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IMPERFECT INVENTORY MODEL FOR TRENDED DEMAND UNDER RADIO FREQUENCY IDENTIFICATION AND TRADE CREDIT

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Abstract: In this paper, models on concepts of radio frequency identification adoption (RFID), imperfect products reworking system, and trade credit for deteriorating inventory with / without utilizing the preservation investment technology are derived, which makes this article, a unique one. Estimation of optimal values of RIFD levels of investment for ordering, operating, just in time efficiencies along with production cycle time, and preservation investment are carried out. The rate of market demand is quadratic in nature based on time and is suitable for the items for which demand rises primarily, and then after it begins to decline. This form of demand is applicable to a vast range of items like garments, fashion accessories, electronics, etc.. The model is further divided into two cases based on demand rate and products reworking of imperfect quality items. Further, in each case, four subcases based on credit period and time of production cycle are analysed. The main objective of the inventory problem is to calculate total manufacturing cost in each subcase. The classical optimization technique is utilized for calculating the optimal values of decision variables. For the validation of developed models in each case, numerical examples are demonstrated, then using the concept of eigen-values of a Hessian matrix, we have proved the convex nature of the systems total cost for the case which has the minimum total cost. Also the decision variable sensitivity analysis is done by altering the inventory parameters for generating fruitful managerial insights. The model derived in this article can be applied in supply chain management of packaged food products/seasonal food products/milk products like butter, cheese, etc., where the tags for RFID are applied to track eatable/milk items of during delivery and storing.

Also, if the model deals with a product of improper production, then it undergoes the reworking process.

Keywords: Deterioration, Trade Credit, Radio Frequency Identification Adoption, Preservation Investment, Time Dependent Demand Rate, Reworking System for Imperfect Production.

MSC: 90B85, 90C26.

1. INTRODUCTION

An economic production quantity (EPQ) model is commonly utilized for solving a problem related to inventory. Also, we know that in a practical scenario, it is not always possible to produce a perfect item. In order to deal with these situations, where imperfect items are produced, various research work has been conducted demonstrating the effect of an imperfect production process on EPQ model.

Initially, on adopting the concept of products imperfect quality in EPQ/ EOQ formulae, Salameh and Jaber [31] stretched the usual EPQ/EOQ inventory models. Then, many contributions dealing with rework process are carried out by; Hayek and Salameh [15], Chan et al. [5], Jamal et al. [19], Konstantaras et al.[22], Yoo et al. [47], Wahab and Jaber[46], Tsao et al. [43], Konstantaras et al. [23], Sinha [38], Jaber et al. [18], Zhou [49]. Most of the research work includes an assumption of fixed ordering cost and production cost, but it could be on considering the merits of radio frequency identification (RFID) technology. Utilizing of RFID technology, the efficiency increases, labor costs declines, inventory information accuracy inproves, and manufacturing processes simplifies. Therefore, ordering cost and production costs are reduced with RFID. There is an impact of RIFD on operations management, which was analyzed by various researchers, Ustundag and Tanyas [45], Shin and Eksioglu [35], Leung et al. [25] and Szmerekovsky and Zhang [39], Szmerekovsky et al. [40], Lee and Lee [24] , Zhang et al. [48], Choy et al. [7], Cui et al. [10], Tsao et al. [43], Tao et al. [41], Kohli and Peng [21]. The demand rate can be supposed as a function fluctuating based on time, level of stock, and price linked with selling of items or together.

Min and Zhou [28], Silver et al. [36] Afterwards various research scholars like; Chung et al. [8, 9], Bose et al. [4], Hariga [14], Silver [37], Shah et al. [33] and Shah et al. [34] assumed the nature of demand rate as fluctuating in forms of linear, quadratic exponential etc. However, considering the practical scenario, to uplift the ordered quantity, supplier grants a trade credit to the manufacturer. Firstly, Haley and Higgins [13] introduced a model with the allowable delay in payments. Then, further studies considering this concept were carried-out by Kingsman [20], Goyal [12], Aggarwal and Jaggi [2] modified Goyals [12] model, Mahata and Goswami [27], Mishra et al. [29], Teng et al. [42], Shah and Shah [32], Lin et al. [26], Arcelus et al. [3], Abad and Jaggi [1], Chang [6], etc.

To reduce the effect of deterioration, the preservation technology investment is utilized by various researchers, Hsu et al. [17], Dye and Yang [11], Pal et al. [30], He and Huang [16].

2. NOTATIONS AND ASSUMPTIONS

2.1. Notations

Parameters

P_R	Rate of reworking of imperfect quality items in units/year(in dollars)
P	Rate of production in units/year
$PIEE$	Rate of production for imperfect quality items in units/year
C	Cost of production/item (in dollars)
$Dt(t)$	Demand rate at time t in units/year
a	Scale demand, where $a > 0$
b	Linear variation of demand with respect to time, where $0 < b \leqslant 1$
c	Quadratic variation of demand, where $0 < c \leqslant 1$
k	Imperfect quality products produced percentage
OrE	Efficiency associated with ordering
Cor	Level of investment for efficiency associated with ordering
J	Efficiency associated with JIT
Cj	Level of investment for efficiency associated with JIT
OpE	Efficiency associated with operating
Cop	Level of investment for efficiency associated with operating
T	Cycle Time (in years)
Q	Total number of products produced throughout a round(in units)
I_R	Level of inventory when reworking of imperfect quality done (in units)
I_o	Level of inventory as soon as original production is accomplished (in units)
C_r	Cost of repairing per item of imperfect quality (in dollars)
C_s	Setup cost per item for each production round (in dollars)
h	Annual holding cost of imperfect items/item (in dollars)
h_R	Annual holding cost for imperfect products undergoes reworking(in dollars)
P_p	Selling price associated with perfect quality products (in dollars)
Θ_o	Deterioration co-efficient
A_p	Cost associated with material purchasing/item (in dollars)
I_e	Earned rate of interest per dollar/year (in dollars)
I_c	Charged rate of interest rate per dollar/year (in dollars)
I_m	Rate of interest charged accumulated for items in stock(in dollars)
ξ	Co-efficient of preservation investment
α	Mark up for efficiency associated with ordering
β	Mark up for efficiency associated with JIT
γ	Mark up for efficiency associated with operating
M	Credit period offered by supplier to manufacturer(in years)

2.2. Assumptions

1. Shortages are not allowed.

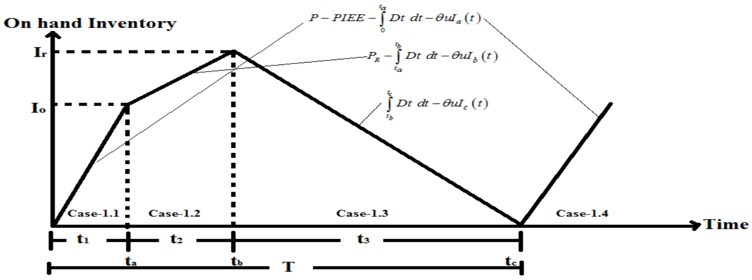

Figure 1: Perfect Inventory Level

2. The rate of market demand is represented by a function of time $Dt(t) = a(1 + bt - ct^2)$.
3. The imperfect quality products percentage is a known constant.
4. The rework for all imperfect product can be done with a repair cost.
5. The rate of rework for imperfect product is a predefined constant. The items undergoing repairing process are similar to the original items.
6. The manufacturer has offered a credit period$M$ by the supplier. The wholesale price per unit of the items traded throughout the credit period is deposited in an account with interest rate I_e . With the completion of this period, the credit is paid and manufacturers takes the payment of charged interest at rate I_m for the products in stock.
7. Let $\Theta_u = \Theta_o, 0 \leqslant \Theta_o \leqslant 1$ be the deterioration co-efficient, in the situation where there is no utilization of preservation technology and let $\Theta_u = \Theta_o \exp^{-\xi u}$ is the co-efficient of deterioration , in case when there is an utilization of preservation technology.

3. MATHEMATICAL FORMULATION OF THE MODEL

On the basis of the rework and rate of market demand relationship, splitting model as in Case 1, where $P_R > \int_0^T Dtdt$ and Case 2, where $P_R < \int_0^T Dtdt$
Case 1 $P_R > \int_0^T Dtdt$ without preservation
Figure 1 and Figure 2 respectively, demonstrate the perfect and imperfect levels of inventories in case 1.

The rate of production of imperfect item is demonstrated PIEE =kP (1)

Also, the rate of production of perfect quality products is always higher than or equal to the addition of the market demand rate and defective products rate of production,

$$\text{P-PIEE-}\int_0^T Dtdt \geqslant 0 \Rightarrow 0 \leqslant k \leqslant (1 - \int_0^T (Dtdt)/P) \quad (2)$$

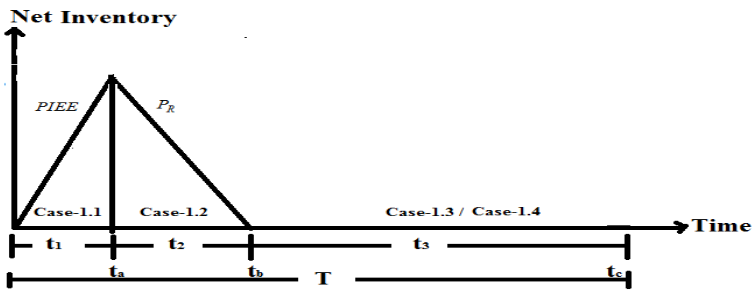

Figure 2: Imperfect Inventory Level

$$T = t_1 + t_2 + t_3 \qquad and \qquad T = Q/(\int_0^T Dtdt) \tag{3}$$

The production time is $t_1 = I_o/(P - PIEE - \int_0^T Dtdt)$ (4)

Initial Inventory level is $I_o = (P - PIEE - \int_0^T Dtdt)(Q/P)$ (5)

The rework time is $t_2 = PIEE\ (Q/(PP_R)) = Q_k/P_R$ (6)

The highest level of inventory level is given by

$$I_R = (1 - (\int_0^T Dtdt(PIEE + P_R))/(PP_R))Q \tag{7}$$

Thus, $t_3 = ((I_R)/(\int_0^T Dtdt)) = Q((1/\int_0^T Dtdt) - (PIEE + P_R)/PP_R)$ (8)

Therefore,$t_a = t_1 = Q/P$ (9)

$t_b = t_1 + t_2 = Q/P + Qk/P_R = Q/P + Qk/P_R$ (10)

$$t_c = t_1 + t_2 + t_3 = Q/P + Qk/P_R + I_R/(1/\int_0^T Dtdt)$$

$$t_c = (Q/P + Qk/P_R + Q/\int_0^T Dtdt - QPIEE + QP_R/PP_R \tag{11}$$

Below stated differential equations demonstrate inventory level of perfect items

$$dI_a/dt = P - PIEE - \int_0^{t_a} Dtdt - \theta_u I_a, \qquad 0 \leqslant t \leqslant t_a \tag{12}$$

$$dI_b/dt = P_R - \int_{t_a}^{t_b} Dtdt - \theta_u I_b, \qquad t_a \leqslant t \leqslant t_b \tag{13}$$

$$dI_c/dt = -\int_{t_b}^{t_c} Dtdt - \theta_u I_c, \qquad t_b \leqslant t \leqslant t_c \tag{14}$$

Utilizing boundary conditions,$I_a(0) = 0; I_a(t_a) = I_b(t_a); I_b(t_b) = I_{max} = I_c(t_b)$;
$I_c(t_c) = 0$; for solving differential equations demonstrated in equations (A1) to

(A3) without preservation, and equations (A10) to (A12) in appendix with preservation. By using appendix equations (A2), (A3) and,$I_b(t_b) = I_{max} = I_c(t_b)$ obtaining highest inventory level I_{max} given by equation (A4) in appendix. The below stated differential equations are level of inventory of imperfect items

$$\mathrm{dI}_d/dt = PIEE, \qquad 0 \leqslant t \leqslant t_a \tag{15}$$

$$\mathrm{dI}_e/dt = P_R, \qquad t_a \leqslant t \leqslant t_b \tag{16}$$

Utilizing the conditions:$I_d(0) = 0, I_e(t_c) = 0$, after solving we get,

$$\mathrm{I}_d(t) = tPIEE, \qquad 0 \leqslant t \leqslant t_a \tag{17}$$

To have positive inventory with no shortages then,

$$\mathrm{I}_e(t) = P_R(t - t_b), \qquad t_a \leqslant t \leqslant t_b \tag{18}$$

The below stated components plays a major role in computing total cost of the system:

$$\text{Prooduction cost per year,} PC = CQ \tag{19}$$

$$\text{Repair cost per year,} RC = C_r Qk \tag{20}$$

$$\text{Setup cost per year, } SC = C_s \tag{21}$$

$$\text{Holding cost,} HC = h(\int_0^{t_a} I_a dt + \int_{t_a}^{t_b} I_b dt + \int_{t_b}^{t_c} I_c dt) + h_r(\int_0^{t_a} I_d dt + \int_{t_a}^{t_b} I_e dt) \tag{22}$$

3.1. RIFD Investment cost

RFID improves the efficiency of a manufacturer including the following efficiencies described as in Lee and Lee (2010), derived as stated below

Ordering efficiency

$$\text{OrE=N1+(G1-N1)(e}^{(\alpha Cor)}), \qquad 0 \leqslant N1 \leqslant G1 \leqslant 1 \tag{23}$$

$G1$ is lowermost efficiency and $N1$ is uppermost efficiency associated with Cor.

JIT efficiency

$$\text{JiT=L1+(U1-L1)(e}^{(\beta C_j)}), \qquad 0 \leqslant L1 \leqslant U1 \leqslant 1 \tag{24}$$

$U1$ is lowermost efficiency and $L1$ is uppermost efficiency associated with C_j.

Operating efficiency

$$\text{OpE=E1+(A1-E1)}(\text{e}^{(\gamma Cop)}), \qquad 0 \leqslant E1 \leqslant A1 \leqslant 1 \tag{25}$$

$A1$ is lowermost efficiency and $E1$ is uppermost efficiency associated with Cop. Bifurcating case 1 on credit period and replenishment cycle length into the subcases.
Subcase 1.1 $0 \leqslant L1 \leqslant M \leqslant t_a$
The rate of interest charged per year is

$$\begin{array}{c} TI_{p1}=A_pI_c(\int_M^{t_a} t(P-PIEE-Dt)dt+ \\ \int_{t_a}^{t_b}(Dt(t_c-t_b)-(t_b-t)(P_R-Dt))dt \\ +\int_{t_b}^{t_c} Dt(t_c-t)dt+\int_M^{t_a} \end{array} (PIEEt)dt + \int_{t_a}^{t_b} (t_b - t)P_R dt \tag{26}$$

The rate of interest earned per year is

$$\text{TI}_{e1} = P_pI_e\int_0^M tDtdt \tag{27}$$

Subcase 1.2 $t_a \leqslant M \leqslant t_b$
The rate of interest charged per year is

$$\text{TI}_{e2} = A_pI_c\int_M^{t_b}(Dt(t_c - t_b) - (t_b - t)(P_R - Dt))dt + \int_{t_b}^{t_c} Dt(t_c - t)dt + \int_M^{t_b}(t_b - t)P_R dt \tag{28}$$

The rate of interest earned per year is

$$\text{TI}_{e2} = P_pI_e\int_0^M tDtdt \tag{29}$$

Subcase 1.3 $t_b \leqslant M \leqslant t_c$
The rate of interest charged per year is

$$\text{TI}_{p3} = A_pI_c\int_M^{t_c}(Dt(t_c - t))dt \tag{30}$$

The rate of interest earned per year is

$$\text{TI}_{e3} = P_pI_e\int_0^M tDtdt \tag{31}$$

Subcase 1.4 $T < M$
The rate of interest charged per year is

$$\text{TI}_{p4} = 0 \tag{32}$$

The rate of interest earned per year is

$$\mathrm{TI}_{e4} = P_p I_e \int_0^M tDtdt + P_p I_e \int_T^{M-T} tDtdt \qquad (33)$$

Thus, annual total cost per unit time is given by equations (A5) and (A14)

Case 2 $P_R > \int_0^T Dtdt$ with preservation Let $\Theta_u = \Theta_o \exp^{-\xi u}, 0 \leqslant \Theta_o \leqslant 1$ is the co-efficient of deterioration, in case if preservation technology is utilized. The inventory level $I(t)$ at any time, t, could be calculated by equations (12) to (16) as stated in case 1. With respect to each cost in case-1, a preservation-technology investment cost $PTI = ut$ is involved for calculating the total cost for the case, presented in appendix equation (A13).

Case 3 $\int_0^T Dtdt > P_R$ without preservation Similarly, as in Case 1, the total cost can be stated using Cases 1.1 to 1.4.

Case 4 $\int_0^T Dtdt > P_R$ with preservation With respect to each cost in Case-1, the cost associated with preservation-technology investment $PTI = ut$ is inserted for calculating the total cost of the same case. So, the total cost per unit time in each case as demonstrated in equations (A6) to (A9) in Appendix. Therefore, to minimize the total cost shown in each case, Calculating the below stated partial derivatives and hence, equating them to zero;

$$\frac{\partial TC}{\partial T} = 0; \qquad (34a)$$

$$\frac{\partial TC}{\partial T} = 0 \quad and \quad \frac{\partial TC}{\partial u} = 0 \qquad (34b)$$

Only in case of preservation investment technology

In order to test convexity of total cost of obtained set of solutions, we implement following algorithm, Step 1 Allotting the various inventory parameters some specific hypothetical values. Step 2 Calculating the solutions by solving simultaneous equations described in Equation (34a) or (34b), utilizing the mathematical software Maple 18. Step 3 Calculating eigen values of following Hessian matrix H at the point of optimality, which is obtained from Equation (34a) or (34b), $H = \begin{bmatrix} \frac{\partial^2 TC}{\partial T^2} & \frac{\partial^2 TC}{\partial T \partial u} \\ \frac{\partial^2 TC}{\partial u \partial T} & \frac{\partial^2 TC}{\partial u^2} \end{bmatrix}$

- In case, if each and every eigen value of matrix H is positive, it is a positive-definite matrix. Then, the total cost is a convex down and stop.

4. NUMERICAL EXAMPLES AND SENSITIVITY ANALYSIS

4.1. Numerical Examples

Example 1: Case 3 $\int_0^T Dtdt > P_R$ without preservation

Considering the specified values:

$a = 10000, b = 0.2, c = 0.5, \theta_o = 0.5, h = 20, h_R = 30, P = 20000, C = 20, C_s = 5000, C_r = 8, k = 0.05, PIEE = 80, I_c = 0.15, M = 0.1, I_e = 0.2, A_p = 80, P_p = 80, G1 = 1, N1 = 0.3, U1 = 1, L1 = 0.3, A1 = 1, E1 = 0.5, \alpha = 0.001, \beta = 0.2, \gamma = 0.2, P_R = 1000$

$Solution:$

$T = 0.11231, Cor = 714.2857, Cop = 4.9807, Cj = 4.9696, Q = 127, Totalcost = 48278.80005, \int_0^T Dtdt = 1133.4013, Therefore, P_R = 1000 < \int_0^T Dtdt = 1133.4013.$

Example 2: Case 4 $P_R < \int_0^T Dtdt$ with preservation
Considering the specified values:
$a = 10000, b = 0.2, c = 0.5, \theta_o = 0.5, h = 20, h_R = 30, P = 20000, C = 20, C_s = 5000, C_r = 8, k = 0.05, PIEE = 80, I_c = 0.15, M = 0.1, I_e = 0.2, A_p = 80, P_p = 80, G1 = 1, N1 = 0.3, U1 = 1, L1 = 0.3, A1 = 1, E1 = 0.5, \alpha = 0.001, \beta = 0.2, \gamma = 0.2, P_R = 1000, \xi = 0.3$

$Solution:$

$T = 0.1123, u = 2.1981, Cor = 714.2857, Cop = 4.9807, Cj = 4.9988, Q = 127,$
$Totalcost = 48276.9109, \int_0^T Dtdt = 1133.6839, Therefore, P_R = 1000 < \int_0^T Dtdt = 1133.6839.$

Example 3: Case 1 $P_R > \int_0^T Dtdt$ without preservation
Considering the specified values:
$a = 10000, b = 0.2, c = 0.5, \theta_o = 0.5, h = 20, h_R = 30, P = 20000, C = 20, C_s = 5000, C_r = 8, k = 0.05, PIEE = 80, I_c = 0.15, M = 0.1, I_e = 0.2, A_p = 80, P_p = 80, G1 = 1, N1 = 0.3, U1 = 1, L1 = 0.3, A1 = 1, E1 = 0.5, \alpha = 0.001, \beta = 0.2, \gamma = 0.2, P_R = 1500$

$Solution:$

$T = 0.11231, Cor = 714.2857, Cop = 4.9807, Cj = 4.9682, Q = 127, Totalcost = 48295.9764, \int_0^T Dtdt = 1132.1932, Therefore, P_R = 1500 > \int_0^T Dtdt = 1132.1932.$

Example 4: Case 2 $P_R > \int_0^T Dtdt$ with preservation
Considering the specified values:
$a = 10000, b = 0.2, c = 0.5, \theta_o = 0.5, h = 20, h_R = 30, P = 20000, C = 20, C_s = 5000, C_r = 8, k = 0.05, PIEE = 80, I_c = 0.15, M = 0.1, I_e = 0.2, A_p = 80, P_p = 80, G1 = 1, N1 = 0.3, U1 = 1, L1 = 0.3, A1 = 1, E1 = 0.5, \alpha = 0.001, \beta = 0.2, \gamma = 0.2, P_R = 1500, \xi = 0.3$

$Solution:$

$T = 0.1122, u = 2.2824, Cor = 714.2857, Cop = 4.9807, Cj = 5.0005, Q = 127,$
$Totalcost = 48293.75516, \int_0^T Dtdt = 1132.5106, Therefore, P_R = 1500 > \int_0^T Dtdt = 1132.5106.$

4.2. Convexity of Total Cost function:

It can be observed from the numerical examples that the average total cost is minimum in case 4 with preservation technology. Therefore, by utilizing algorithm, we check convexity of the total cost, as shown in Figure 4, we computing the optimum solution and did going the sensitivity analysis of the decision variables by altering the inventory parameters−20 percentage to 20 percentage for this case only. Figure 3 represents the graph of total cost verses the length of replenishment cycle in case 3 without preservation. Hessian matrix in case 4 with preservation is

$$\begin{bmatrix} \frac{\partial^2 TC}{\partial T^2} & \frac{\partial^2 TC}{\partial T \partial u} \\ \frac{\partial^2 TC}{\partial u \partial T} & \frac{\partial^2 TC}{\partial u^2} \end{bmatrix} = \begin{bmatrix} 3.821110255X10^6 & -22.59045311 \\ -22.59045311 & 0.8702763048 \end{bmatrix}$$

Eigen values of the Hessian matrix are $\lambda_1 = 0.87014 > 0, \lambda_2 = 3.821110X10^{-6} > 0$

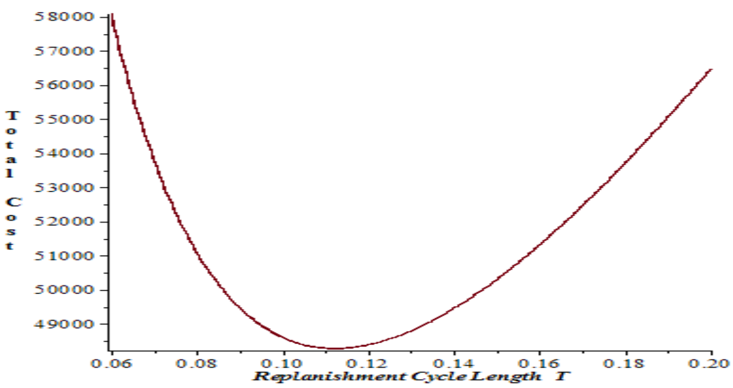

Figure 3: Total Cost vs Cycle length in case 3 without preservation

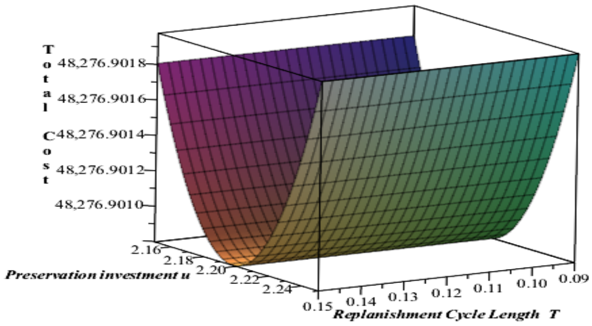

Figure 4: Convexity of cost function in case-4 with preservation

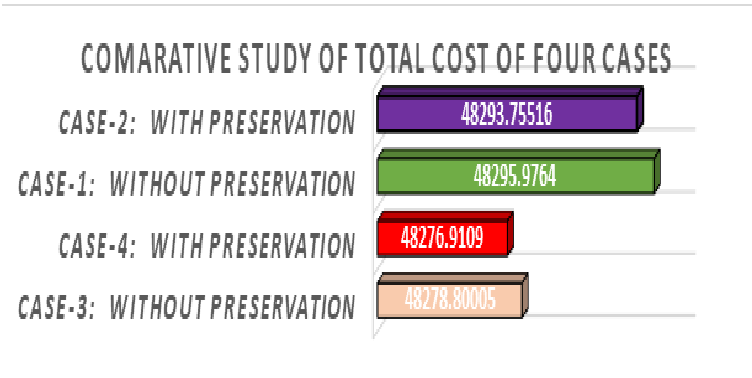

Figure 5: Comparative study of total cost function of four cases

4.3. Sensitivity Analysis

This section consist of the sensitivity analysis of the optimal inventory polices with respect to various inventory parameters. The values of various decision variables on fluctuating the inventory parameters from case-4 in the range -20 percentage to 20 percentage is demonstrated in Table-1, which extracts the below stated observations;

Sensitivity analysis of the annual rate of reworking of imperfect products (P_R)

With respect to increase in the annual rate of reworking of imperfect products, the level of investment for efficiency in JIT increases by lowering cycle time also there is an increase in the cost of preservation investment and the systems total cost hikes within the interval $[0, T]$.

Sensitivity analysis of the annual rate of production(P)

The variation in annual production rate results in level of investment for efficiency in operating, the level of investment for efficiency in JIT, the ordered quantity, the preservation investment cost decreases and cycle length shorten. Also, with the declination in total demand rate within the interval $[0, T]$, which is a desirable virtue for the system.

Sensitivity analysis of annual rate of production of imperfect quality $(PIEE)$

With the variation in the annual rate of production of imperfect quality, with the declination in total demand rate within the interval $[0, T]$, the systems total cost hikes slightly.

Sensitivity analysis of the production cost per product (C)

The variation in production cost per item results in the level of investment for efficiency in operating, increases initially and then start to decrease. The level of investment for efficiency in JIT, ordered quantity, preservation investment cost declines, by shortening the replenishment cycle length. But due to the declination in total demand rate within the interval $[0, T]$, the total cost increases initially and then decrease.

Sensitivity analysis of Scale demand (a)

When scale demand is altered, investment level for JIT efficiency, preservation investment cost decreases. The length of replenishment cycle cut-shorts due to increment in total demand rate within interval $[0, T]$ and total cost rises rapidly with the variation of scale demand.

Sensitivity analysis of Linear variation of demand with respect to time (b)

With the fluctuation in the linear variation of demand with respect to time, the cycle length shortens. The level of investment for efficiency in JIT increases. The preservation investment cost decreases initially and then increase but the systems total cost uplifts in this case with rise of total demand rate within interval $[0, T]$.

Sensitivity analysis of quadratic variation of demand (c)

There is a lengthening of the replenishment cycle length which occurs due to increase in demand rate within the interval $[0, T]$ and in preservation investment cost which decreases the total cost of the system.

There is an increase in level of investment for efficiency in operating, the level of investment for efficiency in JIT increases then decreases slightly, and shortens cycle length. There is a decrease in preservation investment cost and the total demand rate within the interval and the total cost rises rapidly with the variation of imperfect products production percentage.

Sensitivity analysis of cost of repairing of imperfect quality per item (C_r)

When the cost associated with repairing of imperfect quality is varied the ordered quantity, preservation investment cost, demand rate within the interval decreases, along with the shortening of the length of cycle and So, systems total cost rises.

Sensitivity analysis of setup cost per item for each production run (C_s)

There is an increment in level of investment for efficiency in ordering, level of investment for efficiency in operating, level of investment for efficiency in JIT, ordered quantity, preservation investment cost, and demand rate within the interval along with the increment in length of cycle and hence, systems total cost rises.

Sensitivity analysis of the annual holding cost of imperfect products per item (h)

With the variation of cost associated with holding the imperfect products is varied, there is a decrement in investment level for operating efficiency, ordered quantity. The level of investment for efficiency in JIT increases initially and then decrease with shortening of cycle length. The rate of demand within $[0, T]$ interval decreases and So, systems total cost rises.

Sensitivity analysis of annual holding cost of imperfect items reworked/year(h_R)

When the annual holding cost of imperfect products undergoing reworking process per item is varied, there is a fluctuation in the rate of demand within $[0, T]$ interval. So, systems total cost oscillates.

Sensitivity analysis of selling price of perfect quality items(P_p)

A reduction is seen with respect to the variation of selling price of perfect quality items in the various inventory parameters like; investment level of operating efficiency, ordered quantity, preservation investment cost, demand rate within the interval$[0, T]$ along with the shrinking of cycle length resulting in the drop of total cost of the system. The level of investment for efficiency in JIT increases.

Sensitivity analysis of deterioration co-efficient (θ_o)

The level of investment for efficiency in JIT efficiency, rate of demand within the interval $[0, T]$ decreases with the variation of the deterioration coefficient. The preservation investment cost increases. But the systems total cost hikes.

Sensitivity analysis of credit period offered by supplier to manufacturer(M)

A reduction is seen with respect to variation of credit period offered by supplier to manufacturer in various inventory parameters like ; investment level of operating efficiency, ordered quantity, the investment level for JIT efficiency, preservation investment cost, rate of demand within $[0, T]$ interval along with the shrinking of the length of replenishment cycle resulting in drop of systems total cost.

Sensitivity analysis of cost associated with purchasing of Material per item(A_p)

A decrement is seen in level of investment for efficiency in operating, level of investment for efficiency in JIT, ordered quantity, preservation-investment cost, the demand rate within the interval $[0, T]$, along with the shortening of length ofcycle and hence, systems total cost rises with respect to variation of cost associated with purchasing of material per item.

Sensitivity analysis of annual rate of interest earned per dollar(I_e)

When the annual rate of interest earned per dollar is varied, there is a decrement in various inventory parameters like; the level of investment for efficiency in operating efficiency, the preservation investment cost, the investment for JIT efficiency, ordered quantity, along with the rate of demand within the$[0, T]$ interval. Hence, the total cost of the system reduces.

Sensitivity analysis of annual rate of interest charged per dollar(I_c)

All inventory parameters decrease with respect to the variation in annual rate of interest charged per dollar but systems total cost rises.

Sensitivity analysis of markup for ordering efficiency (α)

There is a decrease in level of investment for efficiency in ordering, level of investment for efficiency in operating, level of investment for efficiency in JIT, ordered quantity, length of cycle, rate of demand within the interval$[0, T]$. Also, systems the preservation investment cost decreases then increase total cost drops with markup for ordering efficiency.

Sensitivity analysis of markup for JIT efficiency(β)

When the markup for JIT efficiency is varied, there is a decrement in various inventory parameters like; the investment for JIT efficiency, preservation investment cost along with the rate of market demand within $[0, T]$ interval. Hence, the total cost of the system reduces.

Sensitivity analysis of markup for operating efficiency (γ)

There is a decrease in investment level for operating efficiency, preservation investment cost, the demand rate within the interval $[0, T]$, systems total cost drops by variation of markup for operating efficiency.

Sensitivity analysis of markup for preservation investment cost (ξ)

There is a decrease in preservation investment cost, the demand rate within the interval$[0, T]$ and hence, the systems total cost drops by variation of markup for preservation investment cost.

Inv. Par.	Decision Var.	-20 Per	-10 Per	0 Per	10 Per	20 Per
P_R	Cor	714.2857	714.2857	714.2857	714.2857	714.2857
	Cop	4.9807	4.9807	4.9807	4.9807	4.9807
	Cj	4.9988	4.9988	4.9988	5.007	4.9919
	Q	127	127	127	127	127
	T	0.1124	0.1123	0.1123	0.1123	0.1122
	u	2.1328	2.1694	2.1981	3.4187	3.2803
	$\int_0^T Dtdt$	1134.59	1134.08	1133.68	1133.44	1133.15
	TC	48263.95	48271.21	48276.90	48282.25	48285.83
P	Cor	714.2857	714.2857	714.2857	714.2857	714.2857
	Cop	4.9826	4.9817	4.9807	4.9797	4.9786
	Cj	4.9990	4.9989	4.9988	4.9988	4.9987
	Q	141	134	127	120	114
	T	0.1182	0.1152	0.1123	0.1095	0.10
	u	2.2618	2.2360	2.1981	2.1493	2.0900
	$\int_0^T Dtdt$	1193.8365	1163.1823	1133.6839	1104.8194	1076.23
	TC	50149.2890	49240.7116	48276.9042	47265.6766	46211.45
$PIEE$	Cor	714.2857	714.2857	714.2857	714.2857	714.2857
	Cop	4.9807	4.9807	4.9807	4.9807	4.9807
	Cj	4.9988	4.9988	4.9988	4.9988	4.9988
	Q	127	127	127	127	127
	T	0.1123	0.1123	0.1123	0.1123	0.1123
	u	2.1981	2.1981	2.1981	2.1981	2.1981
	$\int_0^T Dtdt$	1133.6853	1133.6846	1133.6839	1133.6833	1133.6826
	TC	48276.9018	48276.8815	48276.9042	48276.9174	48276.9267
C	Cor	714.2857	714.2857	714.2857	714.2857	714.2857
	Cop	4.9794	4.9817	4.9807	4.9797	4.9786
	Cj	4.9989	4.9989	4.9988	4.9988	4.9987
	Q	148	134	127	120	114
	T	0.1213	0.1152	0.1123	0.1095	0.1066
	u	2.3988	2.2360	2.1981	2.1493	2.0900
	$\int_0^T Dtdt$	1224.7650	1163.1823	1133.6839	1104.8194	1076.2361
	TC	44744.5685	49240.6998	48276.9042	47265.6695	46211.4408
a	Cor	714.2857	714.2857	714.2857	714.2857	714.2857
	Cop	4.9807	4.9807	4.9807	4.9807	4.9807
	Cj	4.9988	4.9988	4.9988	4.9988	4.9922
	Q	127	127	127	127	127
	T	0.1255	0.1184	0.1123	0.1070	0.1024
	u	2.3719	2.2852	2.1981	2.1104	3.2876
	$\int_0^T Dtdt$	1014.2924	1075.8852	1133.6839	1188.2003	1239.9041
	TC	43678.8863	46058.3295	48276.9042	50356.8693	52316.1641
b	Cor	714.2857	714.2857	714.2857	714.2857	714.2857
	Cop	4.9807	4.9807	4.9807	4.9807	4.9807
	Cj	4.9989	4.9989	4.9988	4.9988	5.006
	Q	127	127	127	127	127
	T	0.1125	0.1124	0.1123	0.1122	0.1121
	u	2.2008	2.1993	2.1981	2.1972	3.1348
	$\int_0^T Dtdt$	1133.5614	1133.6223	1133.6839	1133.7463	1133.8771
	TC	48232.9931	48254.9766	48276.9042	48298.7589	48321.0700
c	Cor	714.2857	714.2857	714.2857	714.2857	714.2857
	Cop	4.9807	4.9807	4.9807	4.9807	4.9807
	Cj	4.9988	4.9988	4.9988	4.9988	4.9988
	Q	127	127	127	127	127
	T	0.1122	0.1123	0.1123	0.1123	0.1124
	u	2.1968	2.1974	2.1981	2.1987	2.1994
	$\int_0^T Dtdt$	1133.5234	1133.6035	1133.6839	1133.76459	1133.8453
	TC	48284.6302	48280.7776	48276.9042	48273.0323	48269.1258

Inv. Par.	Decision Var.	-20 Per	-10 Per	0 Per	10 Per	20 Per
h	Cor	714.2857	714.2857	714.2857	714.2857	714.2857
	Cop	4.9809	4.9808	4.9807	4.9806	4.9805
	Cj	4.9909	4.9987	4.9988	4.9922	4.9990
	Q	128	128	127	126	126
	T	0.1129	0.1126	0.1123	0.1120	0.1117
	u	3.2610	2.0794	2.1981	3.2853	2.3758
	$\int_0^T Dtdt$	1139.9502	1136.7613	1133.6839	1130.7058	1127.6380
	TC	48136.1150	48206.4178	48276.9042	48347.4320	48416.8089
h_R	Cor	714.2857	714.2857	714.2857	714.2857	714.2857
	Cop	4.9807	4.9807	4.9807	4.9807	4.9807
	Cj	4.9988	4.9988	4.9988	4.9988	4.9988
	Q	127	127	127	127	127
	T	0.1123	0.1123	0.1123	0.1123	0.1123
	u	2.1982	2.1982	2.1981	2.1982	2.1982
	$\int_0^T Dtdt$	1133.7148	1133.7379	1133.6839	1133.7071	1133.7302
	TC	48276.4787	48276.1527	48276.9042	48276.5874	48276.2511
P_p	Cor	714.2857	714.2857	714.2857	714.2857	714.2857
	Cop	4.9818	4.9813	4.9807	4.9801	4.9795
	Cj	4.9922	4.9920	4.9988	4.9988	4.9988
	Q	135	131	127	123	119
	T	0.1156	0.1140	0.1123	0.1106	0.1089
	u	3.2321	3.2545	2.1981	2.1532	2.1050
	$\int_0^T Dtdt$	1167.3491	1150.6676	1133.6839	1116.5167	1099.0907
	TC	49695.9986	48991.8418	48276.9042	47551.6156	46815.0845
Θ_o	Cor	714.2857	714.2857	714.2857	714.2857	714.2857
	Cop	4.9807	4.9807	4.9807	4.9807	4.9807
	Cj	4.9991	4.9990	4.9988	4.9911	4.9903
	Q	127	127	127	127	127
	T	0.1123	0.1123	0.1123	0.1123	0.1123
	u	1.9089	2.0695	2.1981	3.3175	3.3581
	$\int_0^T Dtdt$	1133.7665	1133.7257	1133.6839	1133.7053	1133.6610
	TC	48275.3785	48276.1705	48276.9042	48278.0577	48278.7667
M	Cor	714.2857	714.2857	714.2857	714.2857	714.2857
	Cop	4.9837	4.9824	4.9807	4.9783	4.9750
	Cj	4.9989	5.007	4.9988	4.9987	4.9984
	Q	151	139	127	113	98
	T	0.1223	0.1177	0.1123	0.1060	0.0987
	u	2.4193	3.3379	2.1981	2.0171	1.7463
	$\int_0^T Dtdt$	1235.6985	1188.6515	1133.6839	1069.8531	995.4067
	TC	52580.0018	50594.4574	48276.9042	45578.6571	42426.5159
A_p	Cor	714.2857	714.2857	714.2857	714.2857	714.2857
	Cop	4.9823	4.9815	4.9807	4.9798	4.9789
	Cj	4.9989	4.9921	4.9988	4.9988	4.9987
	Q	139	133	127	121	116
	T	0.1174	0.1148	0.1123	0.1098	0.1074
	u	2.3188	3.2428	2.1981	2.1316	2.0605
	$\int_0^T Dtdt$	1185.2931	1159.3103	1133.6839	1108.5840	1083.9271
	TC	48073.6816	48187.4682	48276.9042	48342.9732	48385.1466
I_e	Cor	714.2857	714.2857	714.2857	714.2857	714.2857
	Cop	4.9818	4.9813	4.9807	4.9801	4.9789
	Cj	4.9922	4.9920	4.9988	4.9988	4.9987
	Q	135	131	127	123	116
	T	0.1156	0.1140	0.1123	0.1106	0.1074
	u	3.2321	3.2545	2.1981	2.1532	2.0605
	$\int_0^T Dtdt$	1167.3491	1150.6676	1133.6839	1116.5167	1083.9271
	TC	49696.0524	48991.8436	48276.9042	47551.6168	48385.1689

Inv. Par.	Decision Var.	-20 Per	-10 Per	0 Per	10 Per	20 Per
I_c	Cor	714.2857	714.2857	714.2857	714.2857	714.2857
	Cop	4.9823	4.9815	4.9807	4.9798	4.9789
	Cj	4.9989	4.9921	4.9988	4.9988	4.9987
	Q	139	133	127	121	116
	T	0.1174	0.1148	0.1123	0.1098	0.1074
	u	2.3188	3.2428	2.1981	2.1316	2.0605
	$\int_0^T Dtdt$	1185.2931	1159.3103	1133.6839	1108.5840	1083.9271
	TC	48073.6828	48187.4907	48276.9042	48342.9614	48385.1689
β	Cor	714.2857	714.2857	714.2857	714.2857	714.2857
	Cop	4.9807	4.9807	4.9807	4.9807	4.9807
	Cj	6.2597	5.5642	4.9988	4.5525	4.1659
	Q	127	127	127	127	127
	T	0.1123	0.1123	0.1123	0.1123	0.1123
	u	3.4333	3.4245	2.1981	3.4115	2.1978
	$\int_0^T Dtdt$	1134.0313	1133.8839	1133.6839	1133.6695	1133.5085
	TC	48288.8027	48282.6060	48276.9042	48273.5862	48269.4782
γ	Cor	714.2857	714.2857	714.2857	714.2857	714.2857
	Cop	6.2199	5.5318	4.9807	4.5295	4.1532
	Cj	4.9988	4.9988	4.9988	4.998	4.9988
	Q	127	127	127	127	127
	T	0.1123	0.1123	0.1123	0.1123	0.1123
	u	2.1988	2.1984	2.1981	2.1979	2.1977
	$\int_0^T Dtdt$	1133.9439	1133.7996	1133.6839	1133.5892	1133.5102
	TC	48287.9944	48281.8422	48276.9042	48272.8759	48269.5313
ξ	Cor	714.2857	714.2857	714.2857	714.2857	714.2857
	Cop	4.9807	4.9807	4.9807	4.9807	4.9807
	Cj	4.9988	4.9914	4.9988	4.9922	4.9988
	Q	127	127	127	127	127
	T	0.1123	0.1123	0.1123	0.1123	0.1123
	u	2.3910	3.6464	2.1981	2.9763	1.9911
	$\int_0^T Dtdt$	1133.6647	1133.7539	1133.6839	1133.7464	1133.6945
	TC	48277.4199	48277.7067	48276.9042	48277.0611	48276.4859
C_s	Cor	642.8571	682.5396	714.2857	740.2597	761.9047
	Cop	4.9742	4.9779	4.9807	4.9989	4.9846
	Cj	4.9983	4.9987	4.9988	4.9828	4.9989
	Q	95	111	127	143	159
	T	0.0971	0.1050	0.1123	0.1191	0.1255
	u	1.6752	1.9838	2.1981	2.3556	2.4764
	$\int_0^T Dtdt$	978.9464	1059.5028	1133.6839	1202.8801	1268.0401
	TC	41728.8755	45140.7166	48276.9042	51196.9725	53941.6718
k	Cor	714.2857	714.2857	714.2857	714.2857	714.2857
	Cop	4.9807	4.9807	4.9807	4.9807	4.9808
	Cj	4.9919	4.9989	4.9988	4.9988	4.9988
	Q	127	127	127	127	127
	T	0.1124	0.1123	0.1123	0.1123	0.1122
	u	3.2787	2.2254	2.1981	2.1703	2.1420
	$\int_0^T Dtdt$	1134.5917	1134.0996	1133.6839	1133.2796	1132.8862
	TC	48220.3978	48248.5046	48276.9042	48305.1524	48333.1863
C_r	Cor	714.2857	714.2857	714.2857	714.2857	714.2857
	Cop	4.9807	4.9807	4.9807	4.9807	4.9807
	Cj	4.9919	4.9988	4.9988	4.9988	4.9988
	Q	127	127	127	127	126
	T	0.1125	0.1124	0.1123	0.1122	0.1121
	u	3.2755	2.2002	2.1981	2.1960	2.1940
	$\int_0^T Dtdt$	1135.3749	1134.4956	1133.6839	1132.8740	1132.0658
	TC	48209.2955	48242.8815	48276.9042	48310.9068	48344.8617
α	Cor	803.5714	758.3774	714.2857	672.9634	634.9206
	Cop	4.9818	4.9812	4.9807	4.9802	4.9798
	Cj	4.9922	4.9920	4.9988	4.9988	4.9916
	Q	135	131	127	124	121
	T	0.1158	0.1139	0.1123	0.1109	0.1097
	u	3.2299	3.2555	2.1981	2.1617	3.3147
	$\int_0^T Dtdt$	1168.9684	1149.9891	1133.6839	1119.6671	1107.5366
	TC	49764.3910	48963.1827	48276.9042	47684.7334	47169.7185

Table 1: Sensitivity analysis of optimal variables with respect to various inventory parameters

5. CONCLUSION AND FUTURE SCOPE

This article proposes an inventory model based on radio frequency identification adoption (RFID), reworking of imperfect products, and trade-credit for deteriorating inventory with / without utilizing the preservation investment technology. Estimation of the optimal values of RIFD levels of investment for efficiencies in ordering, operating, just-in-time along with production cycle time and preservation investment. The demand function fluctuates with respect to time. The classical optimization technique is utilized for calculating the optimal values. We demonstrated validity of the developed models on numerical examples. Then using the concept of eigen-values of a Hessian matrix, convexity of the systems total cost for the case 4: $\int_0^T Dtdt > P_R$ with preservation, which has the systems minimum total cost. Also, the sensitivity analysis of optimal variables is done by fluctuating the inventory parameters for generating fruitful managerial insights for this case. Also, some possible future directions for research related to this model are: 1. To reduce the systems total cost efforts for investments in advertisement and/or servicing can be utilized. 2. Learning-effects and/or some discounts on purchasing price may be considered. 3. Shortages can be considered.

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6. APPENDICES

Appendix 1: The levels of inventory and systems total-cost in Case 1 and Case 3 (Without Preservation)

$$I_a(t) = \left\{ \begin{array}{l} \frac{1}{6}\left[\frac{2act_a{}^3 - 3abt_a{}^2 - 6at_a + 6P - 6PIEE}{\theta o}\right] \\ -\frac{1}{6}\left[\frac{2act_a{}^3 - 3abt_a{}^2 - 6at_a + 6P - 6PIEE}{\theta o}\right] e^{-\theta uT} \end{array} \right\}, 0 \le t \le t_a \quad (A1)$$

$$I_b(t) = \frac{1}{6\theta o}\left(\begin{array}{l} 2act_a{}^3 e^{-\theta o(-t_b+t)} - 2act_b{}^3 e^{-\theta o(-t_b+t)} \\ +3abt_b{}^2 e^{-\theta o(-t_b+t)} \\ -2act_a{}^3 + 2act_b{}^3 + 3abt_a{}^2 \\ -3abt_b{}^2 + 6I_{\max}\theta o e^{-\theta o(-t_b+t)} \\ -6at_a e^{-\theta o(-t_b+t)} + 6e^{-\theta o(-t_b+t)} at_b \\ -6e^{-\theta o(-t_b+t)} PR + 6at_a - 6at_b + 6PR \end{array} \right) \quad t_a \le t \le t_b \quad (A2)$$

$$I_c(t) = -\frac{a\left(\begin{array}{l} -2ct_b{}^3 + 2ct_c{}^3 \\ +3bt_b{}^2 - 3bt_c{}^2 \\ +6t_b - 6t_c \end{array} \right)\left(-1+e^{-\theta o(-t_c+t)}\right)}{6\theta o} \quad t_b \le t \le t_c \quad (A3)$$

$$I_{\max} = \frac{-a\left(\begin{array}{l} -2ct_b{}^3 + 2ct_c{}^3 \\ +3bt_b{}^2 - 3bt_c{}^2 \\ +6t_b - 6t_c \end{array} \right)\left(-1+e^{-\theta o(t_b-t_c)}\right)}{6\theta o} \quad (A4)$$

$$TC_{1i} = \frac{1}{T}\left[\begin{array}{l} Cr.OpE.Q.k + C.OpE.Q + Cs.OrE + HC.JiT \\ +Cj + Cop + Cor - TIe_i + TIp_i \end{array} \right] \quad (A5)$$

$$for\ i = 1, 2, 3, 4$$

$$TC_{21} = \frac{1}{T}\left[\begin{array}{l} Cr.OpE.Q.k + C.OpE.Q + Cs.OrE + HC.JiT \\ +C_j + Cop + Cor - Pp.Ie.\int_0^M t\,Dt\,dt \\ +Ap.I_c.\left[\begin{array}{l} \int_M^{t_a} t.(P - PIEE - Dt)\,dt \\ +\int_{t_a}^{t_b} \begin{array}{l} (Dt.(t_c - t_b) \\ -(t - t_b).(Dt - P_r) \end{array} dt \\ +\int_{t_b}^{t_c} (Dt.(t_c - t)\,dt \\ +\int_M^{t_a} PIEE.t\,dt + \int_{t_a}^{t_b} (t_b - t).P_r\,dt \end{array} \right] \end{array} \right] \quad (A6)$$

$$TC_{22} = \frac{1}{T}\left[\begin{array}{l} Cr.OpE.Q.k + C.OpE.Q + Cs.OrE + HC.JiT \\ +C_j + Cop + Cor - Pp.Ie.\int_0^M t.\,Dt\,dt \\ +Ap.Ic.\int_M^{t_b} Dt.(tc - tb) - (t - tb).(Dt - \mathrm{Pr})\,dt \\ +\int_{t_b}^{t_c} Dt.(tc - t)\,dt + \int_M^{t_b} (tb - t).P_r\,dt \end{array} \right] \quad (A7)$$

$$TC_{23} = \frac{1}{T}\left[\begin{array}{l} Cr.OpE.Q.k + C.OpE.Q + Cs.Or.E + HC.JiT + C_j + Cop \\ +Cor - Pp.Ie.\int_0^M t.Dt\,dt + Ap.Ic.\int_M^{t_c} Dt.(tc - t)\,dt \end{array} \right] \quad (A8)$$

$$TC_{24} = \frac{1}{T}\left[\begin{array}{l} C_r.OpE.Q.k + C.OpE.Q \\ +C_s.OrE + HC.JiT + Cj + Cop + Cor \\ -Pp.Ie.\int_0^M t.Dt + Pp.Ie.\int_T^{M-T} t.Dt\,dt \end{array} \right] \quad (A9)$$

Appendix 2:The inventory levels in Case 2 and Case 4

(Witht Preservation)

Let $\left(u^2\xi^2 - 2u\xi + 2\right) = X$

$$I_a(t) = \frac{1}{3\theta oX}\left(\begin{array}{l} 2e^{-\frac{1}{2}\theta oXt} acta^3 - 3abt_a{}^2 e^{-\frac{1}{2}\theta oXt} - 2act_a{}^3 + 3abt_a{}^2 + 6at_a - 6P \\ -6at_a e^{-\frac{1}{2}\theta oXt} + 6Pe^{-\frac{1}{2}\theta oXt} - 6PIEEe^{-\frac{1}{2}\theta oXt} + 6PIEE \end{array} \right) \quad (A10)$$

$$I_b(t) = \frac{1}{3\theta oX}\left(\begin{array}{l} 3I_{\max} e^{-\frac{1}{2}\theta oX(-t_b+t)} u^2\xi^2\theta o + 2act_a{}^3 e^{-\frac{1}{2}\theta oX(-t_b+t)} \\ -2act_b{}^3 e^{-\frac{1}{2}\theta oX(-t_b+t)} - 6I_{\max} u\xi\theta o e^{-\frac{1}{2}\theta oX(-t_b+t)} \\ -3abt_a{}^2 e^{-\frac{1}{2}\theta oX(-t_b+t)} + 3abt_b{}^2 e^{-\frac{1}{2}\theta oX(-t_b+t)} - 2act_a{}^3 \\ +2act_b{}^3 + 3abt_a{}^2 - 3abt_b{}^2 + 6I_{\max}\theta o e^{-\frac{1}{2}X(-t_b+t)} \\ -6at_a e^{-\frac{1}{2}\theta oX)(-t_b+t)} + 6at_b e^{-\frac{1}{2}\theta oX(-t_b+t)} \\ -6PRe^{-\frac{1}{2}\theta oX(-t_b+t)} + 6\,at_a - 6\,at_b + 6\,PR \end{array} \right) \quad (A11)$$

$$I_c(t) = \frac{a\left(-2ct_b{}^3 + 2ct_c{}^3 + 3bt_b{}^2 - 3bt_c{}^2 + 6t_b - 6t_c\right)\left(-1+e^{-\frac{1}{2}\theta oX(-t_c+t)}\right)}{3\theta oX} \quad (A12)$$

$$TC_i(T,u) = \frac{1}{T}\left[\begin{array}{l} Cr.OpE.Q.k + C.OpE.Q + Cs.OrE + HC.JiT + C_j \\ +Cop + Cor - TIe_i + TIp_i + PTI \end{array} \right] \quad (A13)$$

$$TC(T) = \left\{ \begin{array}{ll} TC_{i1} & , 0 \le M \le t_a \\ TC_{i2} & , t_a \le M \le t_b \\ TC_{i3} & , t_b \le M \le t_c \\ TC_{i4} & , T < M \end{array} \right. \quad for\ i = 1, 2 \quad (A14)$$

Appendix 3: The level of Investment for efficiencies in ordering and operating in all four cases

(With /Without Preservation)

$Cor = \left(\frac{1}{\alpha} - \frac{1}{\alpha^2 C_s(G1-N1)}\right)$ A15)

$$\text{Cop}=\left(\frac{1}{\gamma} - \frac{1}{T\gamma^2}\cdot\frac{1}{\left(\left(-\frac{1}{3}acT^3+\frac{1}{2}abT^2+aT\right)(A1-E1)(Cr\,k+C)\right)}\right) \quad (A16)$$

Appendix 4: The Investment level for JIT efficiency in Case-1/ Case-3

(Without Preservation)

$$Cj = \frac{1}{\beta} - \frac{1}{\frac{h\beta^2(U1-L1)}{6\theta o^2}(S)} \quad (A17)$$

Where, $S=S_1 + S_2 + S_3 + S_4 + S_5$

Let $\left(-\frac{1}{3}acT^3 + \frac{1}{2}abT^2 + aT\right) = Y$

$$S_1 = \left[\begin{array}{l} \frac{2acT^4Y^4\theta o}{P^4} + \frac{2acT^3\theta oY^3 e^{\left(-\theta oT\frac{Y}{P}\right)}}{P^3} - \frac{3abT^3\theta oY^3}{P^3} - \frac{3abT^2Y^2 e^{\left(-\theta oT\frac{Y}{P}\right)}}{P^2} - \frac{2acT^3Y^3}{P^3} \\ + \frac{3abT^2Y^2}{P^2} - \frac{6aT^2\theta oY}{P^2} - \frac{6aTYe^{\left(-\theta oT\frac{Y}{P}\right)}}{P} + 6\theta oTY \\ - \frac{6PIEE\theta oTY}{P} + 6Pe^{\left(-\theta oT\frac{Y}{P}\right)} - 6PIEEe^{\left(-\theta oT\frac{Y}{P}\right)} + 6aT\frac{Y}{P} - 6P + 6PIEE \end{array}\right]$$

$$S_2 = \left[\begin{array}{l} \left(2T^3ac - 3T^2ab - 6Ta\right) - \frac{2acT^4Y^4\theta o}{P^4} + 3abT^3\theta o\frac{Y^3}{P^3} - 2e^{\left(-\theta oT\frac{Y}{P}+T\theta o\right)}acT^3Y \\ \left(\frac{1}{P} + \frac{k}{PR}\right)^3 + 3e^{\left(-\theta oT\frac{Y}{P}+T\theta o\right)}abT^2Y^2\left(\frac{1}{P} + \frac{k}{PR}\right)^2 - 2acT^4Y^4\left(\frac{1}{P} + \frac{k}{PR}\right)^4\theta o \\ + 3abT^3Y^4\left(\frac{1}{P} + \frac{k}{PR}\right)^3\theta o - 2acT^3Y^3 \\ \frac{e^{\left(-\theta oT\frac{Y}{P} - TY\left(\frac{1}{P}+\frac{k}{PR}\right)\right)}}{P^3} + 4acT^3Y^3\left(\frac{1}{P} + \frac{k}{PR}\right)^3 e^{\left(-\theta oT\frac{Y}{P} - TY\left(\frac{1}{P}+\frac{k}{PR}\right)\right)} \\ 3abT^2Y^2 \\ \frac{e^{\left(-\theta oT\frac{Y}{P} - TY\left(\frac{1}{P}+\frac{k}{PR}\right)\right)}}{P^2} - 6abT^2Y^2\left(\frac{1}{P} + \frac{k}{PR}\right)^2 e^{\left(-\theta oT\frac{Y}{P} - TY\left(\frac{1}{P}+\frac{k}{PR}\right)\right)} \end{array}\right]$$

$$S_3 = \left[\begin{array}{l} +2e^{\left(-\theta o\left(TY\left(\frac{1}{P}+\frac{k}{PR}\right)-T\right)\right)}acT^3Y^3\left(\frac{1}{P} + \frac{k}{PR}\right)^3 - 3e^{\left(-\theta o\left(TY\left(\frac{1}{P}+\frac{k}{PR}\right)-T\right)\right)} \\ abT^2Y^2\left(\frac{1}{P} + \frac{k}{PR}\right)^2 + 6PRe^{\left(-\theta oT\frac{Y}{P} - TY\left(\frac{1}{P}+\frac{k}{PR}\right)\right)} + 2acT^3\frac{Y^3}{P^3} - 3abT^2\frac{Y^2}{P^2} + 6aT^2\theta o\frac{Y^2}{P^2} \\ +6e^{\left(-\theta oT\frac{Y}{P}+T\theta o\right)}aTY\left(\frac{1}{P} + \frac{k}{PR}\right) + 6aT^2Y^2\left(\frac{1}{P} + \frac{k}{PR}\right)^2\theta o - 6PR\theta oTY\left(\frac{1}{P} + \frac{k}{PR}\right) \\ +6PRT\theta o\frac{Y}{P} + \frac{\frac{6aTY}{P}e^{\left(\frac{\frac{-\theta oTY}{P}+T\theta oY\left(\frac{1}{P}+\frac{k}{PR}\right)}{P}\right)}}{P} - 12aTY\left(\frac{1}{P} + \frac{k}{PR}\right)e^{\left(\frac{-\theta oTY+T\theta oY\left(\frac{1}{P}+\frac{k}{PR}\right)}{P}\right)} \\ -6e^{\left(-T\theta oY\left(\frac{1}{P}+\frac{k}{PR}\right)-T\right)}aTY\left(\frac{1}{P} + \frac{k}{PR}\right) + 6abT^2Y^2\left(\frac{1}{P} + \frac{k}{PR}\right)^2 \end{array}\right]$$

$$S_4 = \left[\begin{array}{r} -4acT^3Y^3\left(\frac{1}{P} + \frac{k}{PR}\right)^3 + 6aTe^{\left(\frac{-\theta oTY}{P} - T\theta oY\left(\frac{1}{P}+\frac{k}{PR}\right)\right)} + 6e^{\left(-\theta o\left(TY\left(\frac{1}{P}+\frac{k}{PR}\right)-T\right)\right)}aT \\ -6e^{\left(-\theta o\left(\frac{TY}{P}-T\right)\right)}aT - 2e^{\left(\frac{-\theta oTY}{P} - T\theta oY\left(\frac{1}{P}+\frac{k}{PR}\right)\right)}acT^3 + 3e^{\left(\frac{-\theta oTY}{P} - T\theta oY\left(\frac{1}{P}+\frac{k}{PR}\right)\right)}abT^2 \\ -2e^{\left(-T\theta oY\left(\frac{1}{P}+\frac{k}{PR}\right)-T\right)}acT^3 + 3e^{\left(-T\theta oY\left(\frac{1}{P}+\frac{k}{PR}\right)-T\right)}abT^2 + 12aTY\left(\frac{1}{P} + \frac{k}{PR}\right) - \frac{6aTY}{P} \\ -3e^{\left(-\theta o\left(\frac{TY}{P}-T\right)\right)}abT^2 + 2e^{\left(-\theta o\left(\frac{TY}{P}-T\right)\right)}acT^2 - 3e^{\left(-\theta o\left(\frac{TY}{P}-T\right)\right)}abT^2 + 2e^{\left(-\theta o\left(\frac{TY}{P}-T\right)\right)}acT^3 \\ + \frac{2acT^4Y^4\left(\frac{1}{P} + \frac{k}{PR}\right)^3\theta o}{P} - \frac{3abT^3Y^3\left(\frac{1}{P} + \frac{k}{PR}\right)^2\theta o}{P} - 6PR \\ - \frac{12aT^2Y^2\left(\frac{1}{P} + \frac{k}{PR}\right)\theta o}{P} + \frac{2acT^4Y^4\left(\frac{1}{P} + \frac{k}{PR}\right)\theta o}{P^3} - \frac{3abT^3Y^3\left(\frac{1}{P} + \frac{k}{PR}\right)\theta o}{P^2} \end{array}\right]$$

$$S_5 = \left[\begin{array}{l} +h_r\left(\begin{array}{l} \frac{1}{2}\frac{PIEET^2Y^2}{P^2} + \frac{1}{2}PR\left(T^2Y^2\left(\frac{1}{P} + \frac{k}{PR}\right)^2 - \frac{T^2Y^2}{P^2}\right) \\ -TY\left(\frac{1}{P} + \frac{k}{PR}\right)PR\left(TY\right)\left(\frac{1}{P} + \frac{k}{PR}\right) - \frac{TY}{P}\right) \end{array}\right) \\ +a\left(-2cT^3Y^3\left(\frac{1}{P} + \frac{k}{PR}\right)^3 + 3bT^2Y^2\left(\frac{1}{P} + \frac{k}{PR}\right)^2 - 3bT^2 + 6TY\left(\frac{1}{P} + \frac{k}{PR}\right) - 6T\right) \\ \left(\theta oTY\left(\frac{1}{P} + \frac{k}{PR}\right) - T\theta o + e^{\left(-\theta oY\left(\frac{1}{P}+\frac{k}{PR}\right)-T\right)} - 1\right) \end{array}\right]$$

Appendix 5:The Investment level for JIT efficiency in Case 2/ Case 4

(With Preservation)

$$Cj = \frac{1}{\beta} - \frac{1}{\left(\beta^2(U-L)\right)}\cdot\frac{1}{R}\,(A18)$$

Where, $R = h(1/3)(1/\theta_o{}^2u^2\xi^2 - 2u\xi)U_1 + U_2$

$U_2 = U_{21} + U_{22} + U_{23} + U_{24}$

$$U_1 = \left[\begin{array}{l} \frac{4acT^3Y^3 e^{\frac{1}{2}\left(-\theta o\left(u^2\xi^2-2u\xi+2\right)TY\right)}}{P^3} - \frac{6abT^2Y^2 e^{\frac{1}{2P}\left(-\theta o\left(u^2\xi^2-2u\xi+2\right)TY\right)}}{P^2} \\ + \frac{4acT^4Y^4\theta o}{P^4} - \frac{6abT^3Y^3\theta o}{P^3} + \frac{12aTY}{P} 12TY\theta o - \frac{12PIEE\,TY\theta o}{P} \\ - \frac{12aTYe^{\frac{1}{2P}\left(-\theta o\left(u^2\xi^2-2u\xi+2\right)TY\right)}}{P} - \frac{12aT^2Y^2\theta o}{P^2} - \frac{4acT^3Y^3}{P^3} \\ + \frac{6abT^2Y^2}{P^2} - \frac{6aT^2Y^2u^2\xi^2\theta o}{P^2} + 6TYu^2\xi^2\theta o - \frac{6PIEE\,TYu^2\xi^2\theta o}{P} \\ + \frac{12aT^2Y^2u\xi\theta o}{P^2} - 12TYu\xi\theta o + \frac{12PIEE\,TYu\xi\theta o}{P} \\ +12\ \text{PIEE}\ +12\ \text{P}e^{\frac{1}{2P}\left(-\theta o\left(u^2\xi^2-2u\xi+2\right)TY\right)} \\ \text{-12PIEE}e^{\frac{1}{2P}\left(-\theta o\left(u^2\xi^2-2u\xi+2\right)TY\right)} - 12P + \frac{2acT^4Y^4u^2\xi^2\theta o}{P^4} \\ + \frac{6abT^3Y^3u\xi\theta o}{P^3} - \frac{4acT^4Y^4u\xi\theta o}{P^4} - \frac{3abT^3Y^3u^2\xi^2\theta o}{P^3} \end{array}\right]$$

$$U_{21} = \left[\begin{array}{l} 4acT^3u\xi e^{\left[\begin{array}{c} \frac{u^2\xi^2\theta oTY}{2P}+\frac{1}{2}TY\left(\frac{1}{P}+\frac{k}{PR}\right)u^2\xi^2\theta o \\ +\frac{TYu\xi\theta o}{P} \\ -TY\left(\frac{1}{P}+\frac{k}{PR}\right)u\xi\theta o-\frac{TY\theta o}{P}+\theta oT \end{array}\right]} - 6abT^2u\xi e^{\left[\begin{array}{c} \frac{u^2\xi^2\theta oTY}{2P}+\frac{1}{2}TY\left(\frac{1}{P}+\frac{k}{PR}\right)u^2\xi^2\theta o \\ +\frac{TYu\xi\theta o}{P} \\ -TY\left(\frac{1}{P}+\frac{k}{PR}\right)u\xi\theta o-\frac{TY\theta o}{P}+\theta oT \end{array}\right]} \\ - 2acT^3u^2\xi^2 e^{\left[\begin{array}{c} \frac{u^2\xi^2\theta oTY}{2P}+\frac{1}{2}TY\left(\frac{1}{P}+\frac{k}{PR}\right)u^2\xi^2\theta o \\ +\frac{TYu\xi\theta o}{P} \\ -TY\left(\frac{1}{P}+\frac{k}{PR}\right)u\xi\theta o-\frac{TY\theta o}{P}+\theta oT \end{array}\right]} + \frac{4acT^4Y^4\theta o}{P^4} - \frac{6abT^3Y^3\theta o}{P^3} \\ - 12aTe^{\frac{1}{2}\left(-\theta o\left(u^2\xi^2-2u\xi+2\right)\left(T\frac{Y)}{P}-TY\left(\frac{1}{P}+\frac{k}{PR}\right)\right)\right)} - 12aTe^{\left(-\theta oTY\left(\frac{1}{P}+\frac{k}{PR}\right)-\theta oT\right)} \\ + 12aTe^{\left[\begin{array}{c} \frac{u^2\xi^2\theta oTY}{2P}+\frac{1}{2}TY\left(\frac{1}{P}+\frac{k}{PR}\right)u^2\xi^2\theta o \\ +\frac{TYu\xi\theta o}{P} \\ -TY\left(\frac{1}{P}+\frac{k}{PR}\right)u\xi\theta o-\frac{TY\theta o}{P}+\theta oT \end{array}\right]} - 6abT^2e^{\frac{1}{2}\left(-\theta o\left(u^2\xi^2-2u\xi+2\right)\left(T\frac{Y}{P}-TY\left(\frac{1}{P}+\frac{k}{PR}\right)\right)\right)} \\ + 4acT^3e^{\frac{1}{2}\left(-\theta o\left(u^2\xi^2-2u\xi+2\right)\left(T\frac{Y}{P}-TY\left(\frac{1}{P}+\frac{k}{PR}\right)\right)\right)} \\ + 6aTu^2\xi^2 - 12aTu\xi + 4acT^3e^{-\theta o\left(TY\left(\frac{1}{P}+\frac{k}{PR}\right)-T\right)} \\ - 6abT^2e^{-\theta o\left(TY\left(\frac{1}{P}+\frac{k}{PR}\right)-T\right)} - 24aTY\left(\frac{1}{P}+\frac{k}{PR}\right) \end{array}\right]$$

$$U_{22}=\left[\begin{array}{l}
+\,6abT^2 e^{\left[\begin{array}{c}\frac{u^2\xi^2\theta oTY}{2P}+\frac{1}{2}TY\left(\frac{1}{P}+\frac{k}{PR}\right)u^2\xi^2\theta o\\ +\frac{TYu\xi\theta o}{P}\\ -TY\left(\frac{1}{P}+\frac{k}{PR}\right)u\xi\theta o-\frac{TY\theta o}{P}+\theta oT\end{array}\right]}-4acT^3 e^{\left[\begin{array}{c}\frac{u^2\xi^2\theta oTY}{2P}+\frac{1}{2}TY\left(\frac{1}{P}+\frac{k}{PR}\right)u^2\xi^2\theta o\\ +\frac{TYu\xi\theta o}{P}\\ -TY\left(\frac{1}{P}+\frac{k}{PR}\right)u\xi\theta o-\frac{TY\theta o}{P}+\theta oT\end{array}\right]}\\
+\,\frac{12aTY}{P}+4acT^3Y^3\left(\frac{1}{P}+\frac{k}{PR}\right)^3\\
e^{\left[\begin{array}{c}\frac{u^2\xi^2\theta oTY}{2P}+\frac{1}{2}TY\left(\frac{1}{P}+\frac{k}{PR}\right)u^2\xi^2\theta o\\ +\frac{TYu\xi\theta o}{P}\\ -TY\left(\frac{1}{P}+\frac{k}{PR}\right)u\xi\theta o-\frac{TY\theta o}{P}+\theta oT\end{array}\right]}\\
+\,4acT^4Y^4\left(\frac{1}{P}+\frac{k}{PR}\right)^4\theta o-6abT^3Y^3\left(\frac{1}{P}+\frac{k}{PR}\right)^3\theta o\\
-\,\frac{6abT^2Y^2e^{\frac{1}{2}\left(-\theta o\left(u^2\xi^2-2u\xi+2\right)\left(T\frac{Y}{P}-TY\left(\frac{1}{P}+\frac{k}{PR}\right)\right)\right)}}{P^2}\\
+\,12abT^2Y^2\left(\frac{1}{P}+\frac{k}{PR}\right)^2e^{\frac{1}{2}\left(-\theta o\left(u^2\xi^2-2u\xi+2\right)\left(T\frac{Y}{P}-TY\left(\frac{1}{P}+\frac{k}{PR}\right)\right)\right)}\\
+\,\frac{4acT^3Y^3e^{\frac{1}{2}\left(-\theta o\left(u^2\xi^2-2u\xi+2\right)\left(T\frac{Y}{P}-TY\left(\frac{1}{P}+\frac{k}{PR}\right)\right)\right)}}{P^3}\\
-\,8acT^3Y^3\left(\frac{1}{P}+\frac{k}{PR}\right)^3e^{\frac{1}{2}\left(-\theta o\left(u^2\xi^2-2u\xi+2\right)\left(T\frac{Y}{P}-TY\left(\frac{1}{P}+\frac{k}{PR}\right)\right)\right)}\\
-\,6aTY\left(\frac{1}{P}+\frac{k}{PR}\right)u^2\xi^2+12aTu\xi Y\left(\frac{1}{P}+\frac{k}{PR}\right)\\
-\,4acT^3Y^3\left(\frac{1}{P}+\frac{k}{PR}\right)^3+6abT^2Y^2\left(\frac{1}{P}+\frac{k}{PR}\right)^2\\
e^{-\theta o\left(TY\left(\frac{1}{P}+\frac{k}{PR}\right)-T\right)}\\
-\,4acT^3u\xi e^{\frac{1}{2}\left(-\theta o\left(u^2\xi^2-2u\xi+2\right)\left(T\frac{Y}{P}-TY\left(\frac{1}{P}+\frac{k}{PR}\right)\right)\right)}\\
+\,6abT^2u\xi e^{\frac{1}{2}\left(-\theta o\left(u^2\xi^2-2u\xi+2\right)\left(T\frac{Y}{P}-TY\left(\frac{1}{P}+\frac{k}{PR}\right)\right)\right)}\\
+\,2acT^3u^2\xi^2e^{\frac{1}{2}\left(-\theta o\left(u^2\xi^2-2u\xi+2\right)\left(T\frac{Y}{P}-TY\left(\frac{1}{P}+\frac{k}{PR}\right)\right)\right)}\\
-\,3abT^2u^2\xi^2e^{\frac{1}{2}\left(-\theta o\left(u^2\xi^2-2u\xi+2\right)\left(T\frac{Y}{P}-TY\left(\frac{1}{P}+\frac{k}{PR}\right)\right)\right)}\\
+\,2acT^3u^2\xi^2e^{-\theta oY\left(\frac{1}{P}+\frac{k}{PR}\right)-T}-3abT^2u^2\xi^2e^{-\theta o\left(TY\left(\frac{1}{P}+\frac{k}{PR}\right)-T\right)}\\
-\,4acT^3u\xi e^{-\theta o\left(TY\left(\frac{1}{P}+\frac{k}{PR}\right)-T\right)}+6abT^2u\xi e^{-\theta oY\left(\frac{1}{P}+\frac{k}{PR}\right)-T}\\
-\,6abT^2Y^2\left(\frac{1}{P}+\frac{k}{PR}\right)^2+6abT^2u\xi e^{-\theta oY\left(\frac{1}{P}+\frac{k}{PR}\right)-T}\\
+\,3T^2abu^2\xi^2+4T^3acu\xi-6T^2abu\xi-12a\ TY\left(\frac{1}{P}+\frac{k}{PR}\right)\\
-\,12aT^2\theta oY^2\left(\frac{1}{P}+\frac{k}{PR}\right)^2+12\ PR\ T\theta oY\left(\frac{1}{P}+\frac{k}{PR}\right)
\end{array}\right]$$

$$U_{23}=\left[\begin{array}{l}
+\,3abT^2u^2\xi^2e^{\left[\begin{array}{c}\frac{u^2\xi^2\theta oTY}{2P}+\frac{1}{2}TY\left(\frac{1}{P}+\frac{k}{PR}\right)u^2\xi^2\theta o+\frac{TYu\xi\theta o}{P}\\ -TY\left(\frac{1}{P}+\frac{k}{PR}\right)u\xi\theta o-\frac{TY\theta o}{P}+\theta oT\end{array}\right]}\\
-\,6aTu^2\xi^2e^{-\theta o\left(TY\left(\frac{1}{P}+\frac{k}{PR}\right)-T\right)}+12aTu\xi e^{-\theta o\left(TY\left(\frac{1}{P}+\frac{k}{PR}\right)-T\right)}\\
-\,2T^3acu^2\xi^2e^{\left[\begin{array}{c}\frac{u^2\xi^2\theta oTY}{2P}+\frac{1}{2}TY\left(\frac{1}{P}+\frac{k}{PR}\right)u^2\xi^2\theta o+\frac{TYu\xi\theta o}{P}\\ -TY\left(\frac{1}{P}+\frac{k}{PR}\right)u\xi\theta o-\frac{TY\theta o}{P}+\theta oT\end{array}\right]}
\end{array}\right]$$

$$
U_{24} = \left[\begin{array}{l}
+6\ \mathrm{PR}\ \mathrm{T}u^2\xi^2\theta o\left(-\frac{1}{3}acT^3+\frac{1}{2}abT^2+aT\right)\left(\frac{1}{P}+\frac{k}{PR}\right) \\
+12aT^2\mathrm{u}\xi\theta o\left(-\frac{1}{3}acT^3+\frac{1}{2}abT^2+aT\right)^2\left(\frac{1}{P}+\frac{k}{PR}\right)^2 \\
-12\ \mathrm{PR}\ \mathrm{Tu}\xi\theta o\left(-\frac{1}{3}acT^3+\frac{1}{2}abT^2+aT\right)\left(\frac{1}{P}+\frac{k}{PR}\right) \\
+6aTu^2\xi^2\left(-\frac{1}{3}acT^3+\frac{1}{2}abT^2+aT\right)\left(\frac{1}{P}+\frac{k}{PR}\right) \\
-6abT^2Y^2\left(\frac{1}{P}+\frac{k}{PR}\right)^2 e^{\left[\begin{array}{c}\frac{u^2\xi^2\theta oTY}{2P}+\frac{1}{2}TY\left(\frac{1}{P}+\frac{k}{PR}\right)u^2\xi^2\theta o+\frac{TYu\xi\theta o}{P} \\ -TY\left(\frac{1}{P}+\frac{k}{PR}\right)u\xi\theta o-\frac{TY\theta o}{P}+\theta oT\end{array}\right]} \\
+3abT^2u^2\xi^2 e^{\left[\begin{array}{c}\frac{u^2\xi^2\theta oTY}{2P}+\frac{1}{2}TY\left(\frac{1}{P}+\frac{k}{PR}\right)u^2\xi^2\theta o+\frac{TYu\xi\theta o}{P} \\ -TY\left(\frac{1}{P}+\frac{k}{PR}\right)u\xi\theta o-\frac{TY\theta o}{P}+\theta oT\end{array}\right]} \\
-6aTu^2\xi^2 e^{\left[-\frac{1}{2}\theta o\left(u^2\xi^2-2u\xi+2\right)\left(\frac{TY}{P}-TY\left(\frac{1}{P}+\frac{k}{PR}\right)\right)\right]} \\
+12\ \mathrm{a}\ \mathrm{T}\ \mathrm{u}\xi e^{\left[-\frac{1}{2}\theta o\left(u^2\xi^2-2u\xi+2\right)\left(\frac{TY}{P}-TY\left(\frac{1}{P}+\frac{k}{PR}\right)\right)\right]} \\
+12aT+6abT^2-4acT^3+\frac{12aT^2Y^2\left(\frac{1}{P}+\frac{k}{PR}\right)u^2\xi^2\theta o}{P} \\
-\frac{24aT^2\mathrm{u}\xi\theta oY^2\left(\frac{1}{P}+\frac{k}{PR}\right)}{P}-\frac{2acT^4\mathrm{u}^2\xi^2\theta oY^4\left(\frac{1}{P}+\frac{k}{PR}\right)}{P^3} \\
+\frac{3abT^3u^2\xi^2\theta oY^3\left(\frac{1}{P}+\frac{k}{PR}\right)}{P^2}+\frac{4ac\mathrm{u}\xi\theta oT^4Y^4\left(\frac{1}{P}+\frac{k}{PR}\right)}{P^3} \\
-\frac{6abT^3\mathrm{u}\xi\theta oY^3\left(\frac{1}{P}+\frac{k}{PR}\right)}{P^2}-\frac{2acT^4u^2\xi^2\theta oY^4\left(\frac{1}{P}+\frac{k}{PR}\right)^3}{P}-\frac{4acT^3Y^3}{P^3} \\
+\frac{6abT^2Y^2Y\left(\frac{1}{P}+\frac{k}{PR}\right)}{P^2} \\
-12\mathrm{aTu}\xi e^{\left[\begin{array}{c}\frac{u^2\xi^2\theta oTY}{2P}+\frac{1}{2}TY\left(\frac{1}{P}+\frac{k}{PR}\right)u^2\xi^2\theta o \\ +\frac{TYu\xi\theta o}{P} \\ -TY\left(\frac{1}{P}+\frac{k}{PR}\right)u\xi\theta o-\frac{TY\theta o}{P}+\theta oT\end{array}\right]}+6aTu^2\xi^2 e^{\left[\begin{array}{c}\frac{u^2\xi^2\theta oTY}{2P}+\frac{1}{2}TY\left(\frac{1}{P}+\frac{k}{PR}\right)u^2\xi^2\theta o \\ +\frac{TYu\xi\theta o}{P} \\ -TY\left(\frac{1}{P}+\frac{k}{PR}\right)u\xi\theta o-\frac{TY\theta o}{P}+\theta oT\end{array}\right]}
\end{array}\right]
$$