\frac{(m+1)D}{(1-\alpha)x-D}(1-\alpha)xt_1\right.\\
&-\left[\frac{(n+m+2)D}{(1-\alpha)x}-\frac{(m+1)D}{(1-\alpha)x-D}\right]Dt_1\\
&\left.-\alpha\frac{(n+m+2)Dt_1}{(1-\alpha)}\right]\left[n-\frac{(n+m+2)D}{(1-\alpha)x}\right]\\
&+\frac{h}{2}Dt_1t_2\left[n-\frac{(n+m+2)D}{(1-\alpha)x}\right]\left[n-\frac{(n+m+2)D}{(1-\alpha)x}+1\right]\\
&-i(n+m+2)t_2-\frac{b}{2}Dt_1^2(m+1)^2
\end{aligned}
\tag{24}
$$

3.7. Expected Total Profit

The expression for expected total profit is expressed as follows:

$$
\begin{aligned}
&E[TP(m,n)]\\
&=s(1-E[\alpha])\frac{(n+m+2)Dt_1}{(1-E[\alpha])}+vE[\alpha]\frac{(n+m+2)Dt_1}{(1-E[\alpha])}\\
&\quad-A-(c+d)\frac{(n+m+2)Dt_1}{(1-E[\alpha])}-h\frac{(n+m+2)D^2t_1^2}{(1-E[\alpha])}\frac{(m+1)}{(1-E[\alpha])x-D}\\
&\quad+\frac{h}{2}(1-E[\alpha])xt_1^2\left[\frac{(m+1)D}{(1-E[\alpha])x-D}\right]^2\\
&\quad-ht_1\left[\frac{(n+m+2)D}{(1-E[\alpha])x}-\frac{(m+1)D}{(1-E[\alpha])x-D}\right]\\
&\quad\cdot\left[\frac{(n+m+2)Dt_1}{(1-E[\alpha])}-\frac{(m+1)D}{(1-E[\alpha])x-D}(1-E[\alpha])xt_1\right]\\
&\quad+\frac{h}{2}Dt_1^2\left[\frac{(n+m+2)D}{(1-E[\alpha])x}-\frac{(m+1)D}{(1-E[\alpha])x-D}\right]^2\\
&\quad+\frac{h}{2}Dt_1^2\left[\frac{(n+m+2)D}{(1-E[\alpha])x}-\frac{(m+1)D}{(1-E[\alpha])x-D}\right]^2\\
&\quad+\frac{h}{2}Dt_1^2\left[n-\frac{(n+m+2)D}{(1-E[\alpha])x}\right]^2\\
&\quad-ht_1\left[\frac{(n+m+2)Dt_1}{(1-E[\alpha])}-\frac{(m+1)D}{(1-E[\alpha])x-D}(1-E[\alpha])xt_1\right.\\
&\quad\left.-\left[\frac{(n+m+2)D}{(1-E[\alpha])x}-\frac{(m+1)D}{(1-E[\alpha])x-D}\right]Dt_1-E[\alpha]\frac{(n+m+2)Dt_1}{(1-E[\alpha])}\right]\\
&\quad\cdot\left[n-\frac{(n+m+2)D}{(1-E[\alpha])x}\right]
\end{aligned}
$$

$$
\begin{aligned}
&- \frac{h}{2}Dt_1^2 - h\frac{(n+m+2)Dt_1}{(1-E[\alpha])}t_2\frac{(m+1)D}{(1-E[\alpha])x-D} \\
&+ \frac{h}{2}(1-E[\alpha])xt_1t_2\frac{(m+1)D}{(1-E[\alpha])x-D}\left[\frac{(m+1)D}{(1-E[\alpha])x-D}+1\right] \\
&- ht_2\left[\frac{(n+m+2)Dt_1}{(1-E[\alpha])} - \frac{(m+1)D}{(1-E[\alpha])x-D}(1-E[\alpha])xt_1\right] \\
&\cdot\left[\frac{(n+m+2)D}{(1-E[\alpha])x} - \frac{(m+1)D}{(1-E[\alpha])x-D} - 1\right] \\
&+ \frac{h}{2}Dt_1t_2\left[\frac{(n+m+2)D}{(1-E[\alpha])x} - \frac{(m+1)D}{(1-E[\alpha])x-D}\right] \\
&\cdot\left[\frac{(n+m+2)D}{(1-E[\alpha])x} - \frac{(m+1)D}{(1-E[\alpha])x-D} - 1\right] \\
&- ht_2\left[\frac{(n+m+2)Dt_1}{(1-E[\alpha])} - \frac{(m+1)D}{(1-E[\alpha])x-D}(1-E[\alpha])xt_1\right. \\
&\left. - \left[\frac{(n+m+2)D}{(1-E[\alpha])x} - \frac{(m+1)D}{(1-E[\alpha])x-D}\right]Dt_1 - E[\alpha]\frac{(n+m+2)Dt_1}{(1-E[\alpha])}\right] \\
&- ht_2\left[\frac{(n+m+2)Dt_1}{(1-\alpha)} - \frac{(m+1)D}{(1-\alpha)x-D}(1-\alpha)xt_1\right. \\
&\left. - \left[\frac{(n+m+2)D}{(1-\alpha)x} - \frac{(m+1)D}{(1-\alpha)x-D}\right]Dt_1 - \alpha\frac{(n+m+2)Dt_1}{(1-\alpha)}\right] \\
&\cdot\left[n - \frac{(n+m+2)D}{(1-\alpha)x}\right] \\
&+ \frac{h}{2}Dt_1t_2\left[n - \frac{(n+m+2)D}{(1-\alpha)x}\right]\left[n - \frac{(n+m+2)D}{(1-\alpha)x}+1\right] \\
&- i(n+m+2)t_2 - \frac{b}{2}Dt_1^2(m+1)^2
\end{aligned}
\tag{25}
$$

3.8. *Expected Total Profit per unit time*

Using Renewal reward theorem

$$E[TPU(m,n)] = \frac{E[TP(m,n)]}{E[T]}$$

$$
\begin{aligned}
&E[TPU(m,n)] \\
&\quad = sDt_1 + \frac{vE[\alpha]Dt_1}{(1-E[\alpha])} - \frac{A}{(n+m+2)} - \frac{cDt_1}{(1-E[\alpha])} \\
&\quad - \frac{dDt_1}{(1-E[\alpha])} - \frac{hDt_1^2}{(1-E[\alpha])}\frac{(m+1)D}{(1-E[\alpha])x-D} \\
&\quad + \frac{h}{2(n+m+2)}(1-E[\alpha])xt_1^2\left[\frac{(m+1)D}{(1-E[\alpha])x-D}\right]^2
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{ht_1}{(n+m+2)}\left[\frac{(n+m+2)Dt_1}{(1-E[\alpha])}-\frac{(m+1)D}{(1-E[\alpha])x-D}(1-E[\alpha])xt_1\right]\\
&\cdot\left[\frac{(n+m+2)D}{(1-E[\alpha])x}-\frac{(m+1)D}{(1-E[\alpha])x-D}\right]\\
&+\frac{hDt_1^2}{2(n+m+2)}\left[\frac{(n+m+2)D}{(1-E[\alpha])x}-\frac{(m+1)D}{(1-E[\alpha])x-D}\right]^2\\
&\cdot\frac{ht_1}{(n+m+2)}\left[\frac{(n+m+2)Dt_1}{(1-E[\alpha])}-\frac{(m+1)D}{(1-E[\alpha])x-D}(1-E[\alpha])xt_1\right.\\
&\left.-\left[\frac{(n+m+2)D}{(1-E[\alpha])x}-\frac{(m+1)D}{(1-E[\alpha])x-D}\right]Dt_1-E[\alpha]\frac{(n+m+2)Dt_1}{(1-E[\alpha])}\right]\\
&\cdot\left[n-\frac{(n+m+2)D}{(1-E[\alpha])x}\right]+\frac{hDt_1^2}{2(n+m+2)}\left[n-\frac{(n+m+2)D}{(1-E[\alpha])x}\right]^2\\
&-\frac{hDt_1^2}{2(n+m+2)}-\frac{hDt_1t_2}{(1-E[\alpha])}\frac{(m+1)D}{(1-E[\alpha])x-D}+\frac{h}{2(n+m+2)}\\
&-\frac{ht_2}{(n+m+2)}\left[\frac{(n+m+2)Dt_1}{(1-E[\alpha])}-\frac{(m+1)D}{(1-E[\alpha])x-D}(1-E[\alpha])xt_1\right]\\
&\left[\frac{(n+m+2)D}{(1-E[\alpha])x}-\frac{(m+1)D}{(1-E[\alpha])x-D}-1\right]\\
&+\frac{hDt_1t_2}{2(n+m+2)}\left[\frac{(n+m+2)D}{(1-E[\alpha])x}-\frac{(m+1)D}{(1-E[\alpha])x-D}\right]\\
&\cdot\left[\frac{(n+m+2)D}{(1-E[\alpha])x}-\frac{(m+1)D}{(1-E[\alpha])x-D}-1\right]-\frac{ht_2}{(n+m+2)}\\
&\cdot\left[\frac{(n+m+2)Dt_1}{(1-E[\alpha])}-\frac{(m+1)D}{(1-E[\alpha])x-D}(1-E[\alpha])xt_1\right.\\
&-\left[\frac{(n+m+2)D}{(1-E[\alpha])x}-\frac{(m+1)D}{(1-E[\alpha])x-D}\right]Dt_1\\
&\left.-E[\alpha]\frac{(n+m+2)Dt_1}{(1-E[\alpha])}\right]-\frac{ht_2}{(n+m+2)}\left[\frac{(n+m+2)Dt_1}{(1-E[\alpha])}\right.\\
&-\frac{(m+1)D}{(1-E[\alpha])x-D}(1-E[\alpha])xt_1\\
&-\left[\frac{(n+m+2)D}{(1-E[\alpha])x}-\frac{(m+1)D}{(1-E[\alpha])x-D}\right]Dt_1\\
&\left.-E[\alpha]\frac{(n+m+2)Dt_1}{(1-E[\alpha])}\right]\left[n-\frac{(n+m+2)D}{(1-E[\alpha])x}\right]\\
&+\frac{hDt_1t_2}{2(n+m+2)}\left[n-\frac{(n+m+2)D}{(1-E[\alpha])x}\right]\\
&\cdot\left[n-\frac{(n+m+2)D}{(1-E[\alpha])x}+1\right]-it_2-\frac{bDt_1^2(m+1)^2}{2(n+m+2)} \qquad (26)
\end{aligned}
$$

4. OPTIMALITY AND SOLUTION PROCEDURE

The first-order necessary conditions for optimality (considering m, n as continuous variables) are as follows

$$\frac{\partial E[TPU(m,n)]}{\partial m}=0 \quad \text{and} \quad \frac{\partial E[TPU(m,n)]}{\partial n}=0\,. \tag{27}$$

Further, the sufficient conditions for optimality are given as

$$\left(\frac{\partial^2}{\partial m \partial n}E\left[TPU\left(m,n\right)\right]\right)^2-\left(\frac{\partial^2}{\partial m^2}E\left[TPU\left(m,n\right)\right]\right)\left(\frac{\partial^2}{\partial n^2}E\left[TPU\left(m,n\right)\right]\right)\leq 0$$

and

$$\left(\frac{\partial^2}{\partial m^2}E\left[TPU\left(m,n\right)\right]\right)\leq 0 \;\left(\frac{\partial^2}{\partial n^2}E\left[TPU\left(m,n\right)\right]\right)\leq 0 \tag{28}$$

Since these derivatives are highly non-linear and complex in nature, concavity of the profit function has been proved graphically (refer to Figure 2).

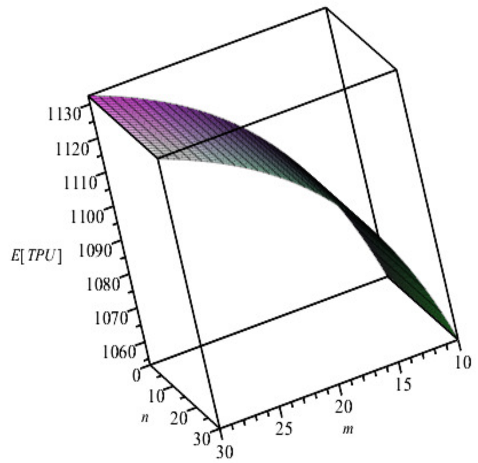

Figure 2: Profit function

In order to obtain the optimal values of m and n, the following algorithm is proposed

Algorithm

(i) Solve the maximization problem for 'm' and 'n' (as continuous variables) using common mathematical tools like Lingo, Solver etc.

(ii) Finalize a range for both 'm' and 'n' such that

$$m_1 < m < m_2 \;\text{ and }\; n_1 < n < n_2$$

where $m_1 = m - 5, \quad m_2 = m + 5$
and $n_1 = n - 5, \quad n_2 = n + 5$

(iii) Let A be the set containing all the discrete values from the range of 'm' and B be the set containing all the discrete values from the range of 'n', i.e.,

$$A = \{m_1, m_1{+}1, m_1{+}2, \ldots, m_2{-}1, m_2\} \text{ and } B = \{n_1, n_1{+}1, n_1{+}2, \ldots, n_2{-}1, n_2\}$$

Take the cartesian product of A and B,

$$C = A \times B = \{(m, n) : m \in A \text{ and } n \in B\}$$

(iv) Using equation (26) calculate $\max E[TPU(m, n)]$ where $(m, n) \in C$

The values of m and n giving this maximum value will provide the optimum values of 'm' and 'n'. The optimum values of dependent decision variables Q and B can further be calculated from equations (6) and (4) respectively.

5. NUMERICAL ANALYSIS

This section illustrates the formulated mathematical model using a numerical example. The following input parameter values taken to illustrate the model are from Das *et al.* [2] model 1

D = 150 units/day, K = Rs. 150/order, λ = 0.5/unit, h = 1.5/unit/day, b = 1.2/unit, i = Rs. 4.5/day,

Other parameter values are: s = Rs. 60/unit, c = Rs. 30/unit, v = Rs. 20/unit, X = 300 units/day, $t_1 = 0.5$, $t_2 = 0.5$.

It has been assumed that the defect proportion (α) follows Uniform distribution with probability density function as

$$f(\alpha) = \begin{cases} 25, & 0.04 \le \alpha \le 0.08 \\ 0, & \text{otherwise} \end{cases}$$

Using algorithm, the optimal values are: $n^* = 17$ and $m^* = 10$, $\theta_1 = 13$, $\theta_2 = 3$ with the corresponding order size $Q^* = 4486$ units/cycle, $E[TPU^*(m, n)] = 1091$/day and $B^* = 851$ units/cycle respectively.

However, when the idle time concept is not taken into consideration, the parameters are the same as that of Wee [23] paper. The results of comparing both the models are recorded in Table 1.

Description	*Cycle time*	*Order quantity*	*Backorders*	*Total profit value*
With idle time	29 days	4486 units	851 units	1091 \$/day
No. idle time	37 days	5986 units	1126 units	2158 \$ / day

Table 1: Comparison between idle time and no idle time

It is evident from the above table that if the concept of idle time is incorporated, the real picture changes, suggesting a change in the ordering policy of the retailer. The optimal ordering quantity decreases along with backordering quantity implying a decrease in cycle time. The above-mentioned figures reveal that though the profit is higher without consideration of idle time, still these results are misleading as the ideal scenario of endless working hours of human labor is hypothetical and impossible to achieve. By incorporating the concept of idle time, there does come an inevitable idle time cost of security, telephone, etc. which can be reduced via fast replenishment of items as is also advised to the retailer through shorter cycle length. Hence, instead of having a theoretical concept which showcases higher profit values, it is more relevant to incorporate practicality of idle time concept even though it comes at a relatively higher cost.

6. SENSITIVITY ANALYSIS AND MANAGERIAL INSIGHTS

The values of various parameters like different cost parameters, percentage of defective items, etc. are not in control of the decision maker. The parameter values change owing to the change in business and financial conditions. Thus, it gets important to analyze the effect of variations in several parameters on the optimal decision. Obtained results are summarized in Table 1.

Parameter	*Values*	n	m	θ_1	θ_2	B	TPU
α	0.04	17	11	13	3	918	1099
	0.06	17	10	13	3	851	1091
	0.08	16	9	12	2	786	1082
	0.1	15	9	12	2	723	1073
c	25	20	12	15	3	1006	1291
	30	17	10	13	3	851	1091
	35	13	8	11	2	695	890
	40	10	6	8	2	539	689
λ	0.0	17	11	13	3	866	1111
	0.5	17	10	13	3	851	1091
	1.5	16	10	12	2	820	1051
	2.5	15	9	11	2	757	971
h	1.0	25	14	17	5	1135	1095
	1.5	17	10	13	3	851	1091
	2.0	12	8	10	21	680	1086
	2.5	10	7	9	1	567	1081
i	0	17	10	13	3	852	1092
	4.5	17	10	13	3	851	1091
	10.5	17	10	13	2	850	1089
	16.5	17	10	13	2	848	1088

Table 2: Influence of change in various parameter values

Based on sensitivity analysis from Table 2, following are the major insights

(i) When the percentage of defectives (α) rises, the optimal values of n and m are observed to decrease, i.e. the cycle length decreases. Consequently, the optimal ordering quantity (Q^*) along with backorder quantity (B^*) and total profit (TPU^*) decrease. In other words, it is economical for the retailer to keep inventory for a shorter time and hence a decrease in optimal order quantity is suggested to lower the effect of increased defectives in the lot. If the number of defectives in further lots does not decrease, the retailer is advised to change the supplier as the latter fails to improve the quality of his products. The lowering of shortage levels indicates that not many people are ready to wait for purchasing in the scenario of greater imperfect quality items. The fall in profit values show that retailer cannot compromise with the quality of the products as these have adverse effect on the sale of products.

(ii) When the cost price of an item (c) decreases, the optimal values of n, m with ordering quantity (Q^*), backorder quantity (B^*) and total profit (TPU^*) increase. This is quite logical as in such a case when the retailer is purchasing on lower rates, he will try to purchase larger quantities which will consume more time to deplete. Thus, the cycle length also shows increasing trend. With more items in hand, duration of inspection time is also bound to increase. Also, higher amount of shortages can be fulfilled with larger order, thus shortages do exhibit increasing trend.

(iii) With decrease in inspection cost (d), the optimal values of n, m along with ordering quantity (Q^*), backorder quantity (B^*) and total profit (TPU^*) increase. The optimal values of θ_1, θ_2 is also observed to increase, i.e. the cycle length and inspection duration also increase. This is quite intuitive as when the retailer is not investing in the inspection process, it will bring lower efficiency in terms of quality of inspection and competent workers. So, the inspection time and resultantly cycle length is assured to increase. In order to utilize decrement in inspection cost, purchase of a larger batch size would be profitable.

(iv) As the value of holding cost (h) increases, the optimal ordering quantity (Q^*) together with backordering quantity (B^*) and cycle length ($n+m+2$) decrease. Thus, the retailer is advised to make frequent orders to deal with the impact of increased cost while holding. Thus, a decrease in cycle length is evident. Further, a fall in inspection time is observed as corresponding to decreased order size. Apparently, total profit (TPU^*) decreases.

(v) When the idle time cost (i) decreases, the optimal ordering quantity (Q^*) increases with backordering quantity (B^*) and total profit (TPU^*). The security charge, telephone charge, etc. beyond working hours may be considered as the idle time cost. When the system is put to rest, these actions keep on going. So, it is advisable to the retailer to minimize the wastage of resources. Hence, when these costs decrease, a larger lot is preferred over a smaller one. This simultaneously increases the level of maximum allowable shortages. Resultantly, a marginal increment in cycle length $(n+m+2)$ and inspection time $(\theta_1 + \theta_2)$ is also observed.

7. CONCLUDING REMARKS

The proposed study analyses a combination of different problems faced by the retail industries viz. impact of idle time, managing of imperfect quality items, and handling of customers in case of stock-outs. The unavoidable opening and closing time of the retail stores are the vital aspects of the study as they exhibit the natural way these stores work. When the idle times are left unaccounted, this may lead to long-term system failures as non-stop working of resources, and labor will ultimately lead to decreased efficiencies. Furthermore, it is unviable to assume that the retail stores always receive perfect quality from their suppliers. So, the presence of defectives is managed with a careful inspection process which helps in maintaining quality standards in today's cut-throat competition. All these attributes in deriving the results closer to reality. Incorporation of idle time advocates a decrease in optimal order size and the profit. The study shows that idle time has a noteworthy impact on the total system operating costs, thus ignoring it while modeling any inventory scenario makes the results questionable.

8. FUTURE RESEARCH GUIDANCE AND LIMITATIONS

The present model can be attempted for obtaining exact timing of cash flows. Thus, it can be extended for finite planning horizon under inflationary conditions. It would also be insightful to study the proposed scenario for different types of trade credit policies. Variations in demand patterns would also add practicality to the present study. The model can further be studied under the supportive environmental conditions viz. carbon emission, green supply chain, RFID, etc.

The model is restricted to the scope of limited storage space for the manufacturer, and also for the compensation of extra costs invested for smooth functioning of the inventory systems.

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