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INVENTORY AND PRICING DECISIONS FOR AN IMPERFECT PRODUCTION SYSTEM WITH QUALITY INSPECTION, REWORK, AND CARBON-EMISSIONS

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Abstract: The present paper considers the effect of imperfect quality items on a production system which further undergoes inspection and rework. The demand of the product is price reliant. Two situations to handle the imperfect items are analyzed: selling them at a reduced price and reworking them. The demand is assumed to meet with perfect products in either case. Further, the study incorporates the carbon-emissions borne during production of goods and their holding in the inventory system. The model aims

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at maximizing the profit function by jointly optimizing mark-up price and production quantity. To demonstrate model characteristics, numerical and sensitivity analysis are also presented.

Keywords: Imperfect-production, Rework, Price-sensitive Demand, Quality-inspection, Carbon-emissions.

MSC: 90B05, 90B30, 90B25, 13P25.

1. INTRODUCTION

The core of any business lies around the inventory and pricing decisions. In today's rapid changing world, a balance between these two verdicts should be maintained. The area of imperfect quality is highly explored by various researchers, [30] demonstrated that as the setup costs are reduced, it can prove to be beneficial for overall production system as it improves the quality control, [31] studied the imperfect manufacturing system on the optimal production run time, [2] developed different models taking into account common cycle and time-changing lot size approach. In the same year, [33] extended the basic inventory framework for defectives. An integrated model under the incurrence of imperfect items was given by [17]. [7] presented a production run scenario with imperfect items under the presence of shortages. [37] and [34] developed volume flexible inventory models for imperfect production processes.

Despite the emergence of new methods, procedures, and techniques, the area of imperfect quality items is open for new strategies. Management of imperfect quality items is still a challenging task. In order to keep track of the defectives in a produced lot, it is important to keep checking on the produced goods by employing an efficient screening process. Screening process is a vital step for any business, thus it becomes important to implement it wisely. A pioneer research that contributed to this area was given in [13], where the authors developed an efficient production design in order to cater modern production environments. A model that presents production process as consisting of various stages followed by a possible screening is given in [43]. The model with imperfect production process and screening was further studied in [12] and [14]. [26], further elucidating the topic of imperfect production system for deciding whether and when to apply an inspection on the defectives. [27], [24] proposed strategies to manage defectives either through salvaging or reworking.

The imperfect items are managed through one or more of the following: the items are either vented at a bargained price, the items are disposed of or the items may be reworked. The production processes are well-designed to manufacture goods with least number of defectives. Thus, the imperfect items are not totally scrap but are reworkable. Various models proposing rework of imperfect goods are given by numerous researchers. [29] encouraged the quality issues to be taken into account in the inventory modeling; the authors depict a situation in which the final product will get delivered to the end customer only if the whole lot is certified in terms of quality. A note on the same was given in [3]. A production inventory framework was given in [35] under imperfect manufacturing. Some significant models that dealt with the reworking of defectives under imperfect manufacturing environment was proposed in [5], [4], [6] and [15]. The economic lot size model under the condition of imperfect production, varying stages of production, and defective management through rework was given in [36]. Consideration of imperfect production, shortages, machine-failure, preventive & corrective maintenance along with the reliability parameter in the inventory model was given in [40]. In [39], authors dealt with a production inventory model for discrete and continuous demands along with imperfect manufacturing, defectiveness, and reworking.

The demand of the product is considered constant by many researchers, which is quite unrealistic when compared to the real-market scenarios. Some of the pioneer research that gave inventory models with varying demand patterns are [8], [1], [25], [28], [46], [47]. Later,in [18], [19], and [20], a model for selling-price reliant demand with deteriorating imperfect quality items under credit-financing wes constructed.

The advancements and developments are rapid in today's world, so as the rising environmental issues regarding growing climate risks. The carbon-emissions due to various processes in the business should be addressed responsibly. Numerous environment experts and practitioners suggest companies to adopt green strategies as being beneficial economically and environmentally. To name a few who contributed to this direction, [16], [9], [38], [41], [42], [44], [48], [49], and [50]. Lately, in [10], authors constructed a sustainable and integrated supply chain model with investment in setup cost regarding carbon emissions. Later, in [45], the sustainable inventory modelling for imperfect quality products under deterioration and carbon emissions was explored. Recently, [11], [22], [21], and [23] put forth a sustainable supply chain scenario with features like defect management, two stage credit policy, carbon-emissions, and more. The review of the literature, and also, the research gaps, are depicted in Table 1.

Papers	**Imperfect production**	**Quality inspection**	**Rework**	**Price dependent demand**	**Carbon emission**
[2]	Yes	Yes	No	No	No
[33]	Yes	Yes	No	No	No
[15]	Yes	Yes	Yes	No	No
[7]	Yes	Yes	No	No	No
[5]	Yes	Yes	Yes	No	No
[4]	Yes	Yes	Yes	No	No
[36]	Yes	Yes	No	No	No
[27]	Yes	Yes	Yes	No	No
[20]	Yes	Yes	No	Yes	No
[45]	Yes	Yes	No	No	Yes
[24]	Yes	Yes	Yes	Yes	No
[13]	Yes	Yes	Yes	No	Yes
Present paper	Yes	Yes	Yes	Yes	Yes

Table 1: Literature Survey for a rework inventory system

In present study, an EPQ model is explored under imperfect quality environment that jointly optimizes the production quantity and mark-up price, towards maximizing profit values. The model incorporates the effect of carbon-emissions and price-sensitive demand with two scenarios analyzed. Managing of imperfect quality items by salvaging them at a lower price, in the first scenario. Considering reworking of imperfect items, for the second scenario, it is assumed that all defectives are reworkable and no items are salvaged. The latter presumption holds true in current business world because the production processes are designed to deliver the desired, but if there is any imperfection, its extent is not so tough. For instance, the luxurious and expensive goods are not scrapped but are always reworked as scrapping them would lead to undue and high expenditure. Examples of products generally repaired are: air conditioning units, components of ceiling fan, imperfect alignment of steering wheels, etc. In our second model, all the imperfect goods are considered to be repairable. Shortages are not allowed. The model addresses the following aspects: (i) Demand is satisfied through perfect items only; (ii) demand is price-sensitive; (iii) the effect of carbon-emissions during the production of goods and while carrying them in the inventory is incorporated; (iv) the imperfect items once accumulated are salvaged (first scenario)or they are reworked (second scenario).

2. NOTATIONS

γ Rate of producing items (/ unit time)
γ_1 Rate of reworking imperfect items (/ unit time)
$D(s_p)$ Rate of demand (/ unit time)
c Cost of manufacturing (/ unit)
c_1 Rework cost (/ unit)

β	Rate of producing imperfect items (/ unit time)
β_1	Cost of screening goods during production (/ item)
β_2	Cost of screening per item after completion of producing goods
H	Inventory carrying cost (/unit/unit time)
H_1	Inventory carrying cost $H_1 > H$ of imperfect items that are reworked (/unit/unit time)
A	Setup cost (fixed)
p	Random fraction of imperfect goods, with p.d.f. $f(p)$
$f(p)$	Probability density functions of p
s_p	Mark-up price of perfect goods, $s_p > z$(/unit)
t_1	Production time, $t_1 = Q/\gamma$.
t_2	Time to inspect.
T	Production cycle
z	Discounted vending price of imperfect goods (/ unit)
p_e	Average of carbon emission cost from producing the goods
w_e	Average of carbon emission cost from storing items in the inventory ($/unit/year)
x	Screening rate (/unit/unit time)
Q	Production batch size (/ cycle)

3. ASSUMPTIONS

- Backorders are not allowed.
- The rate of demand $D(s_p)$; is price-sensitive and hence follows the function $D(s_p) = a - bs_p$ where a and b are constants
- Cost of screening is higher during the manufacturing as compared to after manufacturing, i.e. $\beta_1 > \beta_2$.
- Demand and production rate follows: $\gamma > D(s_p)$.
- Demand is satisfied from perfect goods only.
- Inspection rate and demand rate follows: $x > D(s_p)$
- Storage cost of defectives that are reworked is higher than that of perfect goods.
- Carbon emission takes place while producing the goods and while holding them.

4. MATHEMATICAL MODELING

A model is considered in which manufacturing takes place at a rate γ, and demand takes place at rate $D(s_p)$, $\gamma > D(s_p)$, with production rate $\gamma - D(s_p)$. The demand gets satisfied through perfect items only, thus during the production, a screening is performed before selling out to the market. The manufactured lot delivers a portion p of imperfect goods, with a known p.d.f. $f(p)$. Also, after the completion of manufacturing process, the inspection of the left units is performed at the rate x, where $x > D(s_p)$. Two models are discussed that enables dual options for the decision makers to manage the defectives. The first model assumes that the firm does not have the infrastructure for reworking of defective goods, thus, in this case defectives are salvaged at v ($< s$). However, in the second model, it is supposed that the manufacturer satisfies the technical constraints to perform reworking of goods, thus, defectives are managed through an efficient rework process that restores the defectives to their original condition.

4.1. Model 1 (without rework)

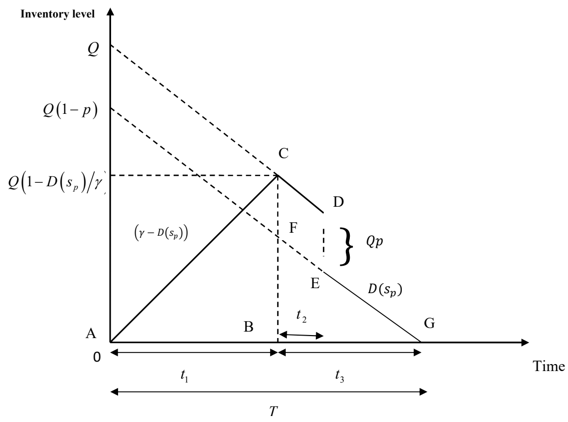

Figure 1: Representation of inventory over time

While the production process, the demand is satisfied only through perfect goods, (see [27]), thus, in $[0, t_1]$ the number of units inspected during

$$t_1 = [D(s_p) + D(s_p)p + D(s_p)p^2 + \cdot]t_1 = \frac{D(s_p)}{1-p}t_1 \tag{1}$$

On completion of the manufacturing process t_1, the number of imperfect goods recognized are the total number of units inspected during $[0, t_1]$, see (1), minus the demand during this interval.

The number of defectives on completion of

$$t_1 = t_1 \left[\frac{D(s_p)}{1-p} - D(s_p)\right] = \frac{pD(s_p)}{1-p}\frac{Q}{\gamma} \tag{2}$$

The uninspected inventory after the manufacturing process ends t_1 equals the maximum inventory level, $Q(1 - D(s_p)/\gamma)$ minus the number of imperfect items recognized on completion oft_1, as in (2). The same is represented in Figure 1.

The unscreened on-hand inventory at

$$t_1 = Q\left(1 - \frac{D(s_p)}{\gamma}\right) - \frac{pD(s_p)}{1-p}\frac{Q}{\gamma} \tag{3}$$

At t_1, uninspected inventory in $[0, t_1]$ is inspected at the rate x. Further, it is verified with ease that the total number of defectives in a cycle, Qp, is the amount of the imperfect goods obtained while $[0, t_1]$, $\frac{pD(s_p)}{1-p}\frac{Q}{\gamma}$, and those obtained while the inspection time t, $p\left[Q\left(1-\frac{D(s_p)}{\gamma}\right)-\frac{pD(s_p)}{1-p}\frac{Q}{\gamma}\right]$.

Two conditions are required to be satisfied:

1. To prevent shortages while manufacturing of goods, the number of perfect goods produced must satisfy the demand during the production, i.e. $N\left(Q,p\right)\geq D(s_p)t_1$, which entails the condition

$$p \leq 1 - D(s_p)/\gamma \tag{4}$$

The uninspected inventory after completion of manufacturing is given in (3), and needs t_2 time units for inspection. Thus, t_2 is given as:

$$t_2 = \frac{Q\left(1-\frac{D(s_p)}{\gamma}\right)-(pD(s_p)(1-p))\left(\frac{Q}{\gamma}\right)}{x} \tag{5}$$

Let t_3 is the time from when manufacturing terminates, i.e. $t_3 = T - t_1$. Then t_3 can be written as

$$t_3 = \frac{Q(1-D(s_p)/\gamma)-Qp}{D(s_p)} \tag{6}$$

2. The bound on the inspection duration is mandatory. Certainly, $t_2 < t_3$ is required,that implies the following condition after some adjustments, to hold true for the screening rate, x

$$x > \frac{D(s_p)(1-D(s_p)/\gamma)-p(D(s_p))^2/(1-p)}{1-\frac{D(s_p)}{\gamma}-p} \tag{7}$$

Let $TR(Q, s_p)$ denotes the total revenue,obtained thorough selling of perfect and defective goods.

- Sales of perfect goods

$$= s_pQ(1-p) \tag{8}$$

- Revenue obtained by salvaging the imperfect goods

$$= zQp \tag{9}$$

- The total revenue is obtained from (8) and (9) as:

$$TR(Q, s_p) = s_pQ(1-p) + zQp \tag{10}$$

Let $TC(Q, s_p)$ be the total cost which comprises of the costs due to setup, production cost with carbon-emissions, screening before and after production, and storage cost.

The cost components are given as:

- Setup cost

$$= A \tag{11}$$

- Production cost

$$= cQ \tag{12}$$

- Cost of inspecting goods during production

$$= \beta_1 \frac{D(s_p)}{(1-p)} \frac{Q}{\gamma} \tag{13}$$

- Cost of inspecting goods after production

$$= \beta_2 Q \left[(1 - D(s_p)/\gamma) - \frac{pD(s_p)}{\gamma(1-p)} \right] \tag{14}$$

- The average inventory can be calculated by summing up the areas under ABC, $CDEF$, and BGF (see Figure 1)

$$= H \left[\frac{1}{2} t_1 Q \left(1 - D(s_p)/\gamma\right) + \frac{1}{2} t_3 Q \left(\frac{D(s_p)}{\gamma} - p \right) + t_2 Q p \right] \tag{15}$$

- Carbon-emissions cost incurred owing to the production and holding of goods in the inventory

$$= p_e Q + w_e \left[\frac{1}{2} t_1 Q \left(1 - D(s_p)/\gamma\right) + \frac{1}{2} t_3 Q \left(\frac{D(s_p)}{\gamma} - p \right) + t_2 Q p \right] \tag{16}$$

Total cycle time is obtained as $T = Q(1-p)/D(s_p)$. Thus, the total cost/cycle, $TC(Q, s_p)$, is given as:

$$\begin{aligned} TC(Q, s_p) = {} & A + (p_e + c)Q + \beta_1 \frac{D(s_p)}{(1-p)} \frac{Q}{\gamma} + \beta_2 Q \left[(1 - D(s_p)/\gamma) - \frac{pD(s_p)}{\gamma(1-p)} \right] \\ & + (H + w_e) \left[\frac{Q^2 \left(1 - D(s_p)/\gamma - p\right)^2}{2D(s_p)} + \frac{Q^2 \left(1 - D(s_p)/\gamma\right)}{2\gamma} \right. \\ & \left. + \frac{Q^2 p \left(1 - D(s_p)/\gamma - \frac{pD(s_p)}{\gamma(1-p)}\right)}{x} \right] \end{aligned} \tag{17}$$

The total profit function is calculated as:

$$TP(Q,s_p) = s_pQ(1-p) + zQp - \left[A + (c+p_e)\,Q + \beta_1 \frac{D(s_p)}{(1-p)}\frac{Q}{\gamma} + \beta_2 Q\left\{(1-D(s_p)/\gamma) - \frac{pD(s_p)}{\gamma(1-p)}\right\} + (H+w_e)\left\{\frac{Q^2\,(1-D(s_p)/\gamma - p)^2}{2D(s_p)} + \frac{Q^2\,(1-D(s_p)/\gamma)}{2\gamma} + \frac{Q^2p\left(1-D(s_p)/\gamma - \frac{pD(s_p)}{\gamma(1-p)}\right)}{x}\right\}\right] \tag{18}$$

The expected total profit per cycle $ETPU(Q,s_p)$ with respect to P:

$$ETP(Q,s_p) = s_pQ(1-E(p)) + zQE(p) - \left[\left\{A + (c+p_e)\,Q + \beta_1 D(s_p)E\left(\frac{1}{(1-p)}\right)\frac{Q}{\gamma} + \beta_2 Q\left\{(1-D(s_p)/\gamma) - \frac{D(s_p)}{\gamma}E\left(\frac{p}{(1-p)}\right)\right\}\right\} + (H+w_e)\left\{\frac{Q^2\,(1-D(s_p)/\gamma - p)^2}{2D(s_p)} + \frac{Q^2\,(1-D(s_p)/\gamma)}{2\gamma} + \frac{Q^2E(p)\left(1-D(s_p)/\gamma - \frac{D(s_p)}{\gamma}E\left(\frac{p}{(1-p)}\right)\right)}{x}\right\}\right] \tag{19}$$

Using the renewal-reward theorem, (see [32]):

$$ETPU(Q,s_p) = \frac{ETP(Q,s_p)}{E\,(T)},$$

where $ETP(Q,s_p) = [Q(1-E(p))]/D(s_p)$. Thus,the expected value of total profit function, $ETPU(Q,s_p)$ is obtained as:

$$ETPU(Q,s_p) = s_p(a-bs_p) + z\frac{(a-bs_p)E(p)}{1-E(p)} - (c+p_e)\,\frac{(a-bs_p)}{1-E(p)} - \beta_1\frac{(a-bs_p)^2}{\gamma(1-E(p))}E\left(\frac{1}{1-p}\right) - \beta_2\frac{(a-bs_p)}{1-E(p)} \cdot \left(1 - \frac{a-bs_p}{\gamma} - \frac{a-bs_p}{\gamma}E\left(\frac{p}{1-p}\right)\right) - A\frac{(a-bs_p)}{Q(1-E(p))} - (H+w_e)\,\frac{Q}{1-E(p)}\left[\frac{E\left\{\left(1-\frac{a-bs_p}{\gamma}-p\right)^2\right\}}{2}\right.$$

$$+\frac{(a-bs_p)\left(1-\frac{a-bs_p}{\gamma}\right)}{2\gamma}$$
$$\left.+\frac{(a-bs_p)\,E(p)\left(1-\frac{a-bs_p}{\gamma}-\frac{a-bs_p}{\gamma}E\left(\frac{p}{1-p}\right)\right)}{x}\right] \tag{20}$$

The necessary conditions of $ETPU(Q, s_p)$ with respect to Q exhibits

$$\frac{\partial ETPU(Q,s_p)}{\partial Q}=0$$

$$Q^*=\sqrt{\frac{A\,(a-bs_p)}{(H+w_e)\left[\begin{array}{l}\frac{E(p^2)}{2}-E(p)-\frac{\left(\frac{a-bs_p}{\gamma}-1\right)(a-bs_p)}{2\gamma}+\frac{E(p)(a-bs_p)}{\gamma}\\+\frac{E(p)(a-bs_p)\left\{\frac{E(p)(a-bs_p)}{\gamma(E(p)-1)}-\frac{a-bs_p}{\gamma}+1\right\}}{x}\end{array}\right]}} \tag{21}$$

$$\frac{\partial^2 ETPU(Q,s_p)}{\partial Q^2}=-\frac{2A\,(a-bs_p)}{Q^3\,(E(p)-1)}<0 \tag{22}$$

and, the necessary condition of $ETPU(Q, s_p)$ with respect to s_p gives the optimal selling price as

$$\frac{\partial ETPU(Q,s_p)}{\partial s_p}=0 \tag{23}$$

$$s_p^*=\frac{1}{b}\left[a-\frac{\left[\begin{array}{l}a(E(p)-1)+b\left\{\beta\beta_2-s_p-vE(p)+\frac{A}{Q(E(p)-1)}\right.\\\left.+\frac{(H+w_e)Q}{\gamma}\left(E(p)+\frac{2\gamma}{x}E(p)-\frac{1}{2}\right)\right\}\end{array}\right]}{\left[\begin{array}{l}(E(p)-1)+\frac{bQ(H+w_e)}{\gamma}\left(\frac{1}{\gamma}+\frac{2E(p)}{x}-\frac{2E(p^2)}{x(E(p)-1)}\right)\\+\frac{2b\beta_2}{\gamma}\left(1-\frac{E(p)}{E(p)-1}\right)+\frac{2b\beta_1}{\gamma(E(p)-1)}\end{array}\right]}\right] \tag{24}$$

Now, for sufficient condition w.r.t. s_p:

$$\frac{\partial^2 ETPU(Q,s_p)}{\partial s_p^2}=-2b-\frac{2b^2\beta_2}{\alpha\,(E(p)-1)}\left(1-\frac{E(p)}{E(p)-1}\right)-\frac{(H+w_e)\,Qb^2}{\alpha}$$
$$\cdot\left\{\frac{1}{\alpha}+\frac{2E(p)}{x}\left(1-\frac{E(p)}{E(p)-1}\right)\right\}<0 \tag{25}$$

4.2. Model 2 (with rework)

In this scenario, the imperfect items get reworked at rate γ_1, with $\gamma_1 < D(s_p)$. The length of time for reworking all the imperfect items is t_3. When the rework process ends, those items are included in the inventory to satiate the demand throughout t_4. Let β be manufacturing rate of imperfect goods.

$$\beta \text{ can be expressed as } \beta = \gamma p \tag{26}$$

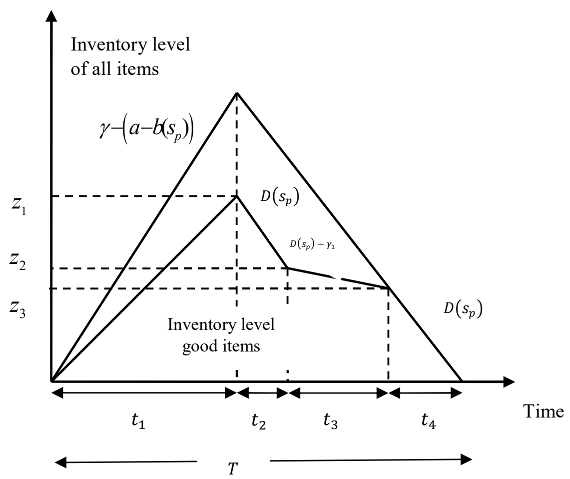

Figure 2: Inventory representation over time when imperfect goods are reworked

The inventory representation of good and imperfect items is given in Figure 2. Here, the inventory tends to rise at $\gamma - D(s_p)$, till the completion of manufacturing process, afterwards, it declines as per demand till the completion of the manufacturing cycle. Figure 2 further depicts the accumulation of perfect goods, which inclines at the rate $\gamma - \beta - D(s_p)$ in $[0, t_1]$. All through $[t_1, t_1 + t_2]$ the inventory of perfect goods is reduced due to demand. During $[t_1 + t_2, t_1 + t_2 + t_3]$, inventory rises through the reworked goods and depletes because of demand, thereby changing the rate to $D(s_p) - \gamma_1$. On completion of t_4, the inventory level of perfect goods declines at the demand rate$D(s_p)$. The inspection time of the uninspected stock at the end of production is found as before, so

$$t_2 = \frac{Q\left(1 - \frac{D(s_p)}{\gamma}\right) - Q\left(pD(s_p)/\left(\gamma(1-p)\right)\right)}{x} \tag{27}$$

In order to prevent backlogs, following condition must be satisfied: $N(Q, t) \geq D(s_p)t_1$, i.e. $p \leq 1 - D(s_p)/\gamma$, which is also given by (4). Further, it is mandatory that the inspection finishes before the completion of the cycle, so $t_2 < t_3 + t_4$. It is to be noted that the remaining cycle $t_3 + t_4$ is also equal to $T - (t_1 + t_2)$, and replacing t_1 and t_2 by their particular expressions, after some adjustments, the following lower bound on x is defined:

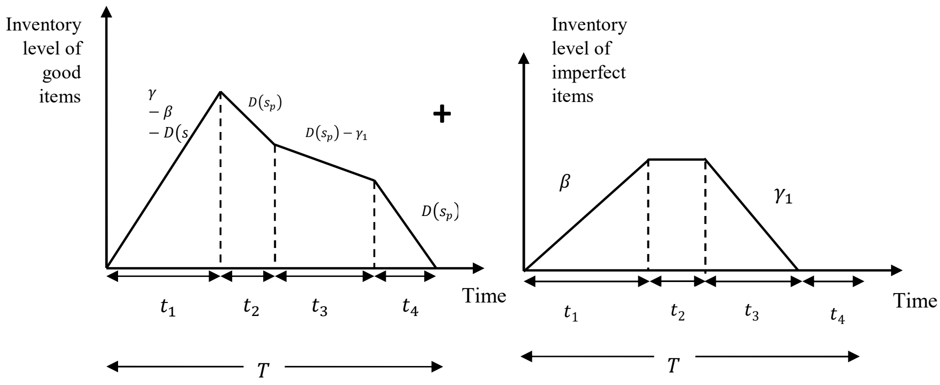

Figure 3: Representation of inventory for good, imperfect and all items

$$x > \frac{\gamma D(s_p)\left[1 - D(s_p)/\gamma - (pD(s_p)/\gamma(1-p))\right]}{\gamma - D(s_p)} \tag{28}$$

At time t_1, the inventory level of perfect goods is z_1, in such a way that

$$t_1 = \frac{Q}{\gamma} = \frac{z_1}{\gamma - \beta - D(s_p)} \tag{29}$$

where,

$$z_1 = Q\left(1 - \frac{D(s_p)}{\gamma} - \frac{\beta}{\gamma}\right) \tag{30}$$

Referring to Figure 3, the reworking time of defectives, t_3, is

$$t_3 = \frac{pQ}{\gamma_1} = \frac{\beta Q}{\gamma\gamma_1} \tag{31}$$

The level of inventory after inspection is z_2, and is given as:

$$\begin{aligned} z_2 &= z_1 - D(s_p)t_2 \\ &= Q\left[\left(1 - \frac{D(s_p)}{\gamma} - \frac{\beta}{\gamma}\right) - \frac{D(s_p)}{x}\left(1 - \frac{D(s_p)}{\gamma} - \frac{pD(s_p)}{\gamma(1-p)}\right)\right] \end{aligned} \tag{32}$$

The second inequity in (22) is determined after substituting z_1 and t_2 with their respective values. Finally, the time which is left after the end of rework process till the end of cycle is calculated as: $t_4 = z_3/D(s_p)$, where z_3 is obtained as

$$z_3 = z_2 - D(s_p)t_3$$

$$= Q\left[\left(1-\frac{D(s_p)}{\gamma}-\frac{\beta}{\gamma}\right)-\frac{D(s_p)}{x}\left(1-\frac{D(s_p)}{\gamma}-\frac{pD(s_p)}{\gamma(1-p)}\right)-D(s_p)\frac{\beta}{\gamma\gamma_1}\right] \tag{33}$$

The total revenue is obtained by selling perfect and reworked goods, which is the sales of total produced goods

- Total revenue $= s_pQ$ (34)

The cost components remain the same as (9), (10), (11), and (12) as in Model 1, except for the rework, inventory carrying and carbon emissions cost associated with production and holding of goods:

- Cost incurred due to the rework of defective goods $= c_1pQ$ (35)

Inventory carrying cost is obtained by summing the inventory carrying of the perfect and imperfect goods (see Figure 3)

- Holding cost $= H\left[\frac{z_1t_1}{2}+\frac{(z_1+z_2)t_2}{2}+\frac{(z_2+z_3)t_3}{2}+\frac{z_3t_4}{2}+\frac{t_1^2\beta}{2}+t_1t_2\beta\right] + H_1\frac{\gamma_1t_3^2}{2}$ (36)

- Carbon-emission cost incurred due to production and holding of goods

$$= p_eQ+w_e\left[\frac{z_1t_1}{2}+\frac{(z_1+z_2)t_2}{2}+\frac{(z_2+z_3)t_3}{2}+\frac{z_3t_4}{2}+\frac{t_1^2\beta}{2}+t_1t_2\beta\right] + w_e\frac{\gamma_1t_3^2}{2} \tag{37}$$

Thus, Model 2 gives the following cost function:

$$\begin{aligned}TC(Q,s_p) = {} & A+(c+p_e)Q+c_1pQ+\beta_1\frac{D(s_p)}{(1-p)}\frac{Q}{\gamma} \\ & +\beta_2Q\left[\left(1-\frac{D(s_p)}{\gamma}\right)-\frac{pD(s_p)}{\gamma(1-p)}\right] \\ & +(H+w_e)\left[\frac{z_1t_1}{2}+\frac{(z_1+z_2)t_2}{2}+\frac{(z_2+z_3)t_3}{2}+\frac{z_3t_4}{2}\right. \\ & \left.+\frac{t_1^2\beta}{2}+t_1t_2\beta\right]+(H_1+w_e)\frac{\gamma_1t_3^2}{2}\end{aligned} \tag{38}$$

The profit function for the cycle can be calculated by subtracting cost components from the revenue earned by selling good items:

$$TP(Q,s_p) = s_pQ-\left[A+(c+p_e)Q+c_1pQ+\beta_1\frac{D(s_p)}{(1-p)}\frac{Q}{\gamma}\right.$$

$$+\beta_2 Q\left\{\left(1-\frac{D(s_p)}{\gamma}\right)-\frac{pD(s_p)}{\gamma(1-p)}\right\}\Bigg]$$
$$-(H+w_e)\left[\frac{z_1t_1}{2}+\frac{(z_1+z_2)t_2}{2}+\frac{(z_2+z_3)t_3}{2}+\frac{z_3t_4}{2}\right.$$
$$\left.+\frac{t_1^2\beta}{2}+t_1t_2\beta\right]-(H_1+w_e)\frac{\gamma_1t_3^2}{2} \tag{39}$$

Applying the renewal-reward theorem, [32]

$$ETPU(Q,s_p)=\frac{ETP(Q,s_p)}{E\,[T]}$$

where $E\,(T)=Q/D(s_p)$.

$$\begin{aligned}
&ETPU(Q,s_p)\\
&\quad=s_pD(s_p)-(c+p_e)D(s_p)p\\
&\quad\;-\beta_1D(s_p)\left[1-\frac{D(s_p)}{\gamma}-\frac{D(s_p)}{\gamma}E\left(\frac{p}{1-p}\right)\right]\\
&\quad\;-(H+w_e)Q\left[\frac{D(s_p)}{2\gamma}\left(1-\frac{D(s_p)}{\gamma}-\frac{\beta}{\gamma}\right)\right.\\
&\quad\;+\frac{D(s_p)}{x}\left(1-\frac{D(s_p)}{\gamma}-\frac{D(s_p)}{\gamma}E\left(\frac{p}{1-p}\right)\right)\\
&\quad\;\cdot\left\{\left(1-\frac{D(s_p)}{\gamma}-\frac{\beta}{\gamma}\right)-D(s_p)\frac{1-\frac{D(s_p)}{\gamma}-\frac{D(s_p)}{\gamma}E\left(\frac{p}{1-p}\right)}{2x}\right\}\\
&\quad\;+\left\{\left(1-\frac{D(s_p)}{\gamma}-\frac{\beta}{\gamma}\right)-\frac{D(s_p)}{x}\left(1-\frac{D(s_p)}{\gamma}-\frac{D(s_p)}{\gamma}E\left(\frac{p}{1-p}\right)\right)\right\}\frac{D(s_p)\beta}{\gamma\gamma_1}\\
&\quad\;+\left\{\left(1-\frac{D(s_p)}{\gamma}-\frac{\beta}{\gamma}\right)-\frac{D(s_p)}{x}\left(1-\frac{D(s_p)}{\gamma}-\frac{D(s_p)}{\gamma}E\left(\frac{p}{1-p}\right)\right)-\frac{D(s_p)\beta}{\gamma\gamma_1}\right\}^2/2\\
&\quad\;\left.+\frac{D(s_p)\beta}{2\gamma^2}+\left(1-\frac{D(s_p)}{\gamma}-\frac{D(s_p)}{\gamma}E\left(\frac{p}{1-p}\right)\right)\frac{D(s_p)\beta}{\gamma x}\right]\\
&\quad\;-(H_1+w_e)Q\frac{D(s_p)\beta^2}{\gamma x2\gamma^2\gamma_1}
\end{aligned}\tag{40}$$

4.3. Concavity of the profit function

Optimality settings of the expected profit function are discussed in this section. In lieu of this, the necessary conditions are:

$$\frac{\partial ETPU(Q,s_p)}{\partial Q}=0\frac{\partial ETPU(Q,s_p)}{\partial s_p}=0$$
$$\frac{\partial ETPU(Q,s_p)}{\partial Q}=0$$

$$Q^* = \sqrt{\frac{A}{\frac{(H_1+w_e)p^2}{2\gamma_1} - (H+w_e)\left[\frac{(p+A_2-1)}{2\gamma} + \frac{A_1\left(p+A_2+\frac{(a-bs_p)A_1}{2x}-1\right)}{x}\right]}} \tag{41}$$

$$A_1 = \frac{E(p)\,(a-bs_p)}{\alpha\,(E(p)-1)} - \frac{(a-bs_p)}{\alpha} + 1$$

$$A_2 = \frac{(a-bs_p)}{\alpha}$$

The sufficient conditions for maximizing the profit function are $D_1(Q, s_p) < 0$, $D_2(Q, s_p) > 0$, the Hessian matrix, H, is estimated as:

$$H = \begin{bmatrix} \frac{\partial^2 ETPU(Q,s_p)}{\partial Q^2} & \frac{\partial^2 ETPU(Q,s_p)}{\partial Q \partial s_p} \\ \frac{\partial^2 ETPU(Q,s_p)}{\partial s_p \partial Q} & \frac{\partial^2 ETPU(Q,s_p)}{\partial s_p^2} \end{bmatrix}$$

and

$$D_1 = \frac{\partial^2 ETPU(Q,s_p)}{\partial Q^2},$$

$$D_2 = \det H = \begin{bmatrix} \frac{\partial^2 ETPU(Q,s_p)}{\partial Q^2} & \frac{\partial^2 ETPU(Q,s_p)}{\partial Q \partial s_p} \\ \frac{\partial^2 ETPU(Q,s_p)}{\partial s_p \partial Q} & \frac{\partial^2 ETPU(Q,s_p)}{\partial s_p^2} \end{bmatrix}$$

where D_1 and D_2 being minors of H.

Because of the extremely non-linear nature of the profit function, the sufficiency conditions cannot be proven mathematically, thereby, graphical method is employed to establish concavity and is represented in Figure 4 using MATHEMATICA 11.

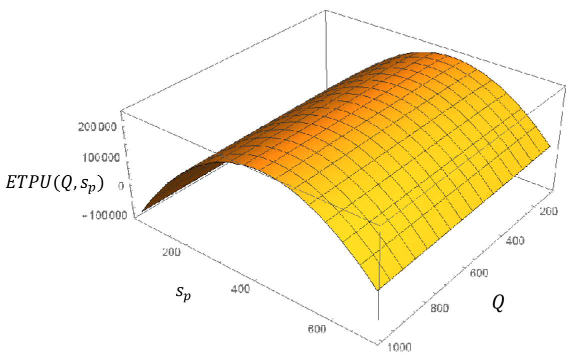

Figure 4: Graphical convexity w.r.t. Q and s_p

5. NUMERICAL ANALYSIS

α	1600 units/year	β_1	\$0.5	a	1800	H_1	\$22/unit/year
z	80\$/unit	β_2	\$0.6	b	2.3	γ_1	100units/year
c	\$104	A	\$1500	p_e	6\$/unit	c_1	\$8/ unit
x	175200 unit	H	\$20/unit/year	w_e	4\$/unit	γ	\$80/ unit time

Table 2: Numerical information

Table 2 gives values of parameters that were used to solve the numerical examples. Fraction of imperfect items is uniformly distributed over $[0, 0.1]$ with p.d.f. as:

$$f(p) = \begin{cases} 10 & \text{for } 0 \le p \le 0.1 \\ 0 & \text{otherwise} \end{cases} \tag{42}$$

Utilizing (28), $E(p) = 0.05$, $E\left(\frac{1}{1-p}\right) = 1.0536$, $E\left(\frac{p}{1-p}\right) = 0.053$ The following results are obtained for the first and second model, respectively.

Models	**Selling price** (s_p)	**Order quantity** (Q)	**Demand** $D(s_p)$	**Profit** $ETP(Q, s_p)$
Model-1	450.32	282.06	764.27	249889
Model-2	448.68	387.39	768.04	253156

Table 3: Comparison of results obtained in Model 1 and Model 2

The models are developed for the imperfect manufacturing process with quality screening and carbon-emissions under price-sensitive demand along with the inclusion and exclusion of rework process. From Table 3, it can be seen that the model with reworking of imperfect items is preferable over the one without the inclusion of rework process.

6. SENSITIVITY ANALYSIS

This section shows the validity and robustness of the developed models. Table 4 gives the sensitivity for the first model, where salvaging of the accumulated defectives is carried out, and in Table 5 sensitivity is presented for the second model in which imperfect items are managed by reworking. This section further presents the observations and important insights for the decision administrators.

Parameter	**Changes parameter**	**Selling price** (s_p)	**Order quantity** (Q)	**Demand** $D(s_p)$	**Profit** $ETP(Q, s_p)$
	30	450.88	236.79	762.97	248261.7
	25	450.61	256.49	763.59	249039.8
$\boldsymbol{H}$	**20**	**450.32**	**282.06**	**764.27**	**249888.8**
	15	449.99	317.16	765.03	250832.3
	10	449.61	369.67	765.89	251911.3
	0.075	450.86	284.95	763.02	249089.6
	0.0625	450.59	283.49	763.65	249494.2
$\boldsymbol{E(p)}$	**0.05**	**450.32**	**282.06**	**764.27**	**249888.8**
	0.0375	450.05	280.65	764.87	250273.6
	0.025	449.8	279.26	765.46	250649.1
	0.75	450.45	282.01	763.97	249787.6
-	0.625	450.38	282.03	762.12	249838.2
$\boldsymbol{\beta_1}$	**0.5**	**450.32**	**282.06**	**764.27**	**249888.8**
	0.375	450.25	282.08	764.43	249939.4
	0.25	450.18	282.11	764.58	249990
	0.9	450.32	282.06	764.27	249768.9
	0.75	450.32	282.06	764.27	249828.8
$\boldsymbol{\beta_2}$	**0.6**	**450.32**	**282.06**	**764.27**	**249888.8**
	0.45	450.32	282.06	764.27	249948.7
	0.3	450.32	282.06	764.27	250008.6
	6	450.44	270.95	763.99	249539.4
	5	450.38	276.33	764.13	249712.3
$\boldsymbol{w_e}$	**4**	**450.32**	**282.06**	**764.27**	**249888.8**
	3	450.25	288.15	764.42	250068.9
	2	450.19	294.65	764.56	250253.1
	9	451.91	281.42	760.61	247481
	7.5	451.11	281.74	762.44	248683.5
$\boldsymbol{p_e}$	**6**	**450.32**	**282.06**	**764.27**	**249888.8**
	4.5	449.52	282.38	766.1	251096.9
	3	448.73	282.7	767.93	252308

Table 4: Sensitivity analysis for the first model

Parameter	**Changes parameter**	**Selling price** (s_p)	**Order quantity** (Q)	**Demand** $D(s_p)$	**Profit** $ETP(Q, s_p)$
	30	448.95	326.89	767.42	252059
	25	448.82	353.33	767.72	252584.5
$\boldsymbol{H}$	**20**	**448.68**	**387.39**	**768.04**	**253156.3**
	15	448.53	433.67	768.38	253789.5
	10	448.36	501.71	768.78	254509.2
	0.075	450.1	340.13	764.77	252343.5
	0.0625	449.36	364.07	766.47	252782
$\boldsymbol{E(p)}$	**0.05**	**448.68**	**387.39**	**768.04**	**253156.3**
	0.0375	448.08	408.17	769.41	253452.5
	0.025	447.62	423.91	770.48	253656.5
	0.75	448.81	387.31	767.74	253059.3
	0.625	448.74	387.35	767.89	253107.8
$\boldsymbol{\beta_1}$	**0.5**	**448.68**	**387.39**	**768.04**	**253156.3**
	0.375	448.62	387.44	768.18	253204.9
	0.25	448.55	387.48	768.33	253253.5
	0.9	448.75	387.35	767.88	253053.4
	0.75	448.71	387.38	767.96	253104.9
$\boldsymbol{\beta_2}$	**0.6**	**448.68**	**387.39**	**768.04**	**253156.3**
	0.45	448.65	387.42	768.11	253207.8
	0.3	448.61	387.45	768.61	253259.2
	6	448.74	372.19	767.89	252914.1
	5	448.71	379.57	767.96	253034
$\boldsymbol{w_e}$	**4**	**448.68**	**387.39**	**768.04**	**253156.3**
	3	448.65	395.73	768.11	253281.2
	2	448.61	404.63	768.19	253408.9
	9	450.18	386.37	764.58	250857.4
	7.5	449.43	386.89	766.31	252005.6
$\boldsymbol{p_e}$	**6**	**448.68**	**387.39**	**768.04**	**253156.3**
	4.5	447.93	387.91	769.76	254309.7
	3	447.18	388.42	771.99	255465.6

Table 5: Sensitivity analysis for the second model

7. OBSERVATIONS AND INSIGHTS

From Tables 4 and 5, following managerial insights are provided:

- By increasing the holding cost, selling price increases, production quantity and demand decreases, thereby, decreasing the total profit. The decision maker may adopt backordering policies or ordering only the requisite amount to cut down on unnecessary stocking charges.
- Next, as the fraction of imperfect items increases, mark up price increases and the production quantity increases so as to compensate the loss due to

scrapped items and the total profit decreases significantly.

- However, in the second model, as the quantity of imperfect items increases, selling price increases but the production quantity is not increased because the imperfect items are reworked and ultimately, all the produced units fulfill the demand. Further, when the screening cost before and after production increases, the total profit tends to decrease in both the models.
- Moreover, as the emission cost rises owing to the production and storage of items, the production quantity and total profit decreases. Higher emission cost suggest the production of only requisite quantities to cut down on escalating emission cost.

8. CONCLUSION

The proposed study focuses on an imperfect production framework where defectives are produced with known probability. The study accommodates the decision makers with two different aspects to handle defectives, depending upon whether or not the manufacturers have the substructure to carry out the rework process. In lieu of this, two models are proposed, in which the first model suggests strategies to manage defectives by salvaging them below the mark up price, which is applicable to the case when the manufacturers do not hold capacity to perform rework of defectives. However, when the manufacturers satisfy the technical constraints to execute the rework process, the defectives are reworked at a constant rate to as-good-as-new state, as in the second model. Thus, out of the two models, the suitable one can be chosen by the decision makers. Further, the demand of the product is considered to be price-reliant. And, due to increasing concern towards the environment, the study considers carbon-emissions when the production process is on-going and while stocking the items. The objective lies in jointly optimizing the production size and the selling price to optimize the profit function. Numerical as well as sensitivity analysis are presented, for showcasing the detailed analysis and managerial implications of the proposed models. Results support enhanced performance of the second model in comparison to the first one as in the second model, the imperfect units do not get salvaged, instead these get reworked and vended at the original amount. The present framework holds applicability to various production firms viz. electronics, textiles, etc.

9. DIRECTIONS FOR FUTURE RESEARCH

Our model has many possibilities for extensions. It can be made more pragmatic by incorporating the effect of inflation and shortages. Disruption during production could be a worthwhile contribution in this line. The model can also be studied under various trade-credit policies.

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