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PRESERVATION TECHNOLOGY INVESTMENT FOR AN INVENTORY SYSTEM WITH VARIABLE DETERIORATION RATE UNDER EXPIRATION DATES AND PRICE SENSITIVE DEMAND

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Abstract: The proposed study addresses a supplier-retailer inventory problem by considering the detrimental impacts of deterioration. The inventory produced by the supplier undergo a machine-shift and hence, produces non-conforming items. The retailer uses the preservation technology to deal with deteriorating items ingeniously where the demand

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is price-sensitive at the buyer's end. Two models are developed through different means viz. Integrated and Bi-level approach and compared so to impart some constructive organizational insights. Sensitivity analysis was done over a numerical example to validate strength of the developed models.

Keywords: Variable Deterioration, Machine-shift, Price-sensitive Demand, Expiration Date, Bi-level Approach, Preservation Technology.

MSC: 90B05.

1. INTRODUCTION

A preliminary research in the area of deteriorating items was carried out by Ghare and Schrader [7]. Some models with constant and Weibull deterioration rate were also developed by Mishra [23], Taleizadeh et al. [32] etc. Owing to efficient management of deteriorating items several researchers discussed the benefits of investment in preservation technology, Dye and Hasieh [3, 4], Hasieh and Dye [13], Singh et al. [34] developed two-stage production model with the investment in preservation technology. Zhang et al. [36] proposed a model involving the idea of preservation technology. Recently, Khanna et al. [17, 18], Giri and Maiti [8, 9], and Mishra et al. [24] have explored this field under various assumptions, e.g., price-sensitive demand.

A production process may not be always reliable. It may change its state "from in-control to out-of-control", which leads to the imperfect production. Porteus [26] developed a model for quality improvement. Lee and Rosenblatt [19, 20] proposed different models for imperfect production process. Later, Ben-Daya [2], Salameh and Jaber [28], Sana [29], Giri and Maiti [8], Chen and Teng [2], and several others investigated the area of inventory under "imperfect-quality" environments. Khanna et al. [17, 18] proposed a modal for imperfect quality items.

Collaborative work is a necessity so several researchers explored the inventory modeling under integrated environments,e.g., see Goyal and Giri [11], Ha and Kim [12], Yang and Wee [35]. Shah and Shah [30], developed models for deteriorating items by using vendor-buyer approach. Manna et al. [22] gave a model for the items that are deteriorating in nature. Further, Mohanty and Shankar [25], Iqbal and Sarkar [14] proposed different models for an integrated system. The integrated problem has been solved under various approaches. Traditionally, it is presumed that the supply chain players have equal powers, but if to get effective management and profit extraction in reallity, it is imperative to follow a leader-follower relationship. Bi-level approach enables decision makers to choose the leader and the follower in an attempt to achieve organization profitability. Gao et al. [5], Ma, et al. [21] etc. established the "bi-level programming" models by exchanging the role of "follower-leader" and "leader-follower".

Though in their disscusion of the integrated inventory modeling several research articles highlighted the equal decision making power of the supply chain players, we agree with the approches where it is imperative to study the behavior of the integrated system under different leadership so as to have a clear picture of up-downs in the total cost. Our present paper develops bi-level and integrated models for an inventory system, with deterioration rate to be variable under the concept of expiration dates and demand of the product as price-sensitive. The key feature of this paper is the lifetime dependent deterioration rate, which makes it novel and adds positively to the existing literature as presented in the Table below. Table 2

Authors	**Demand pattern**	**Deterioration with expiry**	**Preservation technology**	**model Integrated**	**Bi-level approach**
Hsu et al. [15]	Constant	No	Yes	No	No
Sadigh [27]	Advertisement based	No	No	Yes	Yes
Dye [3]	Time dependent	No	Yes	No	No
Tayal et al. [33]	Seasonal	Yes	Yes	Yes	No
Ma et al. [21]	Fuzzy	No	No	Yes	Yes
Zhang et al. [36]	Price dependent	Yes	Yes	Yes	No
Tayal et al. [34]	No	Yes	Yes	Yes	No
Zhang et al. [36]	Price dependent	Yes	Yes	Yes	No
Giri et al. [10]	Price dependent	Yes	Yes	Yes	No
Mishra et al. [24]	Price & stock dependent	Yes	Yes	No	No
This paper	Price dependent	Yes	Yes	Yes	Yes

Table 1: Literature summary

2. NOTATIONS AND ASSUMPTIONS

2.1. Notations

Decision variables

- P Unit selling price for buyer
- T Cycle time
- τ Cost of preservation technology per unit time

Constant parameters

$\theta(t)$	Deterioration rate at time t
$f(\tau)$	Deterioration rate proportion with preservation technology investment
$D(p)$	Demand rate
A	Scale for demand
B	Price-sensitive parameter
P	Production rate of vendor
t_1	Time delay to begin production at the vendor's end
h_b	Inventory carrying cost/unit/unit time at buyer's end
h_v	Inventory carrying cost/unit/unit time at the vendor's end
k_b	Ordering cost per order at buyer's end
k_v	Setup cost per setup at vendor's end
K_d	Deteriorating cost
δP	Defective item production rate, $0 \leq \delta < 1$
$g(t)$	pdf of the time to machine shift
W	Unit wholesale price at vendor's end
c	Unit procurement cost
TPB	Total profit per unit time (buyer)
TPV	Total profit per unit time (vendor)
TPS	Total profit per unit time (supply chain)

2.2. Assumptions

The proposed model is developed under the following assumptions

(i) 1. A "single-buyer, single-vendor supply chain" system for a single-item is considered for a single time period.

(ii) Rate of production is sufficiently greater than the maximum demand rate.

(iii) Since the production rate is sufficiently large as compared to the demand rate, there is no place for shortages.

(iv) Since the vendor's rate of production is sufficiently greater than the buyer's demand rate, then, the vendor takes a time delay t_1 per production run.

(v) During the production, at any random time $t \in [t_1, T]$, the process may shift from an in-control state to an out-of-control state, which leads to the production of defective items.

(vi) The demand rate at the buyer's side is the function of selling-price of the item, i.e., $D(p) = a - bp$, where $a > 0$ and $0 < b < 1$ are the parameters for demand rate such that the demand is always positive.

(vii) The items that are deteriorating in nature have a maximum life period, which is called the expiry date l. Rate of deterioration will tend to unity when the time approaches l. Here $\theta(t) = \frac{1}{1+l-t}$, $0 \leq t \leq T \leq l$.

(viii) The proportion of reducing deterioration rate after using preservation technology-investment is $f(\tau)$, where this function satisfies the conditions $f'(\tau) > 0$, $f''(\tau) < 0$ and $f(0) = 0$ and $f(\tau) = 1 - \frac{1}{1+y_0\tau}$, where $0 < y_0 < 1$

3. MATHEMATICAL MODELING

Consider an inventory situation where the production system at vendor begins the process after a time delay t_d. $E(N)$ number of items produced in each cycle are defective in nature. At the buyer's end the inventory level depletes due to the combined effect of both the demand and deterioration. The nature of the inventory is deteriorating, thus, the buyer spends some amount (τ) as an investment towards preservation technology, which aids in decreasing the rate of deterioration from y_0 to $y_0(\tau)$. The representation of inventory for the vendor and buyer is depicted in Figure 1.

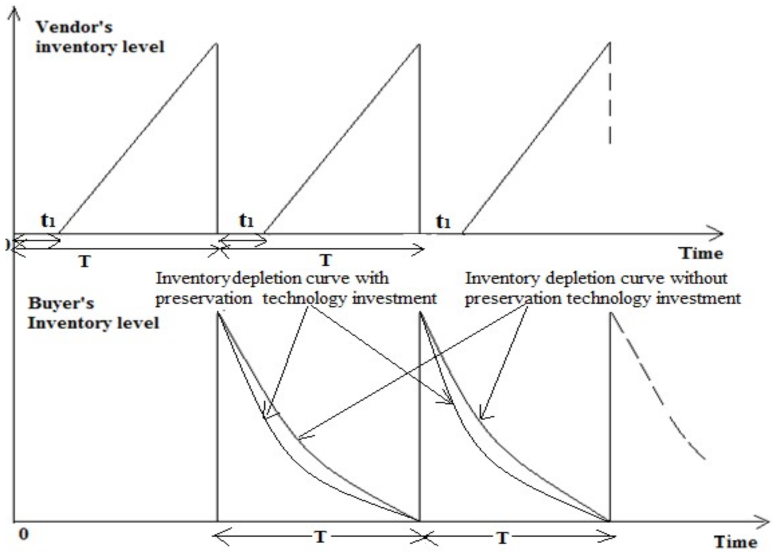

Figure 1: Inventory representation of vendor and buyer

3.1. Buyer's model

$$\frac{dI_b(t)}{dt} + \theta(t)(1 - f(\tau))I_b(t) = -D(p), \quad 0 \le t \le T \tag{1}$$

Considering the boundary condition $I_b(T) = 0$, the solution of Equation (1) is given as:

$$I_b(t) = \frac{D(p)}{f(\tau)}[(1 + l_m - t) - (1 + l_m - T)^{f(\tau)}(1 + l_m - t)^{1-f(\tau)}]. \tag{2}$$

The initial inventory level is

$$I_b(0) = \frac{D(p)}{f(\tau)}[(1 + l_m) - (1 + l_m - T)^{f(\tau)}(1 + l_m)^{1-f(\tau)}] \tag{3}$$

Overall expenditure of the buyer depends upon the following cost components:

- Holding cost (HCB)

$$= \frac{h_b}{T}\int_0^T I(t)dt$$
$$= \frac{h_b D(p)}{T f(\tau)}\Big[\frac{T(2+2l_m-T)}{2}+\frac{(1+l_m-T)^2}{2-f(\tau)}-\frac{(1+l_m)^{2-f(\tau)}(1+l_m-T)^{f(\tau)}}{2-f(\tau)}\Big] \tag{4}$$

- Preservation technology cost (PTB)

$$= \tau \tag{5}$$

- Ordering cost (OCB)

$$= \frac{k_b}{T} \tag{6}$$

- Deterioration cost (DCB)

$$= \frac{K_d D(p)}{T f(\tau)}[(1+l_m)-(1+l_m-T)^{f(\tau)}(1+l_m)^{1-f(\tau)}]-K_d D(p) \tag{7}$$

So, the total cost of the buyer can be determined by the sum of all cost components:

$$TCB(T,\tau,p) = HCB + PTC + OCB + DCB \tag{8}$$

The total revenue of the buyer is given by:

$$TRB(T,\tau,p) = pD(p) \tag{9}$$

So, the total profit of the buyer is

$$TPB(T,\tau,p) = TRB(T,\tau,p) - TCB(T,\tau,p) \tag{10}$$

3.2. Vendor's model

Production process at vendor starts after a delay period t_1, and the process shifts from an "in-control state" to an "out-of-control" state at any random time $t \in [t_1, T]$ and follows a uniform shift distribution having the p.d.f.

$$g(t) = \begin{cases} \frac{1}{T-t_1}, & t_1 \le t \le T \\ 0 \end{cases}$$

The production of defectives causes a defective item cost to the vendor.

Suppose $E(N)$ is the number of items that are defective in nature, produced during each production cycle.

$$E(N) = \int_{t_1}^{T} \delta P(T-t)g(t)dt \tag{11}$$

So, the inventory level is "generated by the following differential equation":

$$\frac{dI_v(t)}{dt} = P - \frac{E(N)}{T-t_1}, \quad t_1 \leq t \leq T, \tag{12}$$

and initial condition is $I_v(t_1) = 0$. Solving the equation with this initial condition, we get

$$I_v(t) = \left(P - \frac{E(N)}{T-t_1}\right)(t-t_1) \tag{13}$$

Now, the cost function for vendor is calculated by using the following components:

- Defective items cost (TCV)

$$= \frac{cE(N)}{T} \tag{14}$$

- Holding cost (THV)

$$= \frac{h_v}{T}\int_{t_1}^{T} I_v(t)dt = \frac{h_v\left(P - \frac{E(N)}{T-t_1}\right)(T-t_1)}{2T} \tag{15}$$

- Setup cost (TSV)

$$= \frac{k_v}{T} \tag{16}$$

The average total cost of the vendor is:

$$TCV(T) = TCV + THV + TSV \tag{17}$$

The total revenue for the vendor is:

$$TRV(T) = \left(\frac{w-c}{T}\right) I_v(T) \tag{18}$$

Hence, the total profit of vendor can be obtained by subtracting equation (17) from (18), we have

$$TPV(T) = TRV(T) - TCV(T) \tag{19}$$

3.3. Integrated supply chain model

Since we are talking about the integrated model then, from the equations (2) and (13), we can calculate the value of t_1 as

$$I_b(0) = I_v(T)$$
$$\Rightarrow \qquad t_1 = T - \frac{2D(p)}{P(2-\delta)f(\tau)}[(1+l_m) - (1+l_m-t)^{f(\tau)}(1+l_m)^{(1-f(\tau))}]$$

The whole profit of the supply chain can be obtained by adding the total profit generated by the vendor and the buyer, given as:

$$TPS(T,\tau,p) = TPB(T,\tau,p) + TPV(T) \tag{20}$$

$$\begin{aligned} &pD(p) - \frac{h_b D(p)}{Tf(\tau)}\left[\frac{T(2+2l_m-T)}{2} + \frac{(1+l_m-T)^2}{2-f(\tau)} - \frac{(1+l_m)^{2-f(\tau)}(1+l_m-T)^{f(\tau)}}{2-f(\tau)}\right] \\ &- \frac{K_d D(p)}{Tf(\tau)}[(1+l_m) - (1+l_m-T)^{f(\tau)}(1+l_m)^{1-f(\tau)}] + K_d D(p) - \frac{k_b}{T} - \tau \\ &+ (w-c)P\left(1 - \frac{t_1}{T}\right) - \frac{wE(N)}{T} - \frac{h_v P}{2}\left(T - 2t_1 + \frac{t_1^2}{T}\right) \\ &+ \frac{h_v E(N)}{2}\left(1 - \frac{t_1}{T}\right) - \frac{k_v}{T} \end{aligned} \tag{21}$$

4. OPTIMALITY

The main aim is to maximize the profit by jointly optimizing the "length of cycle (T)", "investment in preservation technology, (τ)", and "selling price (p)". To prove the optimality of the function, $TPS(T,\tau,p)$, some theoretical results are needed.

Result 1. *"When the selling price p and the preservation technology cost τ are fixed, the profit function $TPS(T,\tau,p)$ is concave with respect to the cycle time T."*

Proof. Refer to "**Appendix BI**".

From the equation BI1, the value of T^* is

$$T^* = \sqrt{\frac{[4k_b + 2Pt_1(2w-2c-w\delta) + h_v P t_1^2(2-\delta) + 4k_v](1+l_m)}{2(1+l_m)h_b D(p) - 2K_d D(p)(f(\tau)-1) + (1+l_m)h_v P(2-\delta)}} \tag{22}$$

□

Result 2. *"When the cycle time T and the selling price p are fixed, the profit function $TPS(T,\tau,p)$ is concave with respect to the preservation technology cost τ."*

Proof. Refer to "**Appendix BII**".

Equation BII1 gives the value of τ^*, which is

$$\tau^* = \frac{1}{y}\sqrt{\frac{TK_d D(p) y}{2(1+l_m)}} - \frac{1}{y} \tag{23}$$

□

Result 3. *"When the cycle time T and the preservation technology cost τ are fixed, the profit function $TPS(T,\tau,p)$ is concave with respect to the selling price p."*

Proof. Refer to "**Appendix BIII**".

From equation BIII1 for the linear demand i.e., $D(p) = a - bp$, value of p is

$$p^* = \frac{hT}{4} + \frac{K_d T(f(\tau)-1)}{4(1+l_m)} \tag{24}$$

□

5. BI-LEVEL MODEL FORMULATION (LEADER-FOLLOWER MODEL)

A realistic approach towards decision making imparts that either the vendor or the buyer is more powerful when it comes to making important decisions. In the present scenario, it is essential to study the model under the leadership of both the players so as to compare which case gives enhanced performance. The two cases are discussed below:

Case 1. ("When vendor is leader and buyer is follower")

In this case, it is assumed that the vendor has more power in decision making and acts as the leader. The model can be formulated as follows:

$$\begin{aligned} &\max TPB(T,\tau,p) = TRB(T,\tau,p) - TCB(T,\tau,p) \\ &\text{subject to: } \max TPV(T) = TRV(T) - TCV(T) \\ &\qquad\qquad \text{and } \begin{cases} T > 0, \\ I_v(T) = I_b(0)t. \end{cases} \end{aligned} \tag{25}$$

The problem given in (25) can be solved by considering firstly the vendor's objective function and optimizing it, and then putting the obtained optimal value in the buyer's objective function. Here, the vendor's problem is solved to obtain the value of T and then, this value of T is substituted in the buyer's objective function so as to achieve the values of τ and p.

The profit function of vendor is given as:

$$TPV(T) = (w-c)P\left(1 - \frac{t_1}{T}\right) - \frac{wE(N)}{T} - \frac{h_v P}{2}\left(T - 2t_1 + \frac{t_1^2}{T}\right)$$

$$+\frac{h_v E(N)}{2}\left(1-\frac{t_1}{T}\right)-\frac{k_v}{T}. \tag{26}$$

The necessary condition for optimality is:

$$\frac{\partial TPV(T)}{\partial T}=0. \tag{27}$$

$$\frac{\partial TPV(T)}{\partial T}=\frac{Pt_1(2w-2c-w\delta)}{2T^2}-\frac{h_v P}{2}\left(1-\frac{t_1^2}{T^2}\right)+\frac{h_v\delta P}{4}$$
$$-\frac{h_v\delta P t_d^2}{4T^2}+\frac{K_v}{T^2}. \tag{28}$$

For sufficiency, second order derivative of the objective function with respect to T is given by $\frac{\partial^2 TPV(T)}{\partial T^2}<0$ (please refer to **Appendix BI**).

The optimal value of T is given as:

$$T^*=\sqrt{\frac{h_v P t_1^2(2-\delta)+4K_v-2Pt_1(2w-2c-\delta w)}{(2-\delta)h_v P}}. \tag{29}$$

By substituting the optimal value of T^*in the buyer's profit function, we get

$$TP_b(T^*,\tau,p)$$
$$=\frac{h_b D(p)}{T^* f(\tau)}\left[\frac{T^*(2+2l_m-T^*)}{2}+\frac{(1+l_m-T^*)^2}{2-f(\tau)}-\frac{(1+l_m)^{2-f(\tau)}(1+l_m-T^*)^{f(\tau)}}{2-f(\tau)}\right]$$
$$+\frac{K_d D(p)}{T^* f(\tau)}[(1+l_m)-(1+l_m-T^*)^{f(\tau)}(1+l_m)^{1-f(\tau)}]-K_d D(p)+\frac{k_b}{T^*}+\tau \tag{30}$$

Now, to establish the optimality of equation (33), the necessary condition that should be satisfied is:

$$\frac{\partial TP_b(T^*,\tau,p)}{\partial\tau}=0,\quad \frac{\partial TP_b(T^*,\tau,p)}{\partial p}=0 \tag{31}$$

Using the approximation of the equations (A.1) and (A.2), we get

$$\frac{\partial TPB(T^*,\tau,p)}{\partial\tau}=-\frac{yK_d D(p)}{2(1+y\tau)^2(1+l_m)}+1=0 \tag{32}$$

i.e.

$$\tau^*=\frac{1}{y}\sqrt{\frac{yK_d D(p)}{2(1+l_m)}}-\frac{1}{y} \tag{33}$$

$$\frac{\partial TPB(T^*,\tau,p)}{\partial p}=\frac{T^* h_b D'(p)}{2}-\frac{K_d D'(p)T^*(f(\tau)-1)}{2(1+l_m)}=0 \tag{34}$$

The above equation gives the value

$$p^*=\frac{hT}{4}+\frac{K_d T(f(\tau)-1)}{4(1+l_m)} \tag{35}$$

For the sufficient condition of optimality, refer to Appendix C.

5.1. Case 2. When buyer is leader and vendor is follower

In this case, it has been presumed that the buyer is more powerful when making decisions, and acts as the leader. The model is formulated as follows:

$$\max TPV(T) = TRV(T) - TCV(T) \tag{36}$$
$$\text{subject to:} \max TPB(T,\tau,p) = TRB(T,\tau,p) - TCB(T,\tau,p)$$
$$\text{and} \begin{cases} T > 0, \\ \tau > 0, \\ p > 0 \\ I_v(T) = I_b(0) \end{cases}$$

Further, the same solution procedure, as in Case 1, can be followed to solve the second case when a buyer is leader and vendor is the follower.

6. NUMERICAL EXAMPLE

The following parameter values should be taken in appropriate units for numerical illustration:
P = 100 units, K_v = \$60, K_b = \$50, c = \$40, w = \$80, $\alpha = 0.05$, h_v = \$1.8, h_b = \$1.6, $b = 0.3$, $a = 120$ units, $y_0 = 0.02$, $\delta = 0.05$, K_v = \$60, K_b = \$60, $l_m = 3$.

(The numerical data are taken from Giri and Maiti [11]).

The integrated model with both the vendor and the buyer sharing equal power demonstrates the following optimal results: Total profit = \$14051.05, length of cycle = 0.622, investment in preservation technology = \$59.133, and the selling price is \$181.953 per unit.

Observations of both the "integrated" and "bi-level models" are listed in the table below:

Approaches	T^*	τ^*	s^*	Vendor's Profit(\$)	Buyer's Profit (\$)	Total profit (\$)
Integrated approach	0.622	059.133	181.953	2527.997	11523.05	14051.05
Bi-level approach (Vendor is leader)	2.497	279.070	203.795	3427.841	11249.75	14677.59
Bi-level approach (Buyer is leader)	0.374	059.259	201.700	2487.288	11679.25	14166.54

Table 2:

The findings of the bi-level model supports vendor's leadership as it generates more profit, compared to both other cases, while comparing all three approaches (Table 2).

7. SENSITIVITY ANALYSIS

l	s	T	u	Total profit
1	182.109	0.519	94.697	13956.45
2	182.011	0.58	72.858	14015.49
3	181.953	0.622	59.133	14051.05
4	181.912	0.653	49.423	14075.54
5	181.882	0.678	42.058	14093.77

b	s	T	u	Total profit
0.1	581.987	0.638	57.623	37986.02
0.2	281.968	0.63	58.387	20018.51
0.3	181.953	0.622	59.133	14051.05
0.4	131.938	0.614	59.862	11083.64
0.5	101.924	0.607	60.572	9316.291

a	s	T	u	Total profit
100	148.717	0.670	54.726	10024.37
110	165.331	0.645	57.014	11954.13
120	181.953	0.622	59.133	14051.05
130	198.580	0.601	61.104	16315.05
140	215.213	0.582	62.945	18746.08

H_b	s	T	u	Total profit
1	181.883	0.659	62.501	14068.26
1.4	181.918	0.64	60.747	14059.53
1.8	181.953	0.622	59.133	14051.05
2.2	181.986	0.606	57.641	14042.8
2.6	182.018	0.591	56.253	14034.77

δ	s	T	u	Total profit
0.01	182.121	0.538	43.73	14013.89
0.015	182.016	0.587	55.819	14034.43
0.02	181.953	0.622	59.133	14051.05
0.025	181.91	0.649	59.673	14064.33
0.03	181.878	0.671	59.151	14075.17

Observations and managerial insights:

- When the expiry date or maximum lifetime (l_m) of the item increases, the total profit increases as the rate of deterioration will be lower thereby, requiring less effort to control it. Further, the selling price decreases and the investment in preservation technology decreases. Thus, products with a longer lifetime do not require preservation strategies. Therefore, if the item has maximum lifetime, it is advisable to invest minimum in the preservation technology.
- An increase in the price-sensitivity parameter (b) implies declining demand, which ultimately decreases the total profit and cycle time. In order to capture demand in such a case, it is suggested to reduce the selling price as it will encourage the customers to purchase the product.
- When the constant scaling parameter (a) increases, the demand increases perpetually, which also increases the total profit and the selling price.
- An increase in the inventory carrying cost (H_b) implies better storage conditions, which ultimately helps in reducing the amount invested in preservation technology, however, due to an increase in the holding cost the total profit will decrease significantly. Thus, storing only the requisite amount in the inventory would be beneficial for avoiding high incurrence in inventory carrying costs.
- With an increase in effectiveness of the preservation technology (δ), the total profit will increase because of reduced deterioration rate, and simultaneously, the investment in preservation technology decreases, too. Hence, the organization should use more effective preservation technology to maximize their profit.

8. CONCLUSION AND FUTURE RESEARCH DIRECTIONS

In this paper, we developed an integrated framework for the main participants of supply chain viz. vendor and buyer. The items in the inventory are deteriorating and at the same time, due to machine shift, imperfect items are produced. The defective items are managed through a defective item cost, and the deteriorating items are acknowledged by investing in the preservation technology. The demand for a product is price-reliant. Two models are proposed so as to have clear functioning of parameters under integrated and bi-level approaches. A numerical analysis, followed by sensitivity analysis, is included to impart features of the developed models. Our model is widely applicable to supply chain firms that deal with the deterioration. It can be extended with fuzzy parameters, or freshness-dependent demand, price and advertisement dependent demand. Another direction of its extension can be the trade-credit policy.

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Appendix A.

To prove optimality the following approximation will be needed

(i) $(1+l_m)-(1+l_m-T)^{f(\tau)}(1+l_m)^{(1-f(\tau))}$

$$= (1+l_m)\left[1-\left(\frac{1+l_m-T}{1+l_m}\right)^{f(\tau)}\right]$$

$$= (1+l_m)\left[1-\left(1-\frac{T}{1+l_m}\right)^{f(\tau)}\right]$$

$$= (1+l_m)\left[1-1+\frac{f(\tau)T}{1+l_m}-\frac{f(\tau)(f(\tau)-1)T^2}{2(1+l_m)^2}+\dots\right]$$

(by neglecting the higher order terms of $\frac{T}{1+l_m}$)

$$= f(\tau)T-\frac{f(\tau)(f(\tau)-1)T^2}{2(1+l_m)} \tag{A.1}$$

(ii) $$\frac{T(2+2l_m-T)}{2}-\frac{(1+l_m-T)^2-(1+l_m-T)^{f(\tau)}(1+l_m)^{(2-f(\tau))}}{2-f(\tau)}$$

$$= T(1+l_m)-\frac{T^2}{2}+\frac{(1+l_m)^2}{2-f(\tau)}\left[\left(\frac{1+l_m-T}{1+l_m}\right)^2-\left(\frac{1+l_m-T}{1+l_m}\right)^{f(\tau)}\right]$$

$$= T(1+l_m)-\frac{T^2}{2}+\frac{(1+l_m)^2}{2-f(\tau)}\left[1-\frac{2T}{1+l_m}+\frac{T^2}{1+l_m}\right]$$

$$= T(1+l_m)-\frac{T^2}{2}+\frac{(1+l_m)^2}{2-f(\tau)}\left[1-\frac{2T}{1+l_m}+\frac{T^2}{1+l_m}-1+\frac{f(\tau)T}{1+l_m}\right.$$
$$\left.-\frac{f(\tau)(f(\tau)-1)T^2}{2(1+l_m)^2}+\dots\right] \tag{A.2}$$

(by neglecting the higher order terms of $\frac{T}{1+l_m}$)

Appendix BI.

The necessary condition for the optimality w.r.t. T is

$$\frac{\partial TPS}{\partial T}=-\frac{D(p)h_b}{2}+\frac{k_b}{T^2}+\frac{K_dD(p)(f(\tau)-1)}{2(1+l)}+\frac{Pt_1(2w-2c-w\delta)}{2T^2}$$
$$-\frac{h_vP}{2}+\frac{h_vPt_1^2}{2T^2}+\frac{h_vP\delta}{4}-\frac{h_vP\delta t_1^2}{4T^2}+\frac{k_v}{T^2}=0 \tag{BI1}$$

and the sufficient condition of optimality is

$$\frac{\partial^2 TPS}{\partial T^2}=-\frac{2k_b}{T^3}-\frac{Pt_1(2w-2c-w\delta)}{T^3}-\frac{h_vPt_1^2}{T^3}(2-\delta)-\frac{2k_v}{T^3}<0 \tag{BI2}$$

Since $(2-\delta)>0$.

Appendix BII.

The necessary condition for the optimality w.r.t. τ is

$$\frac{\partial TPS}{\partial \tau^2}=\frac{yK_dD(p)T}{2(1+l_m)(1+y\tau)^2}-1=0 \tag{BII1}$$

and the sufficient condition of optimality is

$$\frac{\partial^2 TPS}{\partial \tau^2}=-\frac{yK_dD(p)T}{(1+l_m)(1+y\tau)^3}<0 \tag{BII2}$$

Appendix BIII.

The necessary condition for the optimality w.r.t. p is

$$\frac{\partial TPS}{\partial p} = pD'(p) + D(p) - \frac{h_b D'(p)T}{2} + \frac{K_d D'(p)T(f(\tau)-1)}{2(1+l_m)} = 0 \qquad \text{(BIII1)}$$

and the sufficient condition of optimality is

$$\frac{\partial^2 TPS}{\partial p^2} = 2D'(p) < 0, \quad \text{since } D(p) = a - bp, \; D'(p) = -b, \qquad \text{(BIII2)}$$

Appendix C.

$$\frac{\partial^2 TPS}{\partial \tau^2} = -\frac{yK_d D(p)T}{(1+l_m)(1+y\tau)^3} \qquad \text{(C1)}$$

$$\frac{\partial^2 TPS}{\partial p^2} = 2D'(p) \qquad \text{(C2)}$$

$$\frac{\partial^2 TPS}{\partial \tau \partial p} = \frac{yK_d D'(p)T}{2(1+l_m)(1+y\tau)^2} \qquad \text{(C3)}$$

$$\frac{\partial^2 TPS}{\partial p \partial \tau} = \frac{yK_d D'(p)T}{2(1+l_m)(1+y\tau)^2} \qquad \text{(C4)}$$

Here the Hessian matrix $H = \begin{bmatrix} \frac{\partial^2 TPS}{\partial \tau^2} & \frac{\partial^2 TPS}{\partial \tau \partial p} \\ \frac{\partial^2 TPS}{\partial p \partial \tau} & \frac{\partial^2 TPS}{\partial p^2} \end{bmatrix}$ and for sufficient condition for optimality, $\frac{\partial^2 TPS}{\partial \tau^2} < 0$, and $\det H$ should be greater than 0.

$$\begin{aligned}
\det H &= \left(\left(\frac{\partial^2 TPS}{\partial \tau^2} \right) * \left(\frac{\partial^2 TPS}{\partial p^2} \right) \right) - \left(\frac{\partial^2 TPS}{\partial \tau \partial p} \right)^2 \\
&= -\left(\frac{yK_d D(p)T}{(1+l_m)(1+y\tau)^3} * 2D'(p) \right) - \left(\frac{yK_d D'(p)T}{2(1+l_m)(1+y\tau)^2} \right)^2 \\
&\quad \cdot \frac{yK_d bT}{(1+l_m)(1+y\tau)^3} \left[2(a-bp) - \frac{yK_d bT}{4(1+l_m)(1+y\tau)} \right] \\
&= \frac{yK_d bT}{(1+l_m)(1+y\tau)^3} \left[\frac{8(1+x)(1+y\tau)(a-bp) - yK_d bT}{4(1+l_m)(1+y\tau)} \right]
\end{aligned}$$

and since the value of y and b are in between 0 and 1, it will be quite obvious that, $8(1+l_m)(1+y\tau)(a-bp) - yK_d bT > 0$. Hence $\det H > 0$.